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Efficiency improvement of a three-coil WPT system with rectified load based on a selected compensation capacitor

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ABSTRACT

In the existing pieces of literature about the three-coil wireless power transfer (WPT) system, a three-coil structure is proposed to increase the efficiency compared to the two-coil WPT system. The cross-coupling among three coils, compensation capacitor of the repeating coil (RPC) and the rectified load are considered incompletely. The circuit model of a three-coil WPT system with rectified load is developed by taking the cross-coupling of all coils and the compensation capacitor of RPC into consideration in this paper. The relationship among system efficiency, compensation level, and rectifier load is deduced. The detailed mathematical analysis and compensation capacitor selection of RPC are investigated to improve the system efficiency. An experimental study demonstrates that the selected compensation capacitor of RPC achieves a higher efficiency than resonant compensation in a three-coil WPT system with different cross-coupling conditions and rectifier load conditions. System efficiency is promoted from 73.9% (resonant compensation capacitor for RPC) to 84.2% (selected compensation capacitor for RPC).

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KEYWORDS

Wireless power transfer; compensation capacitor selection; rectified load; cross-coupling

1. Introduction

The wireless power transfer (WPT) technology has been developed rapidly these years [1–4]. It can realize the transmission of electric energy across the air compared with the traditional contact mode. Especially the inductive power transfer based on the magnetic-field coupling is the most popular WPT technology [5–8]. The WPT technology has been applied in many areas, such as industrial robots [9,10], electric vehicles [11–13], consumer electronics, implantable equipment, unmanned underwater devices, etc.

Based on the two-coil WPT system, the three-coil [14] or multi-coil [15] structures are proposed to promote the system efficiency when the distance between the transmitting coil and receiving coil increases. In the three-coil structure WPT, the application of a repeating coil (RPC) can reduce the current in the drive circuit [14,16,17]. For the three-coil WPT system with RPC composed of double coils in [18], the constant current and constant voltage modes are realized based on the reconfigurable middle coil. In [19], a switch device is used in the receiving end to select the constant current mode or the constant voltage mode in the WPT system with RPC.

In the existing pieces of literatures about the three-coil WPT system, the cross-coupling between the transmitting coil (TC) and receiving coil (RC) is usually ignored to simplify the analysis of the modelling [14]. In [20] where the cross-coupling and rectifier of load are ignored, the efficiency optimization and power stability of a WPT system charging multiple loads with a relay coil structure are studied, and the frequency configuration and distribution design are proposed. To investigate the comprehensive characteristics of a three-coil WPT system, the cross-coupling among all three coils and rectifier loads should be considered in the modelling and analysis process. On the other hand, RPC is commonly compensated resonantly with a series capacitor [14,20]. The analysis and selection for compensation capacitors of RPC are also rarely discussed to promote system efficiency.

To fill this gap, the efficiency improvement of a three-coil WPT system with rectified load based on a selected compensation capacitor is investigated in this paper. This paper is organized as follows. In Section II, the circuit model of a three-coil WPT system with a rectifier load, considering the cross-coupling of three coils and a compensation capacitor of RPC, is developed. In Section III, the relationship among system efficiency, compensation capacitor of RPC and rectifier load is deduced. The detailed mathematical analysis and compensation capacitor selection of RPC is investigated to improve the system efficiency. In Section IV, the theoretical research is verified by experiments with different cross-coupling and rectifier load conditions. Finally, conclusions are summarized in Section V.
2. Modelling of a three-coil WPT system with a non-resonant repeating coil

The typical circuit diagram of a three-coil WPT system is introduced in Figure 1. The equivalent circuit of the three-coil WPT system with non-resonant compensated RPC considering all cross-couplings is shown in Figure 1. $U_s$ represents the output voltage of a high-frequency inverter. $L_t$ and $R_t$ represent the inductance and internal resistance of TC, respectively. The LCC (inductor-capacitor-capacitor) compensation circuit at the transmission side is composed of $L_f$, $C_f$, and $C_t$. $L_r$ and $R_r$ are the inductance and internal resistance of RC, respectively. $C_r$ represents the compensation capacitance at the receiving side. $R_k$ (the equivalent AC load) in Figure 2 can be converted by $R_{DC}$ (DC load after rectifier in Figure 1) according to $R_k \approx (8R_{DC})/(\pi^2)$ [21]. The inductance and the resistance of RPC are $L_2$ and $R_2$, respectively.

