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USPOREDBA RAZLIČITIH KRITERIJA OPTIMIRANJA PARAMETARA ROBOTSKIH REGULATORA S PROMJENJIVOM STRUKTUROM
COMPARISON OF DIFFERENT CRITERIONS FOR PARAMETER OPTIMIZATION OF VARIABLE-STRUCTURE ROBOT CONTROLLERS

Vesna KRAJČÍ – Domagoj DUŠEVIĆ – Mojmil CECIĆ

Sažetak: *U članku su predstavljeni različiti integralni kriteriji optimiranja sa sljedećim integralnim mjerama kakvoće upravljanja: integral apsolutne vrijednosti pogreške, integral kvadrata pogreške, integral vremenski otežane apsolutne vrijednosti pogreške, integral vremenski otežanog kvadrata pogreške. Te su integralne mjere kakvoće upravljanja korištene u simulacijama za optimiranje parametara regulatora s promjenjivom strukturom kod troosne planarne rotacijske robotske ruke. Optimiranje parametara robotskih regulatora provedeno je simulacijama na računalu u prostoru zglobova robota i u prostoru konfiguracije alata robota, pri čemu su bolji rezultati optimiranja postignuti u prostoru konfiguracije alata robota.*

Ključne riječi:
- robot
- upravljanje s kliznim režimom
- upravljanje s promjenjivom strukturom
- optimizacija

Summary: *This paper presents different integral optimization criterions with the following integral performance indices: integral of absolute value of error, integral of squared error, integral of time multiplied by absolute value of error, integral of time multiplied by squared error. These integral performance indices are used in simulations for parameters optimization of variable-structure controllers of a three-axis planar articulated robot arm. Optimization of robot controller parameters is performed by computer simulations both in robot joint space and in robot tool-configuration space and better optimization results are achieved in robot tool-configuration space.*

Key words:
- robot
- sliding-mode control
- variable-structure control
- optimization

1. UVOD

Za upravljanje nesigurnim ili dinamički vrlo složenim sustavima najčešće se upotrebljava adaptivno ili robustno upravljanje [7]. Iako se adaptivni pristup može primijeniti na šire područje nesigurnosti, robustni regulator je jednostavniji za primjenu i ne treba nikakvo vrijeme za adaptaciju [6], [7]. Robustno upravljanje robotskim manipulatorom je vrlo korisno kada je njegov dinamički model nesiguran, tj. ako robot nosi promjenjivi teret ili je točno računanje dinamike robota presloženo [6], [7].

Jedan tip robustnog upravljanja koji može biti primijenjen za upravljanje mnogim nelinearnim i složenim procesima je postupak upravljanja s promjenjivom strukturom [6], [7]. Jedna od glavnih karakteristika ovog pristupa je u tome što pogreška samo treba biti dovedena do površine

1. INTRODUCTION

For control of uncertain or dynamically very complex systems, either an adaptive control or a robust control is usually used [7]. Although the adaptive approach is applicable to a wide range of uncertainties, the robust controller is simpler for implementation and no time is required for adaptation [6], [7]. Robust control of the robotic manipulator is very useful when its dynamical model is uncertain, i.e. if the robot is carrying a variable load or if the exact evaluation of the robot's dynamics is too complicated [6], [7].

One type of robust control which can be applied to the control of many nonlinear and complex processes is the variable-structure control method [6], [7]. One of the main features of this approach is that the error need only prekapčanja, nakon čega je sustav u kliznom režimu i zbog toga neće biti pod utjecajem nikakvih nesigurnosti modeliranja ili smetnji [7], [12]. Taj regulator s promjenjivom strukturom uklanja nelinearne veze između zglobova robota prisiljavajući sustav ući u klizni režim rada. Ako je zakon upravljanja isprekidan uzduž presjeka kliznih površina, stvorit će trešnju koja može pobuditi nemodeliranu visokofrekvencijsku dinamiku robota [7], [12]. Zbog toga izvorni zakon upravljanja s promjenjivom strukturom treba biti izmijenjen zamjenom funkcije predznaka funkcijom zasićenja [9], [10], [12].

Učinkovitost primijenjenih robotskih regulatora s promjenjivom strukturom može biti poboljšana optimiranjem njihovih parametara prema kriteriju optimiranja sa željenom mjerom kakvoće upravljanja [3], [11], [14].

be brought to the switching surface, after which the system is in sliding mode and therefore it will not be affected by any modelling uncertainties or disturbances [7], [12]. This variable-structure controller eliminates the nonlinear coupling of the robot joints by forcing the system into the sliding mode. If the control law is discontinuous along the intersection of the sliding surfaces, it will create chattering which may excite unmodelled high-frequency robot dynamics [7], [12]. Therefore, the original variable-structure control law has to be modified by replacing the sign function with saturation function [9], [10], [12].

The efficiency of the applied variable-structure robot controllers can be improved by optimization of its parameters according to the optimization criterion with the desired performance index [3], [11], [14].

2. UPRAVLJANJE S PROMJENJIVOM STRUKTUROM I OPTIMIRANJEM

2. VARIABLE-STRUCTURE CONTROL WITH OPTIMIZATION

Za robot se mogu definirati sljedeće varijable stanja sustava: pogreške kuta zakreta osovine *e_i(t)* i brzine *de_i(t)/dt* svakoga i-tog motora robota:

For a robot the following system state variables can be defined: the error of shaft angle $e_i(t)$ and speed $de_i(t)/dt$ of each i-th robot motor:

$$e_i(t) = \theta_{r_i}(t) - \theta_i(t), \quad 1 \le i \le n, \tag{1}$$

$$\dot{e}_i(t) = \dot{\theta}_{r_i}(t) - \dot{\theta}_i(t), \quad 1 \le i \le n. \tag{2}$$

Za varijable sustava i-tog motora robota može biti definirana sljedeća općenita pogreška [13]:

For state variables of the i-th robot motor, the following common error can be defined [13]:

$$\sigma_i(e_i, \dot{e}_i) = K_{\sigma_i} \cdot [\lambda_i \cdot e_i(t) + \dot{e}_i(t)], \quad 1 \le i \le n. \tag{3}$$

Cilj upravljanja je uklanjanje svih pogrešaka motora robota $\sigma_i(t)$:

The goal of the control is the elimination of all robotic motor errors $\sigma_i(t)$:

$$\sigma_i(e_i, \dot{e}_i) = 0, \quad 1 \le i \le n, \tag{4}$$

što je moguće postići upotrebom sljedećeg zakona upravljanja s promjenjivom strukturom i kliznim režimom [1], [8]:

which can be achieved by using the following variable-structure sliding-mode control law [1], [8]:

$$\begin{aligned} u_{R_i}(t) &= u_{1_i}(t) + u_{2_i}(t), \quad 1 \le i \le n, \\ u_{1_i}(t) &= \gamma_i \cdot \operatorname{sgn}\{K_{\sigma_i} \cdot [\lambda_i \cdot e_i(t) + \dot{e}_i(t)]\}, \quad 1 \le i \le n, \\ u_{2_i}(t) &= K_{\sigma_i} \cdot \lambda_i \cdot \dot{e}_i(t), \quad 1 \le i \le n. \end{aligned} \tag{5}$$

