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ABSTRACT
The absolute stability problem of discrete-time time-delayed Lur'e systems with sector bounded nonlinearities is investigated in this paper. Firstly, a modified Lyapunov-Krasovskii functional (LKF) is designed with augmenting additional double summation terms, which complements more coupling information between the delay intervals and other system state variables than some previous LKFs. Secondly, some improved delay-dependent absolute stability criteria based on linear matrix inequality form (LMI) are proposed via the modified LKF and the relaxed free-matrix-based summation inequality technique application. The stability criteria are less conservative than some results previously proposed. The reduction of the conservatism mainly relies on the full use of the relaxed summation inequality technique based on the modified LKF. Finally, two common numerical examples are presented to show the effectiveness of the proposed approach.

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Discrete-time systems; Lur'e systems; LMI; stability; time-varying delays

1. Introduction

Time delay is common in practical engineering and is one of the main reasons for the deterioration or even instability of system performance. Therefore, time-delayed systems have been a hot topic in control theory [1–6]. There are delay-independent stability criteria and delay-dependent stability criteria for time-delayed systems. It is well known that the delay-dependent stability criterion is less conservative than the delay-independence one. So, more and more researchers pay attention to the derivation of delay-dependent stability criteria. And the methods of obtaining the delay dependent stability criterion are constantly updated and developed. At present, the most popular methods include three aspects: one is to seek and improve tight integral or summation inequality techniques; one is to construct LKFs based on tight inequality techniques, which contain as much coupling information between state variables as possible and make full use of the tight inequality techniques; one is to increase the degree of freedom of LMIs. For the improvement of the inequality techniques, the second order Bessel-Legendre inequality is proposed in [7], where some novel hierarchy LMI stability conditions for linear time-delayed systems are obtained; [8] gives an augmented double-integral inequality technique, which can estimate the derivative bounds of the triple-integral terms in the LKF; [9] gives a lower bound lemma via the reciprocally convex approach, which not only achieves performance behaviour identical to approaches based on the integral inequality lemma but also decreases the number of decision variables dramatically up to those based on the Jensen inequality lemma; third-order or high-order Bessel-Legendre inequalities are proposed in [10,11], which can produce tight bounds; and the relaxed quadratic function negative-determination lemmas proposed in [12] can reduce the conservatism of the stability criterion without requiring extra decision variables. The authors of [13] construct an implicit LKF, which provides global asymptotic stability for all delays less than a certain threshold value; the affine parameter-dependent LKF proposed in [14] makes full use of the advantages of convexity properties; the authors of [15,16] use the time-dependent LKF to derive some new stability conditions for time-delayed systems; [17] considers the input-to-state stability of the stochastic impulsive switched time-delayed system via a vector LKF application; some other augmented LKFs can be seen in [18–21]. In addition, some zero equality approaches are applied to increase the degree of freedom of solving LMIs in [22–24]. Some of the above methods are suitable for continuous-time and discrete-time time-delayed systems. The reduction of conservatism of stability criteria for discrete-time time-delayed systems also depends on the construction of LKFs and the development of summation inequality techniques. Recently, some improved stability criteria for discrete-time linear systems with time-varying delays are proposed in [25–30], where some novel free-weighting-based matrix methods and summation

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inequality techniques are used to tighten the upper bounds and the true values of the summations, such as three orthogonal polynomial functions [26], discrete Legendre polynomials-based inequality [30], auxiliary-function-based summation inequalities [25], ect..

The Lur'e system has been one of the hot research topics since it was proposed. Many practical systems can be modelled as such systems, such as, Chua's Circuit and the Lorenz systems, which consist of a feedback connection of a linear dynamical system and a nonlinearity satisfying the sector condition [31]. For continuous-time Lur'e systems with time delays and sector bounded nonlinearities, some absolute stability and robustly absolute stability criteria are given in [32–37] by combining some modified LKFs and improved integral inequality techniques. The authors of [35] first time to investigate the master-slave synchronization for complex-valued delayed chaotic Lur'e systems with constant time delay, where the model and results are more general to treat real valued chaotic Lur'e systems as special cases. A novel LKF is constructed and new sampled-data synchronization criteria of the complex-valued chaotic Lur'e systems are exported. In the case of discrete-time Lur'e systems, many significant stability criteria have been obtained with the development of summation inequality techniques [38–41], where, to the best of the authors' knowledge, the results with the lowest conservatism come from the references [40,41], since each increase of discrete-time delay is at least one. In [40], by dividing the variation interval of the time delays into some subintervals, some new delay-range-dependent robust stability criteria are derived in the form of LMIs via a modified LKF approach. Recently, a modified general free-matrix-based summation inequality has been given in [26], where a newly less conservative stability condition is derived for discrete-time neural networks. However, all the stability criteria based on Lyapunov stability theory are only sufficient conditions and inevitably conservative, so there is still room for further improvement.

Based on the above discussion, the main purpose of this paper is to achieve a less conservative stability condition for the discrete-time time-delayed Lur'e system in two ways: using a free-matrix-based summation inequality and modifying an augmented LKF. Thus, the contribution of this paper can be summarized as follows:

- An improved free-matrix-based summation inequality is used to estimate the upper bounds of the summations in this paper. Compared with the summation inequalities proposed in [26–30,42], two augmented free matrices L_1 and L_2 are employed to involve additional coupling information between some state vectors of the system and themselves. Thus, the summation inequality technique used in this paper is more general than those proposed in [26–30,42].
- To make full use of the free-matrix-based summation inequalitiy in the different delay intervals $[\tau_m, \tau(k)]$ and $[\tau(k), \tau_M]$, an improved LKF is augmented in non-summation term and double summation term, where more information about the different states of the system and time delays than those recently proposed in [38–40,43,44] is applied.
- Combining the improved summation inequality with the augmented LKF, two enhanced absolute stability criteria for the discrete-time Lur'e system with time-varying delays and sector bounded nonlinearities are derived in terms of LMI.

This paper is organized as follows. Section 2 gives the problem statement and provides some definitions, assumptions and lemmas. Section 3 presents the main results. Section 4 shows numerical examples. Conclusions are drawn in Section 5.

