10.31341/jios.46.2.7 UDC 324:575.832
Open Access Original Scientific Paper

Evaluating Compromise in Social Choice Functions

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Abstract

We investigate the notion of compromise in the strict preferential voting setting. We introduce divergence as an inverse measure of compromise in a collection of strict preferential votes. Classical functions of social choice theory are analyzed with respect to divergence. New social welfare functions and new social choice functions with the objective of compromise are defined directly from optimization of divergence and later analyzed with respect to the common desiderata of social choice theory. For a very natural function, a simple divergence minimizer, we prove it satisfies the properties of anonymity, neutrality, consistence, and continuity. Consequently, according to Young's theorem of characterization it follows that this function is a scoring point function. Its scoring point vector is also given. Finally, we discuss the parameter p in the divergence measure which was introduced to address vagueness and fuzziness of compromise and to control for a variety of intended levels of compromise.
Keywords: Social choice function, Social welfare function, Strict preferential voting, Compromise, Borda count, Plurality count, Divergence

1. Background and motivation

In this paper we are investigating the setting where voters vote by expressing strict preferences among candidates (or alternatives). Each voter votes by expressing his (strict) preference – a strict linear (implicitly transitive) order of candidates.

A *social welfare function* (SWF) is a function which aggregates preferences into an outcome preference. Although other definitions exist in literature, we opt for the now standard one introduced by Kenneth Arrow [5]. Similarly, we opt for the definition of the *social choice function* (SCF) as a partial function which aggregates preferences to select a single winner. When such selection is not unique, i.e., following the SCF's method produces *a tie*, we consider the result to be undefined.[1] Ties in

[1] We opt for the definition of SCF as a partial function for the simplicity of the argument. Using the common terminology of the social choice theory, we don't require SCF to satisfy the universal domain property.

elections in practice are handled in a variety of ways but this topic is left out of this paper.

We assume some familiarity with the classical social welfare/choice functions: Borda count (where i-th rank in each preference is awarded a score of $n - i$); Plurality count (where only the number of top rankings matters); and Condorcet method (with "dueling" of all candidate pairs – counting how many times one is placed before the other).

1.1. Motivating example

The following example illustrates the motivation of the paper. Suppose an election is held where 100 voters choose between three candidates: A, B and C. A collection of all votes (preferences) is called a profile.

Suppose our imaginary election produced a profile α which we summarize in Table 1. Numbers on the left are the tally of received preferences in the profile.

51	$A \succ B \succ C$
49	$C \succ B \succ A$

Table 1. A summary of the motivating profile α

Table 1 briefly summarizes that 51 voters in α (a slight majority) have voted $A \succ B \succ C$, whereas the other 49 voters have voted $C \succ B \succ A$.

Who should win this election (with respect to α)? Classical social choice functions agree on the winner (candidate A). Borda count and Plurality count, however, disagree on other ranks. Borda count ranks candidates with respect to their score, where first candidate receives two points, second candidate one point, and third candidate no points, on each preference of the given profile. Therefore, according to Borda count, candidate A scores 102 points, candidate B scores 100 points, and candidate C scores 98 points; yielding Borda linear order of $A \succ B \succ C$. On the other hand, Plurality count ranks candidates with respect to the number of top positions attained in all the preferences in the profile. Candidate A is top ranked in 51 preferences, candidate C in 49 preferences, whereas candidate B is never ranked first. Plurality count produces $A \succ C \succ B$ as an outcome preference for this profile.

Note, however, that 49% of the voters regard candidate A as the worst choice. How legitimate is A's victory when viewed from this perspective? A sensible and intuitive approach would be to try to look for a compromise. Doing so could offer candidate B as a compromise winner of profile α.

We take a simple approach to model the notion of compromise in a given profile by extending the positional analysis to account for all positional information in each vote (preference): i.e., to also take into account how often a candidate placed second, third etc.

