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An Optimization Approach for Pricing Analysis on a Bank Wealth-Management Equity Structured Product

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Abstract: This paper researches on the pricing and design of a certain stock-type structured product. Firstly, a semi-analytic pricing model is deduced by discounting the payoff function of the product. Secondly, the difference between publishers' and investors' required rate of return is explained with market segmentation theory when estimating the pricing model's parameters, which defines the cost and sale price of a product. Finally, with sensitivity analysis, it is concluded that publishers can increase their profits by extending the due date of the product or publishing it with relatively large asset volatility. The study aims to help publishers make reasonable product design and pricing decisions.

Keywords: market segmentation; pricing; sensitivity analysis; structured product

1 INTRODUCTION

Structured products usually refer to financial products that are based on fixed income products, along with one or more derivative contracts having particular markets or indices as their underlying assets. At the moment underlying assets in these derivative contracts may include stock prices (stock indices), commodity indices, interest rates, foreign exchange currencies, etc. Structured products have grown rapidly in international markets since the 1970s, much faster than underlying instruments. Despite the late (compared to other countries) start of structured products in China, with the recent prosperity of the stock market and the continuous improvement of the domestic financial market, they have been developing very quickly as more and richer types of products are available in the markets now, and their embedded options evolve to be more complicated and non-standardized. Among them, trigger-type structured products linked to stock indices have become an important part of the domestic financial product market.

Research on structured products started early oversea and is with depth into pricing models for options inside corresponding products; there are nevertheless few discussions on pricing models for structured products as whole objects. Barrier options are the most common type of non-standardized options in structured products, Milev and Tagliani (2010) [1] studied the numerical pricing problem of discrete double barrier options, assuming that the underlying asset price obeys the geometric Brownian motion, and they derived a numerical solution for discrete double barrier options. Forde and Kumar (2015) [2], under the assumption of stochastic volatility in interest rates, improved the Euclidean barrier option pricing formula based on the jump-diffusion model. Farnoosh (2015) [3] discussed pricings of discrete single barrier options by assuming that dividend yield and volatility are deterministic functions.

For research on the pricing and designing of structured products, oversea scholars are primarily focused on introducing option pricing-related results into the framework of structured product pricing as well as on corresponding empirical analysis; yet the development of the pricing model itself is largely overlooked. Stoimenov and Wilkens (2004) [4], by calculating theoretical prices of particularly structured products with exotic options embedded in the German financial market, demonstrated that such products are generally overpriced in the primary market, while the difference between the theoretical value and the actual price in the secondary market is not significant. Wallmeier and Diethelm (2008) [5] empirically studied the price of a multi-asset conditional principal-protected reverse convertible bond (MBRCs) in the Swiss market and found that the market price of the convertible bond is at least 3.4-6% above its theoretical price and that the overpricing is positively correlated to interest rates. Baule and Tallau (2011) [6] used a stochastic volatility model to price a down-and-out product. They found that the actual price of this product is higher than its theoretical price because the issuer has taken into account the effect of volatility skew. Celerier and Vallee (2013) [7] investigated motivations behind financial products complexity through empirical analysis of a large number of household investors of structured products; they showed that the complexity of structured financial products has gradually increased in recent years, as investors with lower income favour products with higher complexities, which is consistent with the development of confusion strategies out of the profit-seeking nature of banks. Ghent and Torous (2014) [8] examined the relationship between complexities and risks of structured products and showed that higher complexities do not necessarily imply lower risks, and rating agencies are more lenient with complicated products. Soltes and Harcarikova (2015) [9] selected from Tatra Bank a structured deposit with a European barrier option embedded in the Slovakia market for empirical analysis; they reworked and redesigned the product from the perspective of investors.

Domestic structured products consist primarily of bank structured financial products. Since pricing models of structured financial products are usually core technologies of issuing banks, there are relatively few comprehensive and in-depth public kinds of literature or articles on the principles and formulas of structured products. Jiuzhen Huang (2011) [10] discussed three types of double barrier options in the context of the pricing model theory for barrier options, and the result shows that barrier options have obvious advantages in curbing speculative speculation and low-cost hedging. Guoqiang Chu (2014) [11] focused on the dynamic hedging problem of barrier options using the finite difference method for pricing and analyzed parameter characteristics of barrier options, as well as the effectiveness and shortcomings of traditional dynamic hedging strategies on barrier options. Using Tsallis entropy distribution with long-range memory and statistical feedback properties, Pan Zhao (2015) [12] developed a stock price motion model with a jump-anomalous diffusion and used stochastic differentiation and martingale methods to derive pricing formulas and relevant conclusions for European options under the condition of risk neutrality.

