Automatika
Journal for Control, Measurement, Electronics, Computing and Communications

ISSN: (Print) (Online) Journal homepage: https://www.tandfonline.com/loi/taut20

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AUTOMATIKA
2023, VOL. 64, NO. 1, 63–72
https://doi.org/10.1080/00051144.2022.2098747

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ABSTRACT
The parity space method can be applied to detect the fault of the time-invariant control system simply and effectively. However, the parity space method is greatly limited in the time-variant system. Therefore, in this paper, a TS fuzzy system is used to describe this kind of system. The system parameters are obtained based on TS fuzzy rules, and the decoupled fault detection vectors for different channels are designed. Using this vector, the residual representing fault information is extracted directly. Ideally, the fault detection of different channels is decoupled. However, sometimes the fully decoupled detection vector cannot be designed, then a suboptimal decoupled fault detection vector is proposed. Simulation and verification are carried out on a steady-state system and a TS fuzzy system.

ARTICLE HISTORY
Received 28 July 2021
Accepted 3 July 2022

KEYWORDS
TS fuzzy; fault detection; parity space; matrix norm; fuzzy control

1. Introduction

The fault diagnosis of the automatic control system includes many types, such as model-based, experience-based, etc. The model-based approach has many advantages. This method does not need historical data and experience. Compared with the fault diagnosis expert system, this method can avoid the limited knowledge acquisition. The focus of this paper is the parity space method, which is one of the model-based methods.

This method was first studied in reference [1]. Since then, it has been developing, and it has been widely used, such as references [2–4]. There have been some novel achievements in this field. For example, literature [5] adopted stationary wavelet transform for auxiliary design. However, this method is based on the linear time-invariant system . Another reality is that a few studies pay attention to fault decoupling on different channels. This is the motivation for this study.

Practically, most of the control systems are characterized by non-linearity, time-varying, system uncertainty, controller gain disturbance, random disturbance, multiple parameters, etc. They are quite difficult to be described by an accurate mathematical model. Accordingly, fuzzy modelling and fuzzy control are feasible schemes, as described in the literature [6,7]. The fuzzy theory has also been applied to fault diagnosis, such as literature [8–10]. Among many fuzzy systems, the Takagi Sugeno (TS) fuzzy is the only one to get a systematic analysis [11–13]. This method is especially suitable for modelling complex systems. The TS fuzzy model consists of some IF–THEN fuzzy rules. It uses a fuzzy membership function to form a unified mathematical model [14,15]. There are many pieces of research on fault diagnosis of the TS fuzzy system, such as literature [16,17]. These studies put forward good ideas. However, there is no application of the parity vector method in the TS fuzzy model.

In our plan, decoupled parity vectors are designed in the TS fuzzy system. The disturbance term, fault term and control term are considered separately. The input term is regarded as a linear combination of column vectors of the input matrix. The decoupled fault detect vector is designed for each channel independently. The residual for each channel is only sensitive to the fault information on this channel.

2. Failt decoupling vector of the time-invariant system

2.1. Description of the time-invariant system

The discrete state-space model with actuator fault and structural disturbance is shown in formula (1).

$$\begin{cases} \mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}(\mathbf{u}^c(k) + \mathbf{f}(k)) + \mathbf{E}\mathbf{d}(k) \\ \mathbf{y}(k) = \mathbf{C}\mathbf{x}(k) + \mathbf{D}(\mathbf{u}^c(k) + \mathbf{f}(k)) + \mathbf{F}\mathbf{d}(k) \end{cases} \quad (1)$$

In formula (1), $\mathbf{u}(k) = \mathbf{u}^c(k) + \mathbf{f}(k) \in \mathbf{R}^m$ refers to the control input, $\mathbf{x}(k) \in \mathbf{R}^n$ refers to the state variable, $\mathbf{y}(k) \in \mathbf{R}^q$ represents the output. $\mathbf{u}(k)$ represents the real input, $\mathbf{u}^c(k)$ represents the command input and $\mathbf{f}(k)$ represents the input fault; $\mathbf{d}(k) \in \mathbf{R}^r$ refers to the unknown input; $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}, \mathbf{E}, \mathbf{F}$ refers to a known

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coefficient matrix of proper dimensions. The following formula (2)–(4) are obtained by iterating formula (1) repletely.

$$\mathbf{y}(k-\tau) = \mathbf{Cx}(k-\tau) + \mathbf{Du}(k-\tau) + \mathbf{F}d(k-\tau) \tag{2}$$

$$\begin{aligned}\mathbf{y}(k-\tau+1) = {} & \mathbf{CAx}(k-\tau) + \mathbf{CBu}(k-\tau) \\ & + \mathbf{CEd}(k-\tau) + \mathbf{Du}(k-\tau+1) \\ & + \mathbf{Fd}(k-\tau+1)\end{aligned} \tag{3}$$

$$\begin{aligned}\mathbf{y}(k) = {} & \mathbf{CA}^{\tau}\mathbf{x}(k-\tau) + \mathbf{CA}^{\tau-1}\mathbf{Bu}(k-\tau) \\ & + \cdots + \mathbf{CBu}(k+1) + \mathbf{Du}(k) \\ & + \mathbf{CA}^{\tau-1}\mathbf{Ed}(k-\tau) + \cdots \\ & + \mathbf{CEd}(k+1) + \mathbf{Fd}(k)\end{aligned} \tag{4}$$

2.2. Design of the time-invariant fault decoupling vector

$\mathbf{B}\cdot\mathbf{f}(k)$ and $\mathbf{D}\cdot\mathbf{f}(k)$ in formula (1) can be expressed in expanded form, as shown in formula (5):

$$\begin{aligned}&\begin{bmatrix}\mathbf{b}_1 & \mathbf{b}_2 & \ldots & \mathbf{b}_m\end{bmatrix}\begin{bmatrix}f_1(k) & \ldots & f_m(k)\end{bmatrix}^T, \\ &\begin{bmatrix}\mathbf{d}_1 & \mathbf{d}_2 & \ldots & \mathbf{d}_m\end{bmatrix}\begin{bmatrix}f_1(k) & \ldots & f_m(k)\end{bmatrix}^T\end{aligned} \tag{5}$$

where $\begin{bmatrix}\mathbf{b}_1 & \mathbf{b}_2 & \ldots & \mathbf{b}_m\end{bmatrix}$ refers to the column vector in $\mathbf{B}$, $\begin{bmatrix}\mathbf{d}_1 & \mathbf{d}_2 & \ldots & \mathbf{d}_m\end{bmatrix}$ refers to the column vector in $\mathbf{D}$, and $\begin{bmatrix}f_1 & \ldots & f_m\end{bmatrix}^T$ refers to the input fault. $\mathbf{B}\cdot\mathbf{f}(k)$ is regarded as a linear combination of column vectors of $\mathbf{B}$, and $\mathbf{f}(k)$ refers to the weight. If the actuator fault exists, $\mathbf{f}(k)$ must be nonzero. If there is a fault in the first channel, then $f_1(k)$ must not be 0. In this case, formula (1) can also be expressed as formula (6), as follows:

$$\begin{cases}\mathbf{x}(k+1) = \mathbf{Ax}(k) + \mathbf{Bu}^c(k) + \mathbf{B}_1\mathbf{f}_1(k) \\ \qquad\qquad + \mathbf{Ed}(k) + \mathbf{b}_1 f_1(k) \\ \mathbf{y}(k) = \mathbf{Cx}(k) + \mathbf{Du}^c(k) + \mathbf{D}_1\mathbf{f}_1(k) \\ \qquad\qquad + \mathbf{Fd}(k) + \mathbf{d}_1 f_1(k)\end{cases} \tag{6}$$

