Developing a Bi-Level Structure for Evaluation of Regional Bank Branch Managers Focusing on their Consumption

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Abstract. Regional bank branch management is the most important elements of a bank's structure. Each regional bank branch manager (RBBM) manages a large group of branches. In this paper, we develop a bi-level structure for the evaluation of RBBMs. In the developed bi-level structure, RBBMs are positioned at the upper level, and each RBBM has a group of branches located at the lower level. Generally, each RBBM, including their branches, tries to use inputs and produce outputs efficiently. However, each branch performs according to its goals and limited resources. The evaluation is a data envelopment analysis (DEA)-based model that focuses on the bank's consumption perspective. We apply the suggested model to a real-world case study to evaluate five RBBMs, who altogether manage 110 branches in one of the expert banking systems.

Keywords: bi-level structure, regional bank branch managers (RBBMs), data envelopment analysis (DEA), consumption perspective.

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1. Introduction

Data envelopment analysis (DEA) is the main approach for assessing the relative efficiency of decision-making units (DMUs) with multiple inputs and outputs. Charnes et al. [7] formulated this technique as a linear programming model. The first model of DEA is known as the Charnes, Cooper, and Rhodes (CCR) model, under constant returns to scale (CRS) technology. Later, Banker et al. [3] developed the CCR model for variable returns to scale (VRS) and introduced the Banker, Charnes, and Cooper model. Many DEA models have been developed to assess DMUs' performance in many activities in various contexts, such as the health sector, telecommunications, transportation, the electricity industry, and the banking industry. The banking industry is one of the most significant sectors of any economy because it executes essential tasks in the economy, such as equipping deposits, intermediation, facilitating payment flows, and allocating credit. Banks play an essential role in economic stability as a factor in implementing monetary policy. Therefore, the banking system's health and efficiency have always been important. Its unhealthiness and poor performance can cause financial and economic crises. Therefore, bank managers are needed to evaluate the performance of the branches under their supervision and plan to improve their performance continuously.

Traditional DEA models, called black-box models, evaluate DMUs regardless of their internal structure. In the real world, most DMUs have a hierarchical structure such that each unit

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contains several subunits. The black-box models ignore the subunits' independent performance and consider the sum of their outputs, leading to a biased evaluation of units. Banks also have a hierarchical structure. The process of providing services in banks consists of interactive and interdependent steps.

In the literature, units with hierarchical structures have attracted relatively little attention. Some authors have proposed DEA approaches to consider DMUs' internal structure; for instance, Fare and Primont [10] suggested a distance-function model for measuring the multi-plant firms' efficiency, and Kao [14] utilized it to evaluate the efficiency of eight forest districts in Taiwan with a total of thirty-four subordinate working circles. Cook et al. [8] proposed the concept of hierarchical DEA to measure the efficiency score of a series of power plants at various levels. Cook and Green [9] utilized the hierarchical model Cook et al. [8] proposed in their evaluation of eight Canadian power plants consisting of forty subordinate units. Azadeh et al. [1, 2] applied hierarchical models to determine the optimal location of solar plants and wind plants. Kao [15] proposed an extended DEA model that evaluates DMUs with a hierarchical structure in a "black box." In Kao's method, the performance score of DMUs with hierarchical structures is the weighted average of their subunits' efficiency at the hierarchical structure's lower level.

