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ANCIENT ROOTS OF GETALDIĆ'S WORK ON THE DEVELOPMENT OF MATHEMATICAL ANALYSIS AND SYNTHESIS

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Abstract: Ancient methods of analysis and synthesis created in Greek philosophy transformed during the Renaissance and developed within the field of mathematics, which can also be noticed in the works of Marin Getaldić. His successful restorations and reconstructions of the missing works of Apollonius of Perga are a telling example of a transfer of the lost ancient theories widely echoed in the literature of the early modern period. Getaldić was also concerned with the focal area of the Renaissance mathematics—symbolic algebra. With his new methods Getaldić made a step towards the founding of a new mathematical area, analytic geometry. Equally, in the field of natural philosophy (physics), he successfully combined ancient tradition with the modern problem of method, and in his first work *Promotus Archimedes* used experimental and mathematical method, which is an early example of the modern approach to the study of natural sciences.

Keywords: Marin Getaldić, mathematics, philosophy, analysis, synthesis, symbolic algebra, mathematical restorations, transfer of knowledge, problem of method

Introduction

Marin Getaldić (Marino Ghetaldi, Marinus Ghetaldus) (2 October 1568 - 7 April 1626, Dubrovnik) earned a lifetime reputation for being one of the most

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prominent mathematicians of his day, and his works[1] have been the subject of numerous studies.[2] Getaldić's work mainly owes its assessment to his achievements in the affirmation of symbolic algebra, along with his contributions to the founding of a new area of analytic geometry. Viewed from the perspective of the development of science, as well as that of history, philosophy and especially epistemology, one should take into consideration the ancient roots and models on which Getaldić grounded his work. Getaldić's leaning towards ancient tradition had already become apparent during his early schooldays, where in the humanistic atmosphere of his native Dubrovnik he developed his intellectual interests and showed particular appeal for mathematics and natural sciences. Reflections of various influences resulting from the ancient scientific heritage can be traced throughout Getaldić's opus, from his early works based entirely on ancient mathematical

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[1] Getaldić published six works in the field of mathematics and one in physics: *Nonnullae propositiones de parabola nunc primum inuentae & in lucem editae* (Certain propositions on the parabola discovered here for the first time and brought to light). Rome: Apud Aloysium Zannettum, 1603; *Promotus Archimedes seu de variis corporum generibus gravitate et magnitudine comparatis* (Extended Archimedes or On the comparison of weight and volume of the bodies of various type). Rome: Apud Aloysium Zannettum, 1603; *Suplementum Apollonii Galli seu exsuscitata Apollonii Pergaei Tactionum geometriaepars reliqua* (Supplement to Apollonius Gallus or Revived remaining part of the tactile geometry of Apollonius of Perga). Venice: Apud Vincentium Fiorinam, 1607; *Variorum problematum collectio* (Collection of various problems). Venice: Apud Vincentium Fiorinam, 1607; *Apollonius redivivus seu restituta Apollonii Pergaei Inclinationum geometria* (Apollonius revived or Restored geometry of inclination of Apollonius of Perga). Venice: Apud Bernardum Iutam, 1607; *Apollonius redivivus seu restitutae Apollonii Pergaei De Inclinationibus geometriae, Liber secundus* (Apollonius revived or Restored geometry of inclination of Apollonius of Perga, Book Two). Venice: Apud Baretium Baretium, 1613; *De resolutione et compositione mathematica* (On mathematical analysis and synthesis). Rome: Ex Typographia Reurendae Camerae Apostolicae, 1630.

[2] Countless scholars from the early modern period were familiar with Getaldić's work, which they either mentioned in their own works or kept scientific correspondence with the Ragusan scientist. According to extant sources, among the mentioned scholars were Galileo Galilei, François Vietè, Christopher Clavius, Christopher Grienberger, Michel Coignet, Federico Sanminiati, Alexander Anderson, Michelangelo Ricci, Luca Valerie, Paolo Sarpi, Camillo Gloriosi, Gian Vincenzo Pinelli, Kaspar Schott, William Oughtred, Johan Lawson, Pierre Herigone. The list of their works or scientific correspondence in which Getaldić is either mentioned or his works and results are used in whatever manner, is much too long to be cited here and would go well beyond the frame of this article. Some of the works relevant for the study of the ancient roots of Getaldić's work, his achievements and reception in the European context are cited in this article. Apart from his contemporaries, over the centuries to the present day Getaldić's life and work have attracted the attention of many scientists and science historians: Eugen Gelcich, Oton Kučera, Ivan Kazančić, Antonio Favaro, Juraj Majcen, Florio Banfi, Miroslav Vanino, Mirko Dražen Grmek, Ernest Stipanić, Žarko Dadić, Miho Carineo, Andrija Bonifačić, Nikola Čubranić, Jean Grisard, Pier Daniele Napolitani, Ivica Martinović and others. A part of their research relevant to this topic is cited in this article.

tradition to his mature works, in which ancient mathematical methods are contrasted with those of the modern period with an aim to test their power on heterogeneous material.

By developing different types of mathematical analysis and synthesis, the roots of which we find in ancient philosophy and mathematics, Getaldić has produced some of the best and most influential restorations and reconstructions of the missing ancient mathematical works. By so doing, he participated in the transmission of mathematical theories and knowledge from the important works of the antiquity that were lost during the course of the Middle Ages, and thus enabled their reception in the European scientific community of the seventeenth and eighteenth centuries. From the perspective of intellectual history and the history of knowledge transfer, equally noteworthy is Getaldić's role in the development and affirmation of algebraic analysis and symbolic algebra—mathematical area that attracted most scholarly attention in his day, which eventually led to the founding of analytic geometry, followed by a series of other areas through which modern mathematics continued its progress. Therefore, the aim of this article is to elucidate the ancient models to which Getaldić owes his achievements, since he used them as a departure point and framework for creating his own extensive opus, as well as for the work on the development of new methods. This will provide a clearer and more articulate insight into his original contribution to the development of mathematics and the process of knowledge transfer from these areas.

In the elite scientific circles of Europe

Until the age of twenty Getaldić was educated in his native city,[3] after which he assumed government office in the Republic of Dubrovnik.[4] The year 1595 marked a turning point in his life, for it was then that he travelled to London with Marin Gučetić with the purpose of settling the legacy of the latter's uncle, wealthy Ragusan merchant Nikola Gučetić.[5] Getaldić's daily duties allowed him, in his

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[3] Marijana Borić, *Hrvatski velikan Marin Getaldić*. Osijek: Privlačica, 2020: pp. 9-16.

[4] M. Borić, *Hrvatski velikan Marin Getaldić*: pp. 17-18, 61.

[5] Veselin Kostić, *Kulturne veze između jugoslavenskih zemalja i Engleske do 1700. godine*. Beograd: SANU, 1972: pp. 31, 41-43, 64.

spare time, to devote himself to the study of the latest achievements of contemporary science.[6] Basic details of his travels through Europe and his scientific pursuits Getaldić himself provided in his dedication to Gučetić at the beginning of his work *Variorum problematum colectio* (Collection of various problems), from which we learn that his travels through Europe lasted as many as six years, and included the visits to Rome, London, Antwerpen, Paris and Padua.[7]

Crucial impetus for his scientific pursuits Getaldić owes to his encounters with the most influential scientists of that time, such as Michel Coignet in Antwerpen, François Viète and Alexander Anderson in Paris, Galileo Galilei in Padua, Christopher Clavius and Christopher Grienberger in Rome, and others.[8] Of particular significance is Getaldić's encounter with Viète in Paris in 1600, when in his scientific circle he became thoroughly acquainted with the new method of algebraic analysis and symbolic algebra.[9] He fully adopted and improved these methods, and as the greatest achievements of the Renaissance mathematics he presented them in the scientific circle gathered around Galileo Galilei in Padua, and later in Rome, thanks to which in the elite intellectual circles he became reputed as propagator, interpreter and participant in the discovery of new theories and mathematical knowledge that had the power to incite conceptual and epistemological changes in mathematics. New mathematics played one of the key roles in the foundation of modern science and its methodology. By contributing

[6] M. Borić, *Hrvatski velikan Marin Getaldić*: pp. 22-26.

[7] The translation of the dedication has been published in: Marin Getaldić, *Sabrana djela*, ed. Žarko Dadić. Zagreb: Institut za povijest prirodnih, matematičkih i medicinskih nauka JAZU – Izdavački zavod JAZU, 1972: p. 109.

[8] M. Borić, *Hrvatski velikan Marin Getaldić*: pp. 18-34.

[9] Symbolic algebra and algebraic analysis were first introduced by Viète in the work *In artem analyticem isagoge* (Introduction to analytic art), published in 1591. The reprint of the work is published in Viète's complete works: Francisci Vietae, *Opera mathematica*. *In unum Volumen congesta ac recognita, Opera atque studio Francisci a Achooten*. Leiden: Ex officinâ Bonaventuræ & Abrahami Elzeviriorum1646 (Reprint edition: François Viète, *Opera mathematica*. Hildesheim – New York: G. Olms, 1970). In addition, Viète's symbolic algebra and algebraic analysis are presented and analysed in: Jacob Klein, *Greek Mathematical Thought and the Origin of Algebra*. Cambridge, Massachusetts, London: Massachusetts Institute of Technology, 1966: pp. 150-185, 315-353 and H. L. L. Busard, »Viète, François«. *Dictionary of Scientific Biography*, vol. 14. New York: Charles Scribner's Sons, 1981: pp. 18-25, Žarko Dadić, *Povijest znanosti i prirodne filozofije (s osobitim osvrtom na egzaktne znanosti). Knjiga III, Rano novo doba*. Zagreb: Izvori, 2017: pp. 56-67.

actively to the creation of new knowledge and its transmission in the scientific community of the contemporary Europe, as a young man Getaldić had already earned a reputation of an excellent mathematician,[10] enjoying Galilei's close friendship, with whom he exchanged published works and letters to his last day.