Funkcija predznaka u prvom dijelu zakona upravljanja (5) izaziva neželjene visokofrekvencijske oscilacije signala upravljanja, tj. trešnju, što dovodi do velikih gubitaka energije. Za eliminaciju trešnje, funkcija predznaka u tom dijelu zakona upravljanja (5) treba biti zamijenjena funkcijom zasićenja, kako je predloženo u [9], [10], [12]:

The sign function in first part of the control law (5) causes undesirably high-frequency oscillations of the control signal, i.e. chattering, which leads to great energy losses. In order to eliminate chattering, the sign function in this part of the control law (5) has to be replaced by a saturation function, as suggested in [9], [10], [12]:

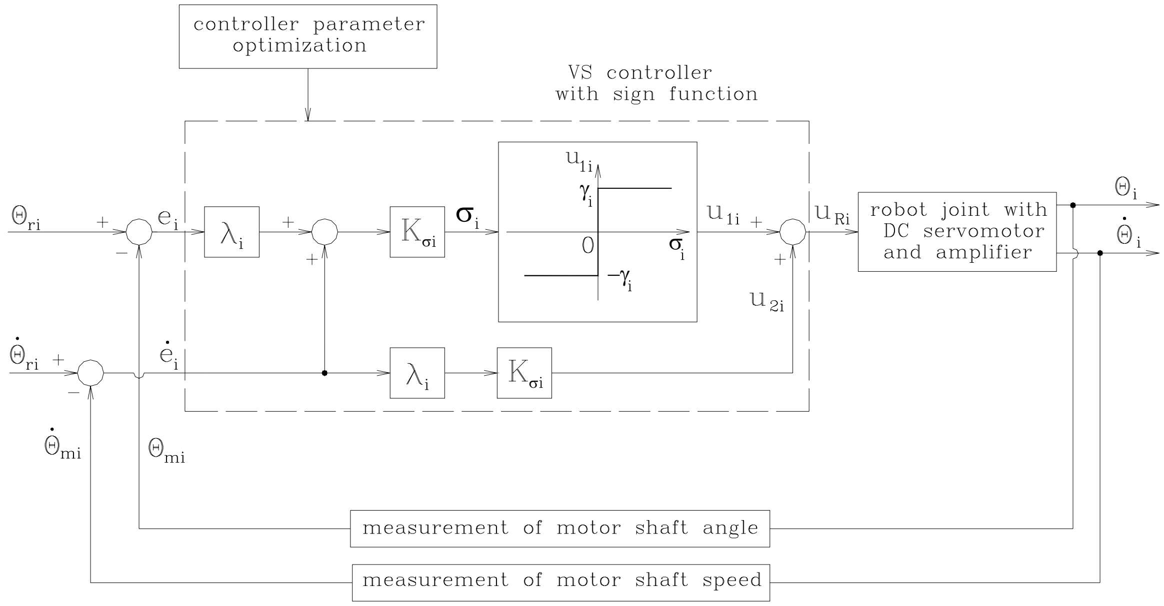
$$u_{1_i}(t) = \gamma_i \cdot sat[\sigma_i(t)] = \left\{ \begin{array}{l} -\gamma_i, \ \sigma_i < -\delta_i \\ \gamma_i \cdot \dfrac{\sigma_i}{\delta_i}, -\delta_i < \sigma_i < \delta_i \\ \gamma_i, \ \sigma_i > \delta_i \end{array} \right\}, \quad 1 \le i \le n, \tag{6}$$

gdje δ_i označava debljinu graničnog sloja za i-ti motor robota, $1 \le i \le n$.

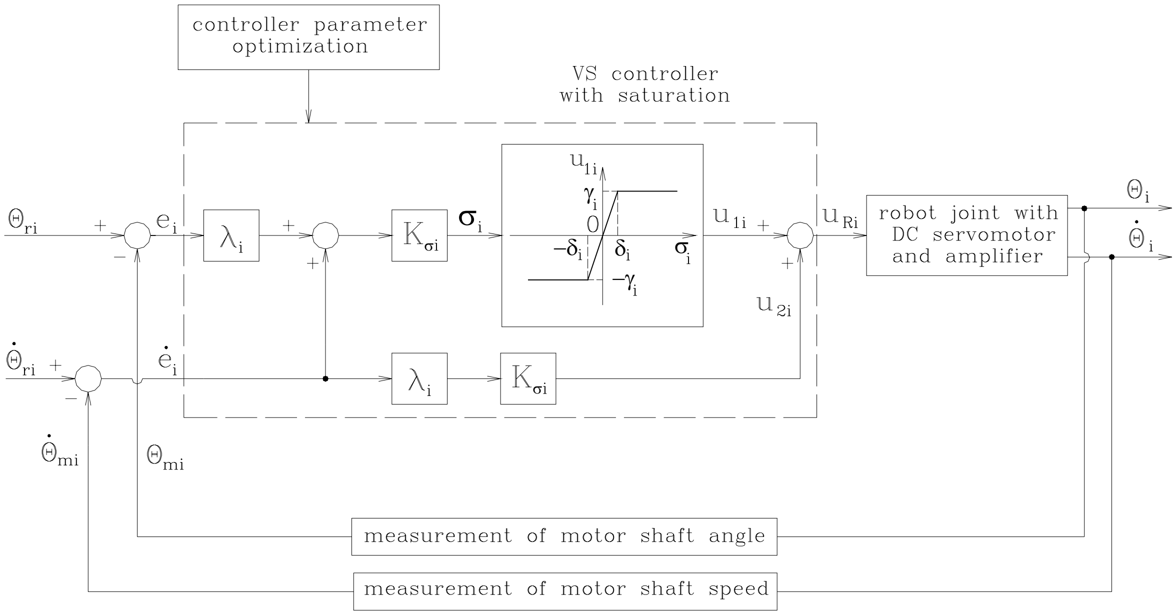
where δ_i denotes the thickness of the boundary layer for the i-th robot motor, $1 \le i \le n$.

Za dobivanje najboljeg slijeđenja trajektorije vrha alata robota, parametre robotskog regulatora s promjenjivom strukturom i kliznim režimom u (5) i (6) treba optimirati prema željenom kriteriju, kako je prikazano na Slici 1 za izvornu metodu upravljanja s funkcijom predznaka i na Slici 2 za izmijenjenu metodu sa zasićenjem.

To get the best robot tool-tip trajectory tracking, the robot variable-structure sliding-mode controller parameters in (5) and (6) have to be optimized according to the desired criterion, as shown in Figure 1 for the original control method with the sign function and in Figure 2 for the modified method with saturation.