Castelli et al. [6] considered DMUs with one-level and two-level hierarchical structures. In one-level structures, each unit is composed of serial stages of parallel subunits with constant returns to scale. For the two-stage situation, a model was proposed by introducing balancing constraints to guarantee degrees of coordination among the hierarchical levels' subunits. Kashim et al. [16] developed Kao's model [15] to measure the efficiency of a university faculty with a hierarchical structure. Furthermore, Gan et al. [11] extended Kao's model [15] for evaluating the performance of the international shipping industry in Taiwan. Wu [19] proposed a bi-level DEA model for evaluating the cost efficiency of systems with one leader and one follower. Zhou et al. [22] evaluated bi-level systems' cost efficiencies, including a single leader at the upper level and multiple followers at the lower level. Yao et al. [21] built on Zhou's work for regional water system vulnerability evaluation. Yang et al. [20] established a bi-level DEA model with multiple followers for evaluating the efficiency of unattended convenience stores. They use weak disability technology to deal with undesirable intermediate measures. Ghasemi et al. [12] considered Farhangian University in Iran a network with a hierarchical structure and tried to calculate the efficiency of the university's campuses using novel models. Pachar et al. [17] proposed a bi-level programming DEA approach to evaluate multiple retail stores' cost efficiency considering a network structure operating in a Stackelberg relationship and defining benchmarks for inefficient stores in India. Hua et al. [13] developed a bi-level DEA cost model to evaluate the efficiency of two subsystems and complete water systems in 10 cities in the Minjiang River Basin. Barat et al. [4] suggested a network DEA-based methodology to address the problem of non-homogeneity in settings where subunits operate in a mixed-network structure. They evaluated layers and the overall system efficiencies.

In this paper, we present a novel DEA model to evaluate the performance of units with bi-level structures that is suitable for units in constant or variable returns to scale technologies. The proposed DEA model focuses on the units' internal operation. It evaluates the units' efficiency by considering the impact of the leader's and the subunits' performance. Our DEA model can be easily extended to multilevel structures. We illustrate the proposed model in a real-world case study of the bank industry, focusing on banks' consumption perspective. From the consumption perspective, banks are considered financial intermediaries. They act as intermediaries by collecting deposits and other liabilities and converting them into interest-bearing assets, such as loans, securities, and other investments. In other words, in this perspective, customer deposits are considered one of the inputs and outputs, including the total of loans and investment services. In the next section, we present the concept of hierarchical structures. In Section 3, we propose the bi-level DEA model. In Section 4, we present a real case study to show the suggested model's application and advantages. Finally, we provide concluding remarks in Section 5.

2. Hierarchical structures

In the real world, many organizations have a hierarchical structure, in which they have units at the first level and each of these units has subunits at their lower level. Also, some subunits in the second level may have several units at the third level, and this hierarchy may continue. For example, a bank may have subdivisions in various provinces of a country, where each province includes several regions and each region includes a number of branches. All resources, including human and budget resources, should be assigned in a hierarchical structure for the best performance (see Figure 1).

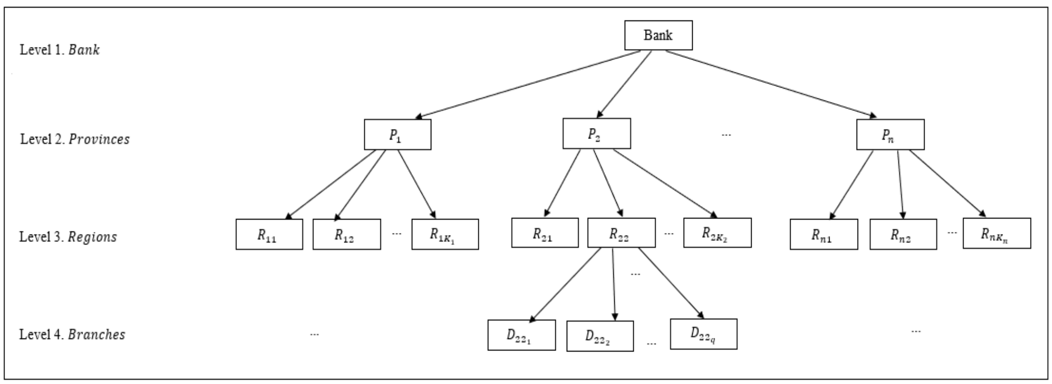

Figure 1: *A hierarchical structure.*

The traditional CCR model treats the system as a "black box" with multiple primary inputs that generate multiple final outputs. Figure 2 shows a "black box" conceptual system. As it shows, the interaction and relationship between the leader and the followers are ignored.