Notation: Throughout this paper, the notations are standard. $\mathbb{Z}$ is the integer set; $\mathbb{R}^n$ denotes the n-dimensional Euclidean space; $\mathbb{R}^{n\times m}$ is the set of all $n \times m$ real matrices; For $P \in \mathbb{R}^{n\times n}$, $P > 0$ (respectively, $P < 0$) mean that P is a positive (respectively, negative) definite matrix. diag$\{a_1, a_2, \ldots, a_n\}$ denotes an n-order diagonal matrix with diagonal elements $a_1, a_2, \ldots, a_n$. e_i $(i = 1, \ldots, m)$ are block entry matrices. For example, $e_2^T = [0\ I\ \underbrace{0 \cdots 0}_{m-2}]$. For a real matrix B and two real symmetric matrices A and C of appropriate dimensions, $\left[\begin{smallmatrix} A & B \\ * & C \end{smallmatrix}\right]$ denotes a real symmetric matrix, where $*$ denotes the entries implied by symmetry. Sym$\{A\} = A + A^T$.

2. Preliminaries

Consider the following discrete-time Lur'e system with time-varying delays and sector-bounded nonlinearities:

$$\begin{cases} x(k+1) = Ax(k) + Bx(k-\tau(k)) + D\omega(k), \\ z(k) = Mx(k) + Nx(k-\tau(k)), \\ \omega(k) = -\varphi(k, z(k)), \\ x(k) = \phi(k), \quad k = -\tau_M, -\tau_M + 1, \ldots, 0, \end{cases} \tag{1}$$

where $x(k) \in \mathbb{R}^n$, $\omega(k) \in \mathbb{R}^m$ and $z(k) \in \mathbb{R}^m$ are the state, input and output vectors of the system, respectively; A, B, D, M and N are real constant matrices with appropriate dimensions; the sequence $\phi(k)$ is the initial condition; $\tau(k)$ is the time-varying delays satisfying $0 \le \tau_m \le \tau(k) \le \tau_M$ with τ_m and τ_M nonnegative integers. The nonlinear functional $\varphi(k, z(k))$ is discrete about k, globally Lipschitz about $z(k)$ and $\varphi(k, 0) = 0$, with satisfying the following conditions:

$$(\varphi(k,z(k)) - K_1 z(k))^T (\varphi(k,z(k)) - K_2 z(k)) \le 0 \quad (2)$$

or

$$(\varphi(k,z(k)))^T (\varphi(k,z(k)) - K z(k)) \le 0, \quad (3)$$

where K_1 and K_2 are real constant diagonal matrices of appropriate dimensions.

Remark 2.1 ([45]): $K = K_2 - K_1$ is a symmetric positive definite diagonal matrix. In other words, the nonlinear function $\varphi(k,z(k))$ satisfying (2) is said to be a sector nonlinear function belonging to $[K_1, K_2]$. If the nonlinear function $\varphi(k,z(k))$ is a sector nonlinear function belonging to $[0, K]$, the nonlinear function $\varphi(k,z(k))$ satisfies (3).

Before deriving the main results, several definitions and lemmas to be used in the subsequent section are given as follows.

Definition 2.1: The Lur'e system is absolutely stable in the sector conditions $[K_1, K_2]$ (or $[0, K]$) if a trivial solution $x(k) = 0$ is globally uniformly asymptotically stable for any nonlinear function $\varphi(k,z(k))$ satisfying the sector-bounded constraint condition (2) (or (3)).

Lemma 2.1 ([26]): *Let $x : [a, b-1] \to \mathbb{R}^n$ be a vector function with a, $b \in \mathbb{Z}$. For a positive definite matrix $R \in \mathbb{R}^{n\times n}$, the following inequalities hold:*

$$-\sum_{i=a}^{b-1} x^T(i)Rx(i) \le -\frac{1}{b-a}\zeta^T \Lambda^T \Xi^T \bar{R} \Xi \Lambda \zeta, \quad (4)$$

$$-\sum_{i=a}^{b-1} \Delta x^T(i)R\Delta x(i) \le -\frac{1}{b-a}\xi^T \Lambda^T \Xi^T \bar{R} \Xi \Lambda \xi, \quad (5)$$

where

$$\zeta = \operatorname{col}\left\{x(b), \sum_{i=a}^{b} x(i), \sum_{i_2=a}^{b}\sum_{i_1=i_2}^{b} x(i_1), \sum_{i_3=a}^{b}\sum_{i_2=i_3}^{b}\sum_{i_1=i_2}^{b} x(i_1)\right\},$$

$$\xi = \operatorname{col}\left\{x(b), x(a), \sum_{i=a}^{b} x(i), \sum_{i_2=a}^{b}\sum_{i_1=i_2}^{b} x(i_1)\right\},$$

$$\Xi = \begin{bmatrix} -I & I & 0 & 0 \\ -I & -I & 2I & 0 \\ -I & I & -6I & 6I \end{bmatrix},$$

$$\Lambda = \operatorname{diag}\left\{I, I, \frac{1}{b-a+1}I, \frac{2}{(b-a+1)(b-a+2)}I\right\},$$

$$\bar{R} = \{R, 3R, 5R\}, \quad \Delta x(i) = x(i+1) - x(i).$$

Lemma 2.2 ([46]): *Let $x : [a, b-1] \to \mathbb{R}^n$ be a vector function with a, $b \in \mathbb{Z}$. For a positive definite matrix $R \in \mathbb{R}^{n\times n}$, any matrices L_1, L_2, H_1, H_2 with appropriate dimensions, a time-varying functions $c(k) \triangleq c_k$ with $a \le c_k \le b$, and vectors β_0, ω_0, the following inequalities hold:*

$$\begin{aligned} -\sum_{i=a}^{b-1} x^T(i)Rx(i) &\le \operatorname{Sym}\left\{\beta_0^T [L_1\Xi \; L_2\Xi]\alpha\right\} \\ &\quad + \beta_0^T \left\{(c_k - a)L_1\bar{R}^{-1}L_1^T + (b - c_k)L_2\bar{R}^{-1}L_2^T\right\}\beta_0, \end{aligned} \quad (6)$$

$$\begin{aligned} -\sum_{i=a}^{b-1} \Delta x^T(i)R\Delta x(i) &\le \operatorname{Sym}\left\{\omega_0^T [H_1\Xi \; H_2\Xi]\varpi\right\} \\ &\quad + \omega_0^T \left\{(c_k - a)H_1\bar{R}^{-1}H_1^T \right. \\ &\quad \left. + (b - c_k)H_2\bar{R}^{-1}H_2^T\right\}\omega_0, \end{aligned} \quad (7)$$

