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Original scientific paper

Optimal Remote Qubit Teleportation Using Node2vec

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Abstract: Much research work is done on implementing quantum teleportation and entanglement swapping for remote entanglement. Due to dynamical topological changes in quantum networks, nodes have to construct the shortest paths every time they want to communicate with a remote neighbour. But due to the entanglement failures remote entanglement establishment is still a challenging task. Also as the nodes know only about their neighbouring nodes computing optimal paths between source and remote nodes is time consuming too. In finding the next best neighbour in the optimal path between a given source and remote nodes so as to decrease the entanglement cost, deep learning techniques can be applied. In this paper we defined throughput of the quantum network as the maximum qubits transmitted with minimum entanglement cost. Much of research work is done to improve the throughput of the quantum network using the deep learning techniques. In this paper we adopted deep learning techniques for implementing remote entanglement between two non-neighbour nodes using remote qubit teleportation and entanglement swapping. The proposed method called Optimal Remote Qubit Teleportation outperforms the throughput obtained by the state of art approach.

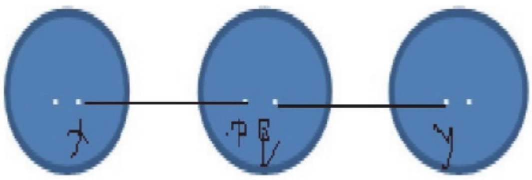
Keywords: bell state measurement result; fidelity; qubits; quantum embedding; remote entanglement; teleportation

1 INTRODUCTION

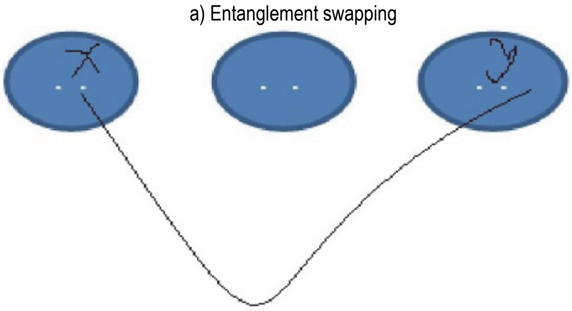
Long Distance communication is the main application of Quantum networks. Quantum teleportation is essential for quantum communications. In long distance communications routing of the qubits is done through sharing the Einstein-Podolsky-Rosen paradox (EPR) pairs. Quantum entanglement is a technique where qubits are sent from node A to node B. The probability that the entanglement link remains success for an amount of time between the nodes depends on the factors like:

1. Distance d between the nodes,
2. Number of intermediate nodes.

Hence during estimating a shortest path, dynamic topology model is quite useful for the nodes for rebuilding the network information. If A and B shared an entanglement then A prepares two qubits and sends one qubit to B using quantum teleportation. Hence through entanglement we can create a one-time used virtual quantum communication link between two parties. Entanglement swapping is a technique where two nodes can share qubits via an intermediate node and remote entanglement on the two links can be established applying entanglement swapping at the intermediate node. In Fig. 1 X and Y share using the virtual link.



a) Entanglement swapping



b) Entanglement swapping

Figure 1 Entanglement swapping and entanglement swapping

First a link is created between X and Z using the memory bits (x, p) This entanglement is shared through intermediate node Z by using the memory bits (p, q) and (y) [20]. The Bell state measurement result at each 1-hop quantum neighbour of a node X and result of node X are calculated. The results are sent to the destination node Y using the classical channel. The destination node performs unitary transformations to recover the original state [1]. Besides finding an optimal path between source and destination, one of the main objectives of any routing techniques is to minimize the number of memory bits used at each node.