Upon his return to Dubrovnik in 1601, Getaldić continued with the experimental work he had started during his travels through Europe. In 1603, in Rome, he published his first works: *Nonnullae propositiones de parabola* (Certain propositions on the parabola), in which, prompted by optical experiments, he conducted mathematical research of the properties of the parabola, along with *Promotus Archimedes seu de variis corporum generibus gravitate et magnitudine comparatis* (Extended Archimedes or On the comparison of weight and volume of the bodies of various type), a physics treatise on the relative ratios of weights inspired by the methodology of Archimedes and Euclid, and arranged systematically into theorems, problems and tables with the results of the measuring carried out with his own hydrostatic scale. He regarded mathematics as a science which most precisely described the real world and believed in the application of experiment as a practical aspect of science, which later requires to be mathematically verified and proven.[11]

Getaldić's mathematical restoration of the ancient works

All Getaldić's works are in one way or another connected with the ancient tradition. Viewed within his entire opus, three works composed as mathematical restorations form a special whole.[12] With this important part of his work, Getaldić achieved a significant transfer of the lost and in the Renaissance unknown ancient mathematical theories and knowledge. Moreover, in his mathematical restorations of the complex and incomplete fragments Getaldić came forward with the first formulations of a couple of lost ancient problems and theorems of relevance for the further development of mathematics. Hence, his restorations widely echoed in the works of the seventeenth and eighteenth century, being variously used by

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[10] M. Borić, *Hrvatski velikan Marin Getaldić*: pp. 58-61, 86-88.

[11] For more details on Getaldić's life, see: M. Borić, *Hrvatski velikan Marin Getaldić*: pp. 3-65.

[12] Mathematical restoration is far more than mere reconstruction and transfer of the lost ancient knowledge exposed to random approach and methods. It is a procedure which includes reinvention of the integral mathematical text of the missing work and the theories expounded in it by using an identical methodological approach that was used in the ancient original.

many mathematicians who either integrated them in full or partially into their own theories and works.[13]

Educated on ancient mathematical tradition, in his restorations Getaldić used ancient Greek mathematical models, geometric analysis and synthesis. He was primarily concerned with the works of Archimedes, Euclid and Apollonius of

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[13] The work *Apollonius redivivus seu restituta Apollonii Pergaei Inclinationum geometria* was widely used in contemporary mathematical literature, and later. An important work that brings Getaldić's restorations of Apollonius' work *On inclinations* is *Cursus mathematicus* (Paris, 1644), written by the French mathematician Pierre Herigone. In this parallel Latin-French edition, he quotes Getaldić's formulations of Apollonius' problems, while the rest of the contents containing solutions, constructions and proofs of problems is presented with the use of Herigone's specific symbolism. This work, along with Viète's restorations, also includes Getaldić's restorations of Apollonius' treatise *Tangencies,* published under the title *Apollonii Pergaei tactionum geometria* (volume I, pp. 915-934), but attributed to Viète, together with Getaldić's restorations of Apollonius' work *On inclinations* (pp. 905-914). Later in the eighteenth century, two English scientists explored Getaldić's restoration of Apollonius' treatise *On inclinations.* Getaldić's restoration also found reception in England during his lifetime, as evidenced by Thomas Harriot (1560-1621), in his manuscript kept in the British Museum (Add. MSS 6784. f. 229). In his investigation of Apollonius' work *On inclinations,* Harriot refers to Getaldić's restoration (see M. Getaldić, *Sabrana djela*: 163). While analysing Anderson's restorations, mathematician Samuel Horsly, author of the work *Apolonii Pergaei inclinationum libri duo* (Oxford, 1770), mentioned Getaldić in his text on two occasions. First, when he asserts that Getaldić, before the very construction in the restorations, applied algebraic analysis which is more useful in finding a constructive solution (Book II, p. 103). In his second reference to Getaldić, he explains the circumstances in which his restoration of *Revived Apollonius* has remained unfinished, and presents and analyses Andersen's *Supplementum Apollonii redivivi* (Book II, p. 113). The fact that he does not mention Getaldić's restoration *Revived Apollonius, Book Two*, suggests that he was probably unfamiliar with that work. Horsly too was engaged in the restorations of Apollonius' works. However, his approach departed significantly from those of his predecessors. By that time, algebraic method had already established itself and had numerous followers. In his solutions of problems Horsly leans on Getaldić's formulations, yet still resorts to Viète's algebraic methods, using algebraic analysis and synthesis. Reuben Burrow, English mathematician who also worked on the restorations of Apollonius' works, mentions Getaldić in his work as well. In 1779 in London he published a book on this topic entitled *A restitution of the geometrical treatise of Apollonius Pergaeus on inclinations*. In the preface he writes that no one had investigated Apollonius' problems so thoroughly as Getaldić and Horsley. Unlike Horsley, who mainly used Getaldić's formulations with some minor changes, Burrow, on the basis of his own research of Pappus' work *Mathematicae collectiones*, independently formulated Apollonius' problems. His formulations differ from those of Getaldić to such an extent that they overlap in only one problem. It concerns Getaldić's Problem II, and Burrow's Problem I. Getaldić and Burrow are the only authors who satisfied the criteria of complete restoration, considering that restoration has to come closer to the original in terms of both contents and methodology. In this respect, all authors who chose the method of reconstruction according to the criterium of a more efficient path to the solution, and if the method did not belong to the tradition of ancient mathematics and as such was not consistently applied, failed to accomplish the restoration in the true sense.

Perga.[14] Only a few of their works survived until the Renaissance in Greek, and some in Latin translation from the Arabic translation of the original, whereas some works have gone completely missing. It was Viète who prompted Getaldić to explore and restore the works of Apollonius. Considering that the original works were rare or completely lost, some mathematicians worked on the reconstruction of the missing works by using extant fragments and citations in the works of younger ancient mathematicians.[15] The contents of Apollonius' works *Tangencies* (περι επαφων, *De tactionibus*) and *On inclinations* (περι νευσεων, *De inclinationibus*) was described in the preface of Book Seven of Pappus' *Mathematicae collectiones* (3rd c. A.D.), and was therefore used as a source for restoration. Mentioned in it were the problems that were being solved and discussed, and from these records it is evident that Apollonius wrote the mentioned works in two volumes.[16]

The treatise *Tangencies* was restored by Viète in the work entitled *Apollonius Gallus seu exsuscitata Apollonii Pergaei* περιεπαφων *geometria* (Paris, 1600.) That same year Getaldić met Viète in Paris and became extensively acquainted with his work. In his treatise Viète managed to reconstruct ten of Apollonius' problems from the mentioned work. This was followed by Getaldić's independent analysis of the preface of Pappus' Book Seven of *Mathematicae collectiones*, whereupon he observed and reconstructed another six problems from Apollonius' work *Tangencies*, in addition to what Viète had previously done.[17] Getaldić further added his own solution of the eighth theorem in Viète's work, as he observed certain flaws in Viète's solution, and finally published his restoration

[14] Apollonius of Perga (3rd c. B.C.) was one of the greatest mathematicians of the antiquity. He studied mathematics in Alexandria, under Euclid's students. He invented the theory of conic sections, expounded in eight books, which is regarded as his most significant work. First four books have survived in the original, books five, six and seven are extant in Arabic translation, while the eighth book is lost. In addition, Apollonius is the author of many works in mathematics and astronomy, which are also missing.

[15] Numerous mathematicians of the Renaissance, and of later periods, tried to reconstruct the works of Apollonius, among whom are the great names of Willebrord Snellius, in full Snell van Royen (1591-1626), Pierre de Fermat (1601-1655), Edmond Halley (1656-1724) and others.

[16] M. Getaldić, *Sabrana djela*: pp. 179, 201-204.