where $\alpha = \operatorname{col}\{\Lambda_1\zeta_1, \Lambda_2\zeta_2\}$ and $\varpi = \operatorname{col}\{\Lambda_1\xi_1, \Lambda_2\xi_2\}$ with

$$\Lambda_1 = \operatorname{diag}\left\{I, I, \frac{1}{c_k-a+1}I, \frac{2}{(c_k-a+1)(c_k-a+2)}I\right\},$$

$$\Lambda_2 = \operatorname{diag}\left\{I, I, \frac{1}{b-c_k+1}I, \frac{2}{(b-c_k+1)(b-c_k+2)}I\right\},$$

$$\zeta_1 = \operatorname{col}\left\{x(c_k), \sum_{i=a}^{c_k} x(i), \sum_{i_2=a}^{c_k}\sum_{i_1=i_2}^{c_k} x(i_1), \sum_{i_3=a}^{c_k}\sum_{i_2=i_3}^{c_k}\sum_{i_1=i_2}^{c_k} x(i_1)\right\},$$

$$\zeta_2 = \operatorname{col}\left\{x(b), \sum_{i=c_k}^{b} x(i), \sum_{i_2=c_k}^{b}\sum_{i_1=i_2}^{b} x(i_1), \sum_{i_3=c_k}^{b}\sum_{i_2=i_3}^{b}\sum_{i_1=i_2}^{b} x(i_1)\right\},$$

$$\xi_1 = \operatorname{col}\left\{x(c_k), x(a), \sum_{i=a}^{c_k} x(i), \sum_{i_2=a}^{c_k}\sum_{i_1=i_2}^{c_k} x(i_1)\right\},$$

$$\xi_2 = \mathrm{col}\left\{x(b), x(c_k), \sum_{i=c_k}^{b} x(i), \sum_{i_2=c_k}^{b}\sum_{i_1=i_2}^{b} x(i_1)\right\}.$$

Remark 2.2: The motivation of proposing the inequalities (6) and (7) are to involve coupling information for additional state variables by additional free matrices, which can relax the conditions of derived criteria. Compared with the summation inequalities proposed in Lemma 2.1 and those in [26–30,42], inequalities (6) and (7) have some advantages. Take inequality (6) for an example:

- Firstly, the inequality (6) is more general than the inequality (4) and Lemma 2 in [26], that is, the inequality (4) and Lemma 2 in [26] are special cases of the inequality (6). Indeed, letting $L_1 = \frac{1}{a-c}\mathrm{col}\{\bar{R} \quad 0\}$, $L_2 = \frac{1}{c-b}\mathrm{col}\{0 \quad \bar{R}\}$ and $\beta_0 = \mathrm{col}\{\Xi\Lambda_1\zeta_1, \Xi\Lambda_2\zeta_2\}$ in (6), the inequality (6) reduces to the inequality (4); letting $\beta_0 = \mathrm{col}\{\omega_{10}, \omega_{11}, \omega_{12}, \omega_{20}, \omega_{21}, \omega_{22}\}$, $L_1 = \mathrm{col}\{\mathrm{diag}\{N_{10}, N_{11}, N_{12}\}, 0\}$ and $L_2 = \mathrm{col}\{0, \mathrm{diag}\{N_{20}, N_{21}, N_{22}\}\}$ in the inequality (6), the inequality (6) reduces to the Lemma 2.2 recently published in [26].
- Secondly, applying the summation inequalities in [26–30,42] to estimate $-\sum_{i=a}^{b-1} x^T(i)Rx(i)$ yields

$$-\sum_{i=a}^{b-1} x^T(i)Rx(i) \leq \lambda_1\zeta_1^T\Lambda_1^T\Xi^T\bar{R}\Xi\Lambda_1\zeta_1 + \lambda_2\zeta_2^T\Lambda_2^T\Xi^T\bar{R}\Xi\Lambda_2\zeta_2. \quad (8)$$

  From (8), it can be found that the obtained quadratic terms $\Xi\Lambda_1\zeta_1$ and $\Xi\Lambda_2\zeta_2$ are only connected with themselves via the coefficient matrix $\bar{R}$, respectively. However, letting $\beta_0 = \mathrm{col}\{\Xi\Lambda_1\zeta_1, \Xi\Lambda_2\zeta_2\}$ in (6) of Lemma 2.2, two free matrices L_1 and L_2 are employed to make $\Xi\Lambda_1\zeta_1$ and $\Xi\Lambda_2\zeta_2$ connect to each other and themselves. Therefore, additional freedom is involved in inequality (6) in comparison to the summation inequalities in [26–30,42].
- The advantages of (7) are similar to those of (6).

3. Main results

In order to more accurately assess the effect of time delays on the stability of the Lur'e system (1), this paper aims to develop new stability criteria with less conservatism. In order to simplify the representation of vectors and matrices, the following notations are used.

$$\tau_{12} = \tau_M - \tau_m, \quad \tau_k = \tau(k), \quad \tau_{km} = \tau(k) - \tau_m,$$
$$\tau_{kM} = \tau_M - \tau(k), s_1(s) = s+1,$$
$$s_2(s) = \frac{(s+1)(s+2)}{2},$$
$$\eta_1(k) = \mathrm{col}\left\{x(k), \sum_{i=k-\tau_m}^{k-1} x(i), \sum_{i=k-\tau_M}^{k-\tau_m-1} x(i), \sum_{i=k-\tau_m}^{k-1}\sum_{j=i}^{k-1} x(j)\right\},$$
$$\eta_2(k) = \mathrm{col}\{x(k), \Delta x(k)\},$$
$$\xi(k) = col\{x(k), x(k-\tau_m), x(k-\tau_k), x(k-\tau_M), \mu_1(k), \mu_2(k), \mu_3(k), \nu_1(k), \nu_2(k), \nu_3(k), \omega(k)\},$$
$$\Delta x(k) = x(k+1) - x(k),$$
$$\mu_1(k) = \sum_{i=k-\tau_k}^{k-\tau_m} \frac{x(i)}{s_1(\tau_{km})},$$
$$\mu_2(k) = \sum_{i=k-\tau_M}^{k-\tau_k} \frac{x(i)}{s_1(\tau_{kM})},$$
$$\mu_3(k) = \sum_{i=k-\tau_m}^{k} \frac{x(i)}{s_1(\tau_m)},$$
$$\nu_1(k) = \sum_{i=k-\tau_m}^{k}\sum_{j=i}^{k} \frac{x(j)}{s_2(\tau_m)},$$
$$\nu_2(k) = \sum_{i=k-\tau_k}^{k-\tau_m}\sum_{j=i}^{k-\tau_m} \frac{x(j)}{s_2(\tau_{km})},$$
$$\nu_3(k) = \sum_{i=k-\tau_M}^{k-\tau_k}\sum_{j=i}^{k-\tau_k} \frac{x(j)}{s_2(\tau_{kM})}.$$

