Different Forms of the Correction Factor Used to Describe Simultaneous Heat and Mass Transfer

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In the literature, different calculation relationships are often presented to describe the same process taking place in the given equipment. This article presents the different forms of the correction factor (Ackermann correction factor) used in describing the processes in which heat and mass transfer occur simultaneously. The form of the equations determining the value of the Ackermann correction factor basically depends on the choice of the orientation of the coordinate system. The article presents the derivation of two forms of equations, on the basis of which the value of this factor is usually determined. Finally, the article also contains the equations that describe the simultaneous transfer of heat and mass. These are used in the design of various types of equipment that are part of the various industrial technologies (e.g., in agricultural, chemical, and food industries). From the point of view of these equations, the article mainly emphasizes that, if it is necessary to work with several equations describing the same process, care should be taken to ensure that these equations are compatible with each other. In this case, compatibility means that there is no need to use a sign correction when substituting the result from one equation into another.

Keywords:

correction factor, Ackermann correction factor, interfacial area, simultaneous heat and mass transfer, corrected form of Newton's law of cooling

Introduction

In many industrial applications, the heat input or output must be ensured. This is done using various industrial equipment¹⁻³. When designing such equipment, it is necessary to know the amount of energy transferred through the boundaries of the balance system that can represent either the whole equipment or only a certain part of it. A special case is the situation where there is a simultaneous transfer of heat and mass through the boundaries of the system (the interfacial area)⁴⁻⁷. In such cases, the amount of energy that passes through the interfacial area can be expressed by an equation analogous to Newton's law of cooling. This equation can be referred to as the corrected form of Newton's law of cooling, in which the Ackermann correction factor appears $(\theta_{\tau})^8$. This factor expresses the effect of mass transfer on the value of the heat transfer coefficient determined according to the equations valid for the cases where there is no simultaneous transfer of heat and mass in the system⁹⁻¹². The form of the equation that determines the value of this correction factor can be different, including its mark^{13–16}. In principle, it is possible to find in the literature an equation defining the value of θ_T , which is written either in the form:

$$\theta_T = \Phi_T / \left[\exp(\Phi_T) - 1 \right] \tag{1}$$

or an equation is given in the form:

$$\theta_T = \Phi_T / \left[1 - \exp(-\Phi_T) \right]$$
 (2)

By analyzing the above equations, it can be easily verified that in the case of Eq. (1), if $\phi_T \rightarrow 0$, then $\lim_{\Phi_T \rightarrow 0} (\theta_T) = \lim_{\Phi_T \rightarrow 0} \left(\frac{\Phi_T}{\exp(\Phi_T) - 1} \right) = \frac{1}{\exp(\Phi_T)} = 1$. Similarly, it can be written for the case when $\phi_T \rightarrow \infty$. In this case, then $\lim_{\Phi_T \rightarrow \infty} (\theta_T) = \frac{1}{\exp(\Phi_T)} = 0$. And last but not least, if $\phi_T \rightarrow -\infty$, then $\theta_T = -\phi_T$. If Eq. (2) is analyzed, the following applies $\lim_{\Phi_T \rightarrow 0} (\theta_T) = \lim_{\Phi_T \rightarrow 0} \left(\frac{\Phi_T}{1 - \exp(-\Phi_T)} \right) =$ $= \lim_{\Phi_T \rightarrow 0} \left(\frac{\Phi_T}{1 - 1/\exp(\Phi_T)} \right) = \exp(\Phi_T) = 1$.

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Similarly, it can be stated for $\phi_T \rightarrow -\infty$. In this case, then $\lim_{\Phi_T \to \infty} (\theta_T) = \exp(\Phi_T) = 0$. Finally, if $\Phi_T \to \infty$, then $\theta_T = -\phi_T$. In general, it means that for the same value of ϕ_{τ} , different values of the Ackermann correction factor can be obtained in terms of the previous equations. It follows that it is important to know which form of the equation is applicable for any given case. It is well known that, if it is necessary to work with an equation unambiguously, it is essential to know the contexts which specify the given equation. In other words, it is necessary to know how the given equation was compiled, as well as the simplifying assumptions under which the relevant equation was derived. This article aims to present the contexts that specify the aforementioned equations determining the value of the Ackermann correction factor. The understanding of the contexts presented in this article (i.e., contexts concerning the simultaneous heat and mass transfer through the interfacial area) is important because the equations reported here are the basis for other variables reported in the literature, as well as the basis for various calculation methods used, for e.g., in the design of equipment. A typical representative of such equipment, in which simultaneous heat and mass transfer occur, is, for example, a drying plant. In a drying plant, the process of drying the wet material occurs, during which heat is simultaneously supplied to the dried material (i.e., heat transfer), and at the same time, moisture is removed from the dried material (i.e., mass transfer). The purpose of drying is to reduce the content of liquid (often water) in the material, which, for example, improves the preservation properties of agricultural and food products, but also improves the biochemical properties of, e.g., grains and seeds. Drying is a very widespread technological process that is also applied in other branches of industry. The drying process can often