Slika 1. Upravljanje s promjenjivom strukturom i funkcijom predznaka te optimiranjem za i-ti motor robota
Figure 1. Variable-structure robot control with sign function and optimization for the i-th robot motor



Slika 2. Upravljanje s promjenjivom strukturom i funkcijom zasićenja te optimiranjem za i-ti motor robota
Figure 2. Variable-structure robot control with saturation function and optimization for the i-th robot motor

Za dobivanje najboljeg ponašanja sustava upravljanja robota, parametri robotskog regulatora trebaju biti odgovarajuće izabrani. Ako je mjera ili indeks kakvoće upravljanja izražen matematički, taj problem može biti riješen za najbolji izbor parametara robotskog regulatora. Nakon optimiranja robotski je sustav optimalan s obzirom na izabrani kriterij [11], [14].

U mnogim je problemima indeks kakvoće upravljanja mjera ili funkcija pogreške $e_{rr}(t)$ između idealnoga i stvarnog stanja. Izražena je pomoću postupka izbora parametara za optimiranje mjere kakvoće upravljanja, prema postojećim fizikalnim ograničenjima. U općenitijim problemima optimalnog upravljanja mjera kakvoće upravljanja sustavom ili indeks kakvoće nije unaprijed postavljen. Umjesto toga izabrana je zamjena pa je mjera kakvoće upravljanja maksimizirana ili minimizirana. Iznos mjere kakvoće upravljanja je nepoznat sve do završetka procesa optimiranja [3].

Izbor odgovarajuće mjere kakvoće upravljanja vrlo je važan. Često korišteni indeksi kakvoće se temelje na integralnim mjerama kakvoće upravljanja, npr. [11], [14]:

- integral apsolutne vrijednosti pogreške:

To get the best performance from the robot control system, the robot controller parameters must be properly selected. If the measure or index of performance is expressed mathematically, the problem can be solved for the best choice of the robot controller parameters. After optimization, the resulting robot system is optimal with respect to the selected criterion [11], [14].

In many problems, the performance index is a measure or function of the error $e_{rr}(t)$ between the ideal and the actual state. It is formulated in terms of the chosen design parameters, so as to optimize the performance index, and it is subject to existing physical constraints. In more general optimal control problems, the system measure of performance, or the performance index, is not fixed in advance. Instead, compensation is chosen so that the performance index is maximized or minimized. The value of the performance index is unknown until the completion of the optimization process [3].

The selection of an appropriate performance index is very important. Commonly used performance indices are based on integral performance measures, such as [11], [14]:

- integral of absolute value of error:

$$J_{IAE} = \int_0^\infty |e_{rr}(t)|dt, \tag{7}$$

- integral kvadrata pogreške:

- integral of squared error:

$$J_{ISE} = \int_0^\infty {e_{rr}}^2(t)dt, \tag{8}$$

- integral vremenski otežane apsolutne vrijednosti pogreške:

- integral of time multiplied by absolute value of error:

$$J_{ITAE} = \int_0^\infty t \cdot |e_{rr}(t)|dt, \tag{9}$$

- integral vremenski otežanog kvadrata pogreške:

- integral of time multiplied by squared error:

$$J_{ITSE} = \int_0^\infty t \cdot {e_{rr}}^2(t)dt. \tag{10}$$

Integralne mjere kakvoće upravljanja J_{IAE}, J_{ISE}, J_{ITAE}, J_{ITSE} trebaju biti minimizirane kako bi se dobili parametri robotskog regulatora za najbolje ponašanje robotskog sustava. Postupak optimiranja može biti proveden u prostoru zglobova robota, ako je funkcija pogreške $e_{rr}(t)$ definirana kao razlika između željenog i stvarnog kuta zakreta osovine motora $e_i(t)$ svakog i-tog motora robota, kako je pokazano u (1). Optimiranje parametara regulatora se također može vršiti i u prostoru konfiguracije alata robota, gdje je funkcija pogreške $e_{rr}(t)$ određena kao razlika između željene i stvarne trajektorije vrha alata robota $e_w(t)$ u sve tri dimenzije:

The integral performance indices J_{IAE}, J_{ISE}, J_{ITAE}, J_{ITSE} have to be minimized to acquire the robot controller parameters for the best robot system performance.

The optimization procedure can be performed in robot joint space, if the error function $e_{rr}(t)$ is defined as the difference between the desired and real motor shaft angle $e_i(t)$ of each i-th robot motor, as shown in (1).

The controller parameter optimization can also take place in robot tool-configuration space, where the error function $e_{rr}(t)$ is determined as the difference of the desired and the actual robot tool-tip trajectory $e_w(t)$ in all three dimensions:

$$e_w(t) = \sqrt{(w_{rx}(t) - w_x(t))^2 + (w_{ry}(t) - w_y(t))^2 + (w_{rz}(t) - w_z(t))^2}, \tag{11}$$

gdje su $w_{rx}(t)$, $w_{ry}(t)$ i $w_{rz}(t)$ x, y i z koordinate željene trajektorije vrha alata robota,a $w_x(t)$, $w_y(t)$ i $w_z(t)$ su x, y i z koordinate stvarne trajektorije.

where $w_{rx}(t)$, $w_{ry}(t)$ and $w_{rz}(t)$ are x-, y- and z-coordinates of the desired robot tool-tip trajectory and $w_x(t)$, $w_y(t)$ and $w_z(t)$ are x-, y- and z-coordinates of the actual trajectory.

Željena trajektorija vrha alata robota prirodno je definirana u prostoru konfiguracije alata robota. Iz te trajektorije vrha alata robota dobivena je odgovarajuća trajektorija zglobova robota i nakon toga je primijenjen izmijenjen postupak robotskog upravljanja s promjenjivom strukturom [7].

Po završetku optimiranja i simulacije robotskog sustava važno je analizirati potrošnju energije po cijeloj trajektoriji vrha alata robota. U tu svrhu se može izračunati ukupna energija svih motora robota prema [2]:

The desired robot tool-tip trajectory is naturally defined in the robot tool-configuration space. From this robot tool-tip trajectory is obtained the corresponding robot joint trajectory and after that the modified variable-structure robot control method is applied [7].

After the completion of optimization and the simulation of the robot system, it is important to analyse energy consumption along the whole robot tool tip trajectory. Therefore, total energy of all robot motors is calculated according to [2]:

$$E=\sum_{i=1}^{n}\left(\int_{0}^{T_s} U_{a_i}\cdot I_{a_i}\cdot dt,\quad U_{a_i}\cdot I_{a_i}>0\right), \tag{12}$$

gdje T_s označava vrijeme prijelaza cijele trajektorije robota, U_{ai} je napon armature i I_{ai} je armaturna struja i-tog motora robota, n je broj osi robota.