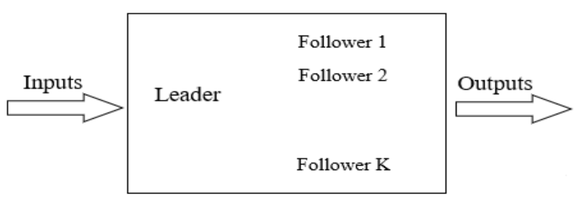

Figure 2: *Structure of a "black box" system.*

The traditional DEA model to evaluate the technical efficiency (TE) of DMU_0, ($0 \in \{1, 2, ..., J\}$) is formulated as

$$TE_0 = Max \frac{uY_0}{vX_0}$$

$$s.t. \quad \frac{uY_j}{vX_j} \leq 1, \quad j = 1, 2, ..., J, \tag{1}$$

$$u \geq \varepsilon > 0\ , \quad v \geq \varepsilon > 0$$

where $\varepsilon > 0$ is the non-Archimedean element and (X_j, Y_j) is the input and output vector of DMU_j, $j \in \{1, 2, ..., J\}$. v and u are the input and output weight vectors.

Also, the black box model for assessing pure efficiency (PE) of DMU_0, $(0 \in \{1, 2, ..., J\})$ can be stated as

$$
\begin{aligned}
PE_0 = Max\ & \tfrac{uY_0 - u_0}{vX_0} \\
s.t.\ \ & \tfrac{uY_j - u_0}{vX_j} \leq 1, \quad j = 1, 2, ..., J, \\
& u \geq \varepsilon > 0\ ,\ \ v \geq \varepsilon > 0,\ \ u_0\, free\ in\ sign
\end{aligned}
\tag{2}
$$

As the black-box models show, a DMU's efficiency is defined based on the aggregation of its and its subunits' performance. Therefore, a unit may be efficient although many of its subunits are not. Also, the real performance of an organization with a hierarchical structure and the relationship between the leader and the followers has not been considered in the conventional DEA models. In multi-level structures, the leader acts and the followers react to the leader's action. After receiving inputs from the leader, the followers work toward their goals and show their performance according to their inputs. Then, followers look for the best performance using the least inputs to obtain the most outputs. Therefore, a unit's overall performance depends on the leader's and their followers' optimal performance.

3. Proposed DEA model

In this section, we present a DEA model, considering its internal characteristics. As aforementioned, a unit's performance depends on its and its subunits' performance. Therefore, the efficiency of a unit with a bi-level structure should be measured based on its independent efficiency and its subunits' efficiency.

Consider a system that at the top level has J units, numbered as $1, 2, ..., J$ (Figure 3). Each DMU_j, $j = 1, 2, ..., J$ has R_j sub-DMUs at the lower level. Note that units at the same level do not need to have the same number of sub-units. Here, we consider a bi-level structure, and the proposed model can be generalized to the multilevel structure.

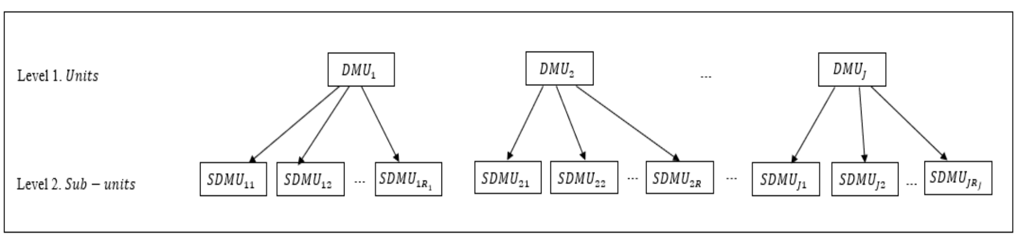

Figure 3: Bi-level structure.

As aforementioned, in organizations with a hierarchical structure, the top management provides inputs to the units at the first level. The managers at the first level consume a part of their inputs to generate their independent final outputs and to produce intermediate outputs to pass to their followers. They allocate the rest of the inputs and intermediate outputs to their followers at the second level. Followers at the second level consume both groups of inputs to produce their final outputs. See Figure 4.