In formula (6), $\mathbf{f}_1(k) = \begin{bmatrix}f_2(k) & \ldots & f_m(k)\end{bmatrix}^T \in \mathbf{R}^{m-1}$. $\mathbf{b}_1$ refers to the first column of $\mathbf{B}$, and $\mathbf{d}_1$ refers to the first column of$\mathbf{D}$.$\mathbf{B}_1$ and $\mathbf{D}_1$ denote the matrices of $\mathbf{B}$ and $\mathbf{D}$ without the first column. Referring to the method of obtaining (2)–(4), the corresponding reasoning is also carried out for (6). As a result, formula (7) is obtained, which decouples the fault information on the first channel.

$$\begin{aligned}\mathbf{Y}_\tau(k) = {} & \mathbf{H}_\tau\mathbf{x}(k-\tau) + \mathbf{H}_\tau^c\mathbf{U}_\tau^c(k) + \mathbf{H}_\tau^d\mathbf{d}_\tau(k) \\ & + \mathbf{H}_\tau^{\mathbf{f}_1}\mathbf{F}_\tau^1(k) + \mathbf{H}_\tau^{f_1}F_\tau^1(k)\end{aligned} \tag{7}$$

In formula (7), the meanings of various matrices are as follows:

$$\mathbf{Y}_\tau(k) = \begin{bmatrix}\mathbf{y}(k-\tau) \\ \mathbf{y}(k-\tau+1) \\ \vdots \\ \mathbf{y}(k)\end{bmatrix}$$

$$\mathbf{H}_\tau^c = \begin{bmatrix}\mathbf{D} & 0 & 0 & 0 \\ \mathbf{CB} & \mathbf{D} & 0 & 0 \\ \vdots & \ddots & \ddots & 0 \\ \mathbf{CA}^{\tau-1}\mathbf{B} & \cdots & \mathbf{CB} & \mathbf{D}\end{bmatrix}$$

$$\mathbf{U}_\tau^c(k) = \begin{bmatrix}\mathbf{u}^c(k-\tau) \\ \mathbf{u}^c(k-\tau+1) \\ \vdots \\ \mathbf{u}^c(k)\end{bmatrix}$$

$$\mathbf{H}_\tau^d = \begin{bmatrix}\mathbf{F} & 0 & 0 & 0 \\ \mathbf{CE} & \mathbf{F} & 0 & 0 \\ \vdots & \ddots & \ddots & 0 \\ \mathbf{CA}^{\tau-1}\mathbf{E} & \cdots & \mathbf{CE} & \mathbf{F}\end{bmatrix}$$

$$\mathbf{d}_\tau(k) = \begin{bmatrix}\mathbf{d}(k-\tau) \\ \mathbf{d}(k-\tau+1) \\ \vdots \\ \mathbf{d}(k)\end{bmatrix} \quad \mathbf{H}_\tau = \begin{bmatrix}\mathbf{C} \\ \mathbf{CA} \\ \vdots \\ \mathbf{CA}^{\tau}\end{bmatrix}$$

$$\mathbf{H}_\tau^{\mathbf{f}_1} = \begin{bmatrix}\mathbf{D}_1 & 0 & 0 & 0 \\ \mathbf{CB}_1 & \mathbf{D}_1 & 0 & 0 \\ \vdots & \ddots & \ddots & 0 \\ \mathbf{CA}^{\tau-1}\mathbf{B}_1 & \cdots & \mathbf{CB}_1 & \mathbf{D}_1\end{bmatrix}$$

$$\mathbf{F}_\tau^1(k) = \begin{bmatrix}\mathbf{f}_1(k-\tau) \\ \mathbf{f}_1(k-\tau+1) \\ \vdots \\ \mathbf{f}_1(k)\end{bmatrix}$$

$$\mathbf{H}_\tau^{f_1} = \begin{bmatrix}\mathbf{d}_1 & 0 & 0 & 0 \\ \mathbf{Cb}_1 & \mathbf{d}_1 & 0 & 0 \\ \vdots & \ddots & \ddots & 0 \\ \mathbf{CA}^{\tau-1}\mathbf{b}_1 & \cdots & \mathbf{Cb}_1 & \mathbf{d}_1\end{bmatrix}$$

$$F_\tau^1(k) = \begin{bmatrix}f_1(k-\tau) \\ f_1(k-\tau+1) \\ \vdots \\ f_1(k)\end{bmatrix}$$

At this point, we can design a fault detection vector $\mathbf{Detect}-\mathbf{Vector}_\tau^1$. With the help of this vector, the fault information is extracted, but the interference and input will not affect the residual. The residual is only affected by the fault.

To design $\mathbf{Detect}-\mathbf{Vector}_\tau^1$, we should first make it meet (8) and (9) as follows

$$\mathbf{DetectVector}_\tau^1 \times (\mathbf{H}_\tau \;\vdots\; \mathbf{H}_\tau^d \;\vdots\; \mathbf{H}_\tau^{\mathbf{f}_1}) = \mathbf{0} \tag{8}$$

$$\mathbf{DetectVector}_\tau^1 \times \mathbf{H}_\tau^{f_1} \neq \mathbf{0} \text{ and } \mathbf{DetectVector}_\tau^1 \neq \mathbf{0} \tag{9}$$

In formulas (8) and (9), $\mathbf{Detect} - \mathbf{Vector}_\tau^1$ is the fault detection vector for the first channel, τ denotes the time window.

Using $\mathbf{Detect} - \mathbf{Vector}_\tau^1$, the residual can be obtained according to (10). The block diagram of residual generation is shown in Figure 1. $\mathbf{Residual}_1$ is only sensitive to the fault arising from the first actuator.

$$\mathbf{Residual}_1 = \mathbf{DetectVector}_\tau^1 \times (\mathbf{Y}_\tau(k) - \mathbf{H}_\tau^c \mathbf{U}_\tau^c(k)) \tag{10}$$

$$\mathbf{Y}_\tau(k) - \mathbf{H}_\tau^c \mathbf{U}_\tau^c(k) = \mathbf{H}_\tau \mathbf{x}(k-\tau) + \mathbf{H}_\tau^d \mathbf{d}_\tau(k) + \mathbf{H}_\tau^{\mathbf{f}_1} \mathbf{F}_\tau^1(k) + \mathbf{H}_\tau^{f_1} F_\tau^1(k) \tag{11}$$

$$\mathbf{Residual}_1 = \mathbf{DetectVector}_\tau^1 \times \mathbf{H}_\tau^{f_1} \times F_\tau^1(k) \tag{12}$$

Finally, $\mathbf{Residual}_1$ satisfies (12). Similarly, the residual associated with the fault of the second channel or the M-th channel needs to be designed separately.