For example, let us take a closer look at rankings of candidate A in all the votes in profile α. A is the top ranked candidate (difference to the top rank is 0) for 51 voters. In the remaining 49 preferences candidate A ranks A places below top (difference to the top rank is 2). We propose to measure and express this "accrued penalty" in α with the following expression

$$51 \cdot 0^p + 49 \cdot 2^p, \qquad \text{for fixed } p > 1. \tag{1}$$

This value reflects the voters' dislike for candidate A (to be selected for a winner) expressed in α. The motivating idea is that the least disliked candidate is a compromise winner. We denote expression (1) with $dvg(A)$. For other candidates we have:

$$dvg(B) = 51 \cdot 1^p + 49 \cdot 1^p = 100,$$
$$dvg(C) = 51 \cdot 2^p + 49 \cdot 0^p = 51 \cdot 2^p.$$

The idea here is that smaller values of dvg reflect a greater level of compromise, with the smallest being the best. Unanimous ranking in a profile produces the lowest possible value of $dvg(\_) = 0$.

Expression (1) with parameter $p > 1$ is inspired by Minkowski's p-metric. For $p = 2$ divergence corresponds to the square of the Euclidean metric, a common and widely applied measure for comparing distances (when square root is not needed).

Raising rank loss to the p-th power in (1) assigns more-than-linear weight to greater distances – the idea here is that strongly disliked options are penalized by design when we are trying to find a a compromise solution. Note that by setting $p = 1$ (linear penalty) we obtain, in essence, the usual Borda count method, but recall that we assume $p > 1$. We will discuss the choice of p in a later section of the paper.

Note that dvg is not a distance metric. We do not measure the distance between preferences like for example Cook and Seiford in [1]. Rather, we introduce divergence as a penalty measure of candidate's rank loss across the voting profile. Nevertheless, our goal is similar: we look for optimization of this measure in the interest of the democratic society (electorate) to obtain the best outcome.

1.2. Notation and definitions

A finite collection of votes is called a profile. Number of occurrences of each vote/preference (number of occurrences) in an election matter. Order of votes in a democratic election does not.

Therefore, we consider any profile α to be a multiset of votes – an unordered collection equipped with the multiplicity function $\mu\alpha$ which counts the occurrences of each element in α. For a tuple of candidates $M = (M_1, M_2, \dots, M_n)$ with $\mathcal{L}_M$ we denote a set of profiles over M.

Compatible profiles $\alpha, \beta \in \mathcal{L}_M$ can be joined as a multiset sum (denoted as $\alpha + \beta$, not to be confused with multiset union not used in this paper). Modeling voting profiles as multisets is not found in the literature but we opt for it here because it provides us with adequate formal elegance and economical notation.

A sum of $k \in \mathbb{N}$ copies of α is also a profile and we denote it as $k \cdot \alpha$. We denote the size of α (number of votes) with $|\alpha|$. Any profile α can be factored as

$$\alpha = \sum_{\sigma_i \in \alpha} k_i \cdot \sigma_i \tag{2}$$

where σ_i are single-vote profiles occurring in α and $k_i = \mu_\alpha(\sigma) \in \mathbb{N}$ are their corresponding multiplicities.

Let $M = (M_1, \dots, M_n)$. Each preference

$$M_{i_1} \succ \cdots \succ M_{i_n}$$

rearranges M's candidates and determines a unique permutation

$$(i_1, i_2, \dots, i_n)$$

(and vice-versa). For example, for $M = (A, B, C)$, preference

$$B \succ C \succ A$$

corresponds to permutation (2 3 1). We abuse this correspondence and consider preferences as permutations when convenient.

A profile α over $M = (M_1, \dots, M_n)$ of size $|\alpha| = n$ can be naturally represented by a matrix. Suppose votes of α are arbitrarily ordered as singleton profiles $\alpha_1, \dots, \alpha_m$. We say that an $m \times n$ matrix $A = [a_{ij}]$ **represents** α if a_{ij} is the rank of the M_j in α_i. Each vote is recorded as a row of A written in the permutation form. Matrix representation of a profile is unique up to a permutation of rows.

Now we can formally define the notion of divergence introduced in (1). However, we immediately generalize and include divergence from other positions (non-leading ranks).

Definition 1.1 (p-divergence from the j-th position). Let $M = (M_1, \dots, M_n)$ be a tuple of candidates and let $\alpha \in \mathcal{L}_M$ be a profile of over M represented by matrix $A = [a_{ij}]$. For (i-th) candidate M_i we define divergence from the j-th position in α as

$$dvg_\alpha(i,j) = \sum_{k=1}^{|\alpha|} \left|\alpha_{k,i} - j\right|^p \tag{3}$$

where $a_{k,i}$ is M_i's rank in k-th preference of α, with $p > 1$.