be found in publications dealing with thermal processes. In reality, however, the drying process is one of the diffusion separation processes. The essence of these processes is precisely the mass transfer from one phase to another. However, the simultaneous heat and mass transfer can also be encountered in the design of condensers in which, for example, vapors condense in the presence of inert gases. In the case of vapor condensation, an inert refers to a substance that does not condense during the process.

Mass transfer can also generally be accompanied by a chemical reaction. However, the equations presented in this article are valid under the condition that there is no chemical reaction in the system. Furthermore, it should always be borne in mind that, if mass transfer occurs in the system through the interfacial area (the mass transfer from one phase to another), the mass transfer is always more or less connected with the energy transfer^{17–21}. This is because each material stream is generally a carrier of energy, which is confirmed by the energy balance equations.

Initial equations for determining the correction factor

In general, as mentioned previously, not only heat transfer, but also mass transfer through the interfacial area can occur in the system. This influences the amount of energy that passes through the interfacial area. The description of such processes can be performed using classical equations, which are, however, corrected by a correction factor. The expression of this factor results from the solution of equations (initial equations) taking into account the fact that, in the system, there is a simultaneous heat and mass transfer through the interfacial area. In this case, the initial equations are: the equation determining the energy flux:

$$\vec{e} = \vec{q} + \vec{N}\vec{h} = -\lambda_t \nabla T + \sum_{i=A}^n \vec{N}_i \vec{h}_i = -\lambda_t \nabla T + \sum_{i=A}^n \vec{N}_i h_i^\circ + \sum_{i=A}^n \vec{N}_i \Delta h_i^{mix}$$
(3)

the equation determining the total molar flux of substance *i*:

$$\vec{N}_{i} = \vec{J}_{i} + x_{i} \sum_{j=A}^{n} \vec{N}_{j} = -cE_{tim} \nabla x_{i} + x_{i} \sum_{j=A}^{n} \vec{N}_{j}$$
(4)

and the equation determining the enthalpy value of the pure substance *i*:

$$h_{i}^{\circ} - h_{iref}^{\circ} = h_{i}^{\circ} - 0 = h_{i}^{\circ} = \int_{T_{ref}}^{T} c_{pl}^{\circ} dT + \int_{p_{ref}}^{p} \left[v_{i}^{\circ} - T \left(\frac{\partial v_{i}^{\circ}}{\partial T} \right)_{p} \right] dp + \Delta_{\beta \to \gamma} h_{i}^{\circ} \left(T_{\beta \to \gamma} \right)$$
(5)

Eq. (3) is the basic equation for describing the technical problems in which heat and mass transfer occur simultaneously. This is an approximate equation that, from the point of view of technical practice, allows to obtain the acceptable results. The meaning of the terms appearing in the equation, as well as its derivation, is described in more detail in the literature^{4-6,8}. The subscript t in the quantity λ emphasizes that this is the value of the variable valid for the case of the turbulent flow of the fluid. In other words, Eq. (3) also applies to the case of turbulent fluid flow, which in practice occurs more frequently than laminar flow. The same is true for the quantity E_{iim} occurring in Eq. (4). Eq. (4) defines the value of \vec{N}_i , the knowledge of which is necessary for working with Eq. (3). Eq. (4) is the general equation. The meaning of the quantities appearing in it is described in more detail, for example, in the literature ^{4–6,22,23}. Eq. (5) represents a classical equation for determining the enthalpy of the pure substance *i*, the knowledge of which is also needed for working with Eq. (3). If the fluid flow is laminar, then, in these equations (i.e., Eq. (3) and Eq. (4)), $\lambda_t = \lambda$ and $E_{tim} = D_{im}$.