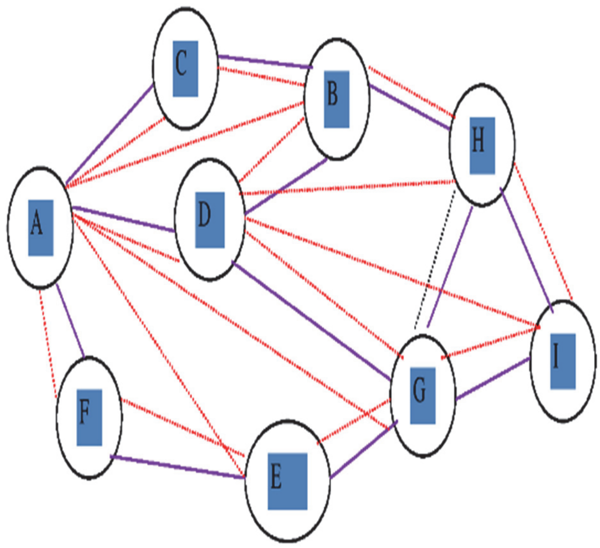


Figure 2 Multihop teleportation

Consider Fig. 2. The thick lines indicate the quantum channels and dotted lines indicate the classical channel. Circles indicate nodes in the network. Let A want to communicate to D. There is no direct quantum channel between them. But a classical channel exists. There is both classical channel and quantum channel between A and its immediate neighbours C, B. After obtaining the virtual quantum link information, nodes apply local measurements to selects the optimal virtual link at that instance of time for it to reach a destination node. Hence A decides which amongst its immediate neighbours to share the EPR pair.

Let C be the optimal node chosen by A. A share EPR pairs with C, A and C perform bell measurements and share them directly with B using the classical channel. Now node B performs the unitary transformations. This method reduces the transmission time too since only the destination is performing the unitary transmissions. Generally, the entanglement swapping can be represented as follows:

$$\begin{aligned}|\varphi\rangle_{h2} = \ & |\varphi\rangle_{12} \otimes |\varphi\rangle_{34} \otimes |\varphi\rangle_{56} \otimes |\varphi\rangle_{78} \otimes |\varphi\rangle_{9,10} \\ = \ & \left(a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle\right)_{12} \otimes \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)_{34} \\ & \otimes \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)_{56} \otimes \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)_{78} \otimes \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)_{9,10}\end{aligned}$$

And its necessary quantum circuit is as shown in Fig. 3.

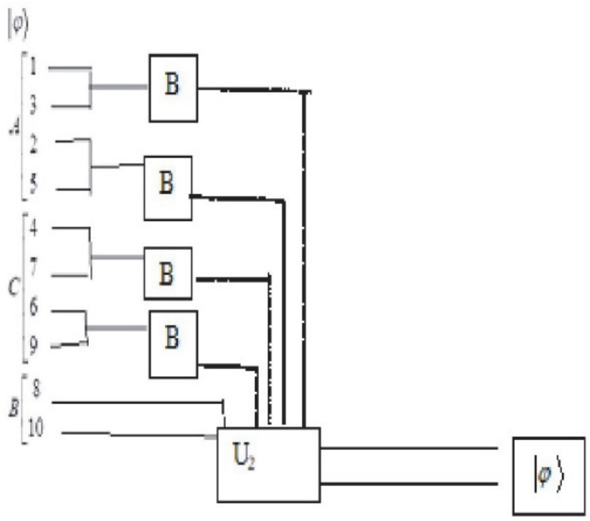


Figure 3 Quantum/circuit for two - hop quantum teleportation

The bell measurement results at B will be in any of the four possible ways

***Bk* (*k* = 1, 2, 3, 4) as follows**

$$B_1 = \left\{\left|\phi^+\right\rangle\left|\phi^+\right\rangle, \left|\phi^+\right\rangle\left|\phi^-\right\rangle, \left|\phi^-\right\rangle\left|\phi^+\right\rangle, \left|\phi^-\right\rangle\left|\phi^-\right\rangle\right\},$$

$$B_2 = \left\{\left|\phi^+\right\rangle\left|\varphi^+\right\rangle, \left|\phi^+\right\rangle\left|\varphi^-\right\rangle, \left|\phi^-\right\rangle\left|\varphi^+\right\rangle, \left|\phi^-\right\rangle\left|\varphi^-\right\rangle\right\},$$