In terms of pricing and designing of structured products, our domestic researches are still in a transitional stage from qualitative introduction to empirical analysis and theoretical research. Qiuping Yang (2013) [13] priced three different products linked to different interest rate reference ranges respectively, the result showed that these products were under-priced to a certain extent, and investors needed to be meticulous. Dongxiang Wang and Yuwen Wang (2015) [14] studied the pricing of a particular triggered interest rate linked financial product under the stochastic interest rate model and established partial differential equations using hedging techniques to eventually give a pricing method for this financial product under the stochastic interest rate model. Guiping Sun (2015) [15] took a structured product with non-standardized options embedded as an example to develop a pricing model and provided a detailed analysis of the sensitivity of the return at issuance and post-issuance risk factors. From the perspective of product issuers, Zishun Ma (2015) [16] used a delta dynamic hedging strategy under the BS model to conduct a comprehensive analysis of the designing, pricing, and risk hedging of structured products, and selected a spread option model and a barrier option model respectively for empirical measurements, to arrive at a product implementation plan that brings relatively stable profit margins for product issuers. Guangcan Zhou (2015) [17] conducted theoretical price determination for three different types of structured financial products of China Merchants Bank (CMB in a sequel): stock indices, commodities, and exchange rates. For the options, part B-S option pricing formula and Monte Carlo simulation are used respectively for pricing. The error of the product pricing is in the end measured by comparing the difference between the theoretical value and the actual value. In summary, there is still a large gap in the research on the designing and pricing of structured products between China and oversea countries. To have deeper insights into the pricing mechanism of structured products, we take an equity structured product of CMB as an example in this paper, and derive a semi-analytic solution formula for the product pricing model by giving the risk-neutral probability density function of the underlying asset when it reaches the barrier level value and, under the assumption of risk-neutrality, using the martingale theory of financial product pricing to find the discount of the expected payoff function at maturity. In the parameter estimation, we take the lead by using the market segmentation theory to explain the reason why the required rate of return for the issuer shall be higher than for investors, to endow a meaningful economic perspective on the existence of a profit margin for such products. Finally, sensitivity analysis is conducted on product parameters. Relevant parameters are optimized to maximize the profit of the issuer. The semi-analytic pricing method for structured products and the dynamic analysis of product parameters derived in this article not only help issuers to master reasonable product designing and pricing approaches, which enhances the competitiveness of structured products and financial markets in China globally, but also provide issuers and risk managers with references for product pricing decisions, helping them to better develop their financial product business and explore more business growth points.

2 PRODUCT DESCRIPTIONS AND THE PRICING MODEL

2.1 Product Descriptions

We will take the 104563 product from "the focus linkage series of the China Merchant Bank" for our pricing analysis; details of this product are listed in the table below:

Table 1 Product description (CMB 104563)

product name	the focus linkage series of the China Merchant Bank: linkage of the stock index performance (CSI 300 index continuous call option with triggering terms), Not Capital Guaranteed Plans (Product Code: 104563)
Maturity time	188 days
expected rate of return	as per the product manifest, floating rates will be paid based on the price of the underlying asset (highest expected annualized rate of return: 14.90%)
start date	2014.09.04
maturity date	2015.03.09
participation rate	100%
barrier price	111% of the opening price□(3 digits after the decimal point)
underlying asset	the CSI 300 closing index value announced by the Shanghai Stock Exchange
capital guarantee rate	not guaranteed
minimum initial purchase	300K shares (1 CNY per share), followed by a 10K shares unit change scale
reimbursement method	Once and in a lump sum after the maturity date

The rate of return for this product is determined by: (1) If the CSI 300 index value exceeded the barrier price during the observation period, then the annualized rate of return upon the maturity date would be set to 5.9%; (2) If the CSI 300 index never exceeded the barrier price during the observation period, but its final value (i.e., the closing index value at the end of the period) were higher than or equal to its initial value (i.e., the opening index value at the beginning of the period), then the annualized rate of return upon the maturity date would be set to 3.9% + min(11%), max(0, rate of increase) where the rate of increase is (final value/initial value − 1)*100%; (3) If the final value were lower than its initial value, then the annualized rate of return upon the maturity date would be set to 3.9%.

It is easy to spot the following features of this product: (1) There is always a profit margin regardless of the market condition-and up to a 14.9% annualized rate of return; (2) High minimum required rate of return-a 3.9% minimum expected annualized rate of return, which is higher than the bank rate for the 1-year fixed deposit; (3) A full participation rate, which means that the profit of the option is equal to the rate of increase of the CSI 300 index.

The product description shows that the rate of return $R(S_t)$ is

$$R(S_T)=\begin{cases} 3.9\% & \left(\overline{S_T}\le H,\, S_T<S_0,\right) \\ 3.9\%+\dfrac{S_T-S_0}{S_0} & \left(\overline{S_T}\le H,\, S_T\ge S_0,\right) \\ 5.9\% & \left(\overline{S_T}>H\right) \end{cases} \quad (1)$$

where S_t is the asset price as a function of time with T, S_0 and S_T being its maturity time, its initial and final value respectively, also $\overline{S_T}$ is $\max S_t (0\le t\le T)$, and H is the barrier price. If we denote K as the exercise price, then $K=S_0$ for this product, while the barrier price $H=S_0\times 111\%$.