Let X_j, $j = 1, 2, ..., J$ be the input of the leader and Y_j, Z_j, $j = 1, 2, ..., J$ be the final and intermediate outputs, respectively. $(X^j_{F_q}, Z^j_{F_q})$, $q \in \{1, 2, ..., R_j\}$ be the input vector of the fol-

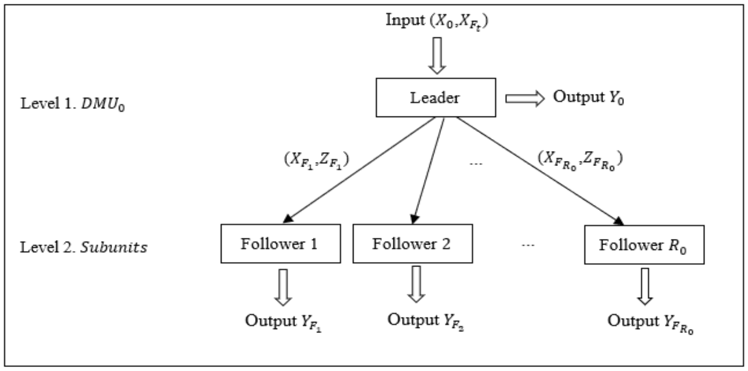

Figure 4: The internal structure of the bi-level unit DMU_0.

lower q in DMU_j and $Y^j_{F_q}$, $q \in \{1, 2, ..., R_j\}$ be the output of the follower q in DMU_j. (u_1, w_1) and v_1 are the input and output weight vectors of the upper level (level 1). u_2 and (v_2, w_2) are the input and output weight vectors of the lower level (level 2). Note that $Z_j = \sum_{q=1}^{R_j} Z_{F_q}$. Based on bi-level structure, the technical efficiency of the DMU_0, $0 \in \{1, 2, ..., J\}$ under an assumption of CRS is defined as follows:

$$\begin{aligned} TE_0 = Max\ & \frac{u_1 Y_0 + w_1 Z_0 + u_2(\sum_{q=1}^{R_0} Y^0_{F_q})}{v_1 X_0 + v_2(\sum_{q=1}^{R_0} X^0_{F_q}) + w_2(\sum_{q=1}^{R_0} Z^0_{F_q})} \\ s.t.\ & \frac{u_1 Y_j + w_1 Z_j + u_2(\sum_{q=1}^{R_j} Y^j_{F_q})}{v_1 X_j + v_2(\sum_{q=1}^{R_j} X^j_{F_q}) + w_2(\sum_{q=1}^{R_j} Z^j_{F_q})} \le 1, \qquad j = 1, 2, ..., J, \\ & u_1 \ge 0\ ,\ \ v_1 \ge 0,\ \ w_1 \ge 0,\ \ u_2 \ge 0\ ,\ \ v_2 \ge 0,\ \ w_2 \ge 0 \end{aligned} \tag{3}$$

where (u_2, v_2, w_2) is the optimal solution of the following lower-level problem:

$$\begin{aligned} Max\ & \frac{u_2(\sum_{q=1}^{R_0} Y^0_{F_q})}{v_2(\sum_{q=1}^{R_0} X^0_{F_q}) + w_2(\sum_{q=1}^{R_0} Z^0_{F_q})} \\ s.t.\ & \frac{u_2(\sum_{q=1}^{R_j} Y^j_{F_q})}{v_2(\sum_{q=1}^{R_j} X^j_{F_q}) + w_2(\sum_{q=1}^{R_j} Z^j_{F_q})} \le 1, \quad j = 1, 2, ..., J, \\ & \frac{u_2 Y^j_{F_q}}{v_2 X^j_{F_q} + w_2 Z^j_{F_q}} \le 1, \quad j = 1, 2, ..., J,\ \ q = 1, 2, ..., R_j, \\ & u_2 \ge 0\ ,\ \ v_2 \ge 0,\ \ w_2 \ge 0 \end{aligned} \tag{4}$$

By adding the constraints $u_2 Y^j_{F_q} / \left(v_2 X^j_{F_q} + w_2 Z^j_{F_q}\right) \le 1$ to the low-level problem, we account for each of the followers' independent performance.