Assumptions and solutions of the initial equations

Based on the previous, the amount of energy passing through a given interfacial area, or the energy flux near the interfacial area, is determined. Solving these equations is simple if the following can be assumed:

a) pressure in the system is constant (i.e., p =constant and therefore $\int_{p_{ref}}^{p} \left[v_i^{\circ} - T \left(\frac{\partial v_i^{\circ}}{\partial T} \right)_p \right] dp = 0$),

b) change of the substances state from β to γ takes place at a temperature T_{ref} (i.e., $\Delta_{\beta \to \gamma} h_i^{\circ}(T_{\beta \to \gamma}) = \Delta_{\beta \to \gamma} h_i^{\circ}(T_{ref})$),

c) substances mixture can be considered ideal $\frac{n}{n}$

(i.e.,
$$\sum_{i=A} N_i \Delta h_i^{mix} = 0$$
),

d) there are steady conditions in the system (i.e., e = constant and $N_i = \text{constant}$),

e) quantities c, λ_i , and E_{tim} can be considered as constants (i.e., c = constant, $\lambda_i = \text{constant}$, and $E_{tim} = \text{constant}$), or these quantities can be replaced by mean values, which are subsequently treated as constants (i.e., $\overline{c} = \text{constant}$, $\overline{\lambda_i} = \text{constant}$, and $\overline{E_{tim}} = \text{constant}$), and

f) energy transfer, as well as mass transfer, takes place only in a direction perpendicular to the interfacial area (i.e., it is a one-dimensional task in which it can be assumed that the transfer takes place, for example, in the direction of the *z*-axis of the Cartesian coordinate system, which is perpendicular to the interfacial area).

According to the mentioned assumptions, Eq. (3) and Eq. (4) can be rewritten in the form:

$$N_{iz}\Big|_{z=w} = -cE_{tim} \frac{dx_i}{dz} + x_i \sum_{j=A}^n N_{jz}\Big|_{z=w} = -cE_{tim} \frac{dx_i}{dz} + x_i \sum_{j=A}^n N_{jw} = N_{iw} = \text{constant}$$
(6)

$$e_z\Big|_{z=w} = -\lambda_t \frac{dT}{dz} + \sum_{i=A}^n N_{iz}\Big|_{z=w} \overline{c}_{pi}^{\circ\gamma} \left(T - T_{ref}\right) + \sum_{i=A}^n N_{iz}\Big|_{z=w} \Delta_{\beta\to\gamma} h_i^{\circ} \left(T_{ref}\right) =$$

$$= -\lambda_t \frac{dT}{dz} + \sum_{i=A}^n N_{iw} \overline{c}_{pi}^{\circ\gamma} \left(T - T_{ref}\right) + \sum_{i=A}^n N_{iw} \Delta_{\beta\to\gamma} h_i^{\circ} \left(T_{ref}\right) = e_w = \text{constant}.$$
(7)

In terms of these equations, it is possible to determine the molar and energy flux through the interfacial area, or near the interfacial area (Fig. 1). Eq. (7) follows from Eq. (3), in which h_i° is expressed by the equation:

$$h_{i}^{\circ} = h_{i}^{\circ}(T, p, \gamma) - h_{iref}^{\circ}(T_{ref}, p_{ref} = p, \beta) = h_{i}^{\circ}(T, p, \gamma) - 0 = \int_{T_{ref}}^{T} c_{pi}^{\circ\gamma} dT + \Delta_{\beta \to \gamma} h_{i}^{\circ}(T_{ref}) = \overline{c}_{pi}^{\circ\gamma}(T - T_{ref}) + \Delta_{\beta \to \gamma} h_{i}^{\circ}(T_{ref})$$
(8)

resulting from Eq. (5), which is modified by the above assumptions. The enthalpy change expressed by Eq. (8) is shown schematically in Fig. 2.



Fig. 1 – Simultaneous heat and mass transfer through the interfacial area (assuming the unidirectional transfer of substance i)



Fig. 2 – Change of enthalpy at constant pressure (i.e., p = constant), including the partial changes of enthalpy

Eq. (6) and Eq. (7) can be solved. The first step is to separate the variables to obtain the equations in the form:

$$\int_{x_{i1}}^{x_{i2}} \frac{\mathrm{d}x_i}{\left(N_{iw} / \sum_{j=A}^n N_{jw}\right) - x_i} = -\frac{N_{iw}}{\left(N_{iw} / \sum_{j=A}^n N_{jw}\right)} \int_{z_1}^{z_2} \frac{\mathrm{d}z}{cE_{tim}}$$
(9)

$$\int_{T_1}^{T_2} \frac{\mathrm{d}T}{e_w - \sum_{i=A}^n N_{iw} \overline{c}_{pi}^{\circ\gamma} \left(T - T_{ref}\right) - \sum_{i=A}^n N_{iw} \Delta_{\beta \to \gamma} h_i^{\circ} \left(T_{ref}\right)} = -\int_{z_1}^{z_2} \frac{\mathrm{d}z}{\lambda_t}$$
(10)