[17] In the preface of the work *Suplementum Apollonii Galli* (Supplement to Apollonius Gallus), Getaldić writes (Ghetaldi, *Opera omnia*, 1968: 177(5)): "Therefore, Apollonius Gallus did not revive the entire tactile geometry of Apollonius of Perga, because he omitted six problems that belong to that geometry. But we shall complete it, and hence Apollonius Gallus will not without Apollonius Illyricus revive Apollonius of Perga, who rested by the injustice of time obscured or by the hand of the barbarians buried."

under the title *Apollonius redivivus seu restituta Apollonii Pergaei Inclinationum geometria*, (Venetiis, Apud Bernardum Iutam, 1607).[18]

Considering that for the purpose of his first restoration Getaldić examined the preface of Book Seven of Pappus' *Mathematicae collectiones* containing also the description of Apollonius' second work *On inclinations,* this fact inspired him towards the restoration of that work as well, which he published in two volumes under the titles *Suplementum Apollonii Galli seu exsuscitata Apollonii Pergaei Tactionum geometriae pars reliqua* (Venetiis, Apud Vincentium Fiorinam, 1607) and *Apollonius redivivus seu restitutae Apollonii Pergaei De Inclinationibus geometriae, Liber secundus* (Venetiis, Apud Baretium Baretium, 1613). Getaldić was the first mathematician who formulated Apollonius' problems on inclinations from very complex and distorted Pappus' notations. Therefore, Getaldić's formulations served as basis for later restorations of that work. In Pappus' text Getaldić recognised five problems from Apollonius' work *On inclinations*. As these problems represent a thematic whole, it was probably his intention to publish them in a separate work. However, intensively preoccupied with the duties he performed for the Dubrovnik Republic,[19] in the first book he printed the first four problems with solutions, while the last, fifth problem he merely formulated, although he already had it mainly solved by then. Getaldić's restoration of the work *On inclinations* encouraged the mathematician Alexander Anderson to examine the fifth problem himself, which he did on the basis of Getaldić's formulation, and published in a treatise *Supplementum Apolloni redivivi* (Paris, 1612). However, Anderson applied the method of analysis which Apollonius did not use in the original work. The fifth problem Getaldić restores in full by using the method of geometric synthesis modelled on Apollonius, and published it in a separate work in 1613. Fifth problem is far more complex than the previous

[18] This work Getaldić started and almost completed during his travels through Europe, because in the letter to mathematician Christopher Grienberger, dated 4 September 1604 and addressed from Dubrovnik to Rome, he writes that he prepared it together with his works *Apollonius redivivus* and *Variorum problematum collectio.* Getaldić's letter has been published in: Miroslav Vanino, »Marin Getaldić i isusovci«. *Vrela i prinosi* 12 (1941): pp. 69-86.

[19] Getaldić was elected as tribute ambassador to Constantinople in 1606, where he was to deliver the annual tribute to the sultan. In Constantinople he remained for a year, executing diplomatic duties for the Dubrovnik Republic. During his mission, he measured the city's latitude, and also searched for the Arabic translation of Apollonius' work. Although it was believed to have survived in Constantinople, Getaldić's search gave no result. M. Borić, *Hrvatski velikan Marin Getaldić*: pp. 50-55.

ones, and thus allows a multitude of different cases and mutual positions of the observed geometrical objects.

Getaldić's mathematical restorations are very important in terms of methodology. Similar to his early works, in his restorations he used only geometrical methods adopted from the ancient Greek mathematical tradition. Ancient mathematicians of the earlier period tended to solve geometric problems by means of construction, starting from given quantities and obtained the sought ones, after which that construction was proven. That procedure is synthetic, the construction itself being known as synthesis. Apollonius' tractates Getaldić restored by using no other but the synthetic method, that is, construction. All formulations, proofs and solutions he wrote entirely in keeping with ancient mathematics, so that his reconstructions are not only mere restoration of the contents of the missing works as is the case with some other authors, but are genuine restorations, because Getaldić, methodologically conscious and consistent in his objective, reconstructs the mathematical material in such a manner so as to faithfully follow in the methodological footsteps of Apollonius and his geometry.

Development of mathematics in the spirit of ancient tradition

In order to provide a clearer interpretation of diverse influences and scientific circumstances in which Getaldić created his opus and developed the methods of analysis and synthesis, one should draw attention to the key development phases of mathematics and its interaction with philosophy beginning with the antiquity, in which we find the roots of the first methods of analysis and synthesis. In antiquity the mentioned methods developed in the area of geometry. The eleventh and twelfth centuries saw the first Latin translations of the lost ancient and Arabic original mathematical works. The Middle Ages were followed by the centuries marked by accumulation of various mathematical knowledge, which by the end of the Renaissance culminated in the conceptual change of mathematics, after which the methods of analysis and synthesis transformed and developed in the area of algebra. This great conceptual change came with the emergence of algebraic analysis and Viète's symbolic algebra, which inspired Getaldić to embark upon his major work *De resolutione et compositione mathematica* (On mathematical analysis and synthesis) (Rome, 1630), the first comprehensive handbook of new algebraic analysis. Getaldić's work on mathematical methods should be viewed within the context of Renaissance thought and the problem of method as a characteristic philosophical problem of early modern thought. He wrote his mathematical works in such a way so as to fully emphasise the importance of the methodological approach to the material. Like many of his contemporaries, he used ancient methods of analysis and synthesis as starting point, while in the mature stage of his work Getaldić dedicated his major work to experimenting and comparison of the achievement of different methods, and through it he promoted and developed the new symbolic algebra and the relevant algebraic analysis. With this work he made a considerable step towards the founding of analytic geometry, an area which was an important link in the further development of mathematics, René Descartes (1596-1650) being generally considered its founder.

The mathematics that Getaldić studied at school was, since the antiquity, strictly divided into two areas of arithmetic and geometry, mainly under the influence of Aristotle,[20] and that was how Getaldić also approached it in his early years. Ancient Greeks did not deal with algebra as a separate mathematical area, and algebraic problems were indirectly incorporated into geometry as geometric problems, which, besides geometrically, by their nature may have been interpreted in algebraic form as well. That is why Getaldić, who in the early phase of his work was entirely under the influence of ancient mathematics, notably Euclid's *Elements*, (4th c. B.C.), like virtually all mathematicians until the beginning of the seventeenth century, examined algebraic problems within geometry and solved them with the help of geometric methods and in geometric formulation. While studying mathematics in 1597 with Michael Coignet in Antwerpen, Getaldić mastered deductive method and axiomatics according to the methodology of Euclid's *Elements,* the first work in the history of mathematics which entirely relies and is built on Aristotle's views on axioms, postulates, mathematical notions and definitions, which facilitated the construction of a rigorous axiomatic deductive

[20] Ancient Greek arithmetic and geometry dealt with objects of completely different properties. Arithmetic dealt with discrete quantities, i.e., whole numbers, which potentially can be infinitely continued, but according to ancient mathematical tradition cannot be divided infinitely but up to the number one only. Ancient Greek geometry dealt with continuous geometric quantities (lines, areas and bodies), which had a reversed property in relation to arithmetic quantities and could be divided infinitely. Clear-cut separation of arithmetic from geometry was probably motivated by the discovery of incommensurability and the difficulties arising from the solution of three major problems of the mathematics of ancient Greece: the quadrature of the circle, doubling the cube and angle trisection, with which the conceptions of Pythagorean mathematics have difficulty in dealing. It was believed that all the mentioned problems could be solved by an elementary compass-and-straight edge construction. Despite the fact that many great mathematicians tried to solve these problems for almost twenty centuries after their discovery, none of them has succeeded as all the three problems also included a hidden irrationality, and so did the problem of incommensurability.

mathematical system.[21] Such a high level of understanding and apprehension of the knowledge and methodics of ancient mathematics adopted by Getaldić, was a departure point upon which the Renaissance science sought the foundations of a new method of certain knowledge and a starting point for the foundation of new science. Fundamental works of ancient mathematical and philosophical tradition were translated throughout the Renaissance, which resulted in a new role and position of mathematics at the turn of the sixteenth century, when Getaldić was writing his works. Science tended to act as a torchbearer for human spirit, its discoveries were to profoundly reshape human life, which resulted in an increasing interest in the development of mathematics not only within science, but in its broader application as well. The growing importance of mathematics was at the same time based on its main properties, reliability and certainty. It represented a major revitalisation of mathematical values and knowledge since the antiquity, as they were fairly neglected in medieval Europe.[22] Yet, the continuity of knowledge maintained during the course of the medieval period proved sufficient as the basis for significant changes that were to take place from the twelfth century onwards under the influence of the fruitful merging of Western European and Oriental mathematics, primarily with that of the Arabic mathematical tradition.[23]

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[21] Žarko Dadić, *Povijest ideja i metoda u matematici i fizici*. Zagreb: Školska knjiga, 1992: pp. 38-45.

[22] An illustrative example is the most influential mathematician of the Middle Ages, Boethius Severinus (480-524), who operated on the territory of present-day Italy, and was the paragon to all mathematical researchers until the beginning of the twelfth century. Basing his work on ancient mathematical tradition, he used various sources: Nicomachus' arithmetic, Euclid's *Elements*, Ptolemy's *Almagest* and other ancient works. However, in his work Boethius did not attain the level of the works he consulted. Thus, with respect to Euclid's geometry, he submitted only the formulations of theorems without the proof procedure, which is the essence of mathematical methodology and foundation of modern mathematics and natural sciences in general. Nevertheless, his work is generally regarded as valuable, knowing that he succeeded in maintaining certain continuity through the Middle Ages by transmitting and disseminating mathematical knowledge of the antiquity. For more details on this topic, see: Ž. Dadić, *Razvoj ideja i metoda u matematici i fizici*: p. 59.