Let $t = 2/\left(v_1 X_0 + v_2(\sum_{q=1}^{R_0} X^0_{F_q}) + w_2(\sum_{q=1}^{R_0} Z^0_{F_q})\right)$; thus, we obtain the equivalent linear model of model (3):

$$
\begin{aligned}
TE_0 = Max\ & \tfrac{1}{2}\left(u_1Y_0 + w_1Z_0 + u_2(\textstyle\sum_{q=1}^{R_0} Y_{F_q}^0)\right) \\
s.t.\quad & u_1Y_j + w_1Z_j + u_2(\textstyle\sum_{q=1}^{R_j} Y_{F_q}^j) - v_1X_j - v_2(\textstyle\sum_{q=1}^{R_j} X_{F_q}^j) - \\
& w_2(\textstyle\sum_{q=1}^{R_j} Z_{F_q}^j) \le 0, \quad j = 1, 2, ..., J, \\
& v_1X_0 + v_2(\textstyle\sum_{q=1}^{R_0} X_{F_q}^0) + w_2(\textstyle\sum_{q=1}^{R_0} Z_{F_q}^0) = 2, \\
& u_1 \ge 0\ ,\ \ v_1 \ge 0,\ \ w_1 \ge 0, \\
& u_2 \ge 0\ ,\ \ v_2 \ge 0,\ \ w_2 \ge 0
\end{aligned}
\tag{5}
$$

where (u_2, v_2, w_2) is the optimal solution of the following linear lower-level problem:

$$
\begin{aligned}
& Max\ u_2(\textstyle\sum_{q=1}^{R_0} Y_{F_q}^0) \\
& s.t.\quad u_2(\textstyle\sum_{q=1}^{R_j} Y_{F_q}^j) - v_2(\textstyle\sum_{q=1}^{R_j} X_{F_q}^j) - w_2(\textstyle\sum_{q=1}^{R_j} Z_{F_q}^j) \le 0, \quad j = 1, 2, ..., J, \\
& \qquad u_2Y_{F_q}^j - v_2X_{F_q}^j - w_2Z_{F_q}^j \le 0, \quad j = 1, 2, ..., J,\ \ q = 1, 2, ..., R_j, \\
& \qquad v_2(\textstyle\sum_{q=1}^{R_0} X_{F_q}^0) + w_2(\textstyle\sum_{q=1}^{R_0} Z_{F_q}^0) = 1, \\
& \qquad u_2 \ge 0\ ,\ \ v_2 \ge 0,\ \ w_2 \ge 0
\end{aligned}
\tag{6}
$$

Model (5) is a bi-level linear programming problem whose optimal solution is constrained to solve the lower-level problem.

We solve the lower-level problem and obtain the optimal solution $u_2^*(\sum_{q=1}^{R_0} Y_{F_q}^0)$, for the objective function. Because v_2 and w_2 may have multiple optimal solutions, we add the constraints set of model (6) to the upper-level model (3) such that $u_2(\sum_{q=1}^{R_0} Y_{F_q}^0) = u_2^*(\sum_{q=1}^{R_0} Y_{F_q}^0)$. Using this method, we calculate the unique v_2 and w_2 in such a way that the overall technical efficiency of the DMU_0 is maximized and the low-level efficiency remains at $u_2^*(\sum_{q=1}^{R_0} Y_{F_q}^0)$. Therefore, we have