By integrating these equations and their subsequent modification, the following equations are obtained:

$$N_{iw} = -\frac{E_{tim}c}{(z_2 - z_1)\varphi} (x_{i2} - x_{i1})$$
(11)

$$e_{w} = -\theta_{T} \frac{\lambda_{t} \left(T_{2} - T_{1}\right)}{\left(z_{2} - z_{1}\right)} + \sum_{i=A}^{n} N_{iw} \Delta_{\beta \to \gamma} h_{i}^{\circ} \left(T_{ref}\right) = e_{sen} + e_{lat}$$
(12)

where:
$$\varphi = \left[\frac{(x_{i2} - x_{i1})}{(N_{iw} / \sum_{j=A}^{n} N_{jw})}\right] / \left[\ln\left(\frac{(N_{iw} / \sum_{j=A}^{n} N_{jw}) - x_{i1}}{(N_{iw} / \sum_{j=A}^{n} N_{jw}) - x_{i2}}\right)\right] \text{ and } \theta_{T} = \left[\frac{\left(\frac{(\sum_{i=A}^{n} N_{iw} \overline{c}_{pi}^{\circ \gamma})(z_{2} - z_{1})}{\lambda_{i}}\right) \left(\frac{T_{2} - T_{ref}}{T_{1} - T_{2}}\right)}{1 - \exp\left(\frac{(\sum_{i=A}^{n} N_{iw} \overline{c}_{pi}^{\circ \gamma})(z_{2} - z_{1})}{\lambda_{i}}\right)} - \frac{\left(\frac{(\sum_{i=A}^{n} N_{iw} \overline{c}_{pi}^{\circ \gamma})(z_{2} - z_{1})}{\lambda_{i}}\right) \left(\frac{T_{1} - T_{ref}}{T_{1} - T_{2}}\right) \exp\left(\frac{(\sum_{i=A}^{n} N_{iw} \overline{c}_{pi}^{\circ \gamma})(z_{2} - z_{1})}{\lambda_{i}}\right)}{1 - \exp\left(\frac{(\sum_{i=A}^{n} N_{iw} \overline{c}_{pi}^{\circ \gamma})(z_{2} - z_{1})}{\lambda_{i}}\right)}\right)}$$

Furthermore, the validity of the film theory will be assumed^{3,5,22,23}. Based on this assumption, the following equations apply: $F = E_{tim} / (z_2 - z_1) = E_{tim} / \delta_D$, and $\alpha = \lambda_t / (z_2 - z_1) = \lambda_t / \delta_T$. Regarding the z_2 -coordinate, or z_1 -coordinate, it should be noted that their position, depending on the choice of direction of the coordinate system (as further documented in Fig. 4), is generally different for the case of mass transfer and different for the case of heat transfer. It therefore follows that the values of the quantities δ_D and δ_T will generally be different, i.e., $\delta_D \neq \delta_T$. Considering these equations, Eq. (11) and Eq. (12) will have the following form:

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$$N_{iw} = -\frac{Fc}{\varphi} (x_{i2} - x_{i1}) = -k_x (x_{i2} - x_{i1}) = -\frac{k_x}{c} (c_{i2} - c_{i1}) = -k_c (c_{i2} - c_{i1}), \qquad (13)$$

$$e_w = -\theta_T \alpha \left(T_2 - T_1 \right) + \sum_{i=A}^n N_{iw} \Delta_{\beta \to \gamma} h_i^{\circ} \left(T_{ref} \right) = e_{sen} + e_{lat}$$
(14)