3.1. Absolute stability criterion under constraint condition (3)

Theorem 3.1: *The time-delayed Lur'e system (1) with nonlinear functional $\varphi(k, z(k))$ satisfying the constraint condition (3) is absolutely stable for given $0 < \tau_m \leq \tau_M$, a symmetric positive definite matrix K, if there exist positive definite matrices $P \in \mathbb{R}^{4n\times 4n}$, $(R_i \in \mathbb{R}^{2n\times 2n})$, $(Q_i, Z_i, G_j \in \mathbb{R}^{n\times n})$, and any matrices $L_i \in \mathbb{R}^{7n\times 4n}$, $H_i \in \mathbb{R}^{7n\times 3n}$, $(i = 1, 2; j = 1, 2, 3)$, such that the following inequalities hold for $\tau_k \in \{\tau_m, \tau_M\}$*

$$R_1 + \begin{bmatrix} 0 & G_1 \\ G_1 & G_1 \end{bmatrix} > 0, \quad (9)$$
$$\begin{bmatrix} \Pi(\tau_m) + \Phi & \tau_{12}\beta^T L_2 & \tau_{12}\beta^T H_2 \\ * & -\tau_{12}\bar{\mathcal{R}}_3 & 0 \\ * & * & -\tau_{12}\bar{Z}_2 \end{bmatrix} < 0, \quad (10)$$
$$\begin{bmatrix} \Pi(\tau_M) + \Phi & \tau_{12}\beta^T L_1 & \tau_{12}\beta^T H_1 \\ * & -\tau_{12}\bar{\mathcal{R}}_2 & 0 \\ * & * & -\tau_{12}\bar{Z}_2 \end{bmatrix} < 0 \quad (11)$$

with

$$\Pi(\tau_k) = \Pi_1(\tau_k) + \mathrm{Sym}\{\Pi_2(\tau_k)\} + e_1(Q_1 + G_1)e_1^T$$

$$
\begin{aligned}
&+e_2(Q_2-Q_1+G_2-G_1)e_2^T\\
&+e_3(G_3-G_2)e_3^T-e_4(G_3+Q_2)e_4^T\\
&+[e_1\quad e_s](\tau_m R_1+\tau_{12}R_2)[e_1\quad e_s]^T\\
&+e_s(\tau_m Z_1+\tau_{12}Z_2)e_s^T-\frac{1}{\tau_m}\varsigma_0\bar{\mathcal{R}}_1\varsigma_0^T\\
&-\frac{1}{\tau_m}\gamma_0\Xi^T\bar{Z}_1\Xi\gamma_0^T,
\end{aligned}
$$

$$
\begin{aligned}
\Pi_1(\tau_k)&=\Psi P\Psi^T+\mathrm{Sym}\left\{\Theta(\tau_k)P\Psi^T\right\},\\
\Pi_2(\tau_k)&=\beta^T\left[L_1\quad L_2\right]\left[\varsigma_1\quad \varsigma_2\right]^T\\
&\quad+\beta^T\left[H_1\Xi\quad H_2\Xi\right]\left[\gamma_1\quad \gamma_2\right]^T,\\
\Psi&=[e_s\quad (e_1-e_2)\quad (e_2-e_4)\\
&\qquad s_1(\tau_m)(e_1-e_7)],\\
\Theta(\tau_k)&=[e_1\quad s_1(\tau_m)e_7-e_1\quad s_1(\tau_{km})e_5+s_1(\tau_{kM})e_6\\
&\qquad -e_2-e_3\quad s_2(\tau_m)e_8-s_1(\tau_m)e_1],\\
\Phi&=-2e_{11}e_{11}^T-\mathrm{Sym}\{(e_1M^TK+e_3N^TK)e_{11}^T\},\\
\beta^T&=[e_2\quad e_3\quad e_4\quad e_5\quad e_6\quad e_9\quad e_{10}],\\
\varsigma_0&=[s_1(\tau_m)e_7-e_1\quad e_1-e_2\quad (\tau_m+2)e_8\\
&\qquad -s_1(\tau_m)e_7-e_1\quad e_1+e_2-2e_7],\\
\varsigma_1&=[s_1(\tau_{km})e_5-e_2\quad e_2-e_3\quad (\tau_{km}+2)e_9\\
&\qquad -s_1(\tau_{km})e_5-e_2\quad e_2+e_3-2e_5],\\
\varsigma_2&=[s_1(\tau_{kM})e_6-e_3\quad e_3-e_4\quad (\tau_{kM}+2)e_{10}\\
&\qquad -s_1(\tau_{kM})e_6-e_3\quad e_3+e_4-2e_6],\\
\gamma_0&=[e_1\quad e_2\quad e_7\quad e_8],\quad \gamma_1=[e_2\quad e_3\quad e_5\quad e_9],\\
\gamma_2&=[e_3\quad e_4\quad e_6\quad e_{10}],\\
\mathcal{R}_i&=R_i+\begin{bmatrix}0 & G_i\\ G_i & G_i\end{bmatrix},\quad \mathcal{R}_3=R_2+\begin{bmatrix}0 & G_3\\ G_3 & G_3\end{bmatrix},\\
\bar{\mathcal{R}}_i&=\mathrm{diag}\left\{\mathcal{R}_i,3\mathcal{R}_i\right\},\\
\bar{Z}_i&=\mathrm{diag}\left\{Z_i,3Z_i,5Z_i\right\},\ (i=1,2),\\
\bar{\mathcal{R}}_3&=\mathrm{diag}\left\{\mathcal{R}_3,3\mathcal{R}_3\right\},\\
e_s&=e_1(A-I)^T+e_3B^T+e_{11}D^T.
\end{aligned}
$$