$$
\begin{aligned}
TE_0 = Max\ & \tfrac{1}{2}\left(u_1Y_0 + w_1Z_0 + u_2(\textstyle\sum_{q=1}^{R_0} Y_{F_q}^0)\right) \\
s.t.\quad & u_1Y_j + w_1Z_j - v_1X_j + u_2(\textstyle\sum_{q=1}^{R_j} Y_{F_q}^j) - v_2(\textstyle\sum_{q=1}^{R_j} X_{F_q}^j) - \\
& w_2(\textstyle\sum_{q=1}^{R_j} Z_{F_q}^j) \le 0, \quad j = 1, 2, ..., J, \\
& v_1X_0 = 1, \\
& u_2(\textstyle\sum_{q=1}^{R_j} Y_{F_q}^j) - v_2(\textstyle\sum_{q=1}^{R_j} X_{F_q}^j) - w_2(\textstyle\sum_{q=1}^{R_j} Z_{F_q}^j) \le 0, \quad j = 1, 2, ..., J, \\
& u_2Y_{F_q}^j - v_2X_{F_q}^j - w_2Z_{F_q}^j \le 0, \quad j = 1, 2, ..., J,\ \ q = 1, 2, ..., R_j, \\
& v_2(\textstyle\sum_{q=1}^{R_0} X_{F_q}^0) + w_2(\textstyle\sum_{q=1}^{R_0} Z_{F_q}^0) = 1, \\
& u_2(\textstyle\sum_{q=1}^{R_0} Y_{F_q}^0) = u_2^*(\textstyle\sum_{q=1}^{R_0} Y_{F_q}^0), \\
& u_1 \ge 0\ ,\ \ v_1 \ge 0,\ \ w_1 \ge 0,\ \ u_2 \ge 0\ ,\ \ v_2 \ge 0,\ \ w_2 \ge 0
\end{aligned}
\tag{7}
$$

DMU_0 is termed technically efficient if $TE_0 = 1$. In the proposed TE model, the effect of two levels is considered in assessing the whole performance of a unit. Our model can be expanded to multi-level structures. Also, it can be generalized to the case in which units have variable returns to scale.

With the addition of the free variables u_0 and u_{02} to models (3) and (4), the overall efficiency of the DMU_0, $0 \in \{1, 2, ..., J\}$ under variable returns to scale assumption is obtained. Similarly, with $t = 2/\big(v_1 X_0 + v_2(\sum_{q=1}^{R_0} X_{F_q}^0) + w_2(\sum_{q=1}^{R_0} Z_{F_q}^0)\big)$ and $u_0 = u_{01} + u_{02}$, which in u_{01} is free in sign, the following linear model for assessing the PE of DMU_0, $0 \in \{1, 2, ..., J\}$ is attained:

$$
\begin{aligned}
PE_0 = Max\ \tfrac{1}{2} &\Big(u_1 Y_0 + w_1 Z_0 - u_{01} + u_2(\textstyle\sum_{q=1}^{R_0} Y_{F_q}^0) - u_{02}\Big) \\
s.t.\quad & u_1 Y_j + w_1 Z_j - v_1 X_j - u_{01} + u_2(\textstyle\sum_{q=1}^{R_j} Y_{F_q}^j) - v_2(\sum_{q=1}^{R_j} X_{F_q}^j) - \\
& w_2(\textstyle\sum_{q=1}^{R_j} Z_{F_q}^j) - u_{02} \le 0, \quad j = 1, 2, ..., J, \\
& v_1 X_0 = 1, \\
& u_2(\textstyle\sum_{q=1}^{R_j} Y_{F_q}^j) - v_2(\sum_{q=1}^{R_j} X_{F_q}^j) - w_2(\sum_{q=1}^{R_j} Z_{F_q}^j) - u_{02} \le 0, \quad j = 1, 2, ..., J, \\
& u_2 Y_{F_q}^j - v_2 X_{F_q}^j - w_2 Z_{F_q}^j - u_{02} \le 0, \quad j = 1, 2, ..., J, \quad q = 1, 2, ..., R_j, \\
& v_2(\textstyle\sum_{q=1}^{R_0} X_{F_q}^0) + w_2(\sum_{q=1}^{R_0} Z_{F_q}^0) = 1, \\
& u_2(\textstyle\sum_{q=1}^{R_0} Y_{F_q}^0) - u_{02} = u_2^*(\sum_{q=1}^{R_0} Y_{F_q}^0) - u_{02}^*, \\
& u_1 \ge 0\ ,\ \ v_1 \ge 0,\ \ w_1 \ge 0\ ,\ \ u_2 \ge 0\ ,\ \ v_2 \ge 0,\ \ w_2 \ge 0, \\
& u_{01}\ ,\ \ u_{02}\ \ free\ in\ sign
\end{aligned}
\tag{8}
$$

DMU_0 is pure efficient if $PE_0 = 1$. By considering various relationships that may exist between upper and lower levels, we can expand the proposed model. In fact, the proposed model is flexible to the organizations' real internal structure.