We abbreviate $dvg_\alpha(i, 1)$ with $dvg_\alpha(i)$. In a single-profile context we abbreviate even further with $dvg(i)$.

Divergence values for all pairs i, j are naturally gathered in the matrix form.

Definition 1.2 (Divergence matrix). Let $M = (M_1, \dots, M_n)$ be a tuple of candidates and let $\alpha \in \mathcal{L}_M$ be a profile over M. We define the divergence matrix as

$$D_\alpha = \left[d_{i,j}\right] \quad where \quad d_{i,j} = dvg_\alpha(i,j).$$

	1	2	3
A	196	100	204
B	100	0	100
C	204	100	196

Figure 1. Divergence matrix D_α

Example 1.3 *For a profile* α *over* $M = (A, B, C)$ *summarized in Table 1 (our motivating example) we compute*

$$D_\alpha = \begin{bmatrix} 196 & 100 & 204 \\ 100 & 0 & 100 \\ 204 & 100 & 196 \end{bmatrix}.$$

Recall that rows of D_α correspond to candidates. Values in columns correspond to rank penalties/distances: i-th row of D_α holds divergence values for i-th candidate; j-th column of D_α holds j-th rank divergence of all candidates. We emphasize this even further with the Figure 1 representing D_α. Divergence of 0 reflects unanimous ranking across all votes in α (B was unanimously ranked 2nd in α). Note that $dvg(B) = 100$ (circled) is the lowest value in the first column of D_α which suggests candidate B as a compromise winner.

Lemma 1.4. Let M be a tuple of candidates and let $p > 1$. For $\alpha, \beta \in \mathcal{L}_M$ the following properties hold:

$$dvg_\alpha(i,j) + dvg_\beta(i,j) = dvg_{\alpha+\beta}(i,j), \tag{4}$$

and consequently

$$D_{\alpha+\beta} = D_\alpha + D_\beta \tag{5}$$

$$D_{n\cdot\alpha} = n \cdot D_\alpha \quad for\ n \in \mathbb{N}. \tag{6}$$

Proof: Follows from rearranging the sum in the right-hand side of (3). □

2. Compromise through divergence

Accepting divergence from the top position as an inverse measure of compromise lets us investigate the ability to compromise of classical social choice functions – Borda and Plurality method. This is interesting because Borda count is usually considered as a social choice function capable of electing a compromise winner. This highly contrasts the plurality method, usually considered unable to compromise.

For profiles with $n \leq 3$ candidates the expected result is obtained (cf. [2]): divergence for the Borda winner is always less than or equal to the divergence of the Plurality winner.

But, as is shown in [2], an analogous claim for profiles with $n > 3$ candidates does not hold. Specifically, profiles on $n > 3$ candidates exist where Plurality Count and Borda method produce different winners, but Plurality winner has smaller divergence than Borda winner!

3. Functions based on divergence

3.1. Simple Divergence Minimizer SDM

Divergence measure allows for a new approach to the construction of the social choice and social welfare functions. The simplest, and most natural way to use divergence, is to look for a minimal divergence from the first position. In most situations, we only care about the winner. Therefore, we can define a social welfare function based exclusively on the divergence from the top position – by considering (only) the first column of the divergence matrix D_α.

Definition 3.1. Let $p > 1$ and let $\alpha \in \mathcal{L}_M$ be a profile over $M = (M_1, \dots, M_n)$ such that all the values in the first column of its divergence matrix D_α are unique. Then there exists a permutation $\sigma \in S_n$ such that

$$dvg(M_{\sigma(1)}) < dvg(M_{\sigma(2)}) < \cdots < dvg(M_{\sigma(n)}).$$

We define a social welfare function SDM(α) as a function returning the strict linear order implied by σ:

$$M_{\sigma(1)} \succ M_{\sigma(2)} \succ \cdots \succ M_{\sigma(n)}.$$

Otherwise (for profiles with ties), SDM is undefined.

Intuitive explanation is that SDM returns the (strict) sorting permutation of the first column of divergence matrix D_α.