It follows that the annual payoff function $p(S_t)$ is

$$p(S_t)=\begin{cases} \left(\dfrac{\left(S_T-K\right)^+}{S_0}+3.9\%+1\right)\left[I_{\left(\overline{S_T}\le H\right)}\right] \\ \left(5.9\%+1\right)\left[I_{\left(\overline{S_T}>H\right)}\right] \end{cases} \quad (2)$$

where $\left(S_T-K\right)^+$ is $\max\left(S_T-K,0\right)$, and $I_{(\bullet)}$ denotes the characteristic function.

2.2 Motion of the Underlying Asset Price

The underlying asset of the CMB 104563 structured product is the CSI 300 index, which means that this product is an equity structured product. It is common to assume that the stock price S_t is a generalized wiener process, while H_t, its continuous compound rate of return in time [0, t] can be described by a geometric Brownian motion. Suppose H_t is a Brownian motion with no drift terms, then

$$S_t=S_0e^{H_t}\ \ H_t=\mu t+\sigma W_t$$

where the drift rate $\mu=0$, $\{W_t\}$ is a standard Brownian motion with σ its volatility and S_0 is the current asset price. The change of the stock price then obeys the differential equation:

$$\mathrm{d}S_t=\mu S_0\mathrm{d}t+\sigma S_t\mathrm{d}W_t$$

where the probability density function of S_T is

$$f(S_T)=\frac{1}{S_T\sigma\sqrt{2\pi T}}\exp(\frac{-[\ln(S_T/S_0)-(\mu-\sigma^2/2)T]^2}{2\sigma^2T})$$

and its cumulative distribution function is

$$F(S_T)=\Phi\left(\frac{\ln(S_T/S_0)-(\mu-\sigma^2/2)T}{\sigma\sqrt{T}}\right)$$

Here T is the time duration of the product, $\Phi(.)$ is the cumulative distribution of the standard Gaussian:

$$\Phi(x)=\int_{-\infty}^{x}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}u^2}\mathrm{d}u$$

2.3 The Product Pricing Model

To improve the universality of our model, we will simply ignore the 300K shares (1 CNY per share) minimum initial purchasing term, and instead, base our calculations on the value of 1 share.

We will derive a pricing model by computing the discounted expected value of the payoff function at maturity. Under the risk-neutral assumption, we shall first deduce the risk-neutral probability density function when the underlying asset hits the barrier price, then obtain a pricing model for this structured product using the martingale theory for the pricing of financial products. The risk-neutral probability density function can be estimated using the extreme value distribution function of the Brownian motion and the joint distribution of its extreme value and its final value. The result of this article follows from Robert (2009) et al. [18, 19], where the following four equations were obtained using both the maximum value theory of the Brownian motion and the Girsanov theorem; details of the proof will not be repeated here:

$$P^Q(S_T\le K,\overline{S_T}\le H)=\Phi(d_1)-\left(\frac{H}{S_0}\right)^{\frac{2r}{\sigma^2}-1}\Phi(d_2) \quad (3)$$

$$P^Q(S_T\ge K,\underline{S_T}\ge H)=\Phi(d_3)-\left(\frac{H}{S_0}\right)^{\frac{2r}{\sigma^2}-1}\Phi(d_4) \quad (4)$$

$$\begin{aligned} &E^Q(S_TI_{S_T\le K,\overline{S_T}\le H})= \\ &S_0e^{rT}[\Phi(d_5)-\left(\frac{H}{S_0}\right)^{\frac{2r}{\sigma^2}+1}\Phi(d_6)] \end{aligned} \quad (5)$$

$$\begin{aligned} &E^Q(S_TI_{S_T\ge K,\underline{S_T}\ge H})= \\ &S_0e^{rT}[\Phi(d_7)-\left(\frac{H}{S_0}\right)^{\frac{2r}{\sigma^2}+1}\Phi(d_8)] \end{aligned} \quad (6)$$

In these equations $\overline{S_T}$ is $\max S_t(0\le t\le T)$, $\underline{S_T}$ is $\min S_t(0\le t\le T)$, $I_{S_T\le K,\underline{S_T}\le H}$ denotes the characteristic function, r is the required rate of return, T is the maturity time measured in years, and other variables are the same as we have defined before.