Sve prethodno spomenute integralne mjere kakvoće upravljanja provjerene su računalnim simulacijama na optimiranju parametara izvornog i modificiranog robotskog regulatora s promjenjivom strukturom.

where T_s denotes the whole robot trajectory traverse time, U_{ai} is armature voltage and I_{ai} is armature current for the i-th robot motor, n is the number of the robot axis.

All of the above mentioned integral performance indices are tested by computer simulations on robot variable-structure controller parameters optimization, both for the original and modified robot control methods.

3. REZULTATI SIMULACIJA

Predstavljeni postupci upravljanja robotom s promjenjivom strukturom i optimiranjem provjereni su simulacijama na računalu u programu Matlab. Za provjeru je uzet troosni električki pokretani rotacijski robot u ravnini [13], čiji je vrh alata vođen po pravocrtnoj vertikalnoj trajektoriji od točke A=(0.2 [m], 0 [m]) do točke B=(0.2 [m], 0.4 [m]). Prilikom planiranja trajektorije robota vrijeme prijelaza trajektorije robota T_s je postavljeno na vrijednost T_S=4.44 [s], kako bi se dobilo najbrže moguće kretanje robota s obzirom na ograničenja ubrzanja i brzina robota.

Na početku analize potrebno je odrediti kinematički i dinamički model izabranog robota. Kinematički model ovog troosnog rotacijskog ravninskog robota dobiven je pomoću Denavit–Hartenbergova algoritma pridruživanja koordinatnih sustava svakom segmentu robota [2], [6], [13]. Za rješavanje problema dinamike robota može se koristiti Lagrange–Eulerov [6], [7], [12], [13], ili Newton–Eulerov [6], [13] postupak te se dobiva sljedeći dinamički model robota:

3. SIMULATION RESULTS

The presented variable-structure robot control methods with optimization are tested by computer simulations in the program Matlab. A three-axis electrically driven articulated planar robot [13] was used for testing, whose tool-tip was driven along a straight-line vertical trajectory from point A=(0.2 [m], 0 [m]) to point B=(0.2 [m], 0.4 [m]). During robot trajectory planning, the robot trajectory traverse time T_s is set to the value T_S=4.44 [s], in order to get as fast robot movement as possible regarding robot acceleration and velocity limits.

At the beginning of the analysis, kinematic and dynamic models of the chosen robot have to be determined. The kinematic model of this three-axis articulated planar robot is obtained by use of the Denavit-Hartenberg algorithm of assigning coordinate frames to each robot link [2], [6], [13]. For solving the robot dynamics problem, the Lagrange-Euler [6], [7], [12], [13], or Newton-Euler [6], [13] method can be used and the following dynamic robot model is obtained:

$$\begin{aligned}\tau_1 ={}& \left[\left(\frac{m_1}{3}+m_2+m_3\right)\cdot a_1^2+\left(\frac{m_2}{3}+m_3\right)\cdot a_2^2+\left(m_2+2\cdot m_3\right)\cdot a_1\cdot a_2\cdot\cos(q_2)+J_{m1}\cdot N_{r1}^2\right]\cdot\ddot{q}_1+\\ &+\left[\left(\frac{m_2}{3}+m_3\right)\cdot a_2^2+\left(\frac{m_2}{2}+m_3\right)\cdot a_1\cdot a_2\cdot\cos(q_2)\right]\cdot\ddot{q}_2-\left(m_2+2\cdot m_3\right)\cdot a_1\cdot a_2\cdot\sin(q_2)\cdot\left(\dot{q}_1\cdot\dot{q}_2+\frac{\dot{q}_2^2}{2}\right)+\\ &+g_0\cdot\left[\left(\frac{m_1}{2}+m_2+m_3\right)\cdot a_1\cdot\cos(q_1)+\left(\frac{m_2}{2}+m_3\right)\cdot a_2\cdot\cos(q_1+q_2)\right]+b_1(\dot{q}_1),\end{aligned} \tag{13}$$

$$\begin{aligned}\tau_2 ={}& \left[\left(\frac{m_2}{3}+m_3\right)a_2^2+\left(\frac{m_2}{2}+m_3\right)a_1a_2\cos(q_2)\right]\ddot{q}_1+\left[\left(\frac{m_2}{3}+m_3\right)\cdot a_2^2+J_{m2}\cdot N_{r2}^2\right]\cdot\ddot{q}_2+\\ &+\left(\frac{m_2}{2}+m_3\right)\cdot a_1\cdot a_2\cdot\sin(q_2)\cdot\dot{q}_1^2+g_0\left(\frac{m_2}{2}+m_3\right)a_2\cos(q_1+q_2)+b_2(\dot{q}_2),\end{aligned} \tag{14}$$

$$\tau_3 = J_{m3} \cdot N_{r3}^2 \cdot \ddot{q}_3 + b_3(\dot{q}_3), \tag{15}$$

gdje su q_i, $\dot{q}_i$ i $\ddot{q}_i$ i-te varijable, brzine i ubrzanja zgloba ($1 \le i \le 3$), τ_i je moment i-tog aktuatora, a_i i m_i su duljine i mase i-tog članka robota, J_{mi} je moment inercije i-tog motora, N_{ri} je i-ti omjer redukcije, g_0 je gravitacijska konstanta i $b_i(\dot{q}_i)$ označava trenje koje se suprotstavlja gibanju i-tog zgloba. U simulacijama na računalu upotrijebljen je realni dinamički model robota s viskoznim, dinamičkim i statičkim trenjem [12], [13], zajedno s ograničenjima konstrukcije robota i aktuatora.

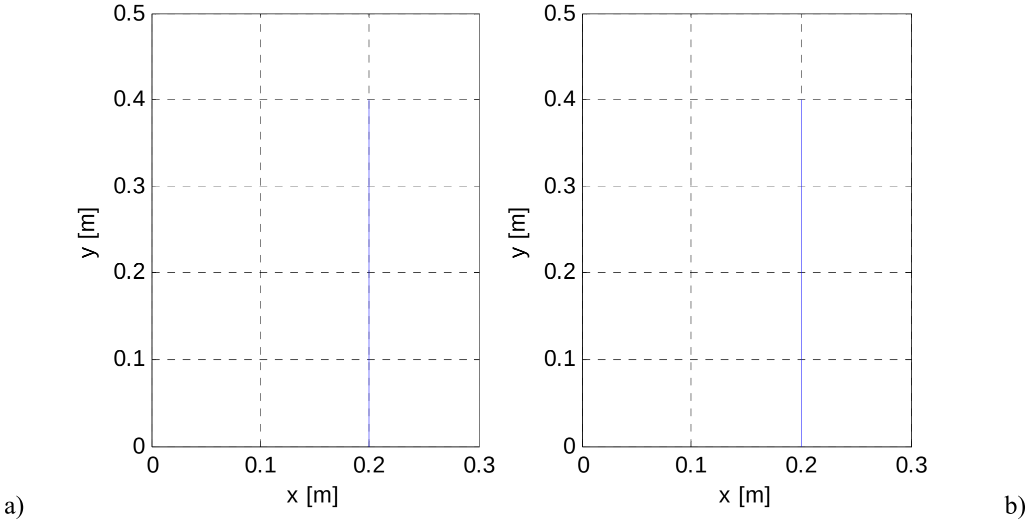
Kako se može vidjeti iz računalnih simulacija [5], klasičan PD-regulator nije dovoljno dobar za zadovoljavajuće upravljanje robotom zbog trenja u zglobovima robota i velikih dinamičkih sila i momenata, čak i kada nema promjene tereta robota. Zbog toga je korištena izmijenjena metoda upravljanja s promjenjivom strukturom i kliznim režimom kako bi se popravila kvaliteta slijeđenja trajektorije vrha alata robota. Stabilnost i robustnost kliznog načina upravljanja je detaljno analizirana u [4].