where:
$$k_x = \frac{Fc}{\varphi} = k_c c \text{ and } \theta_T = \left[\frac{\left(\frac{\left(\sum_{i=A}^n N_{iw} \overline{c}_{pi}^{\circ \gamma} \right)}{\alpha} \right) \left(\frac{T_2 - T_{ref}}{T_1 - T_2} \right) - \left(\frac{\left(\sum_{i=A}^n N_{iw} \overline{c}_{pi}^{\circ \gamma} \right)}{\alpha} \right) \left(\frac{T_1 - T_{ref}}{T_1 - T_2} \right) \exp\left(\frac{\left(\sum_{i=A}^n N_{iw} \overline{c}_{pi}^{\circ \gamma} \right)}{\alpha} \right) \right]}{1 - \exp\left(\frac{\left(\sum_{i=A}^n N_{iw} \overline{c}_{pi}^{\circ \gamma} \right)}{\alpha} \right)}{1 - \exp\left(\frac{\left(\sum_{i=A}^n N_{iw} \overline{c}_{pi}^{\circ \gamma} \right)}{\alpha} \right)}{1 - \exp\left(\frac{\left(\sum_{i=A}^n N_{iw} \overline{c}_{pi}^{\circ \gamma} \right)}{\alpha} \right)}{1 - \exp\left(\frac{\left(\sum_{i=A}^n N_{iw} \overline{c}_{pi}^{\circ \gamma} \right)}{\alpha} \right)}{1 - \exp\left(\frac{\left(\sum_{i=A}^n N_{iw} \overline{c}_{pi}^{\circ \gamma} \right)}{\alpha} \right)}{1 - \exp\left(\frac{\left(\sum_{i=A}^n N_{iw} \overline{c}_{pi}^{\circ \gamma} \right)}{\alpha} \right)}{1 - \exp\left(\frac{\left(\sum_{i=A}^n N_{iw} \overline{c}_{pi}^{\circ \gamma} \right)}{\alpha} \right)}{1 - \exp\left(\frac{\left(\sum_{i=A}^n N_{iw} \overline{c}_{pi}^{\circ \gamma} \right)}{\alpha} \right)}{1 - \exp\left(\frac{\left(\sum_{i=A}^n N_{iw} \overline{c}_{pi}^{\circ \gamma} \right)}{\alpha} \right)}{1 - \exp\left(\frac{\left(\sum_{i=A}^n N_{iw} \overline{c}_{pi}^{\circ \gamma} \right)}{\alpha} \right)}{1 - \exp\left(\frac{\left(\sum_{i=A}^n N_{iw} \overline{c}_{pi}^{\circ \gamma} \right)}{\alpha} \right)}{1 - \exp\left(\frac{\left(\sum_{i=A}^n N_{iw} \overline{c}_{pi}^{\circ \gamma} \right)}{\alpha} \right)}{1 - \exp\left(\frac{\left(\sum_{i=A}^n N_{iw} \overline{c}_{pi}^{\circ \gamma} \right)}{\alpha} \right)}{1 - \exp\left(\frac{\left(\sum_{i=A}^n N_{iw} \overline{c}_{pi}^{\circ \gamma} \right)}{\alpha} \right)}{1 - \exp\left(\frac{\left(\sum_{i=A}^n N_{iw} \overline{c}_{pi}^{\circ \gamma} \right)}{\alpha} \right)}}{1 - \exp\left(\frac{\left(\sum_{i=A}^n N_{iw} \overline{c}_{pi}^{\circ \gamma} \right)}{\alpha} \right)}{1 - \exp\left(\sum_{i=A}^n N_{iw} \overline{c}_{pi}^{\circ \gamma} \right)} \right)}}$$

Regarding variables F and α , it can be stated that these are most often determined experimentally. Their significance is that these quantities replace quantities (in this case λ_T , E_{tim} , δ_T , and δ_D) that are either difficult to measure or not known at all. Film theory assumes that the total resistance to heat transfer (or mass transfer) through the interfacial area is concentrated in a thermal (or concentrate) boundary layer having a thickness δ_T (or δ_D). The boundary layer thickness, which is located above the interfacial area, is generally given by the sum of the thicknesses of the real and fictitious (equivalent) laminar films, and depends on the degree of vorticity of the relevant phase. In the boundary layer, the fluid flow is laminar along the interfacial area. In general, as already stated, $\delta_D \neq \delta_T$ (Fig. 3). The film theory also assumes that equilibrium (the equilibrium state, which is expressed by the equilibrium curve) occurs in the interfacial area. The film theory also allows a qualitative evaluation and comparison of the processes that take place in the equipment, if it is assumed that $\lambda_T = \lambda$ and $E_{tim} = D_{im}$. This assumption subsequently allows obtaining the values of quantities δ_T and δ_D (fictitious values of these



Fig. 3 – Illustration of the conditions near the interfacial area when applying the film model when one of the phases is flowing

quantities), which then give a comprehensive idea of the influence of the conditions on the processes occurring in the equipment.

It follows from Eq. (14) that $e_{sen} = -\theta_T \alpha (T_2 - T_1)$ and $e_{lat} = \sum_{i=A}^n N_{iv} \Delta_{\beta \to \gamma} h_i^{\circ} (T_{ref})$. The quantity e_{sen} refers to the amount of energy required to change the temperature of the substances, it essentially expresses the corrected form of Newton's law of cooling, quantity e_{lat} represents the amount of energy associated with the change of the state of the substances.