[23] The contact of Western Europe with Arabic philosophical and scientific works was essential for the progress of mathematics and physics. Until then, mathematics was studied on Boethius' works, and the original works of Euclid were not used. The first Latin translation of Euclid's *Elements* from Arabic was executed by Adelard of Bath in 1130, while the first revision of Adelard's translation of Euclid's *Elements* was accomplished ten years later by Herman of Dalmatia. This was followed by the translations from the Arabic of many mathematical works written by the Arabic authors. They were based on the Arabic type of mathematics which emerged as a combination of ancient Greek and Indian conceptions of mathematics, which had a profound influence on the Renaissance scientists and their understanding of mathematics. Indian mathematics, unlike Greek, maintained the empirical character, yet developed arithmetical and calculation aspects within calculatory science with specific mathematical procedures. Priority was given to the numerical aspect of the

Arabic mathematics that reached Europe in that period owed its character to the fact that it emerged from a combination of the rigorous ancient Greek methods with the original Oriental mathematics, that of India in particular. Despite the fact that the Arabs adopted certain elements which, viewed generally, did not lead to further progress,[24] by combining Indian arithmetisation of mathematics with ancient Greek rigour, they produced new results and founded new areas of mathematics, which in the early modern period would have crucial impact on the development of mathematics in Europe.

The problem of method

Latin translations of the original Arabic works, as well as the missing ancient mathematical, natural philosophical and philosophical works which survived in translations and interpretations of the Arabic scholars, led gradually between the twelfth and the sixteenth century to an accumulation of a wide-range of new knowledge and changes in the understanding of mathematics[25] and it was upon these foundations

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problem, to which they also deduced geometric objects and quantities, and thus, unburdened by rigorous formal conditions weaved into ancient mathematics, within their understanding obtained useful results and advanced arithmetic and algebra. They developed the positional numeral system, dealt with fractions, negative numbers and accepted the existence of zero, used abbreviations. By merging these two entirely different sources, the Arabs created a successful combination and gave a new impetus to the development of mathematics. They adopted Indian calculation, Indian positional system, calculation of sine and cosine in the numerical sense. From Greeks, the Arabs adopted rigorous deductive system in geometry, in algebra rigorous geometric proof. For more details on this topic, see: Ž. Dadić, *Razvoj ideja i metoda u matematici i fizici*: pp. 55, 56, 63.

[24] Arabic mathematics made a shift backwards to Greek rhetorical notation and Diophantus' avoidance of negative numbers.

[25] A significant impact of these translations is best witnessed in the extended list of the quadrivium of the Latin Christian Europe, and the completion of a final series involving mathematical arts which, along with geometry, astronomy and the theory of music, also included algebra, algorism (*algorismus* – term denoting calculation with the use of Indian numbers) and commercial calculation. The list of works testifies to the important role played by mathematics, especially if we do not confine to strictly mathematical works but include those from other areas which amply rely on mathematics. There emerged a broader interest for theoretical mathematics and astronomy on high level, which is evidently the result of the overall intellectual pursuits of the Renaissance. Beneath that high level, spurred by the social development and new lifestyles, grew a never broader interest for less demanding mathematical subjects. The influence of Arabic mathematics enriched Western European mathematics continuously from the twelfth through the sixteenth century. Unlike ancient scientific tradition, besides whole numbers, fractions were also used, as well as approximative values, surfaces are calculated, and the volumes of geometric bodies, new mathematical procedures are advanced, and different knowledge is accumulated, though the separation of mathematical operations from their object had not yet been obtained nor a higher level of generality.

that Getaldić built his scientific work, developing it between the traditional approach based on ancient heritage on the one hand, and the modern approach to the study of natural sciences on the other. The dawn of the Renaissance witnessed the first signs of the great conceptual change which in mathematics emerged at the turn of the sixteenth century, for which Getaldić should also be credited with his work on the advancement of mathematical methods, and especially with the promotion of the method of algebraic analysis and development of a new mathematical area—symbolic algebra, with which general quantities, *species,*[26] are introduced in mathematics, which can equally be applied to numbers and geometric objects, and introduced a symbolic language instead of the formerly used rhetorical notation.

Getaldić operated at the close of a long Renaissance period of increasing awareness of the method as fundamental philosophical characteristic, and in the beginnings of methodic construction of modern natural science. In this Renaissance actualisation of the problem of method, when there are radical changes regarding methodology, incited by the overall Renaissance will for change and new understanding of reality, in his scientific work Getaldić mainly focuses on the affirmation and development of different mathematical methods and their applications in mathematics, research and the understanding of nature. With his work in mathematics and physics, Getaldić joined a succession of scholars who contributed to the foundation of modern science (Galilei, Kepler, Descartes and others). His work on the development of method, though carried out within different mathematical disciplines, in terms of its significance surpasses the narrow field of mathematics and may be considered within a broader context of the Renaissance problem of method. Renaissance focus on the problem of method emerges from the enquiry into the best, most appropriate and most certain path in finding the truth, and is ultimately motivated by the desire for a better and more efficient understanding and mastering of nature. Infused by the spirit of the epoch, Getaldić too aimed to further develop the mathematical knowledge

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[26] The notion was used by Viète, adopted from the Latin translation of Diophantus' *Arithmetic,* published in Basel in 1575. Plato's philosophy had great influence on the Renaissance science, including that of Viète's interpretation of mathematics, notably in the understanding of the notion of number which he designated with the notion *eidos*, which in Plato's original philosophy stands for *idea*. Diophantus in his *Arithmetic* surpasses Greek understanding of whole numbers, and in his work uses fractions. For the further development of generality, it was necessary to introduce formal language in Diophantus' logic, that is, symbolic numerals instead of definite values. It was not until 1585 that Simon Stevin (1548 – 1620) introduced a new notion of general number, the transformation of which was completed by Viète in 1591 with the introduction of general mathematical symbolism. For a more extensive account, see: J. Klein, *Greek Mathematical Thought and the Origin of Algebra*: pp. 132-149.

of the ancient tradition he had adopted at school, and to transform it into new instruments of knowledge. Prompted by the growing methodic awareness, on the basis of mathematical study of heterogeneous material Getaldić worked on the development of new methods for dealing with theoretical and practical problems. The world of experience as a subject of scientific research, supported by mathematical verification and proof, was analysed in Getaldić's early and only work in physics *Promotus Archimedes*, in an entirely modern approach with the accompanying mathematical methodology, which rejects the former medieval tradition. By using this kind of approach in his earliest work, Getaldić heralded modern approach in the study of natural sciences, which Galileo Galilei, founder of modern physics, postulated some twenty years later. With his approach, in contrast to the methodology inherited from the medieval scholastic system, Getaldić joined the company of the pioneers and exponents of new Renaissance science and philosophy who, treading through the topics of methodology and cognitive theory, sought paths to the problems of reality, creating new views and theories. Mathematics and empiricism, upon which Getaldić bases his research, play a key role in the process of the foundation of modern science. They exist and converge as Renaissance philosophical thoughts in the development of the natural scientific method. Experience becomes the origin of knowledge, its first step which later has to be proven by applying appropriate mathematical procedure and methods. Experience is purposefully placed into a specific function, in interaction with mathematical interpretation it gradually transforms into method, and is given a scientific interpretation.

The methods of analysis and synthesis

The period in which Getaldić wrote his works, turn of the sixteenth century, witnessed an increasing search for new methods of certain knowledge. Apart from the focus on the problem of method, much attention was devoted to finding a path that would lead to individual methodic procedures, that is, ultimately to a single method which would be universal for all sciences. The model for the new method tended to be sought in mathematics, which became an ideal of proof-based science. Within the wide field of mathematics, Euclidean geometry was a much sought-after methodological model. This tendency continued throughout the seventeenth century and later, which is why Getaldić too in a number of his works leaned on Euclid's methodology as an outstanding example of ancient tradition, which he followed not only in terms of approach and form, but also in the choice of material, topics and mathematical problems. Under the influence of Aristotle, in his understanding of mathematics and in many issues of mathematical philosophy Euclid used the methods of geometric synthesis (construction) and analysis in solving geometric problems. In his early works, he did not make notations of the analysis, yet in his later works his notations were modelled after Aristotle.[27]

The concepts of analysis and synthesis have their origins in ancient philosophy.[28] They were applied in ancient Greek mathematics in which as geometric analysis and synthesis they developed in the field of geometry.[29] They were used in solving geometric problems, so that a geometric construction which starts from given quantities and obtains sought quantities is understood as synthesis. A synthetic method of this kind in more complex cases may have been more easily found if the relationship between the required and given quantities had been previously considered. The procedure of problem analysis and the finding of relations between the given and required quantities gave a conclusion which was then used in synthesis, i.e., in the synthetic procedure of geometric construction. We distinguish two types of analyses—theoretical and problematic. The final goal of theoretic analysis is to discover a mathematical conclusion which is formulated in the form of a postulate or theorem, while problematic analysis is

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[27] For a more detailed discussion, see Ž. Dadić, *Povijest ideja i metoda u matematici i fizici*: pp. 46-47, 63-64. In ancient Greece, the problems which by nature led to geometric algebra were solved within geometry and geometric methods. Gradual changes emerged when one of the most important mathematicians and physicists of the late antiquity, Hero (1st c.), introduced into geometric considerations numerical aspect, and solved geometric problems numerically. By doing so, Hero made a step towards algebra, while Diophantus (3rd c.) also conducted methodological transformation of Hero's numerical approach. Diophantus examined equations and solved numerical problems which led towards algebra, and thus transformed the rhetorical notation of ancient Greek mathematics by introducing abbreviations for mathematical notions, while the sentences themselves he reduced to a shorter form, and thus into ancient Greek rhetorical tradition introduced syncopated algebra. However, algebra as a separate mathematical discipline was developed later by the Arabic mathematicians. Although they solved quadratic equations in a most general way, and also introduced classification of the equations of the first and second degree and general procedures of problem solution (*al-jabr, al-mukabala*), they did not use symbolic representation of equations, but rhetorical notation.