3. TS fuzzy of state-space model

TS fuzzy model is described by a set of IF–THEN fuzzy rules. Each rule represents a subsystem, as described like "IF x is M,THEN $y = \mathbf{f}(x)$", where $\mathbf{f}(x)$ denotes the linear function of x. In general, $\mathbf{f}(x)$ refers to a polynomial function of x. The expression of the T-S fuzzy model of the discrete system is shown in formula (13), as follows:

Rule i: If $\theta_1(k)$ is M_{i1}, $\theta_2(k)$ is $M_{i2}, \ldots$, and $\theta_p(k)$ is M_{ip}, then

$$\begin{cases} \mathbf{x}(k+1) = \mathbf{A}_i\mathbf{x}(k) + \mathbf{B}_i\mathbf{u}(k) \\ \mathbf{y}(k) = \mathbf{C}_i\mathbf{x}(k) + \mathbf{D}_i\mathbf{u}(k) \end{cases} \tag{13}$$

In formula (13), i represents the i-th rule, where $i = 1, 2, \ldots, r$, M_{ij} refers to the fuzzy set, $\theta_1(k), \ldots, \theta_p(k)$ represent the antecedent variables, $\mathbf{x}(k) \in \mathbf{R}^n$ refers to the state vector, $\mathbf{u}(k) \in \mathbf{R}^m$ refers to the control vector. The dimension of the coefficient matrix satisfies $\mathbf{A}_i \in \mathbf{R}^{n\times n}$, $\mathbf{B}_i \in \mathbf{R}^{n\times m}$. The complete system is expressed as follows:

$$\begin{cases} \mathbf{x}(k+1) = \sum_{i=1}^r [\mu_i(\theta(k))(\mathbf{A}_i x(k) + \mathbf{B}_i u(k))] \Big/ \sum_{i=1}^r \mu_i(\theta(k)) \\ \mathbf{y}(k) = \sum_{i=1}^r [\mu_i(\theta(k))(\mathbf{C}_i x(k) + \mathbf{D}_i u(k))] \Big/ \sum_{i=1}^r \mu_i(\theta(k)) \end{cases} \tag{14}$$

where $\mu_i(\theta(k)) = \prod_{j=1}^p M_{ij}(\theta_j(k))$, $M_{ij}(\theta_j(k))$ represents the membership degree of the antecedent variable $\theta_j(k)$ in rule i to the fuzzy subset M_{ij}, and $\mu_i(\theta(k))$ represents the activation degree of rule i. The definition is as follows:

$$h_i(\theta(k)) = \mu_i(\theta(k)) \Big/ \sum_{i=1}^r \mu_i(\theta(k)) \tag{15}$$

Formula (14) can be rewritten in the following form:

$$\begin{cases} \mathbf{x}(k+1) = \sum_{i=1}^r h_i(\theta(k))(\mathbf{A}_i\mathbf{x}(k) + \mathbf{B}_i\mathbf{u}(k)) \\ \mathbf{y}(k) = \sum_{i=1}^r h_i(\theta(k))(\mathbf{C}_i\mathbf{x}(k) + \mathbf{D}_i\mathbf{u}(k)) \end{cases} \tag{16}$$

And $h_i(\theta(k)) > 0$, $\sum_{i=1}^r h_i(\theta(k)) = 1$.

4. Fault decoupling vector for the TS fussy system

For the augmented system above-mentioned in (7), its parameters are extracted by the fuzzy rules at time $k-\tau, k-\tau+1, \ldots, k$. The parameters of most systems do not change so fast, and the occurrence of faults is much faster. Therefore, we design fault detection vectors for the system in each short period (for example, time $[k-\tau,\ k]$, time $[k,\ k+\tau]$, time $[k,\ k+2\tau], \ldots$) according to the methods mentioned above. The rules of the fuzzy system are as follows:

$$\begin{cases} \mu_i(\theta(k)) = \prod_{j=1}^p M_{ij}(\theta_j(k)) \\ \mu_i(\theta(k-1)) = \prod_{j=1}^p M_{ij}(\theta_j(k-1)) \\ \ldots \\ \mu_i(\theta(k-\tau)) = \prod_{j=1}^p M_{ij}(\theta_j(k-\tau)) \end{cases} \tag{17}$$

For the fuzzy system, it meets the following requirements:

$$\mu_i^\tau(\theta(k)) = \prod_{j=1}^p M_{ij}(\theta_j(k)) \cdot \prod_{j=1}^p M_{ij}(\theta_j(k-1)) \ldots \times \prod_{j=1}^p M_{ij}(\theta_j(k-\tau)) \tag{18}$$

$$h_i^\tau(\theta(k)) = \mu_i^\tau \theta(k)) \Big/ \sum_{i=1}^r \mu_i^\tau(\theta(k)) \tag{19}$$

$$\begin{cases} \mathbf{x}(k+1) = \sum_{i=1}^r h_i^\tau(\theta(k))(\mathbf{A}_i\mathbf{x}(k) + \mathbf{B}_i\mathbf{u}(k)) \\ \mathbf{y}(k) = \sum_{i=1}^r h_i^\tau(\theta(k))(\mathbf{C}_i\mathbf{x}(k) + \mathbf{D}_i\mathbf{u}(k)) \end{cases} \tag{20}$$

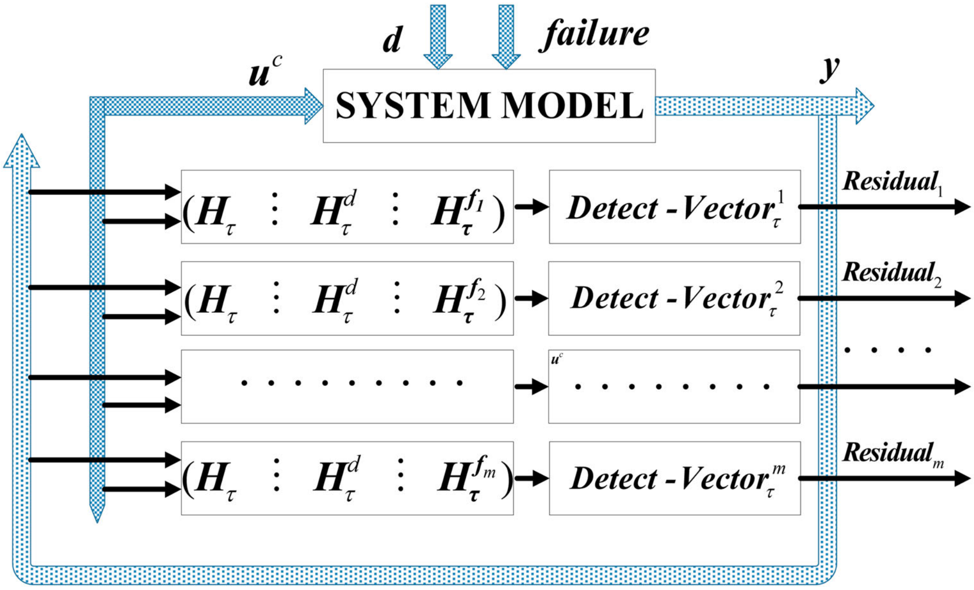

Figure 1. Decoupling residual generator.