$$B_3 = \left\{\left|\varphi^+\right\rangle\left|\phi^+\right\rangle, \left|\varphi^+\right\rangle\left|\phi^-\right\rangle, \left|\varphi^-\right\rangle\left|\phi^+\right\rangle, \left|\varphi^-\right\rangle\left|\phi^-\right\rangle\right\},$$

$$B_4 = \left\{\left|\varphi^+\right\rangle\left|\varphi^+\right\rangle, \left|\varphi^+\right\rangle\left|\varphi^-\right\rangle, \left|\varphi^-\right\rangle\left|\varphi^+\right\rangle, \left|\varphi^-\right\rangle\left|\varphi^-\right\rangle\right\},$$

Much research is not done in the area of teleportation with nodes that do not share entanglement pairs directly. Hence we propose a method to maximize throughput in the quantum networks with limited memory qubits, entanglement link establishing cost and maximum entanglement fidelity.

The unitary transformations $U2$ can be expressed as:

$$I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\left(\sigma_x\right)_8 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$\left(\sigma_x\right)_{10} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\left(\sigma_x\right)_8 \left(\sigma_x\right)_{10} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

In Section II we presented various related studies in the area of remote entanglement. In Section III we presented the proposed method for remote teleportation between non-neighbours and its network model. The proposed method maximizes the throughput in the quantum networks with limited memory qubits, entanglement link establishing cost and maximum entanglement fidelity. In Section IV we presented the simulation results and a comparative study. In Section V we presented Conclusions and future study.

2 RELATED STUDIES

Several methods are invented by researchers for successful implementation of quantum communications using optimal paths. In [1] Marlon David et al., proposed a quantum wireless multihop teleportation method using under decoherence. In [2] Schoute et al., investigated various Shortcuts to quantum network routing. In [3] Gyongyosi, L. and Imre, S studied various entanglement availability differentiation services for the quantum Internet. In [4] Christensen, K. S. et al., implemented a coherent router for quantum networks with superconducting qubits. In [5] Metwally, N. implemented various entanglement routers via a wireless quantum network based on arbitrary two qubit systems. In [6] Xiong, P. Y. et al., implemented a routing protocol for wireless quantum multihop mesh backbone network based on partially entangled GHZ state. In [7] Zhao, X. et al., analysed various efficient shortest paths on massive social graphs. In [8] Zhan, H. T. et al., implemented a Multi-hop teleportation based on W state and EPR pairs. In [9] Gao, X. Q. et al., implemented teleportation of entanglement using a three particle entangled W state. In [10] Li, Z. Z. et al., realized a Multi-user quantum wireless network communication based on multi-qubit GHZ state. In [11] Gyongyosi, L. and Imre, S. implemented Entanglement-Gradient Routing for Quantum Networks. In [12] Verdon, et al., implemented a universal training algorithm for quantum deep learning. In [13] an improved Quantum Stop-Wait Communication Protocol is realized. In [14] Guoan Zhao proposed a quantum secure communication protocol based on Single-photon. In [15] Tao Fang implemented a Controlled Quantum Secure Direct Communication Protocol Based on Extended Three-particle GHZ State Decoy. In [16] Ting Wang, et al., realized a semi-quantum Key distribution protocol based on Bell States. In [17] Daehyon Kim studied normalization of input vectors in deep belief networks (DBNs) for automatic incident detection. In [18] Lei Li presented an extensive review on recent deep learning applications. In [19] Daehyon Kim, proposed a method for application of deep neural network model for automated intelligent excavator. In [20] Hermans, S. L. N. et al., realized a Qubit teleportation between non-neighbouring nodes in a quantum network. In [21] Grover and Leskovec, J. implemented a Node2vec: Scalable feature learning for networks. In [22] Majtey, P. et al., analyzed the relationship between entanglement, energy, and level degeneracy in Two-Electrons Systems. In [23] Fenxiang Fu et al. implemented multi-hop non-destructive teleportation between terminal nodes equipped with limited technology. In [24] Tu Nguyen, N. et al., proposed a multiple-entanglement routing framework for quantum networks. In [25] Ewert, et al., proposed an ultrafast long-distance quantum communication with static linear optics. In [26] Mousolou studied entanglement fidelity and measure of entanglement. In [27] Dai, W., Peng, T. and Win, M. Z. proposed optimal remote entanglement distribution. In [28] Caleffi, M. proposed optimal routing for quantum networks. In [29] Das, S. et al., studied robust quantum network architectures and topologies for entanglement distribution. In [30] Chen, Y.-A. et al., studied an integrated space to-ground quantum communication network over 4600 kilometres.