[28] Viète claimed that Plato was the first in mathematics who discovered the path with which to find the truth, later termed as analysis by Theon. Viète probably came to this conclusion while studying Plato's dialectic which always begins with an opinion in which one assumes that the sought is already known, and then the opinion is refuted as wrong, upon which the truth is concluded. J. Klein, *Greek Mathematical Thought and the Origin of Algebra*: p. 260.

[29] In Book Seven of his major work, most frequently cited as *Mathematical collection*, Pappus describes analysis as the method in which what is sought is considered as known, and starting from that across consequences, that is, a series of conclusions, what is obtained is confirmed as the result of synthesis. Pappus describes analysis as a "reversed solution", i.e., steps to be taken in a reversed order so as to find valid proof. A similar description of analysis has been submitted by his younger contemporary, Theon of Alexandria.

concerned with finding a construction method in certain problems, and defines the path of construction proof. Analysis is carried out and flows in a direction contrary to synthesis. It starts with a presupposition that the required quantities are known, so that through a concluding sequence one could come to a conclusion on the existing relations between the given and required magnitudes. The concept of geometric analysis was the first concept of analysis in mathematics in general, from which all other types of mathematical analysis later developed.[30]

Between ancient tradition and modern methodology

Getaldić's early works and his mathematical restorations were grounded on ancient methods of geometric synthesis and analysis. His early works place him in the corpus of the Renaissance scholars whose admiration for ancient heritage led them towards reinvention of the greatest achievements of ancient Greek science. During the first phase of his scientific work, Getaldić was maturing into a prolific mathematician, preparing to embark upon his major work, *De resolutione et compositione mathematica*, which took him almost twenty years to complete. In it he tested the power and possibilities of new algebraic method in relation to geometric analysis and synthesis from the ancient tradition. A part of the problems that we encounter in his early works Getaldić reconsiders in this work, yet here by using a completely different methodological approach, where he solves these problems within algebraic method.[31] The focal point of his research Getaldić placed on the development and affirmation of different mathematical methods. This is evidenced by the fact that he started to write the two of his most important works *Variorum problematum collectio* and *De resolutione et compositione mathematica*, at the same time with different methodological conceptions, and solved the same mathematical problems in them, demonstrating on the path to solution the diversity of approach and merits of the mathematical methods under consideration.[32]

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[30] With the introduction of general quantities, from the method of geometric analysis developed algebraic analysis within general algebra, followed by analytic observation of curves and mathematical analysis in the broadest sense of the word.

[31] Certain problems from the work *Variorum problematum collectio* and his restorations of the lost treatises *On inclinations* and *Tangencies* by Apollonius of Perga are being repeated.

[32] In the collection of various mathematical problems *Variorum problematum collectio* (Venice 1607), by using different geometric methods he solves the problems of four authors: famous astronomer and mathematician Johannes Müller Regiomontanus (15th c.), along with his contemporaries, influential mathematicians, Roman Jesuits Christopher Clavius and Christopher Grienberg, with whom he kept regular correspondence, and also a Ragusan, Jakov Restić.

Getaldić is fully aware of the far-reaching consequences of the application of general quantities in mathematics and science in general. Apparently, the introduction of general quantities freed mathematical results from their previous form. The power of new method, awaiting to be developed and affirmed, in which Getaldić himself played an important role, led to a ground-breaking change in the development of mathematics. Even if Getaldić had not been engaged in the development of the new area, his work in the field of mathematics within the framework of ancient tradition would per se have been of such high level and rich in original solutions, in both ancient Greek mathematical problems and those in the application to the problems in physics (parabolic mirrors[33] and determination of specific weights[34]), that they alone would have placed him among the prominent mathematicians of the late Renaissance. However, beyond any doubt his greatest contribution Getaldić achieved by shifting away from ancient tradition and its purely geometric understanding of problem, within which, by using geometric methods (analysis and synthesis), he composed his early works. His excellent knowledge of ancient mathematical tradition and geometric analysis and synthesis provided him with a useful insight into the attainments of ancient methods, which proved essential in his pioneering assessment of Viète's *logistica speciosa*.[35] Getaldić fully adopted it, and modelling after Viète, used general quantities, yet in his development of the algebraic method it was merely his starting point, because in his procedure, more clearly than Viète, Getaldić separated the methods of analysis from those of synthesis.

The conceptual change witnessed by mathematics at the time was partly based on the transformations initiated by the method of notation of mathematical texts after the emergence of Latin translations of the original Arabic mathematical works in the twelfth and thirteenth century. It was then that Arabic numerals gradually came into use, with the help of which in certain texts, rhetorical by nature, specific schemes were noted down which enabled a simpler presentation of mathematical expressions and operations. Through the fourteenth, fifteenth and sixteenth centuries mathematical knowledge gradually advanced, some new mathematical symbols were invented, which all together preceded the founding

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[33] See: Juraj Majcen, »Spis Marina Getaldića Dubrovčanina o paraboli i paraboličnim zrcalima«. *Rad JAZU* 223 (1920): pp. 1-43.

[34] Pier Daniele Napolitani, »La geometrizzazione della realtà fisica: il peso specifico in Ghetaldi e in Galileo«. *Bolletino di storia delle scienze matematiche* 8/2 (1988): pp. 139-236.

[35] Starting from Diophantus' procedure, Viète introduces the notion of numerical calculation with general numbers, which he denotes as *logistica speciosa*, in distinction to *logistica numerosa* (J. Klein, *Greek Mathematical Thought and the Origin of Algebra*: p. 165).

of symbolic algebra and was the basis for great changes in the mathematical understanding of the late sixteenth century. What needs to be emphasised for the sixteenth-century mathematics is the fact that prior to the invention of symbolic algebra, in spite of the rapid and powerful discoveries of new algebraic knowledge (new rules and examples of correct procedure, new abbreviations which facilitated mathematical expression), it still remained concrete, considering that the mathematicians of that time confined their thought to a specific problem and concrete object. Abbreviations of syncopated algebra were being advanced and standardised, although algebraic operations were still not abstracted and separated from their concrete objects to which they applied. It was generally considered that the operations and object constituted an indivisible whole, considerations did not go beyond the frame of a specific (concrete) problem, and therefore this period did not yet see the emergence of the notion of formula.[36] In this respect, symbolic algebra introduced a crucial reversal. Greek geometric analysis and synthesis were used in its construction, yet in such a way that by introducing the general quantities referred to as *species*, it was transformed and conducted algebraically within general algebra. The introduced general quantities within the frame of symbolic algebra may just as equally be applied to numbers and geometric objects.[37] Therefore, this new algebra, which operates with general quantities instead of numbers or geometric objects, is pure and general algebra, different from those known until then.[38]

Having understood the meaning and significance of the generality of method introduced by Viète, Getaldić devised his major work as its first, comprehensive and extensive handbook. It is a methodic collection of problems and theorems solved by using new algebraic method on heterogeneous material. The analysis of the work *De resolutione et compositione mathematica* shows that its main contribution lies in the development of the algebraic method itself, although the work contains numerous new, original mathematical results, clearly evidenced

[36] Ž. Dadić, *Povijest ideja i metoda u matematici i fizici*: pp. 77, 88-90.

[37] The first general number was introduced in mathematical practice by Jordanus de Nemore in the thirteenth century, using letter symbol to represent any number, yet his general number related only to numbers and not geometric objects. Therefore, these general numbers could not be used to calculate geometric quantities, such as length, area or volume. In this respect, s*pecies* represent a higher level of generality, because they could equally be applied to numbers and geometric objects. They operate with the shape of things (e.g. letters of the alphabet). That is why their introduction had great influence on the interpretation of the until-then known mathematical results and facilitated the progress of mathematics.

[38] In ancient Greece, algebra developed within the area of geometric problems, while in the mathematics of the Arabs and Diophantus algebra had a numerical character. The Greeks were mainly concerned with geometry and arithmetic, and with algebra only indirectly through geometric problems of such character that they could be interpreted algebraically.

in the examples of problems repeated from older works of the ancient tradition, where it examines geometric problems from the earlier works, along with the theorems of Euclid, Apollonius of Perga, Viète, Regiomontanus and others. Calculation with the use of general quantities led Getaldić towards a new interpretation of the until-then known mathematical results. Getaldić reinterprets the results of geometric problems and conducts algebraic analysis within the area of general algebra. By affirming the new algebraic method on heterogeneous material, Getaldić at the same time proves himself as a consistent disseminator and interpreter of the traditional approach. However, the key difference in relation to the former mathematical approach, based mainly on the admiration of ancient heritage and attempts aimed at the reinvention of the specific notions and methods of the antiquity, emerges in the new understanding of mathematical object, that is, the conception of general number, whose introduction led towards a profound reform not only of algebra but also of mathematics on the whole.