And $h_i^\tau(\theta(k)) > 0$, $\sum_{i=1}^{r} h_i^\tau(\theta(k)) = 1$. Then, the formula of the augmented system is obtained as follows:

$$\mathbf{Y}_\tau(k) = \mathbf{H}_\tau(k)\mathbf{x}(k-\tau) + \mathbf{H}_\tau^c(k)\mathbf{U}_\tau^c(k) + \mathbf{H}_\tau^d(k)\mathbf{d}_\tau(k) + \mathbf{H}_\tau^{\mathbf{f}_1}(k)\mathbf{F}_\tau^1(k) + \mathbf{H}_\tau^{f_1}(k)F_\tau^1(k) \tag{21}$$

The calculation flow of decoupling residual for the fuzzy augmented system described in (21) is shown in Figure 2. As time goes on, the decoupling fault detection vectors are updated on and on. Then the decoupling residual for each channel is updated. The complete residual for each channel is spliced by residual fragments, as shown in Figure 3.

5. Fault decoupling vector of the TS fuzzy system

Equations (8) and (9) are the basis of the above design. There is a problem that may be ignored. The existence of vectors satisfying (8) and (9) also needs to be verified. Assuming $\mathbf{rank}(\mathbf{H}_\tau \; \vdots \; \mathbf{H}_\tau^d \; \vdots \; \mathbf{H}_\tau^{\mathbf{f}_1}) = \varepsilon_\tau$, let's calculate the dimensions of $\mathbf{H}_\tau, \mathbf{H}_\tau^d, \mathbf{H}_\tau^{\mathbf{f}_1}$ and $\mathbf{DetectVector}_\tau^1$. $\mathbf{H}_\tau^{\mathbf{f}_1}$ is $[q(\tau+1)] \times [(m-1)(\tau+1)]$, $\mathbf{H}_\tau$ is $[q(\tau+1)] \times n$, $\mathbf{H}_\tau^d$ is $[q(\tau+1)] \times [r(\tau+1)]$, $\mathbf{DetectVector}_\tau^1$ is $1 \times [q(\tau+1)]$. So formula (8) is true, only if the homogeneous linear equations are solved

$$(\mathbf{H}_\tau \; \vdots \; \mathbf{H}_\tau^d \; \vdots \; \mathbf{H}_\tau^{\mathbf{f}_1})^{\mathrm{T}}\mathbf{DetectVector}_\tau^{1\mathrm{T}} = 0 \tag{22}$$

There is a nonzero solution for (22) if $\varepsilon_\tau < q(\tau+1)$. So, an appropriate τ one should be chosen. Considering the definition of ε_τ, (23) is true:

$$\varepsilon_\tau \leq n + r \times (\tau+1) + (m-1) \times (\tau+1) \tag{23}$$

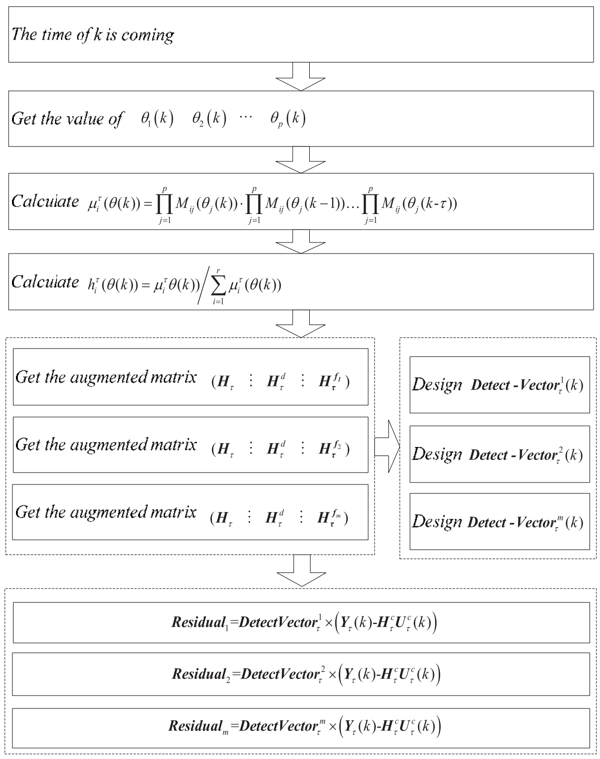

Figure 2. Calculation flow of decoupling residual.

If the dimensions of input, output and interference satisfy formula (24)

$$q - r - m + 1 > 0 \tag{24}$$

Then τ is increased, so

$$\tau > n/(q - r - m + 1) - 1 \tag{25}$$

Then (22) has a nonzero solution and the fault detection vector can be obtained. If (24) is not true, then increasing τ cannot guarantee the existence of the detection

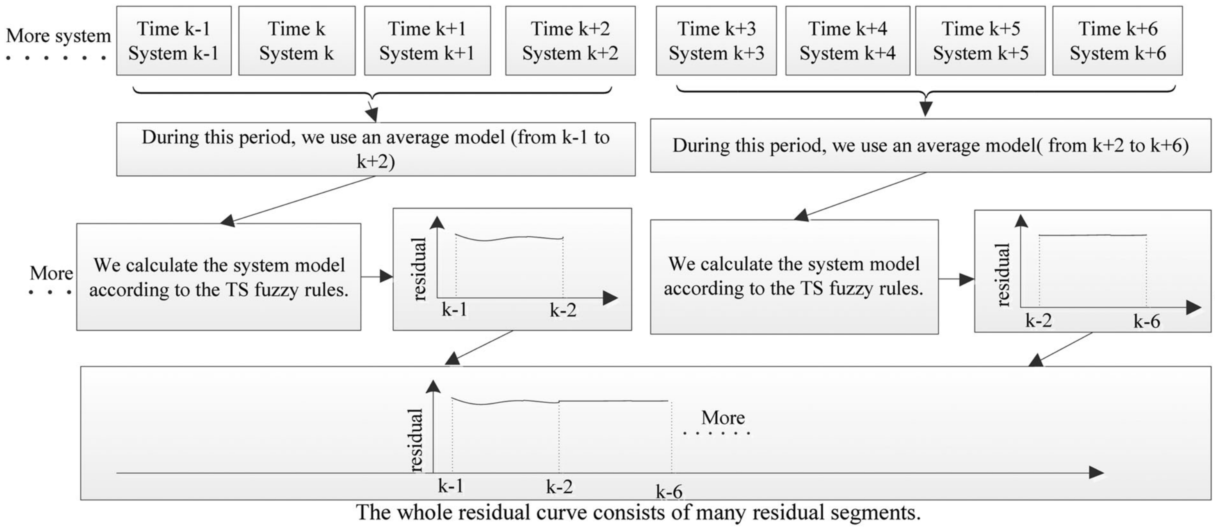

Figure 3. The relationship between the whole residual and the residual segments.

vector. In this case, we need to find a suboptimal solution.