4. Experimental results

In this section, we implement the proposed models in a real-world case study at the bank branch level and then compare the results to those of traditional models. The results can be useful for managerial insights. Here, we solve all models using the GAMS software by Brooke et al. [5].

A real case study

In this section, we consider a real-world case study from a specified bank in Iran and then analyze the results of the implementation of the proposed TE and PE models. This study includes five RBBMs located in Tehran and 110 branches. Table 1 shows the number of branches in each region. The case study's results can be useful for managers to measure the performance of bi-level units and to find out how they can manage any budget to improve the units' efficiency.

Here, each bank branch management consists of a supervisory unit as the leader and a number of branches as followers. The number of followers altogether is 110.

Region	**Branch**
South	23
North	26
Center	17
West	20
East	24

Table 1: *The number of branches for each RBBM.*

In the studies on the performance of the banking industry, there are two main approaches to determining inputs, the production approach and the intermediation approach. In the production approach, banks are producers that use human resources and other physical resources as input to produce various types of deposit accounts and services. Therefore, the outputs of this approach are deposits and services provided to customers. In the intermediation (or consumption) approach, banks are considered intermediaries of financial services. In this approach, the bank accepts deposits from customers and consumes personnel costs and other costs, converts them into loans and services, and earns revenue (see Paradi and Zhu, [18]). In this case study, we considered the consumption perspective to evaluate RBBMs' performance. The leader's input is personnel expenses, and the leader's outputs include services provided as the independent final output and non-operating expenses as intermediate output. The non-operating expenses include the administrative expenses, equipment expenses, expenses of repairs each branch requires, and so on, which are determined at management's discretion considering the number of staff in each branch as well as each branch's size, grading, urgent needs, etc. Therefore, the non-operating expenses are the intermediate output to the follower branches at the lower level.

The followers' inputs at the lower level are deposits, personnel expenses, and non-operating expenses. The followers' outputs are loans and services.

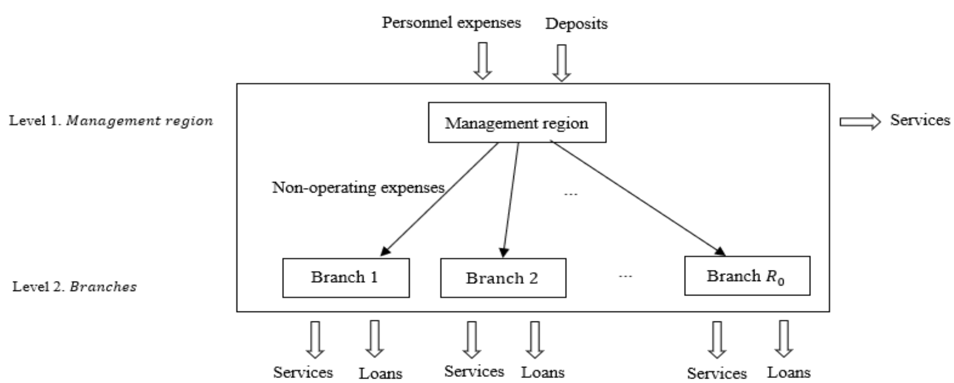

Figure 5: Bi-level structure in a bank.

The input of personnel expenses includes all factors related to a branch's employees: quantitative inputs consist of the hours each staff member works in a month, overtime hours each staff member works, etc. and qualitative inputs, such as personnel knowledge and personnel experiences. All these factors have affected the amount of payment to each staff member, and the total payments to personnel are considered the input of personnel expenses. The input of deposits includes a variety of a branch's methods of raising money. This index is considered the normalized weighted sum of types of accounts according to their value and number of transactions. The output of loans includes all the money given as all kinds of loans and mortgages by a branch. Finally, the output of services is an index that includes all kinds of services a branch presents to its customers.

Tables 2 and 3 present the descriptive statistics of leaders' and followers' inputs and outputs, respectively. The measurement unit of personnel expenses and non-operating expenses is 1,000,000 rials. We normalized other indices and rounded all values and results to two digits.