Za vrijeme podešavanja parametara robotskog regulatora s promjenjivom strukturom, maksimalno dopuštena pogreška slijeđenja trajektorije vrha alata robota postavljena je na vrijednost 0.5 [mm], što je i zadovoljeno prema Slici 3. Također se želi eliminirati trešnja u signalima pogrešaka kutova zakreta osovina motora (Slika 4), kao i u upravljačkim signalima (Slika 5), kako bi se postigla najmanja potrošnja energije. Te zahtjeve mogu ispuniti sljedeći neoptimalni parametri regulatora s promjenjivom strukturom i zasićenjem: $K_{\sigma i}$=0.01, λ_i=13.74, γ_i=3.837, δ_i=0.05, $1 \le i \le 3$. U tom slučaju može se postići sljedeća potrošnja energije E=19.023 [J]. U slučaju upotrebe odgovarajućega robotskog regulatora s funkcijom predznaka količina potrošene energije mnogo je veća E=51.193 [J].

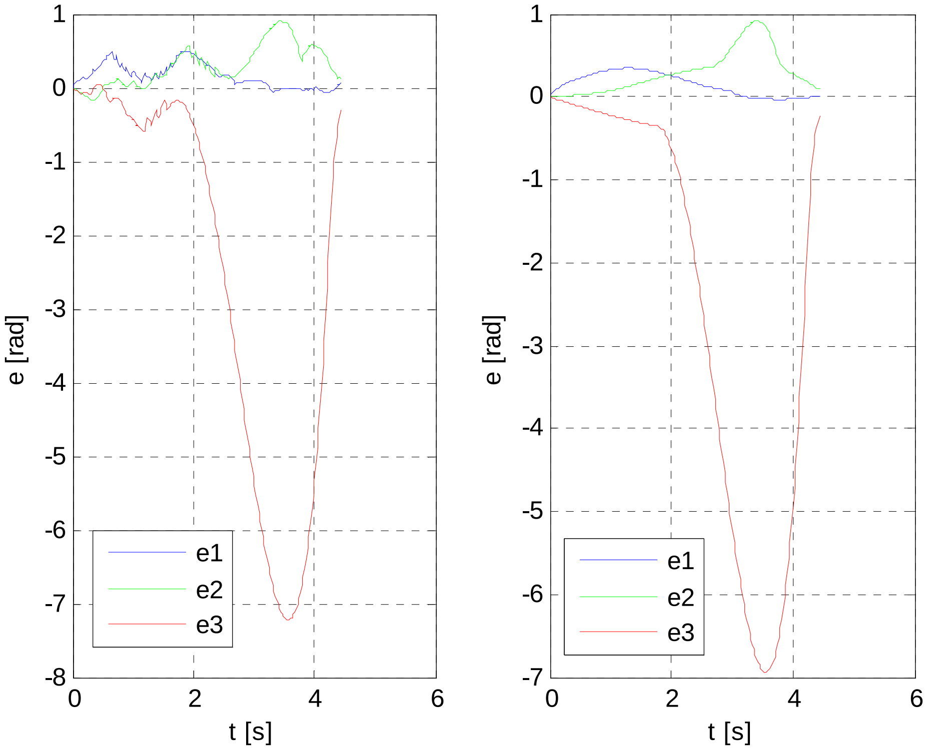
where q_i, $\dot{q}_i$ and $\ddot{q}_i$ are the i-th joint variable, velocity and acceleration respectively ($1 \le i \le 3$), τ_i is the i-th actuator torque, a_i and m_i are length and mass of the i-th robot segment, J_{mi} is moment of inertia for the i-th motor, N_{ri} is the i-th gear ratio, g_0 is gravitational constant and $b_i(\dot{q}_i)$ denotes friction opposing the motion of the i-th joint. In computer simulations, the realistic dynamic robot model with viscous, dynamic and static frictions is used [12], [13], together with robot construction and actuator limits.

As can be seen from computer simulations [5], the conventional PD-controller is not good enough for satisfactory robot control because of robot joint friction and large dynamic forces and torques, even if there is no robot load change. Therefore, the modified variable-structure sliding-mode control method is used to improve the quality of robot tool-tip trajectory tracking. Stability and robustness of sliding-mode control is analysed in detail in [4].

During the adjustment of robot variable-structure controller parameters, the maximum allowed robot tool-tip trajectory tracking error is set to 0.5 [mm], which is fulfilled according to Figure 3. Elimination of chattering in signals of motor shaft angle errors (Figure 4), as in control signals (Figure 5), is also desirable, so that minimal energy consumption can be achieved. These requirements can be fulfilled by the following non-optimal parameters of the variable-structure controller with saturation: $K_{\sigma i}$=0.01, λ_i=13.74, γ_i=3.837, δ_i=0.05, $1 \le i \le 3$. In that case, the following energy consumption E=19.023 [J] can be achieved. In the case of using the corresponding robot controller with sign function, the amount of energy consumption is much larger E=51.193 [J].