$$d_1=\left[\log\left(\frac{K}{S_0}\right)-\left(r-\frac{\sigma^2}{2}\right)T\right]\Big/\sigma\sqrt{T}$$

$$d_2 = \left[\log\left(\frac{KS_0}{H^2}\right) - \left(r - \frac{\sigma^2}{2}\right)T\right] \Big/ \sigma\sqrt{T}$$

$$d_3 = \left[\log\left(\frac{S_0}{K}\right) - \left(r - \frac{\sigma^2}{2}\right)T\right] \Big/ \sigma\sqrt{T}$$

$$d_4 = \left[\log\left(\frac{H^2}{KS_0}\right) - \left(r - \frac{\sigma^2}{2}\right)T\right] \Big/ \sigma\sqrt{T}$$

$$d_5 = \left[\log\left(\frac{K}{S_0}\right) - \left(r + \frac{\sigma^2}{2}\right)T\right] \Big/ \sigma\sqrt{T}$$

$$d_6 = \left[\log\left(\frac{KS_0}{H^2}\right) - \left(r + \frac{\sigma^2}{2}\right)T\right] \Big/ \sigma\sqrt{T}$$

$$d_7 = \left[\log\left(\frac{S_0}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T\right] \Big/ \sigma\sqrt{T}$$

$$d_8 = \left[\log\left(\frac{H^2}{KS_0}\right) + \left(r + \frac{\sigma^2}{2}\right)T\right] \Big/ \sigma\sqrt{T}$$

Since the face value for each share is 1 CNY, the rate of return is computed on this basis. The cost for buying each share can be determined as the discounted expected value of the payoff function at maturity, which consists of all possible future cash inflow, including the listed 1 CNY at the time of purchasing and the expected profit based on the performance of the CSI 300 index, therefore C_n can be computed as

$$e^{rT} C_n = 1 + T \frac{1}{S_0} E^Q\left(\left(S_T - K\right)^+ + 3.9\% S_0\right) \times I_{\left(\overline{S_T} \le H\right)} + E^Q\left(5.9\% S_0 I_{\left(\overline{S_T} > H\right)}\right) \tag{7}$$

After rearranging terms we get

$$\begin{aligned} \frac{S_0}{T}\left(e^{rT} C_n - 1\right) &= E^Q\left[S_T I_{\left(S_T > K, \overline{S_T} \le H\right)}\right] \\ -KE^Q\left[I_{\left(S_T > K, \overline{S_T} \le H\right)}\right] &+ E^Q\left[3.9\% S_0 I_{\left(\overline{S_T} \le H\right)}\right] \\ +E^Q\left[5.9\% S_0 I_{\left(\overline{S_T} > H\right)}\right] &= V_1 - V_2 + V_3 + V_4 \end{aligned} \tag{8}$$

It follows that

$$\begin{aligned} V_1 &= E^Q\left[S_T I_{\left(S_T > K, \overline{S_T} \le H\right)}\right] \\ &= E^Q\left[S_T I_{\left(S_T < H, \overline{S_T} \le H\right)}\right] - E^Q\left[S_T I_{\left(S_T < K, \overline{S_T} \le H\right)}\right] \\ &= S_0 e^{rT}\left[\Phi(d_9) - \left(\frac{H}{S_0}\right)^{\frac{2r}{\sigma^2}+1} \Phi(d_{10})\right] \\ &\quad - S_0 e^{rT}\left[[\Phi(d_5) - \left(\frac{H}{S_0}\right)^{\frac{2r}{\sigma^2}+1} \Phi(d_6)]\right] \end{aligned}$$

$$\begin{aligned} V_2 &= KE^Q\left[I_{\left(S_T > K, \overline{S_T} \le H\right)}\right] \\ &= KE^Q\left[I_{\left(S_T < H, \overline{S_T} \le H\right)}\right] - KE^Q\left[I_{\left(S_T < K, \overline{S_T} \le H\right)}\right] \\ &= K\left[\Phi(d_{11}) - \left(\frac{H}{S_0}\right)^{\frac{2r}{\sigma^2}-1} \Phi(d_{12})\right] \\ &\quad - K\left[\Phi(d_1) - \left(\frac{H}{S_0}\right)^{\frac{2r}{\sigma^2}-1} \Phi(d_2)\right] \end{aligned}$$

$$\begin{aligned} V_3 &= E^Q\left[3.9\% S_0 I_{\left(\overline{S_T} \le H\right)}\right] = 3.9\% S_0 E^Q\left[I_{\left(\overline{S_T} \le H\right)}\right] \\ &= 3.9\% S_0 E^Q\left[I_{\left(S_T \le H, \overline{S_T} < H\right)}\right] \\ &= 3.9\% S_0 \bullet \left[\Phi(d_{11}) - \left(\frac{H}{S_0}\right)^{\frac{2r}{\sigma^2}-1} \Phi(d_{12})\right] \end{aligned}$$

$$\begin{aligned} V_4 &= E^Q\left[5.9\% S_0 I_{\left(\overline{S_T} > H\right)}\right] = 5.9\% S_0 E^Q\left[I_{\left(\overline{S_T} > H\right)}\right] \\ &= 5.9\% S_0\left[1 - P^Q\left(\overline{S_T} \le H\right)\right] \\ &= 5.9\% S_0\left[1 - P^Q\left(S_T < H, \overline{S_T} \le H\right)\right] \\ &= 5.9\% S_0 - 5.9\% S_0 \bullet \left[\Phi(d_{11}) - \left(\frac{H}{S_0}\right)^{\frac{2r}{\sigma^2}-1} \Phi(d_{12})\right] \end{aligned}$$