The comparison of methods

Getaldić's works are grounded on the works of ancient Greek mathematicians, among whom Euclid, Pappus and Diophantus may be singled out. He was also under the influence of Eudoxus' theory of proportions, as well as Archimede's application of logistic methodology, i.e., arithmetic interpretation of geometry. By drawing on ancient tradition and stimulated by algebraic method, Getaldić applied an integration of different tendencies of ancient Greek mathematics, rigorous geometric methods and *logistica*, which implied a routine of simple mathematical calculation and allowed approximative approach.[39] By following closely Viète's *logistica speciosa,* Getaldić introduces general quantities in the considerations of geometric problems, and thus in his last work, *De resolutione et compositione mathematica,* through algebraic method also obtains a change of concept of mathematical object. In order to show a clear difference between the methods, a simple geometric problem will be demonstrated, and for the sake

[39] In addition to the application of geometric methods of ancient Greek mathematics, the beginning of the modern era saw the emergence of yet another tendency— increasing introduction of the numerical aspect and areas of the so-called *logistica* to theoretical mathematics. *Logistica* encompassed practical calculation, whose scientific status was not acknowledged. In dealing with geometric problems which can be interpreted algebraically, Hero also added areas and lengths, which was unacceptable in ancient Greek tradition. By doing so, in contrast to the recent mathematical tradition, he gave priority to the numerical aspect of the problem in relation to its geometric origin. For more on this issue, see Ž. Dadić *Povijest ideja i metoda u matematici i fizici* (1992): pp. 55-56.

of comparison both methods will be applied, geometric and algebraic. Selected as example is the first problem from Book One of Getaldić's work *De resolutione et compositione mathematica*. The problem is relatively simple in relation to far more complex mathematical problems addressed in his work. After the initial formulation of the problem, Getaldić first conducts algebraic analysis, from which he then derives the porism,[40] later used in synthesis.[41] Getaldić formulates Problem I as follows:

Problem I

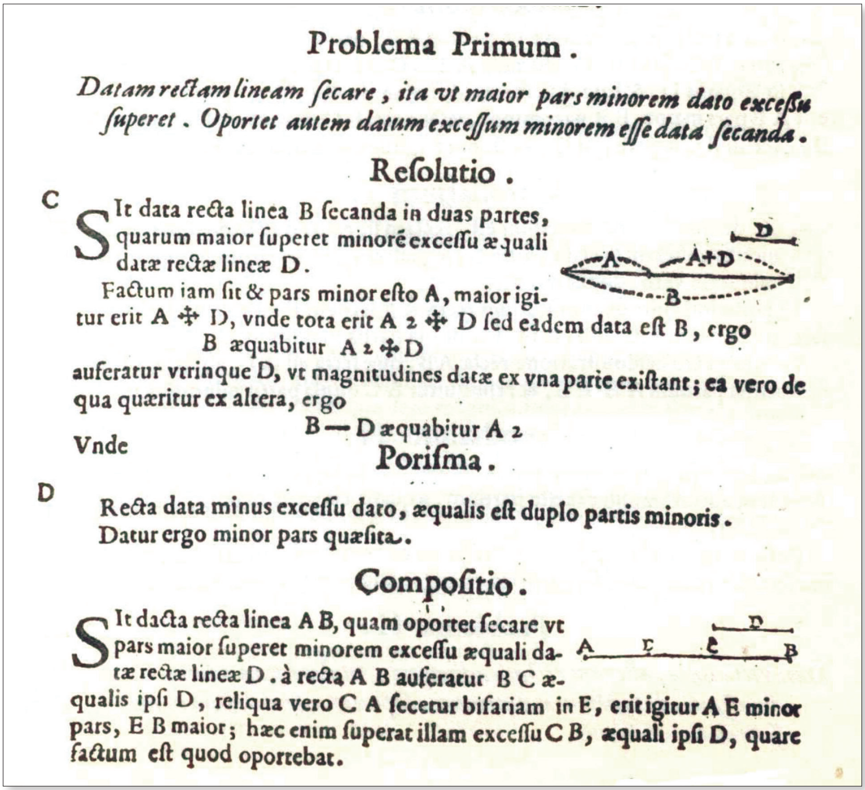
Let the given length be cut in such a way that the larger part exceeds the smaller by the given difference. Let the given difference be smaller than the given length, which ought to be cut.

The problem set in geometric form may be written in the form of a first-degree equation with one unknown. In full keeping with Viète's new algebraic method, Getaldić approaches the problems in such a way that having formulated the problem, he first conducts algebraic analysis. In the analysis itself we can distinguish two steps. In the first step, known as zetetic, geometric objects are presented in

[40] Porism is a notion which is variously interpreted in the history of mathematics, and has its roots in the antiquity. According to Pappus' works, it is a conclusion derived from analysis, while Euclid defines porism as consequence of the synthetical geometric solution of the problem. Porism is a specific type of mathematical theorem, also known as *corollarium* or addition. Etymologically speaking, the notion probably implies certain improvement of a given mathematical problem with the help of these propositions, which are known as 'porisms'. Viète did not use that notion and term in his procedure, and had he done so, it would have followed from the poristic procedure named as such because it leads to the porism. See: J. Klein, *Greek Mathematical Thought and the Origin of Algebra*: pp. 265-266; Oton Kučera, *O Marinu Getaldiću, patriciju dubrovačkom, znamenitom matematiku i fiziku na početku XVII vijeka*, Rad JAZU 117 (1893): pp. 46-47; Ž. Dadić, *Povijest znanosti i prirodne filozofije (s osobitim obzirom na egzaktne znanosti)*: pp. 64-65. In mathematics the meaning of the notion porism changed over time. Getaldić interprets porism as a theorem that follows from the algebraic solution of the problem regardless of the way in which the solution of the given problem would be constructed. As consequence of synthetical solution, instead of the term porism Viète on occasion uses terms consectatium or *corollarium*. The term *corollarium* Getaldić used in the work *Variorum problematum collectio,* also after the synthetical solution of the problem in the sense of consequence, yet the term porism he used in *De resolutione et compositione mathematica* only when the theorem is deduced by analysis.

[41] The obtained solution is not the end of Problem I, as Getaldić develops it further in the book and conducts yet another analysis of the same problem, followed by the porism, and then synthesis. Having performed what was required in Problem I, Getaldić, on the basis of the conclusions deduced from the problem, additionally formulates another two corollaries which he cites, since they are frequently applied in the analyses, notably in the determination of the parts from the sum and difference of the parts.

algebraic form. By doing so, from the given and sought quantities an algebraic equation is formed. Considering that now the observed quantities are general algebraic quantities, they transform in an entirely formal way, irrespective of their geometric starting point. They are thus completely deduced to a final standard form, the so-called canon form. This concludes the first step of algebraic analysis, followed by the second, poristic, in which from the canon form of algebraic equation a conclusion is deduced regarding the relationship between the given and sought quantities. By so doing, algebraic analysis of the problem is completed. The deduced conclusion is referred to as porism, and is applied in the determination of sought quantities, while the method of determination itself falls within synthesis. Getaldić's analysis of the given problem in the rhetorical notation which he used in his work *De resolutione et compositione mathematica,* reads:



Problema Primum.

Datam rectam lineam ſecare, ita vt maior pars minorem dato exceſſu ſuperet. Oportet autem datum exceſſum minorem eſſe data ſecanda.

Reſolutio.

C

SIt data recta linea B ſecanda in duas partes, quarum maior ſuperet minorẽ exceſſu æquali datæ rectæ lineæ D.

Factum iam ſit & pars minor eſto A, maior igitur erit A ✢ D, vnde tota erit A 2 ✢ D ſed eadem data eſt B, ergo

B æquabitur A 2 ✢ D

auferatur vtrinque D, vt magnitudines datæ ex vna parte exiſtant; ea vero de qua quæritur ex altera, ergo

B — D æquabitur A 2

Vnde

Poriſma.

D

Recta data minus exceſſu dato, æqualis eſt duplo partis minoris. Datur ergo minor pars quæſita.

Compoſitio.

SIt dacta recta linea A B, quam oportet ſecare vt pars maior ſuperet minorem exceſſu æquali datæ rectæ lineæ D. à recta A B auferatur B C æqualis ipſi D, reliqua vero C A ſecetur bifariam in E, erit igitur A E minor pars, E B maior; hæc enim ſuperat illam exceſſu C B, æquali ipſi D, quare factum eſt quod oportebat.

Fig. 1. The page from Getaldić's work *De resolutione et compositione mathematica.* (Reproduced from: Marini Ghetaldi, *Opera omnia*, ed. Ž. Dadić: Zagreb: JAZU, 1968: p. 377)

Analysis

Let the given length B be cut into two parts, so that the larger part exceeds the smaller by a difference equal to given length D. May this have already been done and may the smaller part be A, larger part, will then be, A+D, the whole length will then be A_2+D, but that is the given length B, and therefore B will be equal to A_2+D. Let D be subtracted from both ends, so that the given lengths appear on the one end, and the sought one on the other, therefore B–D will be equal to A_2.[42]

Getaldić then formulates a conclusion which holds for given and sought quantities, that is, a porism which he later used in the synthesis:

Porism

Given length minus given difference equals twice the smaller part. Obtained, therefore, is the sought smaller part.

Synthesis

Length AB is given, which ought to be cut, so that the larger part exceeds the smaller by a difference equal to given length D. From length AB let the length BC be subtracted, equal to D itself, and let the remaining CA be cut into E, smaller part will then be AE, and larger EB. Namely this part exceeds that one by the difference CB, which is equal to D itself, and what had to be done was done.