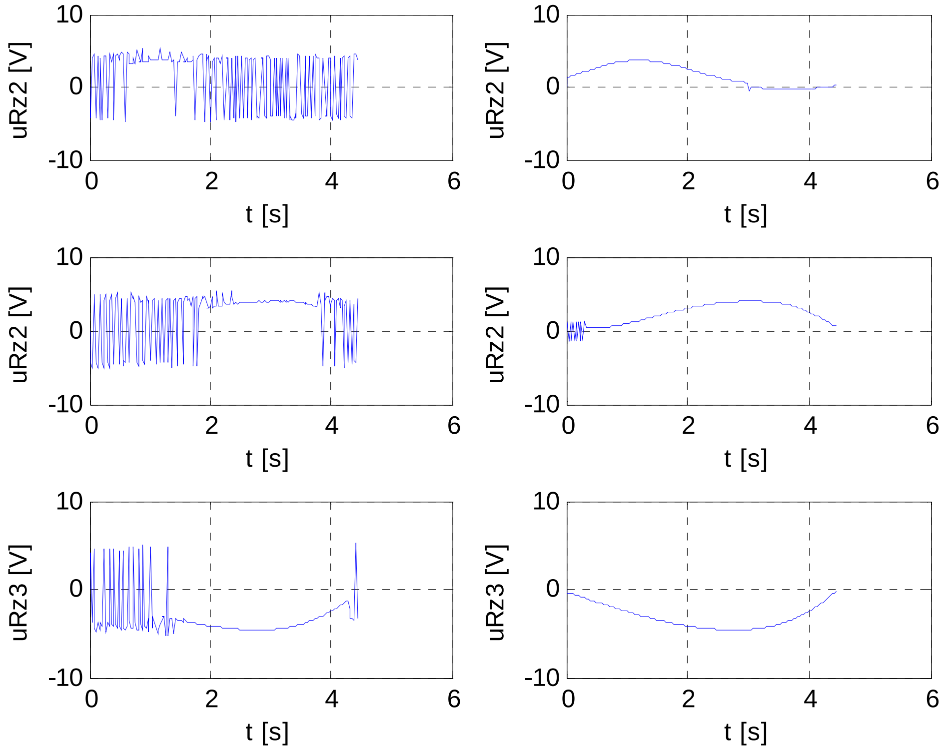


Slika 3. Dobivena trajektorija vrha alata robota uz upotrebu regulatora s funkcijom predznaka (a) i zasićenja (b)
Figure 3. Robot tool-tip trajectory in the case of using controller with sign function (a) and saturation (b)



a) b)

Slika 4. Pogreške kutova zakreta osovina motora uz upotrebu regulatora s funkcijom predznaka (a) i zasićenja (b)
Figure 4. Motor shaft angle errors in the case of using controller with sign function (a) and saturation (b)



a) b)

Slika 5. Ukupni signal upravljanja s promjenjivom strukturom uz upotrebu regulatora s funkcijom predznaka (a) i zasićenja (b)
Figure 5. Total variable-structure control signal in the case of using controller with sign function (a) and saturation (b)

Cilj je procesa optimiranja podešavanje parametara obaju robotskih regulatora s promjenjivom strukturom kako bi se postigla najmanja pogreška slijeđenja trajektorije. U prvom slučaju, optimiranje može biti izvedeno u prostoru zglobova robota prema (1), (7), (8), (9), (10), a u drugom slučaju u prostoru konfiguracije alata robota prema (7), (8), (9), (10), (11). Za vrijeme svih tih optimiranja ukupna količina potrošnje energije duž trajektorije treba biti analizirana.

Na početku je provedeno optimiranje parametara λ_i i γ_i, $1 \le i \le 3$, robotskog regulatora, ako regulator sadrži funkciju predznaka, u prostoru zglobova robota pomoću integrala apsolutne vrijednosti regulacijskog odstupanja (7) s pogreškama slijeđenja trajektorije $e_1(t)$, $e_2(t)$, $e_3(t)$. Rezultati optimiranja dani u Tablicama 1 (za regulator s funkcijom predznaka) i 2 (za regulator sa zasićenjem) nisu zadovoljavajući jer su u svim slučajevima optimiranja pogreške slijeđenja trajektorije vrha alata robota veće od najveće dozvoljene vrijednosti 0.5 [mm] i ukupna je količina potrošene energije povećana (detaljnjiji rezultati dani su u [4]). Funkcija zasićenja u regulatoru malo pomaže u smanjenju e_{wmax} i E, ali ne može ispuniti zahtjeve optimiranja. Zbog toga treba biti provedeno novo optimiranje u prostoru konfiguracije alata robota.

The goal of the optimization process is to adjust the parameters of both variable-structure robot controllers to achieve a minimum of trajectory tracking error. In the first case, optimization can be performed in robot joint space according to (1), (7), (8), (9), (10) and in the second case in robot tool-configuration space according to (7), (8), (9), (10), (11). During all these optimizations, the total amount of energy consumption along the trajectory has to be analysed.

In the beginning, the optimization of robot controller parameters λ_i and γ_i, $1 \le i \le 3$, if the controller has a sign function, is performed in robot joint space by using an integral of the absolute value of error (7) with trajectory tracking errors $e_1(t)$, $e_2(t)$, $e_3(t)$, respectively. The results of optimization, given in Tables 1 (for controller with sign function) and 2 (for controller with saturation), are not satisfactory because in all optimization cases robot tool-tip trajectory tracking errors are larger than the maximum allowed value of 0.5 [mm] and the total amount of energy consumption is increased (more detailed results are given in [4]). The saturation function in the controller helps a little in the decreasing of e_{wmax} and E, but it cannot fulfil the optimization demands. Therefore, a new optimization in robot tool-configuration space has to be performed.

Tablica 1. Optimiranje parametara λ_i i γ_i, $1 \le i \le 3$, regulatora s funkcijom predznaka u prostoru zglobova robota
Table 1. Optimization of parameters λ_i and γ_i, $1 \le i \le 3$, of controller with sign function in robot joint space

criterion	IAE		
i	1	2	3
$K_{\sigma i}$	0.01	0.01	0.01
λ_i	15.4024	147.4361	132.6746
γ_i	3.6138	4.8345	5.1387
e_{wmax} [mm]	1.1751	1.2585	0.1274
E [J]	46.4826	144.8735	142.6051

Tablica 2. Optimiranje parametara λ_i i γ_i, $1 \le i \le 3$, regulatora s funkcijom zasićenja u prostoru zglobova robota
Table 2. Optimization of parameters λ_i and γ_i, $1 \le i \le 3$, of controller with saturation function in robot joint space

criterion	IAE		
i	1	2	3
$K_{\sigma i}$	0.01	0.01	0.01
λ_i	29.8902	91.3362	106.9139
γ_i	8.3484	4.0554	4.706
δ_i	0.05	0.05	0.05
e_{wmax} [mm]	0.5744	0.8898	0.0965
E [J]	39.5277	55.6186	96.9626

Optimiranje parametara regulatora robota u prostoru konfiguracije alata robota je prirodno jer se i pogreška slijeđenja trajektorije robota lako definira u tom prostoru.

Optimization of robot controller parameters in robot tool-configuration space is natural because the robot trajectory tracking error is also easily defined in that space.