In [31] Amer, O. et al., proposed a method for efficient routing for quantum key distribution networks. In [32] Akiba, T. et al., implemented fast exact shortest-path distance queries on large networks by pruned landmark labelling. In [33] Yangming Zhao, et al. proposed a E2E fidelity aware routing and purification for throughput maximization in quantum networks. In [34] Zhao and Qiao, C. implemented a redundant entanglement provisioning and selection for throughput maximization in quantum networks. In [35] Pant, M. et al., implemented a routing entanglement in the quantum internet. In [36] Laszlo Gyongyosi, et al., implemented a decentralized base-graph routing for the Quantum Internet. In [37] Thomas Krauss and Joey McCollum proposed a method for solving the network shortest path problem on a Quantum Annealer. In [38] Jie Wu proposed an adaptive fault-tolerant routing in cube based multi computers using safety vectors. In [40] Thulitha Senevirathna et al., proposed an event-driven source traffic prediction in machine-type communications using LSTM Networks.

In [41] Daehyon Kim proposed Deep Learning Neural Networks for Automatic Vehicle Incident Detection. In [42] Al-akashi Falah proposed a method for improving learning performance in neural networks. In [43] Gao Jun proposed a credible nearest neighbour query in uncertain network. In [44] Bai Xue et al., implemented a new clustering model based on Word2vec mining on Sina Weibo users' Tags. In [45] Yang Xianhui et al., proposed a method for predictive routing for mobile sink routing algorithm. In [46] Chen Yu and Duan Zhemin realized a IP network topology link prediction based on improved local information similarity algorithm. In [47] Chen Wen implemented a continuous reverse k-nearest-neighbour query in dynamic road network. In [48] Hyun Joo Park and Seong Cheol Ki proposed an efficient packet transmission protocol using reinforcing learning in Wireless Sensor Networks. AI techniques are applied in various event predicting/detecting applications.

In [49] Yong Suk Kim et al., studied on a deepfake-based deep learning algorithm for medical data manipulation detection. In [50] Heba M. Afify et al., proposed a multi-images recognition of breast cancer histopathological via probabilistic neural network approach.

3 TELEPORTAION BETWEEN NON-NEIGHBOUR NODES

3.1 Deep Neural Network Model

Let $QN = (V, E, C, K, M)$ denote the undirected quantum network where

a) $|V|$ denotes the set of nodes like sender, receiver and intermediate nodes,
b) K denotes number of nodes in the network and
c) M denotes edges with positive edge cost.

We assumed a quantum network with $d(x, y)L_i = 2^{QEL_i - 1}$ [37, 38] where $d(x, y)$ is the edge distance between x, $y \in V$ and QEL_i is the level of entanglement between x, y. Also we assumed a set S^* containing the edges with an QEL_i where $i = 1, ..., r$ between two nodes with an edge E_j spanned by r intermediate nodes. $|E|$ denotes the set of edges between the nodes with $E_r^* \in S^*$.