Correction factor taking into account the simultaneous heat and mass transfer

In the previous part, quantity θ_{τ} , actually the equation determining the value of this quantity, has been defined. This equation defines a quantity referred to in the literature as the Ackermann correction factor, which takes into account the fact that heat and mass transfer occur simultaneously in the system. However, such an equation is not suitable for practical use because it contains the variables related to the choice of the reference state. It should also be borne in mind that the fore-derived equations (because heat and mass flux are vector variables) were constructed assuming a one-dimensional problem. In this case, the positive direction of the z-coordinate still needs to be defined. Taking this into account, starting from Fig. (4), it is subsequently possible to obtain the equations stated in the introduction of this article (i.e., Eq. (1) and Eq. (2)).

Since the choice of the reference state can be arbitrary, it will further be assumed that $T_{ref} = T_w$, and $\beta = II$ (i.e., the reference state is represented by phase II). It will also be assumed that a phase conversion I \rightarrow II will occur in the system. Taking into account the aforementioned and with respect to Eq. (8), then, when determining the change of enthalpy, the following must apply: $\gamma = I$. If the facts just



Fig. 4 – Representation of the conditions near the interfacial area in the case of simultaneous transfer of mass and heat through the interfacial area resulting from a different choice of the coordinate system orientation

mentioned are taken into account, then, in Eq. (14), the following will hold: $\Delta_{\beta \to \gamma} h_i^{\circ}(T_{ref}) = \Delta_{\Pi \to I} h_i^{\circ}(T_w)$, and $\overline{c}_{pi}^{\circ \gamma} = \overline{c}_{pi}^{\circ 1}$. This applies equally to both cases shown in Fig. (4). However, the further adjustment of Eqs. (13) and (14) already depends on the choice of the coordinate system orientation. According to Fig. 4a, the following applies: $x_{i1} = x_{iw}$, $x_{i2} = x_{if}$, $T_1 = T_w$, and $T_2 = T_f$, which gives the equations in the form:

$$N_{iw} = -k_x \left(x_{if} - x_{iw} \right) \tag{15}$$

$$e_{w} = -\theta_{T}\alpha \left(T_{f} - T_{w}\right) + \sum_{i=A}^{n} N_{iw} \Delta_{\mathrm{II} \to \mathrm{I}} h_{i}^{\circ} \left(T_{w}\right) = e_{sen} + e_{lat}$$
(16)

where:
$$k_x = \frac{Fc}{\varphi} = k_c c$$
, $\varphi = \frac{\left[\frac{\left(x_{if} - x_{iw}\right)}{\left(N_{iw} / \sum_{j=A}^n N_{jw}\right)}\right]}{\left[\ln\left(\frac{\left(N_{iw} / \sum_{j=A}^n N_{jw}\right) - x_{iw}}{\left(N_{iw} / \sum_{j=A}^n N_{jw}\right) - x_{if}}\right)\right]}$, and $\theta_T = \frac{\left(\frac{\left(\sum_{i=A}^n N_{iw} \overline{c}_{pi}\right)}{\alpha}\right)}{\exp\left(\frac{\left(\sum_{i=A}^n N_{iw} \overline{c}_{pi}\right)}{\alpha}\right) - 1}$.

Eqs. (15) and (16) are mutually compatible. This means that they can be arbitrarily combined with each other without applying a sign correction.

According to Fig. 4b, the following applies: $x_{i1} = x_{ij}$, $x_{i2} = x_{iw}$, $T_1 = T_j$, and $T_2 = T_w$, which in turn gives the equations in the form:

$$N_{iw} = -k_x \left(x_{iw} - x_{if} \right) \tag{17}$$

$$e_{w} = -\theta_{T} \alpha \left(T_{w} - T_{f}\right) + \sum_{i=A}^{n} N_{iw} \Delta_{\mathrm{II} \to \mathrm{I}} h_{i}^{\circ} \left(T_{w}\right) = e_{sen} + e_{lat} \qquad (18)$$

where:
$$k_x = Fc/\varphi$$
, $\varphi = \left[\frac{\left(x_{iw} - x_{if}\right)}{\left(N_{iw} / \sum_{j=A}^n N_{jw}\right)}\right] / \left[\ln\left(\frac{\left(N_{iw} / \sum_{j=A}^n N_{jw}\right) - x_{if}}{\left(N_{iw} / \sum_{j=A}^n N_{jw}\right) - x_{iw}}\right)\right]$, and $\theta_T = \frac{\left(\frac{\left(\sum_{i=A}^n N_{iw}c_{pi}\right)}{\alpha}\right)}{1 - \exp\left(-\frac{\left(\sum_{i=A}^n N_{iw}\overline{c}_{pi}\right)}{\alpha}\right)}$.

