AREA OF THE REPUBLIC OF CROATIA

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ABCTRACT. The paper reviews the theoretical and practical issues in the area determination of the Republic of Croatia at the University of Zagreb, Faculty of Geodesy.

1. INTRODUCTION

According to the published sources [1, 2, 18, 20, 21, 22, 25] the area of the continental part of Croatia is equal to 56538 km². However, the different numerical values can be found as well [8,9]. The common characteristic of all mentioned publications is that one can find out neither the source wherefrom the numerical data are taken over, nor the description of the area determination procedure.

2. AREA DETERMINATION ON THE BASIS OF COORDINATES

The geometric method of area determination of territory units in the form of polygon consists in the decomposition of polygon in the simple geometric figures as for instance triangles or quadrilaterals. By measuring the appropriate elements of these figures, and using the known mathematical formulae, the areas of single figures can be calculated, so that their sum gives the total area of polygon.

Herring and Freeman [10] suggested the automation of geometric method by which the polygon is decomposed into triangles, and the area of each triangle is computed by using the known formula: a side times height divided by two. Taking into account that in the mentioned procedure the authors use a digitizer giving the coordinates of digitized points, then it is clear, that they can not directly apply the quoted formula. Before that, they have to compute the length of the side and the length of the appropriate hight of the triangle. Thus, the procedure is unnecessarily complicated. It is amazing that such a clumsy method has been proposed at all (there is also the four pages program listing added), when the relations for computation of the polygon area P directly from the coordinates of its vertices are well known long ago.

However, it is less known that there is a great number of such formulae:

$$2P = \sum (y_{i+1} - y_i)(x_{i+1} + x_i) \tag{1}$$

$$2P = \sum (x_i - x_{i+1})(y_i + y_{i+1}) \tag{2}$$

$$2P = \sum x_i (y_{i+1} - y_{i-1}) \tag{3}$$

$$2P = \sum y_i (x_{i-1} - x_{i+1}) \tag{4}$$

$$2P = \sum (x_i y_{i+1} - y_i x_{i+1}) \tag{5}$$

$$2P = \sum (x_i \Delta y_i - y_i \Delta x_i) \tag{6}$$

$$2P = \sum x_i (\Delta y_i + \Delta y_{i-1}) \tag{7}$$

$$2P = -\sum y_i (\Delta x_i + \Delta x_{i-1}) \tag{8}$$

$$2P = \sum (\Delta y_i \Delta x_i + 2x_i \Delta y_i) \tag{9}$$

$$2P = -\sum (\Delta y_i \Delta x_i + 2y_i \Delta x_i) \tag{10}$$

Each of the formulae has as many items as the polygon has vertices. The coordinate differences are introduced by

$$\Delta x_i = x_{i+1} - x_i, \quad \Delta y_i = y_{i+1} - y_i . \tag{11}$$

Going round the polygon counter-clockwise, all the formulae (1)-(10) will give the positive value of the area.

It is often considered that if the areas P computed by two formulae, e. g. (1) and (2) are equal, then the area is accurately determined. However, the computed values could be mutually equal as well in the case when the wrong coordinates came in the calculaiion [15]. Therefrom, we can conclude that a special consideration should be given to the checking of coordinates as the input data in the procedure of computation.

With the development of computers, the method of area determination on the basis of coordinates has become more important because it enables the area determination of the extremely irregular shapes, and the coordinates of boundary points can be obtained directly form a map by means of digitization. This method is so effective and quick that it is now the most often used method in offices having the appropriate digitizing device [17].

3. DIGITIZATION

Our own program has been designed for the digitization with the precise large digitizer CALCOMP 9100. The program is assigned to the digitization of single points with the number of attributes, as well as to the digitization of contour lines and boundary lines of territory units. Our first experiences in digitization have been described in the former paper [7].

At the Faculty of Geodesy, the boundary of the Republic of Croatia, the coast line, the islands and the boundaries of county districts have been digitized. The digitization was performed by using the digitizer CALCOMP 9100 and our own program DIGIT, with total of about 19000 digitized points. Due to the division of the county district Split into three new county districts, and the fact that some of boundaries and some of islands were omitted in the first digitization, it has been done additionally.