Let vector x denote a vector with minimum energy. Then use a homogenous quadratic polynomial [39] $H = \arg x \min \{xT, Qx\}$ where $x \in \{0, 1\}$ and $Q \in \mathbb{R}\ N \times N$. In this paper the main objective function is to minimize the homogenous quadratic polynomial. The cost of the communication link depends on high probability rates of establishing successful entanglement links with its neighbour nodes

Let $Ø(x)$ denote the position of node x. Let $Ø(y)$ denote the position of node y. Let u_j denote the j-th neighbor of x with position $Ø(u_j)$. Let v_j denote the j-th neighbor of y with position $Ø(v_j)$. Then $p(Ø(x), Ø(y))$ the probability that QEL_i level of entanglement exists between x and y [37, 38] is defined as:

$$p\left(\varnothing\left(x\right),\varnothing\left(y\right)\right)=\frac{d(\varnothing\left(x\right),\varnothing\left(y\right)^{-k}}{\sum d(\varnothing\left(x\right),\varnothing\left(z\right)}+c_{\varnothing(x),\varnothing(y)} \tag{1}$$

where

$$c_{\varnothing(x),\varnothing(y)}=P_{rQEL_i}\left(E\left(x,y\right)-\frac{d(\varnothing\left(x\right),\varnothing\left(y\right)^{-k}}{\sum d(\varnothing\left(x\right),\varnothing\left(z\right)} \tag{2}$$

where $P_{rL_i}\left(E\left(x,y\right)\right.$ [37, 38] denote the probability that nodes x, y are connected through an $QELi$ level entanglement in the QN.

With a given fidelity F in an entangled link E_h the maximum entangled states required per second at a node Uk $(Q(F)(Uk))$ should be less than the maximally existing entangled states per second $(Q(F)(E^*h)$ $\left(E_h^*\right)$ [38]. That is

$$\sum\nolimits_{k\in K}\sum\nolimits_{f\in £_h} C_{k,h}^{f} Q\left(F\right)\left(Uk\right)\le Q\left(F\right)\left(E*h\right)\left(E_h^*\right) \tag{3}$$

where $£$ denotes the maximally entangled states.

Let C [38] represent the minimized cost if

$$C = \min \sum_{k \in K} \sum_{f \in £_h} \sum_{h \in E} \left(1 - P_{rQEL_i}\left(E_h^*\right)\right) C_{k,h}^f \tag{3}$$

The process is repeated until the destination is reached.

The proposed method is based on a deep neural network model. It is a feed forward neural network which contains an input layer, hidden layer and output layer. The network model is trained after collecting some known set of $ol \le K$ number of optimal positions $p\left(\varnothing(x), \varnothing(y)\right)$ in the graph G. We computed optimal positions from these known positions to remaining nodes in the network to form $ol*(K-1)$ training pairs. We applied four binary operations $\left\{p\left(\varnothing(x)+\varnothing(y)\right)\right\}/2, p\left(\varnothing(x)-\varnothing(y)\right), p\left(\varnothing(x)*\varnothing(y)\right)$ and $p\left(\varnothing(x), \varnothing(y)\right)$ and used Mean Square error MSE [41] to estimate the quality of the predictor for measuring the average of the squares of the difference between the estimator and the estimated. Stochastic gradient descent SGD is used as the optimizer. We applied node2vec embedding's with default number of random walk as $\gamma = 10$.

We set Return parameter (rp) = 1, In-out parameter (ioq) = 1, learning rate = 0.001, embedding_dimensions = 50, num_epochs = 10.

While choosing the intermediate's nodes, an optimal node is preferred. The optimality depends on the following factors

a) maximally entangled [37, 38] states per second with a fidelity F through an entangled link,
b) Minimum cost to establish a quantum link,
c) Minimum failure rates of the virtual links,
d) Optimal memory qubits at intermediate nodes.

3.2 Optimal Remote Qubit Teleportation (OMRQT)

1. For each source destination pairs repeat the following process:
2. For one source x and destination z pair repeat the following until destination is reached
3. Rebuild network topology dynamically Let hop = {1, 2, 3, 4, 5, 6, 7, …, deg(G)} denotes the hop distance
4. For each sender x at position $\varnothing(x)\text{hop} = \phi(x)$ select an entangled pair to the appropriate next node y at position $\varnothing(x)\text{hop+1} = \phi(y)$, and make a Bell State Measurement of the qubit it wishes to send and its qubit of the entangled pair.
5. We assume that node x may appear l times in the process to forward data to the neighbour.
6. Let C [38] represent the cost of establishing a quantum link between x and y and x uses a classical communication network to forward the Bell State Measurement result to node y. That is
7. $C = \min \sum_{k \in K} \sum_{f \in £_h} \sum_{h \in E} \left(1 - P_{rQEL_i}\left(E_h^*\right)\right) C_{k,h}^f$
8. Let C $\phi(x)$, $\phi(y)$ hop represent cost of an entanglement swapping for a node x with neighbour y which is at a hop length hop.
9. Let variable length matrix X denote these entanglement swapping cost C