Proof: Based on the summation inequalities technique of Lemma 2.2, we construct the following improved LKF candidate:

$$
V(k)=\sum_{i=1}^{4}V_i(k) \tag{12}
$$

with

$$
\begin{aligned}
V_1(k)&=\eta_1^T(k)P\eta_1(k),\ V_2(k)=\sum_{i=k-\tau_m}^{k-1}x^T(i)Q_1x(i)\\
&\quad+\sum_{i=k-\tau_M}^{k-\tau_m-1}x^T(i)Q_2x(i),\ V_3(k)\\
&=\sum_{i=-\tau_m}^{-1}\sum_{j=k+i}^{k-1}\eta_2^T(j)R_1\eta_2(j)\\
&\quad+\sum_{i=-\tau_M}^{-\tau_m-1}\sum_{j=k+i}^{k-1}\eta_2^T(j)R_2\eta_2(j),\ V_4(k)\\
&=\sum_{i=-\tau_m}^{-1}\sum_{j=k+i}^{k-1}\Delta x^T(j)Z_1\Delta x(j)\\
&\quad+\sum_{i=-\tau_M}^{-\tau_m-1}\sum_{j=k+i}^{k-1}\Delta x^T(j)Z_2\Delta x(j).
\end{aligned}
$$

The forward differences of $V(k)$ along the trajectory of (1) are, respectively, computed as

$$
\begin{aligned}
\Delta V_1(k)&=\eta_1^T(k+1)P\eta_1(k+1)-\eta_1^T(k)P\eta_1(k)\\
&=\xi^T(k)\left(\Psi P\Psi^T+\mathrm{Sym}\left\{\Theta(h_k)P\Psi^T\right\}\right)\xi(k),
\end{aligned}\tag{13}
$$

$$
\begin{aligned}
\Delta V_2(k)=&\,\xi^T(k)\left(e_1Q_1e_1^T+e_2(Q_2-Q_1)e_2^T-e_4Q_2e_4^T\right)\\
&\times\xi(k),
\end{aligned}\tag{14}
$$

$$
\begin{aligned}
\Delta V_3(k)&=\eta_2^T(k)(\tau_mR_1+\tau_{12}R_2)\eta_2(k)\\
&\quad-\sum_{i=k-\tau_m}^{k-1}\eta_2^T(i)R_1\eta_2(i)\\
&\quad-\sum_{i=k-\tau_M}^{k-\tau_m-1}\eta_2^T(i)R_2\eta_2(i),
\end{aligned}\tag{15}
$$

$$
\begin{aligned}
\Delta V_4(k)&=\Delta x^T(k)(\tau_mZ_1+\tau_{12}Z_2)\Delta x(k)\\
&\quad-\sum_{i=k-\tau_m}^{k-1}\Delta x^T(i)Z_1\Delta x(i)\\
&\quad-\sum_{i=k-\tau_M}^{k-\tau_m-1}\Delta x^T(i)Z_2\Delta x(i).
\end{aligned}\tag{16}
$$

The following equations are obvious for symmetric matrices G_1, G_2 and G_3.

$$
\begin{aligned}
0&=x^T(k)G_1x(k)-x^T(k-\tau_m)G_1x(k-\tau_m)\\
&\quad-\sum_{i=k-\tau_m}^{k-1}\left[\Delta x^T(i)G_1\Delta x(i)+2\Delta x^T(i)G_1x(i)\right],
\end{aligned}\tag{17}
$$

$$
\begin{aligned}
0&=x^T(k-\tau_m)G_2x(k-\tau_m)-x^T(k-\tau_k)G_2x(k-\tau_k)\\
&\quad-\sum_{i=k-\tau_k}^{k-\tau_m-1}\left[\Delta x^T(i)G_2\Delta x(i)+2\Delta x^T(i)G_2x(i)\right],
\end{aligned}\tag{18}
$$

$$0 = x^T(k-\tau_k)G_3x(k-\tau_k) - x^T(k-\tau_M)G_3x(k-\tau_M) - \sum_{i=k-\tau_M}^{k-\tau_k-1}\left[\Delta x^T(i)G_3\Delta x(i) + 2\Delta x^T(i)G_3x(i)\right]. \tag{19}$$

It follows that from $\Delta V_3(k)$ and the above zero equations (17)–(19):

$$\begin{aligned}\Delta V_3(k) &= \eta_2^T(k)(\tau_m R_1 + \tau_{12}R_2)\eta_2(k) + e_1G_1e_1^T \\ &\quad + e_2(G_2-G_1)e_2^T + e_3(G_3-G_2)e_3^T - e_4G_3e_4^T \\ &\quad - \sum_{i=k-\tau_m}^{k-1}\eta_2^T(i)\mathcal{R}_1\eta_2(i) - \sum_{i=k-\tau_k}^{k-\tau_m-1}\eta_2^T(i)\mathcal{R}_2\eta_2(i) \\ &\quad - \sum_{i=k-\tau_M}^{k-\tau_k-1}\eta_2^T(i)\mathcal{R}_3\eta_2(i).\end{aligned} \tag{20}$$

The following $\mathcal{R}_1$- and Z_1-dependent summation inequalities in $\Delta V_3(k)$ and $\Delta V_4(k)$ can be obtained according to (9) and Lemma 2.1.

$$-\sum_{i=k-\tau_m}^{k-1}\eta_2^T(i)\mathcal{R}_1\eta_2(i) \le -\frac{1}{\tau_m}\xi^T(k)\varsigma_0\bar{\mathcal{R}}_1\varsigma_0^T\xi(k), \tag{21}$$

$$-\sum_{i=k-\tau_m}^{k-1}\Delta x^T(i)Z_1\Delta x(i) \le -\frac{1}{\tau_m}\xi^T(k)\gamma_0\bar{Z}_1\gamma_0^T\xi(k). \tag{22}$$

$\mathcal{R}_2 > 0$ and $\mathcal{R}_3 > 0$ can be obtained from (10) and (11). Thus, the free-matrix-based summation inequalities proposed in Lemma 2.2 can be used to estimate the following $\mathcal{R}_2$-, $\mathcal{R}_3$- and Z_2-dependent summation inequalities.