The compensation capacitor $C_2$ is connected in series. $k_1$ and $k_2$ represent the coupling coefficient between TC and RPC and that between RC and RPC, respectively. $k_3$ is the coupling coefficient between TC and RC. The system works at the frequency of $f$. The corresponding angular frequency is $\omega = 2\pi f$.

In Figure 2, $I_f$ is the output current of the inverter. $I_t$, $I_2$ and $I_r$ are the currents of TC, RPC, and RC, respectively. The following circuit equation can be derived based on the mesh current analysis

$$
\begin{align*}
(Z_{L_f} + Z_{C_f})I_f - Z_{C_f}I_t &= U_s - Z_{C_f}I_f + (Z_{L_2} + Z_{C_t} + Z_{C_f} + R_t)I_t \ - Z_{k_1}I_2 - Z_{k_2}I_r = 0 \\
&= -Z_{k_1}I_2 - Z_{k_2}I_r = 0
\end{align*}
$$

According to (1) and (2), the current of TC can be calculated by

$$
I_t = \frac{U_s}{\omega L_f}.
$$

The series resonant compensation circuit is applied on the receiving side, which satisfies

$$
\omega = \frac{1}{\sqrt{L_rC_r}}.
$$

In general, the compensation circuit is designed resonantly according to the operating frequency of the system. The compensation capacitor of RPC is rarely investigated in a three-coil WPT system with a rectifier load. In this paper, the selection of compensation capacitor is discussed in detail considering all cross-couplings among three coils and a rectifier load.

Parts of (1) can be rewritten as

$$
\begin{align*}
\left\{ \begin{array}{l}
(R_2 + j(\omega L_2 - 1/(\omega C_2))) \cdot I_2 - Z_{k_2} \cdot I_r = U' \\
-Z_{k_2} \cdot I_2 + \left( R_r + \frac{8R_{DC}}{\pi^2} \right) \cdot I_r = U''
\end{array} \right.
\end{align*}
$$

The current at the receiving side can be deduced by

$$
\begin{align*}
I_r &= \frac{U''(R_2 + j(\omega L_2 - 1/(\omega C_2))) + U'Z_{k_2}}{R_r + \frac{8R_{DC}}{\pi^2}(R_2 + j(\omega L_2 - 1/(\omega C_2))) - Z_{k_2}^2} \\
&= \frac{U_r}{L_f} \cdot \left( R_r + \frac{8R_{DC}}{\pi^2} \right) \left( R_2 + \omega^2 k_2^2 L_2 L_r \right) \\
&\quad + j(\omega L_2 - 1/(\omega C_2)) \left( R_r + \frac{8R_{DC}}{\pi^2} \right). 
\end{align*}
$$

The compensation topology on the transmitting side is LCC type and the parameter [22] satisfies
The current of RPC can be represented by
\[ I_2 = \frac{U}{L_f} \left( R_r + j(\omega L_r - 1/(\omega C_2)) \right) \]

where the efficiency analysis factor \( \chi_1 \) is given by
\[ \chi_1 = \frac{A_1 \delta^2 + A_2}{B_1 \delta^2 + B_2 \delta + B_3} \]

(15)

According to (11) the compensation capacitor \( C_2 \) and the cross-coupling \( k_1, k_2, k_3 \), will affect the output power and system efficiency. The algebraic analysis and optimization will be proposed to analyse the influence of non-resonant compensation degree on system efficiency in the next part.