Optimiranje parametara λ_i i γ_i, $1\leq i\leq 3$, robotskog regulatora u prostoru konfiguracije alata robota najprije je provedeno u slučaju funkcije predznaka i nakon toga u slučaju zasićenja, upotrebom svih spomenutih integralnih kriterija optimiranja (7), (8), (9), (10) s pogreškom slijeđenja trajektorije $e_w(t)$. U toku tih optimiranja, sve upravljačke petlje robota imale su zbog jednostavnosti iste parametre regulatora λ_i i γ_i, $1\leq i\leq 3$. Iz rezultata optimiranja prikazanih u Tablici 3 (za regulatore s promjenjivom strukturom i funkcijom predznaka) i Tablici 4 (za regulatore s promjenjivom strukturom i zasićenjem) može se zaključiti da sva optimiranja provedena u prostoru konfiguracije alata robota, bez obzira na korišten integralni kriterij optimizacije, ispunjavaju zahtjev najveće dozvoljene pogreške slijeđenja trajektorije vrha alata robota. Rezultati su vrlo slični za različite integralne mjere kakvoće upravljanja. U usporedbi s rezultatima dobivenima za neoptimalne početne uvjete parametara regulatora s promjenjivom strukturom najveća pogreška slijeđenja trajektorije vrha alata robota smanjena je više od 5 puta, dok je ukupna količina potrošene energije povećana manje od 3 puta (za funkciju predznaka) ili čak ostala ista (za funkciju zasićenja). Zamjena funkcije predznaka u zakonu upravljanja robotom funkcijom zasićenja ponovo donosi drastično smanjenje e_{wmax} (3 puta) i E (čak 7 puta).

Optimization of robot controller parameters λ_i and γ_i, $1\leq i\leq 3$, in robot tool-configuration space is firstly performed in the case of sign function and after that in the case of saturation, by using all of the aforementioned integral optimization criterions (7), (8), (9), (10) with the trajectory tracking error $e_w(t)$. During these optimizations, all robot control loops have the same controller parameters λ_i and γ_i, $1\leq i\leq 3$, for simplicity reasons. From the results of optimization shown in Table 3 (for variable-structure controllers with sign function) and Table 4 (for variable-structure controllers with saturation) it can be concluded that all optimizations performed in robot tool-configuration space, regardless of the integral optimization criterion used, fulfil the demand for the maximal allowed robot tool-tip trajectory tracking error. The results are very similar for different integral performance indices. In comparison with the results obtained for non-optimal starting conditions of variable-structure controller parameters, maximal robot tool-tip trajectory tracking error is decreased by more than 5 times, while the total amount of energy consumption is increased by less than 3 times (for sign function) or even remains equal (for saturation function). The replacement of the sign function in the robot control law by the saturation function brings again the drastic decrease of e_{wmax} (3 times) and E (even 7 times).

Tablica 3. Optimiranje parametara λ_i i γ_i, $1\leq i\leq 3$, regulatora s funkcijom predznaka u prostoru konfiguracije alata robota, s istim regulatorima s promjenjivom strukturom u svim upravljačkim petljama robota

Table 3. Optimization of parameters λ_i and γ_i, $1\leq i\leq 3$, of controller with sign function in robot tool-configuration space, with the same variable-structure controllers in all robot control loops

criterion	IAE	ISE	ITAE	ITSE
$K_{\sigma i}$	0.01	0.01	0.01	0.01
λ_i	48.993	47.0721	50.4704	46.2262
γ_i	5.3971	5.5054	5.3732	5.6514
e_{wmax} [mm]	0.082424	0.080636	0.091166	0.085878
E [J]	136.0034	138.9721	136.1722	143.831

Tablica 4. Optimiranje parametara λ_i i γ_i, $1\leq i\leq 3$, regulatora s funkcijom zasićenja u prostoru konfiguracije alata robota, s istim regulatorima s promjenjivom strukturom u svim upravljačkim petljama robota

Table 4. Optimization of parameters λ_i and γ_i, $1\leq i\leq 3$, of controller with saturation function in robot tool-configuration space, with the same variable-structure controllers in all robot control loops

criterion	IAE	ISE	ITAE	ITSE
$K_{\sigma i}$	0.01	0.01	0.01	0.01
λ_i	29.0307	32.4718	32.21	32.7121
γ_i	4.3996	3.9797	3.982	3.9786
δ_i	0.05	0.05	0.05	0.05
e_{wmax} [mm]	0.026881	0.026499	0.026785	0.026228
E [J]	19.9519	19.5967	19.5869	19.6103

Optimiranje parametara λ_i i γ_i, $1\leq i\leq 3$, robotskog regulatora napravljeno je ponovo u prostoru konfiguracije alata robota, ali s različitim optimalnim parametrima regulatora λ_i i γ_i, $1\leq i\leq 3$, u svakoj od triju upravljačkih petlji robota.

Optimization of robot controller parameters λ_i and γ_i, $1\leq i\leq 3$, is done again in robot tool-configuration space, but with different optimal controller parameters λ_i and γ_i, $1\leq i\leq 3$, in each of three robot control loops.

Za dobivanje optimalnih vrijednosti parametara regulatora u svim trima upravljačkim petljama robota parametri robotskih regulatora u prvoj (λ_1 i γ_1) i drugoj (λ_2 i γ_2) petlji optimirani su u prostoru konfiguracije alata robota, dok su parametri trećega robotskog regulatora s promjenjivom strukturom (λ_3 i γ_3) optimirani u prostoru zglobova robota, jer valjanje alata robota ne utječe na položaj vrha alata robota u radnoj ravnini, nego samo na njegovu orijentaciju. Rezultati ove složenije metode optimiranja su dani u Tablici 5 (za regulatore s promjenjivom strukturom i funkcijom predznaka) i Tablici 6 (za regulatore s promjenjivom strukturom i zasićenjem). Iz tih tablica i detaljnijih rezultata danih u [4] za ovaj složeniji oblik optimiranja, koji zahtijeva mnogo više vremena za dovršenje, može se vidjeti da ta složenost u slučaju planarnog robota ne poboljšava kvalitetu slijeđenja trajektorije u prostoru konfiguracije alata robota, osim kvalitete pokreta valjanja (dano u [4]), ali s posljedicom u podvostručenju količine potrošene energije u slučaju optimalnih parametara robotskih regulatora s promjenjivom strukturom i zasićenjem.

In order to get the optimal values of controller parameters in all three robot control loops, the parameters of robot controllers in the first (λ_1 and γ_1) and the second (λ_2 and γ_2) loop are optimized in robot tool-configuration space, while parameters of the third variable-structure robot controller (λ_3 and γ_3) are optimized in robot joint space, because the robot tool roll does not affect the position of robot tool-tip in the working plane, only its orientation. The results of this more complex optimization method are given in Table 5 (for variable-structure controllers with sign function) and Table 6 (for variable-structure controllers with saturation). From these tables along with the more detailed results given in [4] for this more complex type of optimization, which needs much more time for completion, it can be seen that this complexity in the case of the planar robot does not improve the quality of trajectory tracking in robot tool-configuration space, except for the quality of roll motion (shown in [4]), but with the consequence of doubling the amount of energy consumption in the case of robot optimal variable-structure controllers with saturation.