As can be seen from previous definition, (partial) social welfare function SDM is defined for cases without ties. We note that ties occur rarely, especially for non-integer values of p.

We define a social choice function SDM_1 as a function which selects a unique candidate with minimal divergence as a winner (when such a candidate exists).

Of course, when both SDM and SDM_1 are defined, we can regard SDM_1 as a restriction of SDM.

Properties of SDM_1 and SDM

Let us consider which properties or axioms of social choice theory SDM_1 satisfies.

In social choice theory, symmetric treatment of voters is called *anonymity*, and symmetric treatment of candidates, *neutrality*. Anonymity and neutrality are natural equity conditions expected from democratic social choice and social welfare functions.

It is clear from Definition 3.1 that social choice (welfare) function SDM produces the same result if preferences in a profile are permuted, which makes it anonymous. It is also clear that SDM treats all candidates equally: that is, if we permute positions of candidates in all preferences, then candidates will be permuted in the same way in result ordering of a SDM, making it neutral.

Therefore, the following proposition holds.

Proposition 3.2. For all $\mathrm{p} > 1$ functions SDM and SDM_1 are anonymous and neutral.

We consider additional properties of SDM: *consistency* and *continuity*. Young has shown that these properties are crucial for characterization of position scoring social choice functions (see [9]).

Definition 3.3 (Consistency) Social choice function F satisfies consistency if for all compatible profiles α_1, α_2 for which $F(\alpha_1) = F(\alpha_2)$ also

$$F(\alpha_1 + \alpha_2) = F(\alpha_i) \quad for\ i = 1{,}2.$$

Definition 3.4 (Continuity) Social choice function F satisfies continuity if for compatible profiles α_1 and α_2 there exists $n \in \mathbb{N}$ such that

$$F(n \cdot \alpha_1 + \alpha_2) = F(\alpha_1).$$

(The idea here is that a sufficiently large $n \in \mathbb{N}$ will sway the F-outcome of the $(n \cdot \alpha_1 + \alpha_2)$ profile to match the outcome of α_1.)

Proposition 3.5. Social choice function SDM_1 satisfies consistency and continuity for all $p > 1$.

Proof of consistency. Let $\alpha_1, \alpha_2 \in \mathcal{L}_M$ be such that

$$\mathrm{SDM}_1(\alpha_1) = \mathrm{SDM}_1(\alpha_2).$$

Therefore, minimal values m_1 and m_2 of the first columns of D_{α_1} and D_{α_2} occur at the same position (in the i-th row). Since

$$D_{\alpha_1+\alpha_2} = D_{\alpha_1} + D_{\alpha_2}$$

by Lemma 1.4, it follows that the minimal value in the first column of $D_{\alpha_1+\alpha_2}$ is $m_1 + m_2$, also in the i-th row. Therefore

$$\mathrm{SDM}_1(\alpha_1 + \alpha_2) = \mathrm{SDM}_1(\alpha_i) \quad for\ \ i = 1{,}2.$$

Proof of continuity. Let us denote $\alpha = k \cdot \alpha_1 + \alpha_2$. Let m_i be the minimal value in the first column of D_{α_i}, for $i = 1, 2$.

We may assume (without the loss of generality) that m_1 is in the first row of D_{α_1} and m_2 occurs in some other row of D_{α_2} (otherwise consistency yields the result). So we have first candidate as the winner for α_1 and some other for α_2.

From Lemma 1.4 we recall that

$$D_\alpha = k \cdot D_{\alpha_1} + D_{\alpha_2}.$$

Therefore the first column of divergence matrix D_α can be written down as

$$k \cdot \begin{bmatrix} m_1 \\ m_1 + a_2 \\ \vdots \\ m_1 + a_n \end{bmatrix} + \begin{bmatrix} m_2 + b_1 \\ m_2 + b_2 \\ \vdots \\ m_2 + b_n \end{bmatrix} \qquad with\ a_i > 0, b_i \geq 0.$$