where

$$d_9 = \left[\log\left(\frac{H}{S_0}\right) - \left(r + \frac{\sigma^2}{2}\right)T\right] \Big/ \sigma\sqrt{T}$$

$$d_{10} = \left[\log\left(\frac{S_0}{H}\right) - \left(r + \frac{\sigma^2}{2}\right)T\right] \Big/ \sigma\sqrt{T}$$

$$d_{11} = \left[\log\left(\frac{S_0}{H}\right) - \left(r - \frac{\sigma^2}{2}\right)T \right] \bigg/ \sigma\sqrt{T}$$

$$d_{12} = \left[\log\left(\frac{S_0}{H}\right) - \left(r - \frac{\sigma^2}{2}\right)T \right] \bigg/ \sigma\sqrt{T}$$

Merging and sorting terms in V_1, V_2, V_3, V_4, we obtain that the product cost is

$$\begin{aligned}
C_n &= e^{-rT} + T \cdot \Phi(d_9) - T \cdot \left(\frac{H}{S_0}\right)^{\frac{2r}{\sigma^2}+1} \Phi(d_{10}) \\
&- T \cdot \Phi(d_5) + T \cdot \left(\frac{H}{S_0}\right)^{\frac{2r}{\sigma^2}+1} \Phi(d_6) \\
&- \frac{TKe^{-rT}}{S_0} \left[\Phi(d_{11}) - \left(\frac{H}{S_0}\right)^{\frac{2r}{\sigma^2}-1} \Phi(d_{12}) \right] \\
&+ \frac{TKe^{-rT}}{S_0} \left[\Phi(d_1) - \left(\frac{H}{S_0}\right)^{\frac{2r}{\sigma^2}-1} \Phi(d_2) \right] \\
&- T \cdot 2.0\% e^{-rT} \bullet \left[\Phi(d_{11}) - \left(\frac{H}{S_0}\right)^{\frac{2r}{\sigma^2}-1} \Phi(d_{12}) \right] \\
&+ T \cdot 5.9\% e^{-rT}
\end{aligned} \tag{9}$$

This is the semi-analytic solution to the pricing model of the corresponding structured product. The joint distribution of the maximum and the final value of the Brownian motion is used here since there is an up-and-out barrier option embedded in this product. A similar pricing model can be obtained, with the same approach, by using the joint distribution of the minimum and the final value of the Brownian motion for products with down-and-out options. Alternatively, by combing these two types of joint distribution, product values for equity structured products with double barrier options can also be derived. We omit further details on other types of products here.

3 PARAMETER ESTIMATE AND THE RESULT OF THE PRICING ANALYSIS

3.1 Parameter Estimate for the Discount Rate Based on the Market Segmentation Theory

Upon estimating parameters in the equation particular attention should be paid to the selection of the yield to maturity, i.e., the discount rate r. Traditional researchers usually adopt identical discount rates for investors and issuers, which leads to identical valuations of products under the risk-neutral assumption, consequently, issuers would be unable to profit from issuing products, and the economic meaning for the existence of a structured product's market could then not be effectively explained.

Giordano and Siciliano (2015) [20] have argued that the product should be priced so that it is acceptable to investors. Therefore we will consider product values based on the risk-neutral assumption as the hedging cost required by the issuer to issue the structured product, while the product values obtained by investors are considered as their acceptable prices, which in particular is not the same as selling prices. To profit, the issuer may design different product terms to distinguish and maximize the difference between prices and costs.

In Wedel and Kamakura (2012) [21] it was mentioned that the "market segmentation theory" completely isolates and mutually separates bond markets by maturities, in the sense that long-term and short-term bonds are traded in different segmented markets, and reach equilibriums independently. Based on the market segmentation theory, we propose that legislation, investment barriers, technology, time, and other relevant factors make issuers (as institutional investors) and ordinary investors in two different financial markets. Due to reasons such as information asymmetry they shall, and will choose different required rates of return upon their estimations of product values.

Suppose that r_1, r_2 respectively require rates of return for issuers and ordinary investors. The market segmentation theory shows that $r_1 = r_2$ the risk-free rate cannot hold, as the difference between the acceptable price (selling price) and product cost is explainable only if $r_1 > r_2$, and only then will the further existence of a structured product's market be meaningful. Concrete reasons for r_1 larger than r_2 include:

(1) If $r_1 = r_2 =$ a risk-free rate, and assuming risk neutrality when estimating the value of the product, then the cost of issuance equals the selling price; consequently, the issuer will not be able to profit, which makes the existence of structured products meaningless.

(2) As institutional investors, issuers are more capable of obtaining higher returns due to their larger capital size, better timely access to information, lower barriers to the investment market, more investment experience and technical guidance, as well as the ability to both short-term arbitraging and long-term funding planning; thus their expected returns from financial products are higher.