Assuming the suboptimal fault detection vector $*\mathbf{DetectVector}_\tau^1$ is a transverse vector of left null space of $\mathbf{H}_\tau$, $\mathbf{P} = [\mathbf{p}_1, \mathbf{p}_2, \cdots \mathbf{p}_{q(\tau+1)}] \in \mathbf{R}^{w \times q(\tau+1)}$ is a set of bases of left null space of $\mathbf{H}_\tau \in \mathbf{R}^{[q(\tau+1)] \times n}$, and $\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \ldots, \alpha_w]$ is a set of coefficient vectors, then the following formula is satisfied:

$$* \mathbf{DetectVector}_\tau^1 \mathbf{H}_\tau = \boldsymbol{\alpha}\mathbf{P}\mathbf{H}_\tau = 0 \tag{26}$$

Based on formulas (10)–(12), the residual of suboptimal decoupling fault is obtained, as shown in (27)

$$*\mathbf{Residual}_1 = *\mathbf{DetectVector}_\tau^1(\mathbf{H}_\tau^d \mathbf{d}_\tau(k) + \mathbf{H}_\tau^{\mathbf{f}_1}\mathbf{F}_\tau^1(k) + \mathbf{H}_\tau^{f_1} F_\tau^1(k)) \tag{27}$$

Define $*\mathbf{H}_\tau = [\mathbf{H}_\tau^d : \mathbf{H}_\tau^{\mathbf{f}_1}]$, $\mathbf{U}(k) = \begin{bmatrix} \mathbf{d}_\tau(k) \\ \mathbf{F}_\tau^1(k) \end{bmatrix}$, and then (28) is obtained:

$$*\mathbf{Residual}_1 = *\mathbf{DetectVector}_\tau^1(*\mathbf{H}_\tau \mathbf{U}(k) + \mathbf{H}_\tau^{f_1} F_\tau^1(k)) \tag{28}$$

The two-norm ratio performance index η is defined, as shown in (29), and the ideal fault detection vector can be obtained by minimizing η. After the equivalent transformation (30) and (31), the formula (32) is obtained.

$$\mu = ||(*\mathbf{DetectVector}_\tau^1)(*\mathbf{H}_\tau)||_2^2 / ||(*\mathbf{DetectVector}_\tau^1)(\mathbf{H}_\tau^{f_1})||_2^2 \tag{29}$$

$$\mu = ||(\boldsymbol{\alpha}\mathbf{P})(*\mathbf{H}_\tau)||_2^2 / ||(\boldsymbol{\alpha}\mathbf{P})(\mathbf{H}_\tau^{f_1})||_2^2 \tag{30}$$

$$\mu = [(\boldsymbol{\alpha}\mathbf{P})(*\mathbf{H}_\tau)][(\boldsymbol{\alpha}\mathbf{P})(*\mathbf{H}_\tau)]^T / [(\boldsymbol{\alpha}\mathbf{P})(\mathbf{H}_\tau^{f_1})][(\boldsymbol{\alpha}\mathbf{P})(\mathbf{H}_{\tau\,\tau}^{f_1})]^T \tag{31}$$

$$[\mathbf{P}(*\mathbf{H}_\tau)(*\mathbf{H}_\tau)^T\mathbf{P}^T - \mu\mathbf{P}\mathbf{H}_\tau^{f_1}(\mathbf{H}_\tau^{f_1})^T\mathbf{P}^T]\boldsymbol{\alpha}^T = 0 \tag{32}$$

Assuming that $\mathbf{P}(*\mathbf{H}_\tau)(*\mathbf{H}_\tau)^T\mathbf{P}^T = \boldsymbol{\omega}$, $\mathbf{P}\mathbf{H}_\tau^{f_1}(\mathbf{H}_\tau^{f_1})^T \mathbf{P}^T = \boldsymbol{\varphi}$, (32) can be converted into (33):

$$\boldsymbol{\omega}\boldsymbol{\alpha}^T = \mu\boldsymbol{\varphi}\boldsymbol{\alpha}^T \tag{33}$$

The problem of solving generalized eigenvalues is described in formula (33). The solution process is as follows:

(1) Solve the formula $\det(\boldsymbol{\omega} - \mu\boldsymbol{\varphi}) = 0$. If there is a solution, the solution satisfying formula (33) can be found;
(2) Let $*\mu = \min_i \lambda_i(\boldsymbol{\omega}, \boldsymbol{\varphi})$, where $*\mu$ refers to the optimal value of the performance index, and the corresponding optimal eigenvector $(*\boldsymbol{\alpha})^T$ is obtained;
(3) The suboptimal decoupling fault detection parity vector $*\mathbf{DetectVector}_\tau^1 = (*\boldsymbol{\alpha})^T\mathbf{P}$ is calculated, and then the residual error is obtained.

6. Calculation and analysis

6.1. Case 1. Example of decoupled fault vector for steady-state systems

The first example is to show the design of the decoupling vector and the generation of fault residual. To illustrate this point, the fuzzy characteristics of the model are not considered for the time being. According to reference [18], the motion equation of a helicopter in the vertical plane is established as follows

$$x(k+1) = \begin{bmatrix} 0.9996 & 0.0003 & 0.0002 & -0.0037 \\ 0.0005 & 0.9900 & -0.0002 & -0.0406 \\ 0.0010 & 0.0037 & 1.0453 & 1.5644 \\ 0 & 0 & 0.0101 & 1.0524 \end{bmatrix} x(k)$$

$$+\begin{bmatrix} 0.0044 & 0.0018 \\ 0.0353 & -0.0755 \\ -0.0559 & 0.0454 \\ -0.0003 & 0.0002 \end{bmatrix} u(k) + w(k) \tag{34}$$

$$y(k) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} x(k) + v(k) \tag{35}$$

where $x(k) = \begin{bmatrix} \gamma & \omega & \omega_y & \theta \end{bmatrix}$ denotes the state variable, $u(k) = \begin{bmatrix} u_1(k) & u_2(k) \end{bmatrix}^T$ denotes the control variable, $w(k) \in R^4$ is the process noise. In the state variable, γ denotes the longitudinal speed, ω denotes the vertical speed, ω_y denotes the pitch angle rate, θ denotes the pitch angle, γ, ω, ω_y are measurable. The sampling time is 0.01 s.

Faults mainly include slow and fast change faults. Sudden change faults are more obvious, while slow change faults change slowly and are difficult to detect. Therefore, we take the slow change fault, which is difficult to detect as the research object. The calculation takes a certain time when the system is already in a steady state at the beginning. There is a slow change actuator failure in the first channel. The fault detection vector is designed according to the method mentioned earlier. When $n = 1$, calculate $\boldsymbol{H}_1$ as follows

$$\boldsymbol{H}_1 = H_\tau \;\vdots\; H_\tau^d \;\vdots\; H_\tau^{f_1}$$
$$= \begin{bmatrix} 1.0000 & 0 & 0 \\ 0 & -1.0000 & 0 \\ 0 & 0 & 1.0000 \\ 0.9996 & 0.0003 & 0.0002 \\ -0.0005 & -0.9900 & 0.0002 \\ 0.0010 & 0.0037 & 1.0453 \end{bmatrix}$$
$$\times \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -0.0037 & 0.0018 & 0 \\ 0.0406 & 0.0755 & 0 \\ 1.5644 & 0.0454 & 0 \end{bmatrix} \tag{36}$$