To prove continuity we must show that $\mathrm{SDM}_1(\alpha) = \mathrm{SDM}_1(\alpha_1)$, e.g. that first columns of D_α and D_{α_1} have minimal values in the same row (first) for some k. Then, it should hold

$$(k \cdot m_1) + (m_2 + b_1) \leq k \cdot (m_1 + a_j) + (m_2 + b_j), \quad for\ j \geq 2,$$

for some natural number k. This simplifies into

$$k \geq \frac{b_1 - b_j}{a_j} \qquad for\ all\ j \geq 2.$$

As $a_j > 0$ and a unique $b_j = 0$ it follows that $k > 0$. It is sufficient to take

$$k \geq \max_{j>1} \left\lceil \frac{b_1 - b_j}{a_j} \right\rceil$$

to obtain $\mathrm{SDM}_1(\alpha) = \mathrm{SDM}_1(\alpha_1)$. □

From Proposition 3.5, and Young's characterization, it follows that SDM_1 is a position-scoring social choice function. One can easily check that scoring vector for SDM_1 is $(0, -1, -2^p, \dots, -(n-1)^p)$. Normalized[2] version of the SDM_1 scoring vector is

$$s = \left(1, 1 - \left(\frac{1}{n-1}\right)^p, 1 - \left(\frac{2}{n-1}\right)^p, \dots, 0\right).$$

Since SDM_1 is a position-scoring social choice function, with normalized scoring vector s, according to Llamazares and Pena (see [6]), it satisfies a number of desirable properties. One of them is that SDM_1 is not susceptible to the Pareto paradox, or the dominated-winner paradox.

Social choice functions which are susceptible to the Pareto paradox can elect a candidate who is strictly dominated by some other candidate, that is, there is a candidate which is preferred to a winner in all of the voter' preferences. Llamazares and Pena showed that a position scoring SCF with strict scoring vector cannot elect such candidate as a winner.

[2] Normalized form of the scoring vector has 1 on the first, and 0 on the last position.

Social choice function SDM_1 is also immune to the absolute loser paradox. Absolute loser paradox is considered as an especially intolerable paradox. SCF function that is susceptible to this paradox can elect as a winner a candidate who is ranked last by the absolute majority of voters!

According to Llamazares and Pena, position scoring SCF is immune to the absolute loser paradox if its scoring vector satisfies the condition

$$\sum_{i=1}^{n-1} s_i \geq \frac{n}{2}. \tag{7}$$

For $p = 1$ the scoring vector is a finite arithmetic n-sequence with the sum of exactly $\frac{n}{2}$. For $p > 1$ the scoring vector (as an n-sequence) strictly dominates this arithmetic sequence, therefore satisfying the condition (7). Thus, SDM_1 is immune to the absolute loser paradox.

3.2. Utilization of the whole divergence matrix

In this section we review a few attempts to construct new social welfare functions which utilize all of the divergence matrix – divergence from all positions. Probably the simplest way to follow a very naive approach: after selection of the winner (first column), we repeat the selection of the minimal divergence in the remaining rows of the next column, and so on column-by-column. This simple iterative approach, however, produces strange results.

Example 3.6. *Let $p = 2$ and let $\alpha \in \mathcal{L}_M$ be a profile over $M = (A, B, C, D, E)$ summarized in Table 2.*

38	$A \succ B \succ C \succ D \succ E$
10	$B \succ C \succ A \succ D \succ E$
3	$E \succ B \succ A \succ C \succ D$

Table 2. A summary of the profile α from Example 3.6.

For profile α candidate A is the clear winner by Borda, Condorcet and Plurality method. But we have $dvg(A) = 52$ and $dvg(B) = 41$. Therefore SDM_1 does not declare A a winner. But what about the order of other candidates?

Method run is visualized in Figure 2. In i-th column, the lowest remaining value of divergence (from the i-th place) is circled. Therefore, the result of this greedy approach is:

$$B \succ C \succ D \succ E \succ A$$

Intuitively, this is awful. Note that candidate A is placed before D in all preferences. Yet, candidate A ranks last in the final outcome – two places below D! This example clearly demonstrates the lack of *Pareto efficiency* for this method.[3]

In a similar fashion we could try the naive row-by-row optimization. But this quickly runs into problems (cf. Motivating example, where all the candidates best fit in the middle position).[4]

	1	2	3	4	5
A	52	51	152	355	660
B	41	10	81	254	529
C	189	50	13	78	245
D	480	219	60	3	48
E	768	435	204	75	48

Figure 2. Greedy column-by-column minimization with D_α

Our final suggestion is to apply this method cumulatively. After selecting a candidate with the lowest divergence in the first column D_α, we could search for the candidate (row) which minimizes a sum of divergences from the first and second position. And so on.