$$X = \begin{bmatrix} C_{\phi(1),\phi(1_1),1} & C_{\phi(1),\phi(1_2),1} & \cdots & C_{\phi(1),\phi(1_{\text{maxoutdeg}}),1} \\ C_{\phi(2),\phi(2_1),1} & C_{\phi(2),\phi(2_2),1} & \cdots & C_{\phi(2),\phi(2_{\text{maxoutdeg}},1} \\ \cdots & \cdots & \cdots & \cdots \\ C_{\phi(|V|),\phi(|V|_1),1} & C_{\phi(|V|),\phi(|V|_2),1} & \cdots & C_{\phi(|V|),\phi(|V|_{\text{maxoutdeg}}),1} \end{bmatrix}$$

10. Vertex y at position $\varnothing(x)\phi(x_y)$ which is a neighbour of x is chosen as the best 1 – hop neighbour of node x if it satisfies the constraints
- Maximum entanglement fidelity utility [38]
- $\sum_{k \in K} \sum_{f \in £_h} C_{k,h}^f Q(F)(Uk) \le Q(F)(E*h)\left(E_h^*\right)$
- Minimum cost of the entanglement [38] with level Eli

  $$H_C = \left(\sum_{a=1}^{|V|} \min \sum_{b=1}^{\text{maxoutdeg}_a} C_{\phi(a),\phi(a_b),1} \right)$$

  where $C = \min \sum_{k \in K} \sum_{f \in £_h} \sum_{h \in E} \left(1 - P_{rQEL_i}\left(E_h^*\right)\right) C_{k,h}^f$
- Minimum failure rates of the virtual links
- Optimal memory qubits at intermediate nodes.

11. Implement a biased random walk with initial Return parameter (rp) = 1, In-out parameter (ioq) = 1.
12. Apply binary operations $\left\{p\left(\varnothing(x)+\varnothing(y)\right)\right\}/2$, $p\left(\varnothing(x)-\varnothing(y)\right)$, $p\left(\varnothing(x)*\varnothing(y)\right)$ and $p\left(\varnothing(x), \varnothing(y)\right)$.
13. Generate training data set using the random walk with num_epochs = 10.
14. Implement the embedding model with learning rate = 0.001, embedding_dimensions = 50.
15. Train the model and analyse the learning Embeddings using by calculating Mean Square error MSE [41] to estimate the quality of the predictor for measuring the average of the squares of the difference between the estimator and the estimated and stochastic gradient descent SGD as the optimizer.
16. The receiver uses the BMR. The bell measurement results at B will be n any of the four possible values Bk (k = 1, 2, 3, 4) as follows

$$B_1 = \left\{\left|\phi^+\right\rangle\left|\phi^+\right\rangle, \left|\phi^+\right\rangle\left|\phi^-\right\rangle, \left|\phi^-\right\rangle\left|\phi^+\right\rangle, \left|\phi^-\right\rangle\left|\phi^-\right\rangle\right\},$$
$$B_2 = \left\{\left|\phi^+\right\rangle\left|\varphi^+\right\rangle, \left|\phi^+\right\rangle\left|\varphi^-\right\rangle, \left|\phi^-\right\rangle\left|\varphi^+\right\rangle, \left|\phi^-\right\rangle\left|\varphi^-\right\rangle\right\},$$
$$B_3 = \left\{\left|\varphi^+\right\rangle\left|\phi^+\right\rangle, \left|\varphi^+\right\rangle\left|\phi^-\right\rangle, \left|\varphi^-\right\rangle\left|\phi^+\right\rangle, \left|\varphi^-\right\rangle\left|\phi^-\right\rangle\right\},$$
$$B_4 = \left\{\left|\varphi^+\right\rangle\left|\varphi^+\right\rangle, \left|\varphi^+\right\rangle\left|\varphi^-\right\rangle, \left|\varphi^-\right\rangle\left|\varphi^+\right\rangle, \left|\varphi^-\right\rangle\left|\varphi^-\right\rangle\right\},$$

to recover the state of the teleported qubit.