The singular value decomposition of the matrix is carried out to obtain the following

$$S_{H_1} = \begin{bmatrix} 1.9794 & 0 & 0 & 0 & 0 \\ 0 & 1.4139 & 0 & 0 & 0 \\ 0 & 0 & 1.4082 & 0 & 0 \\ 0 & 0 & 0 & 0.7910 & 0 \\ 0 & 0 & 0 & 0 & 0.0528 \end{bmatrix}$$

$$U = \begin{bmatrix} -0.0002 & -0.7064 & 0.0336 \\ 0.0065 & 0.0334 & 0.7086 \\ 0.3371 & -0.0019 & -0.0166 \\ -0.0015 & -0.7062 & 0.0334 \\ 0.0226 & 0.0335 & 0.7036 \\ 0.9412 & -0.0016 & -0.0158 \end{bmatrix}$$

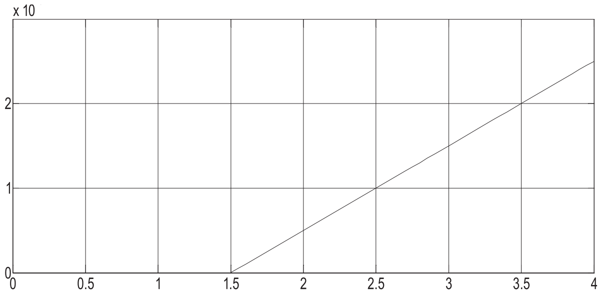

Figure 4. Slow change fault.

$$\times \begin{bmatrix} 0.0021 & 0.0181 & -0.7067 \\ -0.0254 & 0.7041 & 0.0182 \\ -0.9411 & -0.0202 & -0.0024 \\ -0.0011 & -0.0184 & 0.7070 \\ 0.0109 & -0.7091 & -0.0181 \\ 0.3370 & 0.0194 & 0.0021 \end{bmatrix}$$

$$V = \begin{bmatrix} -0.0004 & -0.9989 & 0.0473 \\ -0.0128 & -0.0473 & -0.9979 \\ 0.6673 & -0.0026 & -0.0234 \\ 0.7443 & 0.0010 & 0.0027 \\ 0.0224 & 0.0008 & 0.0373 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\times \begin{bmatrix} 0.0017 & 0.0010 & 0 \\ 0.0201 & -0.0371 & 0 \\ -0.7444 & -0.0011 & 0 \\ 0.6671 & 0.0304 & 0 \\ 0.0204 & -0.9988 & 0 \\ 0 & 0 & 1.0000 \end{bmatrix} \tag{37}$$

Thus, the value ε_τ of $\boldsymbol{H}_1$ is obtained as follows:

$$\varepsilon_\tau = 5 < q \times (\sigma + 1) = 3 \times (1 + 1) = 6 \tag{38}$$

Therefore, there is an optimal decoupling fault vector for the first channel. U_2 is used to construct a fault decoupling vector as follows:

$$\textbf{DetectVector}_\tau^1 = [-0.7067\; 0.0182\; -0.0024\; 0.7070\; -0.0181\; 0.0021] \tag{39}$$

Similarly, the decoupling fault vector of the second channel is constructed as follows:

$$\textbf{DetectVector}_\tau^2 = [-0.08811\; 0.6981\; -0.0198 \times 0.0877\; -0.7046\; 0.0191] \tag{40}$$

Figure 4 shows the slowly changing fault, Figure 5 shows the control input of the first channel, Figure 6 shows the input of the second channel, Figure 7 shows the residual generated in the first channel when there is a fault in the first channel, Figure 8 shows the residual of the second channel when there is a fault in the first channel.

In the above simulation, decoupling is realized. The detection vector of a channel is only sensitive to the fault

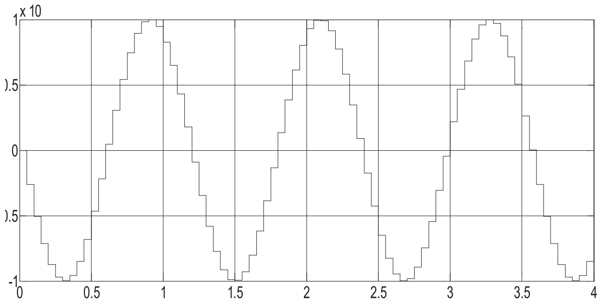

Figure 5. Input of the first channel.

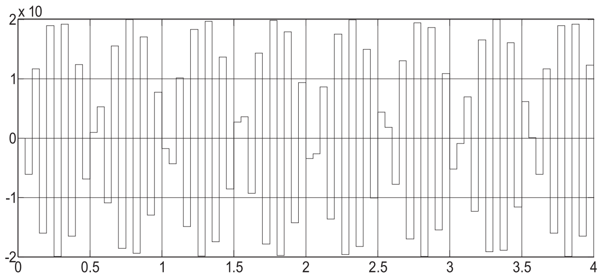

Figure 6. Input of the second channel.

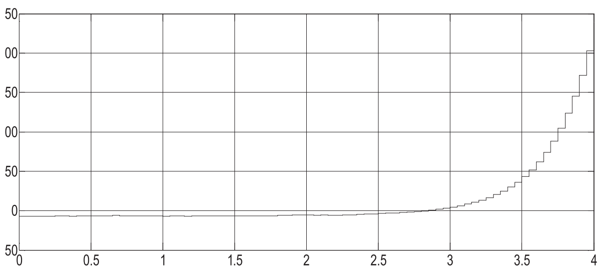

Figure 7. Residual of channel 1 when the fault in channel 1.

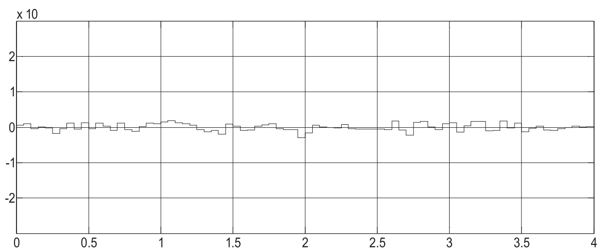

Figure 8. Residual of channel 2 when the fault in channel 1.

of that channel. As shown in Figure 4, when the fault is added to other channels, the residual is almost 0. This indicates that the fault detection vector corresponds to its channel.

In this example, the optimal decoupling vector exits. To simplify the algorithm, the simulation is carried out for the steady system. To fully verify the algorithm in this paper, another example is considered, as shown in case 2. This example takes TS Fuzzy into account, and there is no optimal decoupling vector. So, it is necessary to solve the suboptimal solution.

6.2. Case 2. Example of the decoupled fault vector for TS fuzzy systems

In reference [19], the model of cement calciner is established as follows

$$\begin{aligned} \mathbf{x}(k+1) &= \mathbf{A}_1\mathbf{x}(k) + \mathbf{B}_1\mathbf{u}(k) \\ \mathbf{y} &= \mathbf{C}_1\mathbf{x}(k) \end{aligned} \tag{41}$$

When the furnace temperature $T < 830°\text{C}$, then

$$\mathbf{A}_1 = \begin{bmatrix} 0 & 1 \\ 0 & 1.0040 \end{bmatrix} \mathbf{B}_1 = \begin{bmatrix} 9.7739 & 0.6780 \\ 1.4731 & -0.0950 \end{bmatrix}$$

$$\mathbf{C}_1 = \begin{bmatrix} 1 & 0 \end{bmatrix} \mathbf{D}_1 = [0, 0]$$

When the furnace temperature $840°\text{C} < T$, then

$$\mathbf{A}_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0.9965 \end{bmatrix} \mathbf{B}_2 = \begin{bmatrix} 0.5367 & 1.2868 \\ 0.1576 & -0.0176 \end{bmatrix}$$

$$\mathbf{C}_2 = \begin{bmatrix} 1 & 0 \end{bmatrix} \mathbf{D}_2 = [0, 0]$$

Here, the fault residual of this model is designed (Figure 9).