3. Analysis and selection of a compensation capacitor

The system efficiency can be calculated by
\[ \eta = \frac{8R_{DC}}{\pi^2 R_r \chi_1 + (\pi^2 R_r + 8R_{DC})^2} \]

where \( R_{DC} \) is the DC resistance. \( \sigma = \frac{R_r}{R_{DC}} \).

Based on (3), (7) and (8), we have
\[ \frac{|I_2|^2}{|I_r|^2} = \frac{\pi^4 R_r^2 R_T^2 L_r^2 + \pi^2 R_r^2 \omega^2 k_2^2 L_r^2 + \pi^2 \omega^2 k_3^2 L_r^2 + \pi^2 \omega^2 k_1^2 L_r^2}{\pi^2 R_r^2 L_r^2 + \pi^2 \omega^2 k_3 L_r^2 + \pi^2 \omega^2 k_2 L_r^2} \]

(12)

where \( R_r = \pi^2 R_r + 8R_{DC} \) and \( \delta = (L_2 - 1/(\omega^2 C_2)) / L_0 \).

The output power of the three-coil WPT system with a rectifier load is computed by
\[ P = \frac{8R_{DC} I_r^2}{\pi^2} \]

(9)

The efficiency analysis factor can be expressed as
\[ \chi_1 = \frac{A_1 \delta^2 + A_2}{B_1 \delta^2 + B_2 \delta + B_3} \]

(15)

where \( A_1 = \omega^2 L_0^2 R_T^2 \), \( A_2 = (R_0 R_2 + \pi^2 \omega^2 k_2^2 L_r^2 + \pi^2 \omega^2 k_3^2 L_r^2 + \pi^2 \omega^2 k_1^2 L_r^2) \omega^2 L_0^2 L_r \), \( B_1 = \pi^4 \omega^4 R_0^2 R_T^2 L_r^2 \), \( B_2 = 2\pi^4 \omega^4 R_0^2 R_T^2 L_r^2 \), \( B_3 = \pi^4 \omega^4 k_1^2 k_2^2 L_r^2 L_r \).

The output power can be achieved at the point \( \delta = \delta_1 \) and the minimum value is achieved at the point \( \delta = \delta_2 \). The Vieta theorem tells us that when \( \delta_1 \delta_2 = -\frac{A_2}{B_1} \) and \( \delta_1 < 0 < \delta_2 \), \( \delta_1 \) will lie in the over-compensation region and \( \delta_2 \) will lie in the under-compensation region.

Based on L'Hopital's rule in mathematics, we have \( \lim_{\delta \to -\infty} \chi_1(\delta) = \lim_{\delta \to +\infty} \chi_1(\delta) = \frac{A_1}{B_1} \), the minimum extreme point \( \delta = \delta_2 \) achieved by \( \chi_1 \) is exactly the minimum value.

Therefore, the system efficiency will monotonically decrease for \( \delta \in (-\infty, \delta_1) \), increase for \( \delta \in (\delta_1, \delta_2) \), and decrease for \( \delta \in (\delta_2, +\infty) \). The maximum value of efficiency is achieved at \( \delta = \delta_2 \) and the corresponding variation trend is presented in Table 1.

The theoretical optimal compensation degree \( \delta_2 \) can be obtained by substituting the parameters in (15) and (17). In the RPC circuit, the compensation capacitor
Table 1. Change trend of the efficiency factor and system efficiency.

<table>
<thead>
<tr>
<th>δ</th>
<th>(−∞, δ₁)</th>
<th>δ₁ (δ₁, δ₂)</th>
<th>δ₂ (δ₂, +∞)</th>
</tr>
</thead>
<tbody>
<tr>
<td>χᵢ</td>
<td>MAX</td>
<td>MIN</td>
<td></td>
</tr>
<tr>
<td>η</td>
<td>MIN</td>
<td>MAX</td>
<td></td>
</tr>
</tbody>
</table>

χᵢ ↗ MAX ↘ MIN ↗ MAX ↘ MIN

Table 2. System parameters.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>L₁ (μH)</th>
<th>L₂ (μH)</th>
<th>L₃ (μH)</th>
<th>L₄ (μH)</th>
<th>C (nF)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>43.6</td>
<td>64.4</td>
<td>6.8</td>
<td>17</td>
<td>149</td>
</tr>
</tbody>
</table>

According to (15) and (11), the efficiency factor χᵢ (logarithmic form) and the system efficiency changing with the compensation degree of RPC are shown in Figure 3 (k₃ is not ignored). In Figure 3, the efficiency factor χᵢ is at the minimum value when δ = δ₂ and the system efficiency is at the maximum value, demonstrating the correctness of the algebraic derivation analysis and optimization of the compensation degree.