17. If the receiver is not the destination node z the process continues with step 2.
18. Stop.

4 SIMULATION RESULTS

We assumed the parameters the same as in [33] as follows:

a) N quantum nodes, by randomly connecting them with $2N$ quantum links.
b) Assumed 200 quantum nodes and 50 source destination pairs in the network.
c) The fidelity of a quantum link F_i [37, 38] and quantum link capacity is evenly distributed.
d) each node had 100 units of quantum memory.
e) The threshold fidelity is assumed to be less than 1 [0.9].
f) Network had a max_in degree as 2, 4, 6, 8 and 10.

We implemented a method to achieve maximum network throughput for remote entanglement between two non-neighbour nodes. We used deep learning techniques for implementing remote entanglement between two non-neighbour nodes using remote qubit teleportation and entanglement swapping. Generated training data set using the random walk with num_epochs = 10.

We implemented embedding model with learning rate as 0.001 and embedding_dimensions as 50. We trained the model and analyzed the learning embeddings using by calculating. Mean Square error MSE to estimate the quality of the predictor for measuring the average of the squares of the difference between the estimator and the estimated and stochastic gradient descent SGD as the optimizer. For testing the pairs we employed the same method as used for training the model. We generated approximately 5 percent of training data set pairs as test set pairs.

We compared the simulation work with the E2E Fidelity aware Routing and Purification (EFiRAP) [33] and Redundant entanglement provisioning and selection REPS [34]. We simulated the method with different number of nodes.

Fig. 4 shows that the throughput increases with the number of nodes. But when the network becomes denser the throughput starts decreasing. The throughput achieved by the OMRQT method outperforms the method in [33] since the cost of establishing the link, and link selection error rate is minimized in the OMRQT method due to training the network for link selection using deep neural network model.

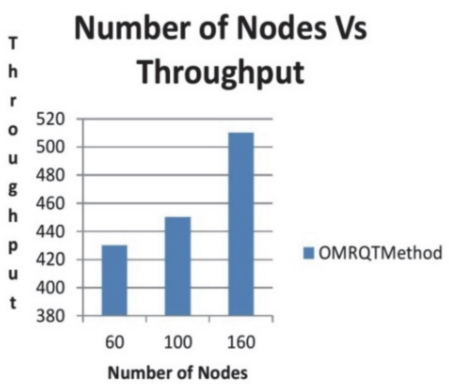


Figure 4 Number of nodes vs throughput

EFiRAP has a throughput less than 400 qbs and REPS had a throughput less than 200qbs for N = 60 where as OMRQT method had a throughput of 430 qbps and EFiRAP had a throughput of 400 qbps and REPS had a throughput of less than 200 qbps when N = 100 whereas OMRQT method had a throughput of 450 qbps. For N = 160 EFiRAP had a throughput of more than 400 qbps and REPS had a throughput less than 100 qbps and OMRQT method had a throughput of 510 qbps.

With the above values, Fig. 4 proves that the OMRQT method attained higher throughput than the EFiRAP and REPS methods.

We compare the network throughput of the OMRQT method with [33, 34]. The entanglement links with fidelity above a threshold will be contributing to the achieved throughput. With the threshold 0.9 we simulated the method and Fig. 5 shows the results. With a link capacity of 10 EFirAP had a throughput of less than 200 qbps, REPS had a throughput of less than 50 qbps whereas the OMRQT method had a throughput of 200 qbps. With the link capacity of 30 EFiRAP had a throughput of 400 qbps, REPS had a throughput of less than 50 qbps and OMRQT method had a throughput of 200 qbps.