	Min	**Max**	**Mean**	**STD**
Inputs:				
Personnel expenses	9663.08	16172.11	12161.59	2408.48
Intermediate outputs:				
Non-operating expenses	1957.60	3910.60	3011.36	884.50
Outputs:				
Services	932.10	2539.00	1466.42	681.43

Table 2: *Data statistics of the RBBMs.*

	Min	**Max**	**Mean**	**STD**
Inputs:				
Personnel expenses	1384.62	13332.97	4413.43	2052.48
Deposits	460.80	1454.40	1154.39	177.92
Intermediate outputs:				
Non-operating expenses	8.68	359.00	136.88	72.01
Outputs:				
Loans	86.80	3590.00	1368.81	720.10
Services	62.53	2861.00	676.10	458.76

Table 3: *Data statistics of the branches.*

We calculate the TE and PE scores for the regions at the black-box and bi-level models. Tables 4 and 5 show the results for all selected regions. In the black-box model, we considered the leader's and followers' total inputs and the sum of the final outputs the leader and followers produce. Here, we used models (1) and (2) to calculate the black-box TEs and PEs. Also, we used models (7) and (8) to calculate the bi-level TEs and PEs.

Regions	**Black-box TE**	**Rank**	**Bi-level TE**	**Rank**
South	0.87	4	0.84	2
North	1	1	1	1
Center	0.99	2	0.69	4
West	0.94	3	0.65	5
East	0.79	5	0.76	3

Table 4: *TE results of the black-box and bi-level models.*

Table 4 shows that in the regions where the regions' performances are similar (North, Center, and West Regions), the proposed model improved discrimination. The improved discrimination provides a better perspective of the regions' ranking. This result is most obvious in the PE scores. Table 5 shows that the black-box model under the VRS assumption organizes regions into three categories, but the proposed model with the VRS assumption organizes them into five categories. Intermediate outputs are part of each region's activities, which are ignored in the conventional model. We added these outputs to the proposed model in the form of logical constraints, which have developed a model with more discrimination power than the conventional model.

Regions	**Black-box PE**	**Rank**	**Bi-level PE**	**Rank**
South	0.91	2	0.86	4
North	1	1	1	1
Center	1	1	0.98	2
West	1	1	0.95	3
East	0.85	3	0.83	5

Table 5: *PE results of the black-box and bi-level models.*

In general, it can be said the main pitfall that can be seen in most cases of evaluation of the upper level is the overestimation of the efficiencies, which leads to inaccurate ranking. The problem's origin is generally the small number of units in the upper level. Our case study clearly shows that the number of regions in the upper level is 5.

In the proposed model in this paper, we added the internal relations of the upper and lower levels, as intermediate outputs and inputs, to the model. These cases generate more discrimination than the black-box model, and it can improve the accuracy of the ranking compared to the black-box model, especially in cases with a small number of units in the upper level.

As the results in Table 4 and especially Table 5 show, the overestimation of the black-box model's results is obvious. The rankings that we obtained based on the overestimation results cannot be reliable, and we can see that for PEs, a complete ranking cannot be obtained. The more discriminatory results of the proposed model lead to a complete ranking for the TEs and PEs.

5. Conclusion

In the real world, most DMUs have a hierarchical structure such that each unit includes several subunits at lower levels. Conventional DEA models ignore the DMUs' internal structure. This negligence may lead to biased evaluation of the units and produce misleading results. We proposed a new DEA model to assess bi-level units, which are hierarchical units with two levels: the leader at the upper level and the followers at the lower level. The proposed model can be easily extended to hierarchical units. Our proposed model provides more meaningful evaluations of DMUs with a hierarchical structure by considering the internal operations between upper and lower levels and intermediate outputs and inputs. We used a real-world case study selected from bank branches to validate the proposed model and compare it to the traditional black-box model. We examined this assessment from the consumption perspective of the bank. The suggested model overcomes the disadvantages of traditional DEA models, such as ignoring the internal operations between upper and lower levels. Also, the proposed model has more discrimination power than the conventional model. More discrimination provides a better perspective of units' ranking. Therefore, it helps managers make accurate assessments of the units under their supervision.

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