Tablica 5. Optimiranje parametara λ_i i γ_i, $1 \le i \le 3$, regulatora s funkcijom predznaka u prostoru konfiguracije alata robota, s različitim optimalnim regulatorima s promjenjivom strukturom u svakoj upravljačkoj petlji robota

Table 5. Optimization of parameters λ_i and γ_i, $1 \le i \le 3$, of controller with sign function in robot tool-configuration space, with different optimal variable-structure controllers in each robot control loop

criterion	IAE			ISE			ITAE			ITSE		
i	1	2	3	1	2	3	1	2	3	1	2	3
$K_{\sigma i}$	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
λ_i	14.0	14.52	132.7	60.14	50.65	132.7	13.69	14.8	132.7	47.1	52.4	132.7
γ_i	3.58	3.97	5.14	3.54	7.31	5.14	3.57	3.97	5.14	3.48	5.48	5.14
e_{wmax} [mm]	0.45511			0.11246			0.4673			0.092758		
E [J]	76.9314			131.305			76.8028			112.0699		

Tablica 6. Optimiranje parametara λ_i i γ_i, $1 \le i \le 3$, regulatora s funkcijom zasićenja u prostoru konfiguracije alata robota, s različitim optimalnim regulatorima s promjenjivom strukturom u svakoj upravljačkoj petlji robota

Table 6 Optimization of parameters λ_i and γ_i, $1 \le i \le 3$, of controller with saturation function in robot tool-configuration space, with different optimal variable-structure controllers in each robot control loop

criterion	IAE			ISE			ITAE			ITSE		
i	1	2	3	1	2	3	1	2	3	1	2	3
$K_{\sigma i}$	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
λ_i	20.8	30.1	106.9	16.5	29.64	106.9	24.13	31.75	106.9	17.1	19.5	106.9
γ_i	6.29	4.01	4.71	8.13	4.23	4.71	5.36	3.99	4.71	7.79	7.04	4.4
δ_i	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
e_{wmax} [mm]	0.028011			0.02757			0.027014			0.030728		
E [J]	43.13234			43.9862			43.1381			66.6271		

4. ZAKLJUČAK

Optimiranje parametara robotskog regulatora s promjenjivom strukturom i zasićenjem daje najbolje rezultate u slijeđenju vrha alata planarnog robota i smanjenju potrošnje energije ako je izvedeno u prostoru konfiguracije alata robota, bez obzira na to koji je integralni kriterij optimiranja korišten.

4. CONCLUSION

Optimization of parameters of robot variable-structure controllers with saturation gives the best results in planar robot tool-tip trajectory tracking and reduction of energy consumption if it is performed in robot tool-configuration space, regardless of which integral optimization criterion is used.

5. POPIS OZNAKA

5. LIST OF SYMBOLS

broj osi (stupnjeva slobode) robota	n	number of robot axis (degrees of freedom)
referentni kut zakreta osovine motora	θ_r - rad	reference motor shaft angle
kut zakreta osovine motora	θ - rad	motor shaft angle
mjereni kut zakreta osovine motora	θ_m - rad	measured motor shaft angle
pogreška kuta zakreta osovine motora	e - rad	motor shaft angle error
referentna brzina vrtnje osovine motora	$d\theta_r/dt$ - rad/s	reference motor shaft speed
brzine vrtnje osovine motora	$d\theta/dt$ - rad/s	motor shaft speed
mjerena brzina vrtnje osovine motora	$d\theta_m/dt$ - rad/s	measured motor shaft speed
pogreška brzine vrtnje osovine motora	de/dt - rad/s	motor shaft speed error
općenita pogreška varijabli stanja motora	σ - rad/s	common motor state variables error
ukupni signal upravljanja s promjenjivom strukturom	u_R - V	total variable-structure control signal
1. dio signala upravljanja s promjenjivom strukturom	u_1 - V	the first part of variable-structure control signal
2. dio signala upravljanja s promjenjivom strukturom	u_2 - V	the second part of variable-structure control signal
parametar regulatora s promjenjivom strukturom	λ - 1/s	variable-structure controller parameter
parametar regulatora s promjenjivom strukturom	K_σ - Vs^2/rad	variable-structure controller parameter
parametar regulatora s promjenjivom strukturom	γ - V	variable-structure controller parameter
debljina graničnog sloja	δ - rad/s	thickness of boundary layer
funkcija pogreške	e_{rr}	error function
integral apsolutne vrijednosti pogreške	J_{IAE}	integral of absolute value of error
integral kvadrata pogreške	J_{ISE}	integral of squared error
integral vremenski otežane apsolutne vrijednosti pogreške	J_{ITAE}	integral of time multiplied by absolute value of error
integral vremenski otežanog kvadrata pogreške	J_{ITSE}	integral of time multiplied by squared error
x-koordinata referentne trajektorije vrha alata robota	w_{rx} - mm	x-coordinate of reference robot tool-tip trajectory
y-koordinata referentne trajektorije vrha alata robota	w_{ry} - mm	y-coordinate of reference robot tool-tip trajectory
z-koordinata referentne trajektorije vrha alata robota	w_{rz} - mm	z-coordinate of reference robot tool-tip trajectory
x-koordinata trajektorije vrha alata robota	w_x - mm	x-coordinate of robot tool-tip trajectory
y-koordinata trajektorije vrha alata robota	w_y - mm	y-coordinate of robot tool-tip trajectory
z-koordinata trajektorije vrha alata robota	w_z - mm	z-coordinate of robot tool-tip trajectory
pogreška slijeđenja trajektorije vrha alata robota	e_w - mm	robot tool-tip trajectory tracking error
maksimum pogreške trajektorije vrha alata robota	e_{wmax} - mm	maximum robot tool-tip trajectory tracking error
ukupna energija svih motora robota	E - J	total energy of all robot motors
ukupno vrijeme prijelaza trajektorije robota	T_s - s	total robot trajectory traverse time
napon armature	U_a - V	armature voltage
struja armature	I_a - A	armature current
varijabla zgloba	q - rad	joint variable
brzina zgloba	$\dot{q}_i$ - rad/s	joint velocity
ubrzanje zgloba	$\ddot{q}_i$ - rad/s^2	joint acceleration
moment aktuatora	τ - Nm	actuator torque
duljina članka robota	a - m	length of robot segment
masa članka robota	m - kg	mass of robot segment
moment inercije motora	J_m - kgm^2	moment of motor inertia
omjer redukcije	N_r	gear ratio
gravitacijska konstanta	g_0 - m/s^2	gravitational constant
trenje zgloba robota	b - Nm	joint friction

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Pregledni članak

Subject review

Adresa autora / Author's address:
Mr. sc. Vesna Krajčí, dipl. ing.
Domagoj Dušević, dipl.ing.
Sveučilište u Rijeci, Tehnički fakultet
Vukovarska 58
HR–51000 Rijeka, Hrvatska
Izv. prof. dr. sc. Mojmil Cecić, dipl.ing.
Sveučilište u Splitu, Fakultet elektrotehnike, strojarstva i brodogradnje
Ruđera Boškovića b.b.
HR-21000 Split, Hrvatska