In the case of Eqs. (17) and (18), in terms of their sign compatibility, the same is true as stated for Eqs. (15) and (16).

Based on the adjustments made, it can be seen that two equations can be written to calculate θ_r

(Ackermann correction factor), i.e.,
$$\theta_T = \frac{\left(\frac{\left(\sum_{i=A}^n N_{iw} \overline{c}_{pi}^{\circ 1}\right)}{\alpha}\right)}{\exp\left(\frac{\left(\sum_{i=A}^n N_{iw} \overline{c}_{pi}^{\circ 1}\right)}{\alpha}\right) - 1}$$
 and $\theta_T = \frac{\left(\frac{\left(\sum_{i=A}^n N_{iw} \overline{c}_{pi}^{\circ 1}\right)}{\alpha}\right)}{1 - \exp\left(-\frac{\left(\sum_{i=A}^n N_{iw} \overline{c}_{pi}^{\circ 1}\right)}{\alpha}\right)}$

These equations are identical to Eq. (1) and Eq. (2), because the following applies in these equations: $\Phi_T = \left(\sum_{i=A}^n N_{iw} \overline{c}_{pi}^{\circ I}\right) / \alpha$. The dependence of θ_T on Φ_T in terms of these equations is shown in Fig. (5). Regarding Eq. (1) and Eq. (2), it can still be noted that, when solving the same problem, the same value of the correction factor (i.e., θ_{τ}) is always obtained on their basis, which is also confirmed by Fig. (5).

Variable θ_r should be regarded as a correction term that takes into account the fact that there is a simultaneous heat and mass transfer in the system. If there is no mass transfer through the interfacial area in the system, the following applies: $\phi_T = 0$ and $\theta_{T} = 1$ (Fig. 5). The same applies to the case where the system is one-component. In such a case, from the point of view of mass transfer, the following applies: $N_{iw} = 0$ (i.e., $\sum_{i=A}^{n} N_{iw} \overline{c}_{pi}^{\circ 1} = 0$), and therefore $\phi_T = 0$ and $\theta_T = 1$.

Conclusion

The present article deals with the correction coefficient, the Ackermann correction factor, which basically corrects the value of the heat transfer coefficient determined for the case that only heat transfer occurs in the system through the interfacial area. If only heat transfer occurs in the system through the interfacial area or the system is one-component, then in such cases $\theta_T = 1$. However, if there is simultaneous heat and mass transfer in the system, this factor should generally be considered. In such a case, it is generally true that $\theta_{x} \neq 1$. Different forms of the Ackermann correction factor can be found in the literature, however, all these forms are based on one equation. Based on it, two forms of equations, commonly reported in the literature, determining the value of the Ackerman correction factor can be obtained, as documented in this paper. When working with this factor, the fact on the basis of which the relevant equation was established must always be taken into account. The same also applies to quantities and equations, the knowledge of which is necessary to determine the value of the Ackermann correction factor (in this case, mainly quantity N_{iw}). The article further emphasizes that if several equations are to be used in the calculation, care must be taken to ensure that these equations are compatible with each other. This allows the results obtained from one equation to be directly substituted into another equation, or to directly combine the relevant equations with each other. In this way, it is possible to avoid the sign



Fig. 5 – Dependence of θ_T on Φ_T by Eq. (1) and Eq. (2) (i.e., $---\theta_T = \frac{\Phi_T}{\exp(\Phi_T) - 1}$, and $--\theta_T = \frac{\Phi_T}{1 - \exp(-\Phi_T)}$)

 J_{i}

п

correction (i.e., correcting the obtained result with a minus sign), which is commonly reported in the literature in the context of the use of certain calculation equations describing the same process. In the context of this article, compatibility means that the result determined on the basis of one equation is directly, without sign correction, substituted into another equation. The compatible equations presented in this article, in the aforementioned sense, are the following pairs of equations: Eq. (15) and Eq. (16), or Eq. (17) and Eq. (18). Any combination of these pairs of equations can be used to calculate the devices, and the results obtained from them will always be the same.

Understanding the equations presented here and the facts that follow from them is important because these equations apply to the design of the equipment in which the simultaneous heat and mass transfer occur through the interfacial area. On the other hand, it is also true that on the basis of these equations, it is possible to evaluate the experimental measurements or define other quantities, which confirms the stated fact regarding the knowledge of the connections leading to the forms of the equations mentioned here. This, of course, applies in general, i.e., for any equation used in the calculation.