It is assumed that the furnace temperature has been changing within $820°\text{C} - 850°\text{C}$, the activation-degree function $830°\text{C} - 840°\text{C}$ can be simply written as follows:

$$h_1(T(k)) = \begin{cases} 1 & T \le 830°\text{C} \\ (840 - T)/10 & 830°\text{C} < T < 840°\text{C} \end{cases},$$

$$h_2(T(k)) = \begin{cases} 0 & T \le 830°\text{C} \\ (T - 830)/10 & 830°\text{C} < T < 840°\text{C} \end{cases}$$

The sampling time is 0.5 s. In 2 min ($k = 0 - 120$), the furnace temperature at the sampling time point is shown in Figure 10. The temperature rises from 813 to 835 within 0–50 s, and then changes slightly around 835.

In this system, the main parameters satisfy:$m = 2$, $n = 2$, $q = 1$, $r = 0$. If $\varepsilon_\tau < q(\tau+1) = \tau + 1$ is satisfied, there is a fully decoupled vector for fault detection. In this case, $\varepsilon_\tau = \mathbf{rank}(\mathbf{H}_\tau \;\vdots\; \mathbf{H}_\tau^d \;\vdots\; \mathbf{H}_\tau^{\mathbf{f}_1})$, the dimensions of $(\mathbf{H}_\tau \;\vdots\; \mathbf{H}_\tau^d \;\vdots\; \mathbf{H}_\tau^{\mathbf{f}_1})$ is $[q(\tau+1)] \times [(m-1)(\tau+1) + n + r(\tau+1)] = (\tau+1) \times (\tau+3)$. Because$\tau + 1 < \tau + 3$, then $\varepsilon_\tau \le \tau + 1$. If $(\mathbf{H}_\tau \;\vdots\; \mathbf{H}_\tau^d \;\vdots\; \mathbf{H}_\tau^{\mathbf{f}_1})$ is full row rank, then $\varepsilon_\tau = \tau + 1$, otherwise $\varepsilon_\tau < \tau + 1$. Based on the previous conclusion, no matter how to expand τ, the existence of a decoupling vector cannot be guaranteed. Therefore, to reduce the amount of calculation, we choose $\tau = 2$ in this case to calculate the suboptimal decoupling vector.

Test1: Step fault is added at the 40 s on channel 1, and the fault is not added on channel 2. The residual generated by the fault detection vector designed for channel 1 is shown in Figure 11

Test2: Step fault is added at the 40 s on channel 1, and step fault is added at the 30 s on channel 2. The residual generated by the fault detection vector designed for channel 1 is shown in Figure 12.

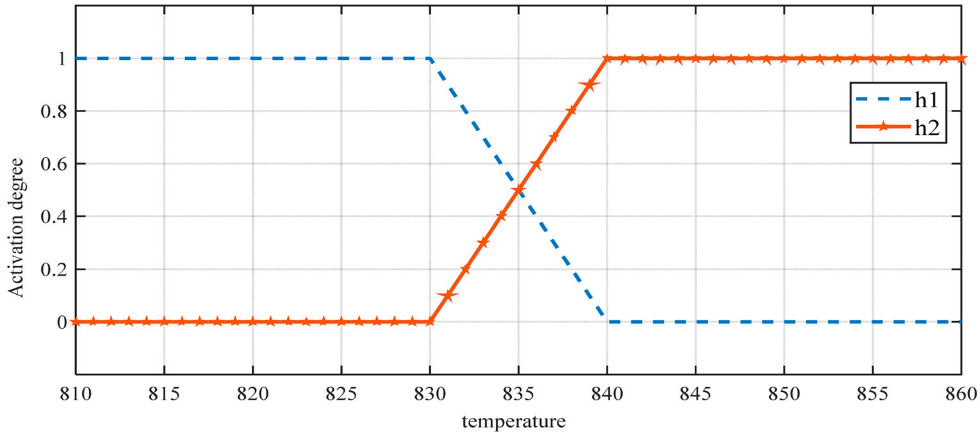

Figure 9. Activation-degree function.

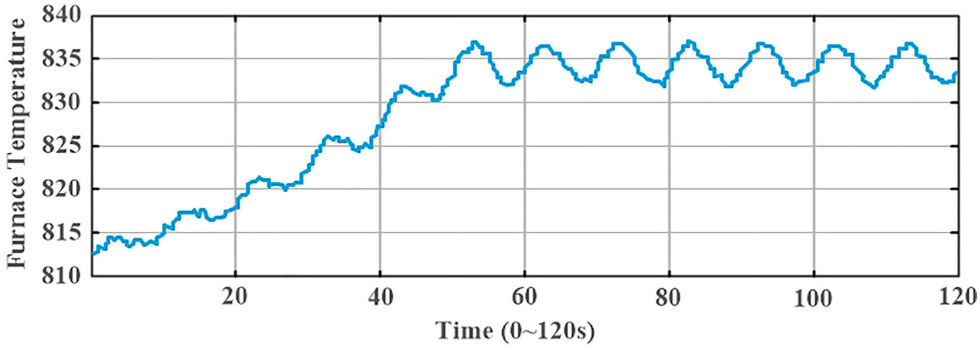

Figure 10. Change of activation degree with furnace temperature.

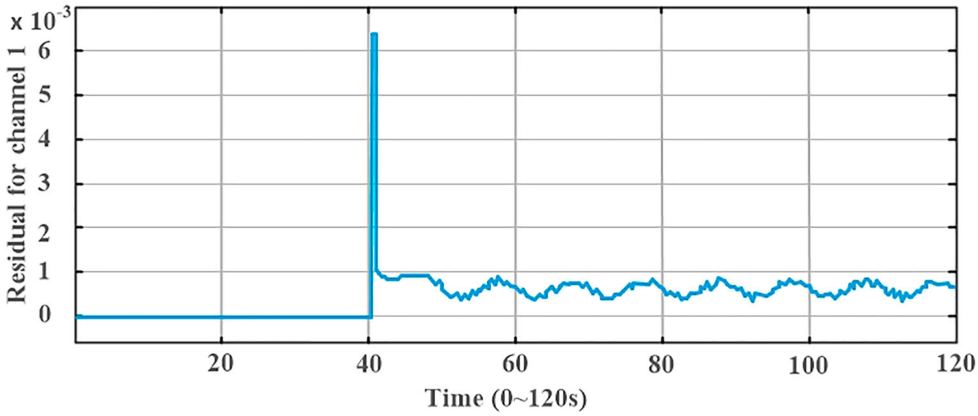

Figure 11. When there is a fault on channel 1 and no fault on channel 2, the residual for channel 1 is calculated.