4. Experimental verifications

The experimental prototype is built, as shown in Figure 4. The high-frequency full-bridge inverter is set up using DSP (TMS320F28335) as the controller and four MOSFETs (IRFP4227PBF). The full-bridge rectifier consists of four diodes (DSEP15-06A) on the receiving side. The system parameters are listed in Table 2. The external length of a side of three square coils is 16 cm. The turns of the transmitting coil are 15. The turns of the repeating coil are 4. The turns of the receiving coil are 15.

After fixing the positions of three coils, mutual inductances are measured using an LCR meter (TH2817C). According to coil inductances, the cross-coupling can be obtained as k₁ = 0.181, k₂ = 0.152, k₃ = 0.0672. According to (15) and (17), the optimal compensation factor δ can be calculated as 0.1963 (@R̄DC = 5Ω) and 0.1188 (@R̄DC = 10Ω). The optimal compensation capacitors are 523.6 and 451.3 nF respectively, while the resonant compensation capacitor is 372.5 nF. The measured results of efficiency and receiving power varying with δ are shown in Figure 5. Figure 6 shows the waveforms of the input voltage, input current, and output voltage when the optimal compensation is applied (@R̄DC = Ω and @R̄DC = 10Ω). The waveforms are measured using the oscilloscope (Tektronix TDS2014C), the current probe (Sunraise SRS6025) and the voltage probes (Cybertek P1300).

Setting R̄DC = 5Ω, different coupling conditions are obtained by changing the positions of the three coils. Based on (15) and (17), the optimal compensation factor δ can be 0.1963 (@A: k₁ = 0.181, k₂ = 0.152, k₃ = 0.0672), 0.1074 (@B: k₁ = 0.125, k₂ = 0.128, k₃ = 0.045) and 0.043 (@C: k₁ = 0.0889, k₂ = 0.0798, k₃ = 0.0247). The corresponding compensation capacitors are 523.6, 442.4 and 397.7 nF.

The measured results of efficiency and receiving power varying with δ are shown in Figure 7.

According to the experiment results under different load and crossing coupling conditions, the compensation capacitor of RPC influences the efficiency of the three-coil WPT system markedly. For case A
in Figure 7, system efficiency is promoted from 73.9% (resonant compensation) to 84.2% (non-resonant compensation optimization). Higher efficiency is achieved under non-resonant compensation optimized by the theoretical analysis compared to resonant compensation. The theoretical analysis and optimization are verified experimentally under different load or cross-coupling conditions.

5. Conclusions
In a three-coil WPT system, the cross-coupling among three coils, the compensation capacitor of the repeating coil (RPC) and the rectified load are not fully considered simultaneously. In this paper, the circuit model of a three-coil WPT system with a rectified load is developed to analyse the general status by taking the cross-coupling of all coils and the compensation capacitor of RPC into consideration. The internal connection among system efficiency, compensation level and rectifier load is derived. According to the proposed modelling and analysis, it is proved that the optimal efficiency is achieved at resonant compensation theoretically when the coupling between TC and RC is ignored. After taking the coupling between TC and RC into consideration, the detailed mathematical analysis and compensation capacitor selection of RPC are investigated to improve the system efficiency. Finally, the experimental study demonstrates that the selected compensation capacitor of RPC achieves a higher efficiency than the resonant compensation in a three-coil WPT system with different cross-coupling conditions and rectifier load conditions.

Disclosure statement
No potential conflict of interest was reported by the author(s).

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