$$\begin{aligned}&-\sum_{i=k-\tau_M}^{k-\tau_m-1}\eta_2^T(i)\mathcal{R}_2\eta_2(i) \\ &\le \xi^T(k)\mathrm{Sym}\left\{\beta^T[L_1 \quad L_2][\varsigma_1 \quad \varsigma_2]^T\right\}\xi(k) \\ &\quad + \tau_{km}\xi^T(k)\beta^TL_1\bar{\mathcal{R}}_2^{-1}L_1^T\beta\xi(k) \\ &\quad + \tau_{kM}\xi^T(k)\beta^TL_2\bar{\mathcal{R}}_3^{-1}L_2^T\beta\xi(k),\end{aligned} \tag{23}$$

$$\begin{aligned}&-\sum_{i=k-\tau_M}^{k-\tau_m-1}\Delta x^T(i)Z_2\Delta x(i) \\ &\le \xi^T(k)\mathrm{Sym}\left\{\beta^T[H_1\Xi \quad H_2\Xi][\gamma_1 \quad \gamma_2]^T\right\}\xi(k) \\ &\quad + \tau_{km}\xi^T(k)\beta^TH_1\bar{Z}_2^{-1}H_1^T\beta\xi(k) \\ &\quad + \tau_{kM}\xi^T(k)\beta^TH_2\bar{Z}_2^{-1}H_2^T\beta\xi(k).\end{aligned} \tag{24}$$

And, it follows from (3) that

$$-2\omega^T(k)\omega(k) - 2\omega^T(k)K[Mx(k)+Nx(k-h(k))] \ge 0. \tag{25}$$

From Equations (13)–(16) and (20)–(25), we have

$$\begin{aligned}\Delta V(k) &\le \xi^T(k)\Big[\Pi(\tau_k) + \Phi + \beta^T \\ &\quad \left(\tau_{km}L_1\bar{\mathcal{R}}_2^{-1}L_1^T + \tau_{kM}L_2\bar{\mathcal{R}}_3^{-1}L_2^T\right)\beta\Big]\xi(k) \\ &\quad + \xi^T(k)\beta^T\left(\tau_{km}H_1\bar{Z}_2^{-1}H_2^T + \tau_{kM}H_2\bar{Z}_2^{-1}H_2^T\right) \\ &\quad \times \beta\xi(k),\end{aligned} \tag{26}$$

which together with Schur complement and (10)–(11) imply that $\Delta V(k) < 0$. Therefore, by Lyapunov stability theorem, it can be guaranteed that the time-delayed Lur'e system (1) is asymptotically stable, which can guarantee the time-delayed Lur'e system (1) is absolutely stable from Definition 2.1. ■

Remark 3.1: In Theorem 3.1, an improved LKF (12) augmented by some terms in $V_1(k)$ and $V_3(k)$ is constructed, for example, $\sum_{i=k-\tau_m}^{k-1}\sum_{j=i}^{k-1}x(j)$ in $V_1(k)$, $\Delta x(k)$ in $V_3(k)$, which are ignored in the LKFs of [38–40,43,44]. So, more coupling information between the different states of the system and time delays than [38–40,43,44] is considered in the LKF (12). Moreover, these augmented terms are just some necessary terms for Lemma 2.2 to make full use of the free-matrix-based summation inequalities proposed in Lemma 2.2 in the different delay intervals $[\tau_m, \tau(k)]$ and $[\tau(k), \tau_M]$. Thus, the less conservatism of Theorem 3.1 proposed in this paper than those recently proposed in [38,40,43,44] lies in the combination of Lemma 2.2 and the modified LKF.

3.2. *Absolute stability criterion under constraint condition (2)*

Now, we consider the nonlinear function $\varphi(k, z(k))$ satisfying (2), that is, $\varphi(k, z(k))$ in the sector $[K_1, K_2]$. We apply the loop transformation suggested in [47] and obtain that the absolute stability of the time-delayed Lur'e system (1) with the nonlinear function $\varphi(k, z(k))$ satisfying (2) is equivalent to that of the following system:

$$\begin{cases} x(k+1) = (A - DK_1M)x(k) + (B - DK_1N) \\ x(k-\tau(k)) + D\omega(k), \\ z(k) = Mx(k) + Nx(k-\tau(k)), \\ \omega(k) = -\varphi(k, z(k)), \end{cases} \tag{27}$$

where the nonlinear function $\varphi(k, z(k))$ is in the sector $[0, K_2 - K_1]$, that is, satisfies (2).

Then, similar to the proof of Theorem 3.1, we can establish the following corollary.

Corollary 3.1: *The time-delayed Lur'e system* (1) *with nonlinear functional* $\varphi(k, z(k))$ *satisfying the constraint* *condition (2) is absolutely stable for given $0 < \tau_m \le \tau_M$, symmetric positive definite diagonal matrices K_1 and K_2, if there exist positive definite matrices $P \in \mathbb{R}^{4n\times 4n}$, $(R_i \in \mathbb{R}^{2n\times 2n})$, $(Q_i, Z_i, G_j \in \mathbb{R}^{n\times n})$, and any matrices $L_i \in \mathbb{R}^{7n\times 4n}$, $H_i \in \mathbb{R}^{7n\times 3n}$, $(i = 1, 2;\ j = 1, 2, 3)$, such that the LMI (9) and the following inequalities hold for $\tau_k \in \{\tau_m, \tau_M\}$*

$$\begin{bmatrix} \tilde{\Pi}(\tau_m) + \tilde{\Phi} & \tau_{12}\beta^T L_2 & \tau_{12}\beta^T H_2 \\ * & -\tau_{12}\bar{\mathcal{R}}_3 & 0 \\ * & * & -\tau_{12}\bar{Z}_2 \end{bmatrix} < 0, \quad (28)$$

$$\begin{bmatrix} \tilde{\Pi}(\tau_M) + \tilde{\Phi} & \tau_{12}\beta^T L_1 & \tau_{12}\beta^T H_1 \\ * & -\tau_{12}\bar{\mathcal{R}}_2 & 0 \\ * & * & -\tau_{12}\bar{Z}_2 \end{bmatrix} < 0 \quad (29)$$