With link capacity of 60 EFiRAP had a throughput of 600 qbps, REPS had a throughput almost equal to 200 qbps and OMRQT, method had a throughput *f* 750 qpbs. With the above values, Fig. 5 proves that the OMRQT method attained higher throughput than the EFiRAP and REPS methods.

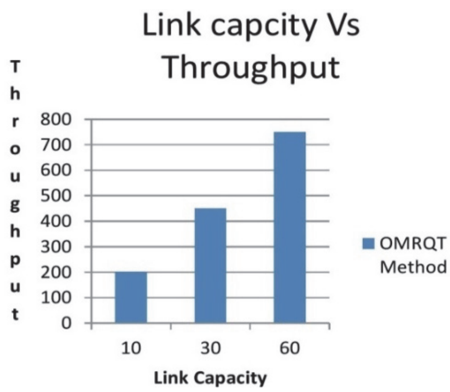


Figure 5 Link capacity vs throughput

Fig. 6 shows that with the increase in the entanglement link capacity, the Channel Utilization increases. We compare the entanglement link capacity of the OMRQT method with [33, 34]. With a link capacity of 10 EFiRAP achieved a channel utilization of less than 40%, REPS had a utilization of less than 100% whereas the OMRQT method had a utilization of 30%.

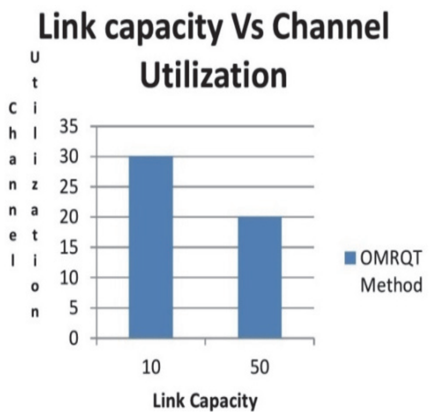


Figure 6 Entanglement link capacity vs channel utilization

With a link capacity of 50 EFirAP achieved a channel utilization of less than 30%, REPS had a utilization of less than 60 whereas the OMRQT method had a utilization of 20%. With the increase in the entanglement paths between each Source and destination edges the throughput of the OMRQT method outperforms the method in [33, 34]. Fig. 7 shows the results.

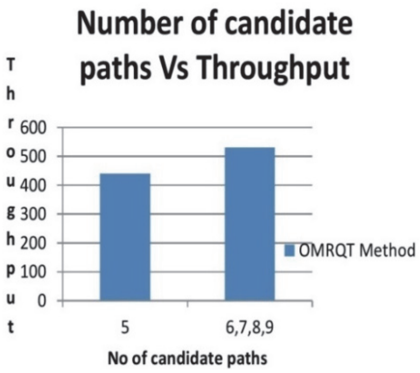


Figure 7 Number of candidate paths vs throughput

We compare the throughput of the OMRQT d method with [33, 34]. With the number of candidate paths as 5 EFirAP had a throughput of 400 qbps, REPS had a throughput of less than 150 qbps, whereas the OMRQT method had a throughput of 440 qbps. With the number of candidate paths more than 6, 7, 8 and 9 EFirAP had a throughput of less than 500 qbps, REPS had a throughput of less than 150 qbps, whereas the OMRQT method had a throughput of 530 qbps.

5 CONCLUSIONS

In this paper we implemented a method to achieve maximum network throughput for remote entanglement between two non-neighbour nodes. Generally, nodes know only about their neighbouring nodes and finding best neighbour node into its optimal path to destination is a challenging task. This path between a given source and remote nodes should decrease the entanglement cost. Hence to predict such optimal links, deep learning techniques are a great choice. We used deep learning techniques for implementing remote entanglement between two non-neighbour nodes using remote qubit teleportation and entanglement swapping. The results proved that the Remote Qubit Teleportation method outperforms the throughput obtained by the existing research mentioned in [33].

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