In order to draw a clearer line between geometric method and that of algebra, I shall repeat Problem I, yet in such a way so as to apply geometric analysis to it. Geometric analysis of Problem I, where the observed quantities are geometric objects, would read as follows:

Given that the problem is analytical, one should again presuppose that what was required has already been done, i.e., that the division of the length has already been done according to the problem conditions. Let the given length be PR and the difference between its parts XY. Let us assume that length PR has already

[42] Getaldić's notation differs from the contemporary one. When he wishes to write A twice, in his notation and symbol representation it is A2.

been divided at point Q as required by the problem statement. Then the larger part QR exceeds the smaller PQ by the given difference XY. What follows from this is that when smaller length PQ is aligned with larger QR (at QP'), then the remainder will be equal to the given difference XY. This completes the geometric analysis of the problem.

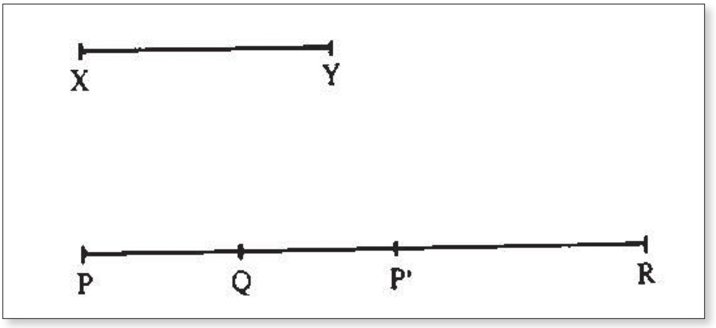


Fig. 2.

On the basis of previously conducted geometric analysis, we conclude in the synthesis that in the process of the construction the given difference XY at P'R should first be aligned with the length PR. Then the derived remainder PP' at point Q is divided into two equal parts. Thus constructed point Q divides the given length as required in the problem statement. By so doing, geometric synthesis, i.e., construction of the problem, is completed.

The difference between these two types of analysis is based on the difference of the mathematical objects used. In algebraic analysis the course of analysis is the same as in geometric analysis, but unknown geometric objects are presented in an even more general form, in the form of species. These objects are first presented in algebraic form. Then algebraic relationships instead of geometric are formulated between them. This results in equations which contain given and sought quantities. These equations then transform formally, regardless of the geometric starting point. They are further deduced to the final, so-called canon form. From this canon form a conclusion is derived regarding given and sought quantities, i.e., some sort of conclusion about them is deduced. Deduced conclusion fully corresponds to that geometric conclusion which followed from geometric analysis. Finally, that conclusion, which follows from algebraic analysis, is used in the synthesis, in the same manner as the conclusion that followed from geometric analysis.

In the algebraic analysis of Problem I, Getaldić presupposes that the observed length B is already divided as stated in the problem. Unlike the previously conducted geometric analysis, the given and sought quantities are no longer considered as geometric object, but as general quantities, in keeping with Viète's algebraic method.[43] With the help of general quantities it is possible to formulate algebraic equation, which in Getaldić's work appears in rhetorical notation.[44] The equation is further transformed to canon form,[45] from which a direct conclusion is deduced about the relationships between given and sought quantities, that is, a porism is given, after which Getaldić comes forward with the synthesis.

Canon form is the final form of algebraic equation, from which a porism is derived, stating that the difference between given length and given difference of the parts is double that of the smaller part (in algebraic notation: B–D = 2A). This concludes the algebraic analysis of the problem under consideration. The synthesis alone, that is, construction to the sought quantity with the help of porism, which in Getaldić's case has been obtained by algebraic analysis, could be performed both geometrically and numerically. If the synthesis is performed numerically, then the unknown quantity A would be determined by including the remaining known numerical values in the canon form of the equation. If the synthesis is performed geometrically, then the construction is performed according to the deduced porism, in the way Getaldić has done in the synthesis of Problem I. From the given length he subtracted the given difference, which according to the porism equals twice the smaller part. He then divided that length into two equal parts, and thus obtained the sought smaller part. In this way he divided the given length according to given conditions of the problem and the conclusions that followed from the porism.

Conclusion

In terms of methodology and concept, Getaldić's diverse scientific works may be divided into two parts. Getaldić's early works may be regarded as reinterpretation of the selected works of ancient tradition with an aim to spread ancient knowledge and theories, but also to provide a deeper insight into these works and advance

[43] Therefore, Getaldić denotes the entire length by B, smaller part by A, and the difference of parts by D.

[44] What follows is that larger part is A+D, and the whole length B = 2A + D.

[45] That is, after the conducted transformations B–D = 2A, the sought quantity being A= (B–D)/2.

them within the context of ancient Greek mathematical methods. In his mature works, however, Getaldić focused on the problem of method. The embryos of the ideas that he adopted on his study trip through Europe, Getaldić developed over the twenty years he spent in Dubrovnik, independently and totally isolated from the advances and developments in the European scientific community of the first decades of the seventeenth century. These investigations he compiled in the capital work *De resolutione et compositione mathematica* (Rome 1630), expounded in five books. Although Getaldić operated in an environment imbued with the influence of the Renaissance and humanism, new ideas and knowledge reached Dubrovnik at a much slower pace than in the Western Europe, the home of modern science in the sixteenth and seventeenth century. During the time of his isolation in Dubrovnik, Getaldić devised new theoretical and practical solutions and original works which had a wide reception in the European scientific community not only during his lifetime, but also later, in the course of the seventeenth and eighteenth century.[46] His example best illustrates how the transmission of knowledge did not only develop from the European centres towards the periphery, for in Getaldić's case that process operated in both directions.

Getaldić worked at a time when the accumulated knowledge of ancient works and the spread of humanistic education surpassed ancient tradition, and when gradually, through methodic transformation, science of the modern period was founded and shaped. It took almost twenty centuries for the ancient mathematical methodology, complemented by the knowledge drawn from the Arabic and Indian mathematical tradition, to change and develop new methods in the approach of new theoretical knowledge and practical solutions. While building his rich opus, Getaldić leaned heavily on the original ancient mathematical methods, which he consistently applied to heterogeneous material. His work is largely based on the works of Greek mathematicians, with emphasis on Pappus and Diophantus, under the influence of Eudoxus' theory of ratios and Archimedes' application of logistic methodology, i.e., arithmetic interpretation of geometry. By doing so, Getaldić in a unique and fruitful manner combines mutually different tendencies of ancient Greek mathematics.

Getaldić's introduction to ancient mathematical heritage and the composition of his early works, grounded exclusively on the use of geometric analysis and synthesis, represents the first and important segment of his development path

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[46] Ž. Dadić, *Povijest znanosti i prirodne filozofije*: pp. 95-104, 111-118.

and shaping. Having become acquainted with Viète's symbolic algebra which operates with general quantities, Getaldić approached systematically the possibilities of symbolic algebra in relation to the earlier traditional ancient mathematical methods, which would have a pivotal role in the further development of modern mathematics, and would gradually lead to the second great conceptual change in the history of mathematics.[47] The change did not only reflect in mathematics, but gave way to the emergence of the new, more simple and more exact interpretations in other sciences as well. New *logistica speciosa,* built on the foundations of ancient mathematical analysis and synthesis, marked the second, mature phase of Getaldić's work. It saw the introduction of the general quantities known as *species* in mathematics, which can equally be applied to numbers and geometric objects. Calculation with the use of *species* (*logistica speciosa*) enabled a new interpretation of the mathematical results. The use of general quantities (relevant to both numbers and geometric objects), changed Greek geometric analysis and synthesis by being performed algebraically within general algebra. To new method Getaldić devoted his major work, *De resolutione et compositione mathematica*, the first comprehensive handbook of new symbolic algebra and algebraic analysis. With his results, Getaldić made a step towards the founding of a new area—analytic geometry, which after two millennia of separation would gradually lead to a reunion of the mathematical areas divided in the antiquity—geometry and algebra.