with

$$\begin{aligned} \tilde{\Pi}(\tau_k) &= \tilde{\Pi}_1(\tau_k) + \mathrm{Sym}\{\Pi_2(\tau_k)\} + e_1(Q_1 + G_1)e_1^T \\ &\quad + e_2(Q_2 - Q_1 + G_2 - G_1)e_2^T \\ &\quad + e_3(G_3 - G_2)e_3^T - e_4(G_3 + Q_2)e_4^T + [e_1 \quad e_s] \\ &\quad (\tau_m R_1 + \tau_{12}R_2)[e_1 \quad \tilde{e}_s]^T \\ &\quad + e_s(\tau_m Z_1 + \tau_{12}Z_2)e_s^T - \frac{1}{\tau_m}\varsigma_0\bar{\mathcal{R}}_1\varsigma_0^T \\ &\quad - \frac{1}{\tau_m}\gamma_0 \Xi^T \bar{Z}_1 \Xi \gamma_0^T, \tilde{\Pi}_1(\tau_k) \\ &= \tilde{\Psi}P\tilde{\Psi}^T + \mathrm{Sym}\left\{\Theta(\tau_k)P\tilde{\Psi}^T\right\}, \tilde{\Psi} \\ &= [\tilde{e}_s \quad (e_2 - e_4) \quad (e_1 - e_2) \\ &\quad s_1(\tau_m)(e_1 - e_7)], \\ \tilde{e}_s &= e_1(A - DK_1M - I)^T \\ &\quad + e_3(B - DK_1N)^T + e_{11}D^T, \tilde{\Phi} \\ &= -2e_1M^TK_1^TK_2Me_1^T \\ &\quad - 2e_3N^TK_1^TK_2Ne_3^T - 2e_{11}e_{11}^T \\ &\quad - \mathrm{Sym}\{e_{11}(K_1 + K_2)(Me_1^T + Ne_3^T) \\ &\quad + e_1M^T(K_2^TK_1 + K_1^TK_2)Ne_3^T\}. \end{aligned}$$

Proof: Just replace equation (20) in the proof of Theorem 3.1 with the following inequality

$$-2\left(\varphi(k,z(k)) - K_1 z(k)\right)^T \left(\varphi(k,z(k)) - K_2 z(k)\right) \ge 0, \quad (30)$$

that is,

$$2\left[\omega(k) + K_1(Mx(k) + Nx(k-\tau_k))\right]^T \left[-\omega(k) - K_2(Mx(k) + Nx(k-\tau_k))\right] \ge 0. \quad (31)$$

■

4. Numerical examples

The following content mainly show the effectiveness of the stability criteria obtained in this paper according to two common classical numerical examples considered regularly in many recent references [38–41,44]. The main method is to solve the LMIs in the relevant stability results via Matlab LMI-Toolbox, so as to obtain the maximum allowable time-delay upper bound value (MADUBs). And the index of the number of decision variables (NoVs) is applied to show the complexity of criteria.

Table 1. MADUBs τ_M for different τ_m (Example 4.1).

Methods\\τ_m	4	6	8	10	12	15	17	18	20	NoVs
[38]	17	18	19	20	21	23	25	25	27	42
[39]	17	18	19	20	21	23	25	25	27	42
[40]	19	20	21	22	23	24	26	26	28	27
[41]	22	23	24	25	25	26	27	27	29	295
Corollary 3.1	22	23	24	26	26	26	28	28	29	459

Example 4.1 (**[38–41]**)**:** The time-delayed Lur'e system (1) with the parameters described as follows:

$$A = \begin{bmatrix} 0.8 & 0 \\ 0.05 & 0.9 \end{bmatrix}, \quad B = \begin{bmatrix} -0.1 & 0 \\ -0.2 & -0.1 \end{bmatrix},$$

$$D = \begin{bmatrix} -0.02 \\ -0.03 \end{bmatrix}, M = \begin{bmatrix} 0.3 & 0.1 \end{bmatrix},$$

$$N = \begin{bmatrix} 0.1 & 0.2 \end{bmatrix}, \quad K_1 = 0.2, \quad K_2 = 0.5.$$

Here, the lower bound values of time-varying delay τ_m are first given, and then the relevant MADUBs values τ_M of the time-varying delay can be obtained by solving LMIs in Corollary 3.1 by Matlab LMI-Toolbox. All of the results are listed in Table 1, where the values obtained by the methods in some most recent references and Corollary 3.1 in this paper are compared. From this table, it can be intuitively seen that the upper bound values of the time-varying delay obtained by Corollary 3.1 in this paper are larger than those in [38–41]. It can be further explained that the stability criterion for the discrete-time Lur'e system (1) obtained by the method in this paper is less conservative than some existing published method.

In order to show that the upper bounds of the time-varying delay in this paper ($\tau_m = 20$ and $\tau_M = 29$) are still within the range of the true values of the maximum upper bound of time delay that guarantees the system stability, Figure 1 gives the simulation results for the Lur'e system (1) with $\varphi(k, z(k)) = (\frac{7}{20} + \frac{3}{20}\sin(k))z(k)$, the initial state $x(0) = [1 \quad -1]^T$ and $\tau(k) = \mathrm{INT}\{\frac{49}{2} + \frac{9}{2}\sin(\frac{k\pi}{16})\}$. Here, $\mathrm{INT}\{\cdot\}$ denotes the integral part of numerical value.

Example 4.2 (**[38,40]**)**:** Consider the time-delayed Lur'e system (1) with the following parameters:

$$A = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.7 \end{bmatrix}, \quad B = \begin{bmatrix} -0.1 & 0 \\ -0.1 & -0.1 \end{bmatrix},$$

$$D = \begin{bmatrix} -0.02 \\ -0.03 \end{bmatrix}, M = \begin{bmatrix} 0.6 & 0.8 \end{bmatrix},$$