(3) For ordinary investors, due to legislation, preferences, and other relevant factors, the costs of achieving free mobility of funds between bonds and securities with different maturities are completely different from issuers. Moreover, considering differences in positions of investors in the financial market, barriers they are facing, and experiences, technologies, and time costs they have, ordinary investors are more likely to make short-term investments. Also due to higher costs and incompleteness in acquiring information, for the same amount of investment, ordinary investors will obtain less return than issuers in real markets, i.e., $r_1 > r_2$, the required rate of return for ordinary investors is less than the rate for issuers.

To summarize, even if both ordinary investors and issuers are capable of realizing the risk-neutral rational person assumption, they will still face different rates of return and make different expectations on product values in assessments, which in turn leaves profit margins in product pricing. As a result, we choose the interest rate for the 6-month loan (6.0% per year, announced in 2014.09) by CMB as the discount rate r_1 for the issuance cost, since the loan is a relatively stable basic business for banks, and issuers can adopt a "pool and (re-)flow" strategy for higher profits, while the interest rate for the 6 months fixed deposit (2.8% per year, announced in 2014.09) by CMB is chosen as the discount rate r_2 for the price acceptable to investors since investors normally have limited means of investment, among which fixed deposits are fairly popular.

Wang (2021) [22] and Hadad (2022) [23] used the same parameter estimation method in their study of structured product pricing, while Auh (2022) [24] tested the validity of such parameter estimation empirically.

3.2 Product Costs and Selling Prices

Following product, parameters can be easily found in the description of the CMB 104563 equity structured product: exercise price $K = S_0 = 2426.22$ (the closing price of CSI 300 is 2426.22 on 2014.09.04), barrier price $H = S_0 \times 111\% = 2693.10$, $T = 188/365$ (taking 365 days in a year). The annual historical volatility of the CSI 300 index is selected as the Long-term volatility $\sigma = 28.57\%$, also $r_1 = 6\%$, $r_2 = 2.8\%$ and the drift rate is assumed to be $\mu = 0$.

Solving the product cost C_n and the acceptable price P_n to investors with Matlab for different discount rates r_1 and r_2 on the risk-neutral assumption, we get $C_n = 0.9961$ and $P_n = 1.0124$, i.e., the value for each share of the structured product is 0.9961 CNY while the acceptable price is 1.0124 CNY per share. Therefore, on top of existing parameters, it is possible to set the price between 0.9961 and 1.0124 to make profits from price differences.

3.3 Analysis of the Product Pricing

The rule of the CMB 104563 product dictates that investors pay at par for buying and get reimbursed in a lump sum at maturity. This indicates that the product is issued at 1 CNY per share in the primary market. According to the pricing model, the theoretical product value, i.e., the cost of the product expected by the issuer, is 0.9961 CNY per share, which is less than its pricing, while the product value expected by investors, i.e., the maximum selling price acceptable to investors, is 1.0124 CNY, which is greater than its pricing. This shows that the current pricing is reasonable under the risk-neutral assumptions for both sides, which not only motivates investment desires but also leaves theoretical profit margins. Nevertheless, reasonable pricing is not necessarily an optimum one, the profit margin of this product can be further extended in the following two ways:

(1) Assuming the same terms and time of issuance, the issuer can make more profits by increasing the price per share, up to 1.0124 CNY.

(2) Assuming the same price and income, the issue can reduce the product's cost to increase the profit margin by adjusting corresponding parameters and the time of issuance. The adjustment of product parameters will hinge on the sensitivity analysis of the product value.

4 SENSITIVITY ANALYSIS

Since the value of the product at the time of issuance is the cost of issuing the product, the product value at the time of issuing directly determines the profit of the issuer if the income of issuing the product is fixed, it is easy to find from the product pricing model and related terms that the product value is affected by the initial price of the underlying asset S_0, the volatility σ, the required rate of rate r, the maturity time T and the knock out barrier price H at the time of issuance. Therefore, it is viable to determine the appropriate time of product issuance and relevant product terms by considering the sensitivities of the above five factors on the product value from the perspective of minimizing the product hedging cost.

4.1 The Initial Price

Assuming that the initial price of the underlying asset S_0 is between 2000 and 3000 with a 10-unit change scale while other variables stay constant, the curve of the product value concerning the initial price of the underlying asset is plotted in Fig. 1.

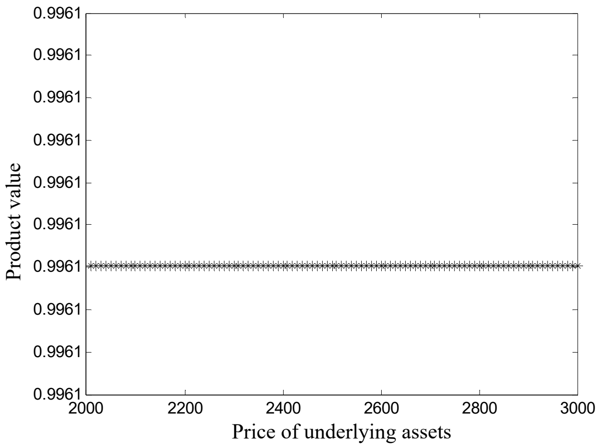

Figure 1 Impact of initial price on product value

The product value is not affected by the initial price of the underlying asset if the required rate of return $r = 2.8\%$ and the volatility $\sigma = 28.57\%$ stays unchanged.