Educated on the works of ancient Greek mathematicians, in which he attained a respectable level of knowledge, Getaldić was among the first to observe the advantages of symbolic algebra, and he adopted it in terms of the problem approach as well as in the form, symbolic representation and expression, having recognised

[47] Mathematics as an empirical discipline developed from as early as the second millennium B.C. in the ancient civilisations of Babylon and Egypt. Mathematics saw the first great conceptual change after the Asian-European exchange of knowledge in the Hellenistic period. Having adopted the mathematics of the ancient peoples of the Orient, which was empirical in nature, the Greeks transformed it theoretically and structurally into a scientific discipline. The Greeks laid the scientific foundations and frame of mathematics as scientific discipline. They introduced abstract mathematical notions, mathematical proof, axiomatic deductive system, separated the areas of arithmetic and geometry, and made a clear distinction between mathematical theory and application, i.e., between theoretical considerations and the routine of mathematical calculation, known as *logistica,* to which they did not acknowledge scientific status considering that it did not deal with abstractions but with concrete objects. By doing so, they defined the main areas and directions of the development of mathematics until the seventeenth century. From the twelfth century onwards, under the influence of Arabic mathematics, numerous new mathematical solutions developed and accumulated, though modelled on ancient mathematical methods and conceptions.

the importance of its application in geometry. However, in performing the analysis procedure Getaldić departed formally from Viète's procedure of the three degrees of analysis (zetetic, poristic and rhetic or exegetic).[48] The third step which includes the equation solution Viète also regarded as analytic procedure, although it concerns synthesis. Getaldić commented on this also in his work *De resolutione et compositione mathematica,* in which he made a clear distinction between analysis and synthesis. Within analysis, Getaldić follows Viète's first two steps: zetetic and poristic. He then formulates a porism, the statement that follows from the equation and which Viète did not emphasise after the poristic procedure.[49] The final, third step, which Viète interprets as part of the analytic procedure, Getaldić designates as a procedure of synthesis which can be performed arithmetically or geometrically in the sense of the rhetic or exegetic procedure. In addition to the aforementioned, of notable methodological importance is Getaldić's innovative scheme known as *Conspectus resolutionis et compositionis*, in which he provides a specific insight into the double-chain logic reasoning of the conducted analysis and synthesis, as two procedures that develop in reverse order. The scheme presents the mutual relationship between the analytic and synthetic procedure, characterised by Getaldić's accentuated tendency to formalise the procedures by using symbolic representation of his day.[50] The scheme shows how symbolic algebra with the introduction of general quantities led to the advancement and application of the symbolic language of mathematics instead of the formerly used rhetorical notation. This paved the way for the geometric problems, regardless of their geometric starting point, to be viewed and formally solved with the help of algebraic analysis within general algebra.

Algebraic analysis tended to be increasingly applied in the solution of geometric problems, for which Getaldić is to be greatly credited. However, he realised that neither the new method, however useful it was and opened new horizons and areas, could in all segments replace geometric method and reject its value. This methodological awareness and the work on the development of different mathematical methods is doubtless the greatest achievement that Getaldić has handed down to us. He used different methods, ancient geometric and the new algebraic. In support

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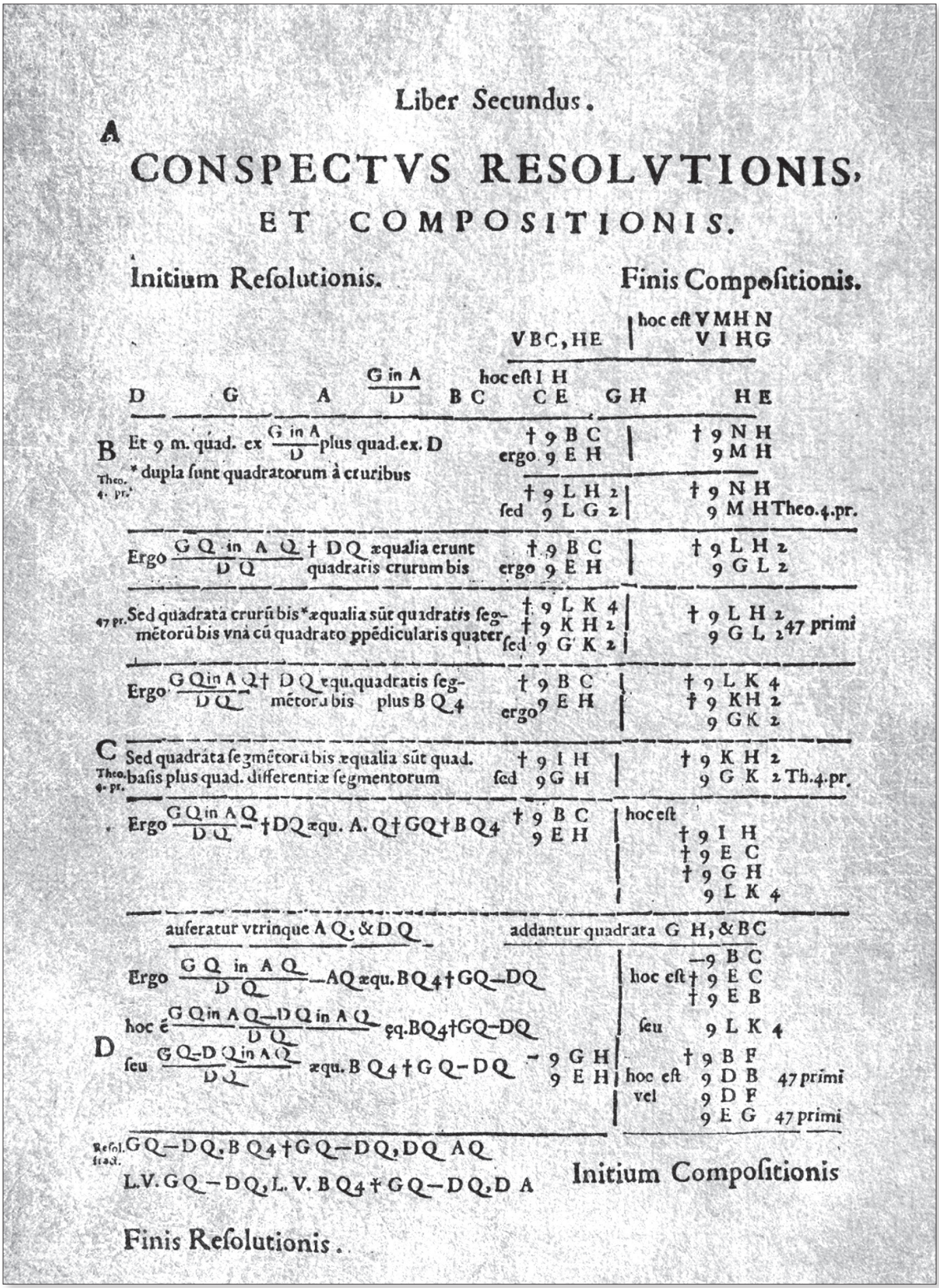
[48] Rhetic procedure is related to numerical solution, and exegetic to geometric quantities.

[49] In his methodological procedure, Getaldić probably follows Pappus' work which he consulted, and having formulated the porism, brings the synthesis, while the third rhetic or exegetic he includes in the synthesis. On method, see: Ž. Dadić, *Povijest znanosti i prirodne filozofije*: pp. 64-65.

[50] See the example in M. Ghetaldi, *Opera omnia*: p. 21.


Liber Secundus.

A

CONSPECTVS RESOLVTIONIS, ET COMPOSITIONIS.

Initium Resolutionis. Finis Compositionis.

VBC, HE | hoc est V M H N / V I H G

D G A G in A / D B C hoc est I H / C E G H H E

B Et 9 m. quad. ex G in A / D plus quad. ex. D + 9 B C ergo 9 E H | + 9 N H 9 M H

Theo. 4. pr. * dupla sunt quadratorum à cruribus + 9 L H 2 sed 9 L G 2 | + 9 N H 9 M H Theo. 4. pr.

Ergo G Q in A Q + D Q / D Q æqualia erunt quadratis crurum bis + 9 B C ergo 9 E H | + 9 L H 2 9 G L 2

47 pr. Sed quadrata crurū bis * æqualia sūt quadratis segmētorū bis vnà cū quadrato ppēdicularis quater + 9 L K 4 + 9 K H 2 sed 9 G K 2 | + 9 L H 2 9 G L 2 47 primi

Ergo G Q in A Q + D Q / D Q æqu. quadratis segmētorū bis plus B Q 4 + 9 B C ergo 9 E H | + 9 L K 4 + 9 K H 2 9 G K 2

C Sed quadrata segmētorū bis æqualia sūt quad. Theo. 4. pr. basis plus quad. differentiæ segmentorum + 9 I H sed 9 G H | + 9 K H 2 9 G K 2 Th. 4. pr.

Ergo G Q in A Q / D Q + D Q æqu. A. Q + G Q + B Q 4 + 9 B C 9 E H | hoc est + 9 I H + 9 E C + 9 G H 9 L K 4

auferatur vtrinque A Q, & D Q addantur quadrata G H, & B C

Ergo G Q in A Q / D Q — A Q æqu. B Q 4 + G Q — D Q | — 9 B C hoc est + 9 E C + 9 E B

hoc ē G Q in A Q — D Q in A Q / D Q æq. B Q 4 + G Q — D Q | seu 9 L K 4

D seu G Q — D Q in A Q / D Q æqu. B Q 4 + G Q — D Q — 9 G H 9 E H | + 9 B F hoc est 9 D B 47 primi vel 9 D F 9 E G 47 primi

Resol. stat. G Q — D Q, B Q 4 + G Q — D Q, D Q A Q

L.V. G Q — D Q, L. V. B Q 4 + G Q — D Q, D A Initium Compositionis

Finis Resolutionis.

Fig. 3. Getaldić's scheme entitled *Conspectus resolutionis et compositionis*
Reproduction from the reprint M. Ghetaldi, *Opera omnia*: p. 405 (41).

of the claim that Getaldić as mathematician had trodden the right methodological path is best testified by the development of mathematics after his day. Despite the possibilities provided by algebraic method, geometric method was used and promoted by many seventeenth-century mathematicians, such as Pascal Blaise (1623-1662), Thomas Hobbes (1588-1679), Isaac Barrow (1630-1677) and others, and it helped achieve numerous important results in the determination of areas and the tangents to the curves, which in the eighteenth century were used in the invention of a new mathematical area—infinitesimal calculus.