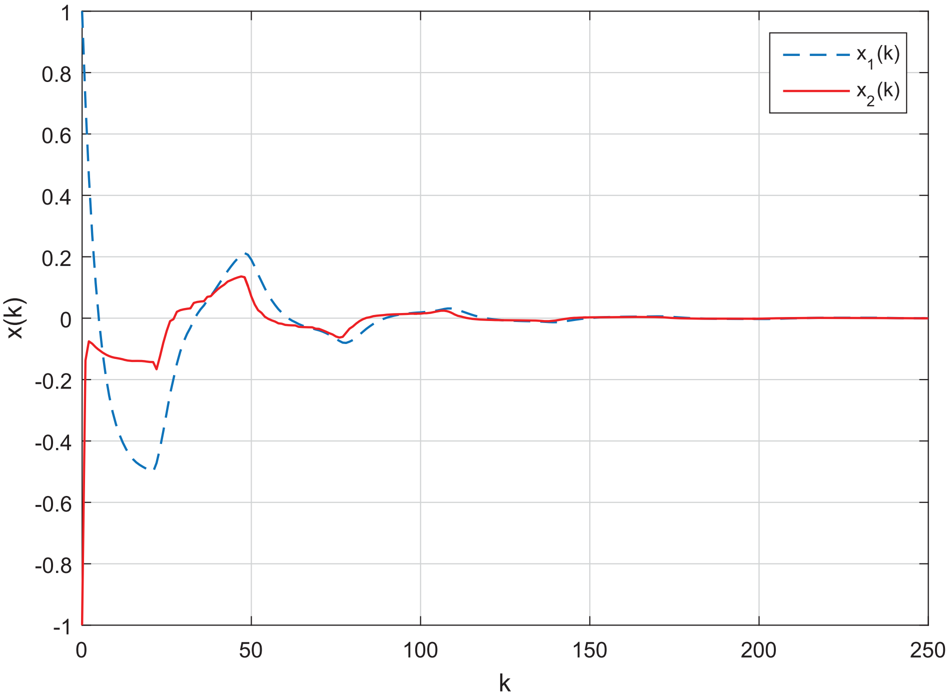

Figure 1. The state response of Lur'e system (1) under the conditions given in Example 4.1.

$$N = \begin{bmatrix} 0 & 0 \end{bmatrix}, \quad K = 0.5.$$

Similar to Example 4.1, for given the lower bound values of time-varying delay τ_m, the MADUBs for τ_M of the time-delayed Lur'e system (1) are listed in Table 2. The different methods and results are still compared in the table. From the table, it can be intuitively seen that the upper bound values of the time-varying delay in this paper are lager than some previous proposed in [38,40,41]. Thus, the stability criterion is less conservative than [38,40,41].

Table 2. MADUBs τ_M for different τ_m (Example 4.2).

Methods\ τ_m	4	6	8	10	12	15	17	18	20	NoVs
[38]	17	19	21	23	25	28	30	31	33	42
[40]	27	29	30	32	34	37	39	40	42	27
[41]	29	31	32	33	35	38	40	41	44	295
Theorem 3.1	29	31	32	34	35	39	41	41	44	459

To confirm that the obtained result ($\tau_m = 20$ and $\tau_M = 44$) is still within the range of the true values of the maximum upper bound of time delay that guarantees the system stability, the simulation result is shown

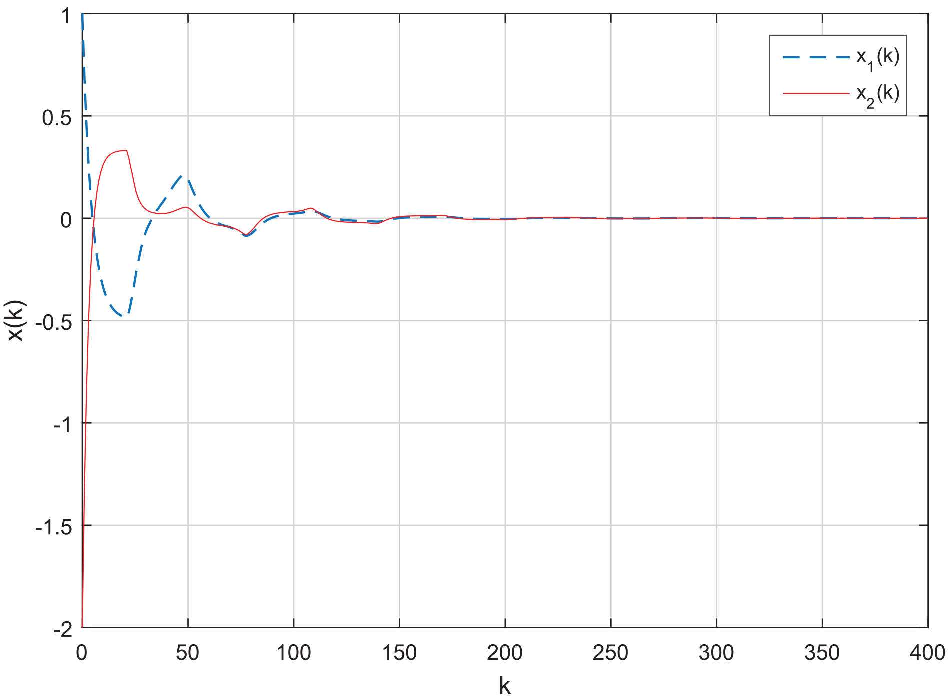

Figure 2. The state response of Lur'e system (1) under the conditions given in Example 4.2.

in Figure 2, which shows that the state responses of the Lur'e system (1) with $\varphi(k, z(k)) = 0.5\sin^2(z(k))$ and $\tau(k) = \text{INT}\{32 - 12\sin(\frac{k\pi}{4})\}$ converge to zero under the initial state $x(0) = [1 \ \ 2]^T$.

Remark 4.1: From the Examples 4.1 and 4.2, it is obvious that Theorem 3.1 and Corollary 3.1 contain 459 decision variables, which are more than [38–41]. Thus, Theorem 3.1 and Corollary 3.1 reduce the conservatism but increases the complexity of the solution.

5. Conclusion

This paper mainly focus on the delay-dependent absolute stability of discrete-time time-delayed Lur'e systems with sector-bounded nonlinearities. A modified LKF with some augmented double summation terms is constructed to make full use of the improved free-matrix-based summation inequality technique. Some less conservatism stability criteria than some recently published are derived as LMIs, which can be solve by Matlab LMI-Toolbox. The reduction in conservatism lies in the combination of Lemma 2.2 and the modified LKF. The effectiveness of the proposed method is illustrated by comparison and discussion in numerical examples.

The derivation method of the stability criterion presented in this paper can be extended to other related time-delayed systems, for example, time-delayed neural networks, time-delayed linear systems, other time-delayed nonlinear systems, and so on. Of course, our research group will also apply relevant theories into practice, as the goal of scientific research.

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