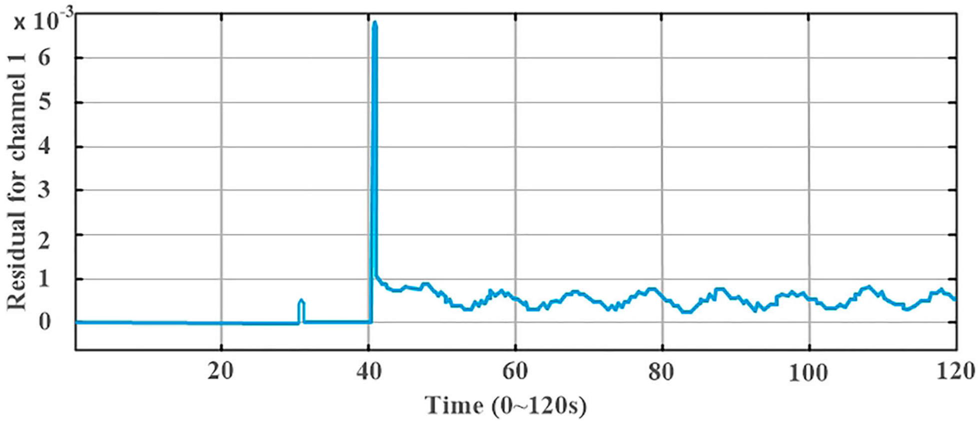

Figure 12. When there is a fault on channel 1 and there is a fault on channel 2, the residual for channel 1 is calculated.

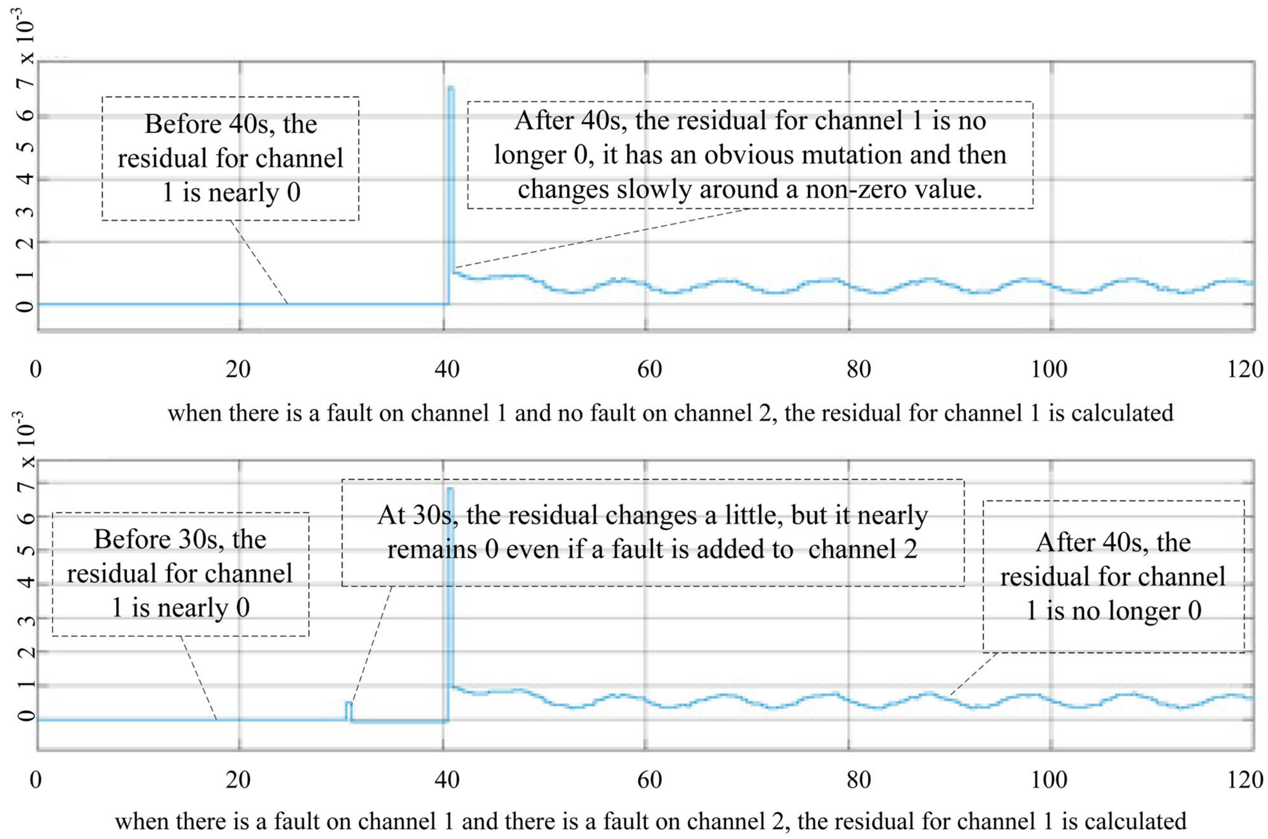

Figure 13. Comparison and description.

Table 1. Comparison of some different fault diagnosis studies.

No.	title	focus	characteristics of the method
1	Application of fully decoupled parity equation in fault detection and identification of DC motors	Parity equation vectors	It detects the Actuators' fault. Robust
2	Failure detection of redundant sensor with reduced-order parity vectors	Reduced-order parity vectors	It detects and isolates the sensor's fault. Fault estimation
3	Robust fault estimation design for discrete-time nonlinear systems by a modified fuzzy fault estimation observer	Fuzzy fault estimation observer	Robust. Fault estimation
4	This paper	Decoupling fault detection vectors. TS fuzzy	Robust. Detect and isolate the actuators' fault

When the fault was not added, as shown in Figure 11, before the 40 s, the residual is zero. When the fault occurs at 40 s, the residual is displayed after a very short delay. In Figure 12, the fault detection vector designed for channel 1 is only sensitive to the fault of channel 1. Even if the fault is added to the second channel at 30 s, it only produces a small fluctuation. Finally, the residual reflects the fault of channel 1 added at 40 s. The comparison and explanation illustrate the process more clearly in Figure 13.

As expected, the two figures are nearly the same. The faults of the two are very different, but the figures are still very similar because they both reflect the residual of channel 1. This proves that the residual of channel 1 is almost independent of the fault of channel 2. The experiment verifies the success of decoupling.

7. Conclusion

In this article, the parity vector of decoupled fault detection was designed for the TS fuzzy system. The best scenario is that a fully decoupled fault detection vector exists. For some systems, this requirement cannot be met. Then, we designed a suboptimal decoupled fault detection vector instead. The most obvious advantage of the method is the decoupling of different channels. Fault diagnosis is very concerning topic in recent years. Many results on fault detection are published. The focus of these studies is very different. To make the characteristics of this paper clearer, here is a simple comparison shown in Table 1.

The research in this paper may be more similar to the research of No. 1 in Tab. 1. However, No. 1 only focuses on the determined system. To deal with uncertain systems, TS fuzzy is introduced in this paper. In addition, the suboptimal decoupling fault detection vector proposed in this paper greatly expands the applicability of the method.

Disclosure statement

No potential conflict of interest was reported by the author(s).

Funding

This work was supported by Innovation and Entrepreneurship Program for College Students; Key R & D Plan Project of Shannxi Province (Grant No. 2022GY-135).

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