4.2 The Volatility

Assuming that the volatility σ is between 0.1 and 0.6 with a 0.01 unit change scale while other variables stay constant, the curve of the product value concerning the volatility of the underlying asset is plotted in Fig. 2.

If the required rate of return remains constant, then as the volatility increases, the product value will first decrease and then increase. For example, if the required rate of return is 6.0%, the barrier price is $H = S_0 \times 111\%$, and the volatility increases from 0.1, then the product value starts at 1.0001 and decreases to a minimum of 0.9961 when the volatility reaches 0.22-0.30, afterward it rises again and reaches 0.9967 when the volatility is 0.5, but overall speaking, the speed of increase in product value is less than its speed of decreasing. This shows that the product value is high only if the volatility of the underlying asset is low because it is more likely for the rate of return to be near its maximum 14.9% if the volatility is low. As the rate of return is 5.9% in the knock-out range, which is higher than the 3.9% rate of return (which is also its minimum) in the barrier range, the product value can rebound and still increase even after hitting its bottom value if the volatility is high. However, since there is a large gap between the 5.9% rate of return after knocking out and the possible maximum 14.9% rate of return by staying in the barrier range, the increasing scale is limited at high volatility.

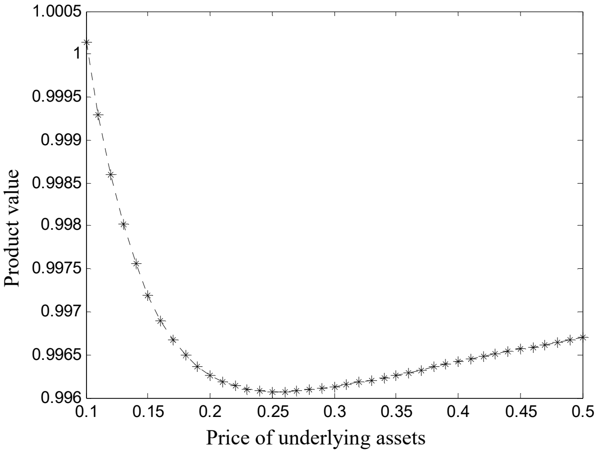

Figure 2 Impact of volatility on product value

4.3 The Required Rate of Return

Assuming that the required rate of return r is between 2% and 10% with a 0.1% unit change scale while other variables stay constant, the curve of the product value concerning the rate of return (the discount rate) is plotted in Fig 3. It shows that if the volatility and other variables remain constant, then the product value decreases as the required rate of return increases, with almost a constant trend of decreasing, i.e., the negative slope does not change. For example, when the volatility is 0.2875, the product value drops from 1.0165 to 0.9761 as the required rate of return rate increases from 2% to 10%.

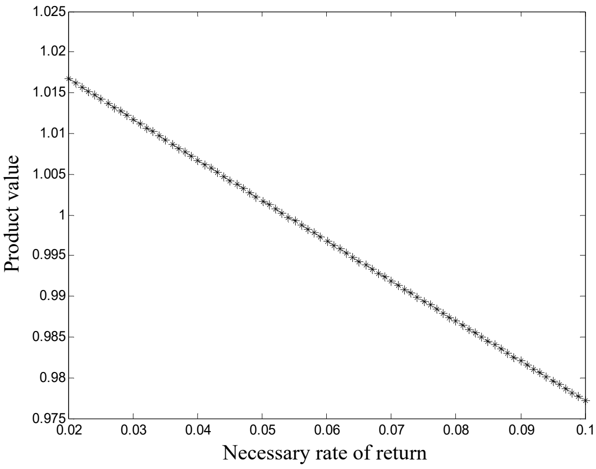

Figure 3 Impact of RRR on product value

4.4 The Maturity Time

Assuming that the maturity time is between 1 and 365 days (based on statistics of structured products sold by CMB) with a 1-day unit change scale, the curve of the product value concerning the maturity time is plotted in Fig. 4.