The map of country districts of Croatia at the scale of 1 : 1 000 000 has been reproduced in the Institute of Cartography at the Faculty of Geodesy in Zagreb in 1979. The map projection is Gauß-Krüger with the central meridian 16°30'. This map projection has been proposed for the territory of Croatia by Frančula [3, 4]. In order to decrease the maximal linear distortions, the linear scale of 0.9997 has been introduced at the central meridian. Doing this, the maximum linear distortions at the outermost meridians run up to about 40 cm a kilometre, which at the distance of 500 km is equal to 150-200 m or 0.15-0.20 mm at the scale of 1 : 1 000 000. The area distortion in that case runs from -0.06% at the central meridian, up to about 0.08% at the outermost meridians. It is obvious, that linear and area distortions are practically unperceivable at the whole territory, i. e., the distortions remain within the limits of geometrical accuracy of the map.

4. COORDINATE TRANSFORMATION

In any transformation of coordinates we make a difference between the determination of transformation parameters and the transformation of individual points. The transformation parameters can be defined in different ways. If one has several points with known coordinates in both coordinate systems, then the least squares method is the most frequently applying procedure. We also apply this method in determing the parameters of the affine or Helmert transformation from the local digitizer system onto the coordinate system of the Gauß-Krüger map projection. The unknown parameters have been estimated on the basis of greater number of corresponding points (intersections of meridians and parallels).

5. ACCURACY ESTIMATION OF DIGITIZED POINTS

The errors occuring in the procedure of digitization can be divided into:

1) errors of the data source
2) errors of the digitizing device
3) errors of the pointing
4) errors of method of digitization.

The result of all individual errors determines the absolute or external accuracy of digitization. If we are not able to examine the impact of individual errors, then we need to know how to establish the accuracy estimation in some other way.

For instance, if we would have a pair of theoretically identical points, one of which would have error-free coordinates, while the other point would have its coordinates obtained by digitization, then the difference of the mentioned coordinates is a real error. On the basis of the greater number of such deviations, we would be able to say something about the quality of digitization.

However, there is usually a problem, because there are no points with their theoretical coordinates along the digitized line elements (state boundary, coast line, islands etc.). But, such points can be determined in a similar way to the one used at the Faculty of Geodesy in Zagreb in the framework of finding the intersections of aircorridors with the state boundary [16, 13, 6, 12].

6. AREA COMPUTATION WITH THE ACCURACY ESTIMATION

Under the territory unit we understand a part of land or sea bounded by the boundary line which is closed, i. e. which starts and ends in the same point. The area of territory unit is the area of its projection onto the reference, or in our case the Bessel ellipsoid. Let us suppose that the Gauß-Krüger projection of a territory unit is a closed polygon in the plane, then we can compute its area by using the formula [23]:

$$P = \frac{1}{2} \sum_{i=1}^{n} (y_{i+1} - y_i)(x_{i+1} + x_i)/p_i \qquad (12)$$

where x_i, y_i are cartesian coordinates of the polygon vertices ($x_1 = x_n$, $y_1 = y_n$), and p_i the factor of correction due to map projection.

The accuracy estimation of area computed by the formula (12) with $p_i = 1$ can be determined on the basis of the expression [19]:

$$m_P^2 = \frac{1}{8} m_T^2 \sum_{i=1}^{n} [(y_{i+1} - y_{i-1})^2 + (x_{i+1} + x_{i-1})^2] \quad , \qquad (13)$$

where m_T is the mean square error of a single point. Thereby, we assume that the coordinates are not correlated and that the mean square error of abscissas and ordinates are mutually equal.

7. RESULTS ACHIEVED ON THE BASIS OF DIGITIZING THE MAP AT THE SCALE OF 1 : 1 000 000

By using the formula (12) we computed the area of 90 largest Croatian islands and of 94 territory units at the continental part of Croatia. After the appropriate classification, the final values of county district areas have been achieved and published in [5]. In order to get a better insight in the distortion of area due to map projection, the areas are computed without the corrections ($p_i = 1$) and with the corrections. The absolute value of the maximum relative error of area computed with and without the correction was about 0.1 %. The total area of the continental part of the Republic of Croatia has been computed as the sum of all county district areas and makes 56488 km^2. It is 50 km^2 less then the most frequent quoted value.

In the paper [14] there is a review of obtained accuracy estimation for the area of islands and for the area of continental part of Croatia on the basis of the mean square error of points belonging to the boundary lines. The mean square error of the total area of the continental part of Croatia is about 13 km^2.

It is interesting, that the islands Krk and Cres are of approximately the same area, and that there are no scientific arguments for declaring Krk as the largest island in the Adriatic sea. Our results are in accordance with the recent investigations of other authors [24, 11].

REMARK

All numerical values quoted in this paper are not definitive and not in the official use.

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