The processes that involve the simultaneous transfer of heat and mass are applied not only within industrial technologies, but also in everyday life. The purpose of these processes (e.g., the drying mentioned in the introduction) is generally to separate the substances (substance) from the substance system in order to obtain a system with the required composition.

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Nomenclature

Latin letters

- c total molar concentration, mol m⁻³
- c_i molar concentration of a substance *i*, mol m⁻³
- D_{im} diffusivity coefficient for a multicomponent system, m² s⁻¹
- E_{tim} total diffusivity coefficient for a multicomponent system, m² s⁻¹
- *e* energy flux, W m⁻²
- mass transfer coefficient based on the molar concentration difference in the case of bidirectional mass transfer, m s⁻¹
- h enthalpy, J mol⁻¹

- molar flux of a substance *i*, mol m⁻² s⁻¹
- k_c mass transfer coefficient based on the molar concentrations difference, m s⁻¹
- k_x mass transfer coefficient based on the molar fractions difference, mol m⁻² s⁻¹
- N total molar flux of a substance, mol m⁻² s⁻¹
- N_i total molar flux of a substance *i*, mol m⁻² s⁻¹
 - number of substances in the mixture passing through the interfacial area, –
- *p* total pressure, Pa
- q heat flux, W m⁻²
- T absolute temperature, K
- v molar volume, m³ mol⁻¹
- x_i molar fraction of a substance i, –
- *z* distance from the interfacial area or the distance in the direction of the *z*-coordinate, m
- $I \rightarrow II change of the state (phase) I to state (phase) II$

Greek letters

 α – heat transfer coefficient related to all substances in the mixture, W m⁻² K⁻¹

 Δh^{mix} – mixing enthalpy, J mol⁻¹

- $\Delta_{\beta \to \gamma} h_i^{\circ} \left(T_{\beta \to \gamma} \right) \text{ enthalpy change during the phase change of the pure substance } i \text{ from phase } \beta \text{ to phase } \gamma \text{ at the temperature } T_{\beta \to \gamma}, \text{ J mol}^{-1}$
- $\Delta_{\beta \to \gamma} h_i^{\circ} \left(T_{ref} \right) \text{enthalpy change during the phase} \\ \text{change of the pure substance } i \text{ from } \\ \text{phase } \beta \text{ to phase } \gamma \text{ at the temperature} \\ T_{ref} \text{ J mol}^{-1}$
- $\Delta_{II \to I} h_i^{\circ}(T_w) \text{enthalpy change during the phase} \\ \text{change of the pure substance } i \text{ from } \\ \text{phase II to phase I at the temperature} \\ T_w, \text{ J mol}^{-1}$
- δ_D mass boundary layer thickness, m
- $\delta_{\scriptscriptstyle T}$ thermal boundary layer thickness, m
- ϕ rate factor, –
- φ factor taking into account the type of concentration diffusion, –
- λ thermal conductivity, W m⁻¹ K⁻¹
 - total thermal conductivity, W m⁻¹ K⁻¹
 - correction factor, –

Mathematical operators

- d differential of a given quantity
- Δ difference of a given quantity
- ∇ nabla or del operator, m⁻¹
 - $\left(\frac{\partial f}{\partial x}\right)$

λ,

θ

 partial derivative of the function *f* with respect to *x* at constant *z*

Overlines

| \vec{X} | – vector |
|----------------|---|
| \overline{X} | - average value or partial quantity |
| $X _{z=w}$ | near the interfacial area |

Superscripts

| X° – quantities | belonging to | pure substance |
|--------------------------|--------------|----------------|
|--------------------------|--------------|----------------|

- X¹ - quantities belonging to the substance in state I (in phase I)
- Χγ quantities belonging to the substance in state γ (in phase γ)

Subscripts

| A, B | - substances A and B |
|------------------------------|---|
| f | – in the volume of fluid |
| i | - substance i in mixture, $i = A, B, C,$ |
| lat | – latent |
| w | - near the interfacial area |
| ref | - reference state |
| sen | - sensitive |
| Ζ | - Cartesian coordinate, in the z-axis direction |
| 1, 2 | - related to distances 1 and 2, related to cross-sections 1 and 2 |
| $\mathrm{II} \to \mathrm{I}$ | - change of the state (phase) II to state (phase) I |
| $\beta \rightarrow \gamma$ | - change of the state (phase) β to state (phase) γ |
| Т | - quantities related to the heat transfer taking |

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transfer

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