Cumulative greedy approach applied to the profile from Example 3.6 obtains (reasonable)

$$B \succ A \succ C \succ D \succ E.$$

We won't pursue this further in this paper.

[3] Social welfare function is satisfying a Pareto principle, or is Pareto efficient, if on a profile candidate A is dominated by candidate B, then candidate A is placed after candidate B in a resulting preference of the SWF.

[4] Another problem with row-by-row optimization is that if it assumes an optimization done through some order of the rows, it could violate neutrality of the SWF. Such procedure could potentially not equally treat all candidates.

3.3. On the choice of p

The notion of a compromise winner is somewhat vague and fuzzy. Naturally, we have to ask – where do we draw the line? Which profiles justify the selection of non-majority candidate as a compromise winner?

We have introduced the parameter p as an instrument to control the intended level of compromise. The value of p must be decided in the planning phase, according to the demands and specifics of the situation. The decision on the choice of p should be a social decision, dependent on the setting and in line with the ambitions of the particular election or voting-based decision. For example, taking a high value of p (e.g. $p = 4$) makes sense in a situation where a small number of voters has a significant need for compromise. Larger values of p have the effect of greater rank distances contributing considerably more (than linear) to divergence. On the other end of the spectrum, we can imagine a situation with a large electorate where the need for compromise is not among the primary objectives. However, if there is an intent to avoid polarization in the electoral process, a lower value of p (e.g. $p = 1.5$) would be appropriate to account for the appropriate level of compromise.

Ongoing research of the subject (cf. [3]) suggests that choice of $p = 2$ yields some desirable properties. Still, we reiterate that the choice of the value of p remains a social (rather than mathematical/technical) decision.

4. Conclusion

We analyzed and modelled the notion of compromise in social choice theory for the case of strict preferential voting. We introduced the measure of divergence which captures all the positional information from the voting profile and gives above-linear weight to poor rankings of candidates. This is by design – a compromise winner is not supposed to be disliked in any significant part of the electorate.

After establishing divergence as an inverse measure of compromise, we review the classical functions of social choice theory. Common folklore is that Borda count is the classical function able to produce a compromise winner, in stark contrast to plurality count. This is indeed the case for voting profiles with $n = 3$ candidates but does not hold in general for $n \geq 4$ with respect to divergence [2].

In this paper, unlike what is common in the literature of social choice theory, voting profiles are defined as multisets. This formal tweak provides some technical elegance evident in Lemma 1.4 about linearity of divergence. A few proofs on properties of SDM also benefit from this. Some previous results needed rephrasing with respect to revised notation.

Next, from minimization of divergence measure we define a new social welfare function SDM (simple divergence minimizer) and its SCF counterpart SDM_1. We prove that both can, according to Young's theorem, be characterized as a scoring point SCF. Further, we explicitly provide its scoring point vector.

A few approaches to optimization utilizing the whole divergence matrix have been reviewed. Naive approaches fail to meet common SCF requirements. However, a cumulative approach to optimization shows promise, but this is the subject of further research.

Acknowledgements

This work has been supported by Croatian Science Foundation under the project HRZZ-UIP-2017-05-9219 and by the University of Applied Sciences Velika Gorica and Algebra University College.

Appendix A: Notation and abbreviations

α	–	profile, multiset of preferential votes
$\|\alpha\|$	–	size (length) of profile α
M	–	a tuple of candidates, alternatives
$\mathcal{L}_M$	–	a set of finite profiles over M
S_n	–	set of n-permutations
$dvg_\alpha(A)$, $dvg(A)$	–	divergence (from the leading position)
$dvg(A,i)$	–	divergence from i-th rank
W_{BC}	–	winner by Borda count
W_{PC}	–	winner by Plurality count
W_{CND}	–	Condorcet winner
SCF	–	social choice function
SWF	–	social welfare function
SDM	–	Simple divergence minimizer SWF
SDM_1	–	Simple divergence minimizer SCF

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