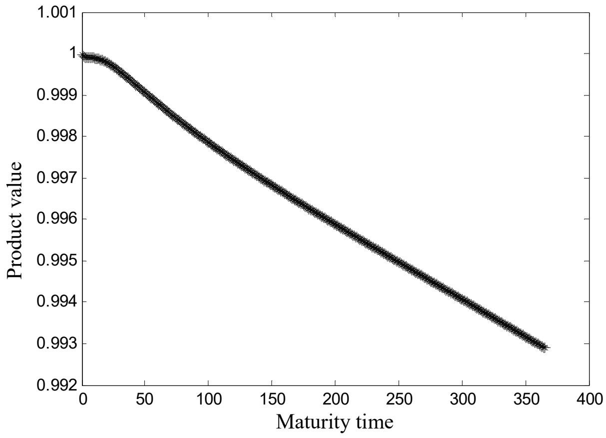

Figure 4 Impact of maturity time on product value

If the volatility $\sigma = 28.57\%$ and the required rate of return 6.0% stay unchanged, then the product value gradually decreases as the maturity time increases. When the maturity time starts to increase at 1-day, the product value first rises (from 1.0000). This is because there is essentially no time value if the maturity time is just around 1 day, as the time is too short for any profit to be not negligible, consequently the product value is close to the buying capital. Afterward, the product value starts to decrease, especially after about 15 days, the product value gradually decreases at an almost constant rate as the maturity time extends, i.e., the negative slope does not change. It eventually drops to 0.9929 when the maturity time is 365 days. This is because volatility is likely to be low if the maturity time is short, thus the time cost is also low and so the values of short-term products change little, while as the maturity time extends, knocking out is more likely to happen and the time cost for initial buying capital also increases, which leads to a decrease in the product value.

4.5 The Barrier Price

If all other factors remain constants, then changes in barrier prices mean changes in maximum possible rates of return for investors. Assuming that the product barrier price is between 1.00 to 1.40 times the initial price S_0 with a 0.001 unit change scale, the curve of the product value concerning the barrier price is plotted in Fig. 5.

If the other four aforementioned variables stay unchanged and suppose that the barrier price $H = n \times S_0$, then as the barrier price increases, the product value first decreases and then increases. For example, if the required rate of return is 6.0%, the volatility is 28.57%, and the maturity time is 188 days, then as the barrier price increases from $H = 1 \times S_0$, the product value will first drop from 0.9990 and hit its minimum 0.9961 when the barrier price reaches around 1.103~1.130 times of the initial price, afterward it climbs up again to 1.0173 when $H = 1.4 \times S_0$. The overall trend of increase is faster than that of decreasing, which is due to the low possibility of knocking out at high barrier prices, thus leading to a higher profit margin and accordingly a higher product value.

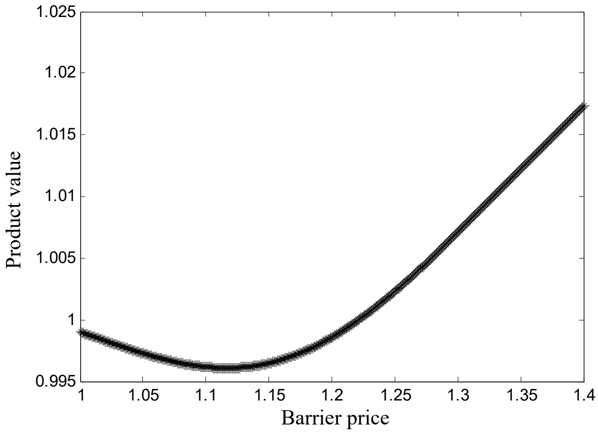

Figure 5 Impact of barrier price on product value

In summary, the required rate of return, as well as the maturity time, are the two most prominent factors, the volatility and the barrier prices also have influences; in contrast, the product value is essentially unaffected by the initial price, which makes it a negligible factor for issuance. Hence for equity structured products, aside from raising the selling prices, means of obtaining more profits include: (1) Picking an issuance time when the underlying asset has high volatility and (2) Extending the maturity time for as long as possible to lower the product cost, and determining the barrier price after all other factors have been decided. As per our calculations, if the volatility and the required rate of return are respectively fixed at $\sigma = 28.57\%$ and $r = 6.0\%$, then it is appropriate to set the barrier price to either 111%, 112%, or 113% of the initial price, yet higher barrier prices imply lower knocking our possibilities, it is optimum to set the barrier price to 111% of the initial price to minimize the cost of issuance.

5 CONCLUSIONS AND PROSPECTS

For the CMB 104563 equity structured product, we have produced a semi-analytic solution to its product pricing and explained using the market segmentation theory that the reason behind the difference between its cost and price is the gap in the required rates of return demanded by the issuer and investors. By analyzing the sensitivity of the product value to its parameters, we showed that the issuer can increase the profit margin by either raising the product price to 1.0124 CNY per share, or extending the product maturity date for as long as possible, or choosing to issue the product in the time of high volatility of the underlying asset.

Nevertheless, shortcomings of this article include: (1) Due to the complicated properties of structured products, we were unable to derive a universal pricing model, nor did we carry out relevant empirical analysis for other types of structured products to verify our pricing strategy. (2) The analysis of product parameters is based on the premise that the profit is maximized when the cost of an incoming product is minimized; other comprehensive factors such as influences of selling prices on profits were not taken into accounts; (3) The risk-neutral assumption does not fully fit behaviour patterns of individual investors. These are all possible directions for future research.

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