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NUMERICAL EVALUATION OF PLANE GRINDING STABILITY

Summary

Regenerative chatter is a serious productivity limitation in machining. The term refers to unstable relative vibrations between the workpiece and the cutting tool that negatively affect almost all cutting operations by chip removal. Grinding is the most commonly used abrasive process. As a result of significant tool wear in grinding, surface regeneration (which causes regenerative chatter) can occur on the workpiece and around the grinding wheel circumference. This study examines only the regenerative mechanism related to the grinding wheel, i.e., the effect of distributed grain dullness in particular, which causes instability in the machining processes. A chatter vibration model was formulated and validated by numerical simulations and experimental data. For the first time, the new model accurately predicts the existence of a region of stability in the grinding process. That new model refutes the previous model stating that grinding cannot be stable considering grinding wheel regeneration.

Key words: *machine tool dynamics, grinding chatter, wheel regeneration*

1. Introduction

Grinding is the oldest machining operation [1]. Being a simple cutting method, grinding has a number of advantages and disadvantages compared to conventional machining operations such as turning and milling. Abrasive processes are known for achieving excellent surface quality and dimensional accuracy, relatively easy cutting of materials that are difficult to process and enormous material removal. Abrasive machining can replace "large chip" machining processes such as milling, planning and turning. It should be noted here that abrasive machining is not the same as precision grinding although they are very similar. In abrasive processing, the goal is not high surface precision, but the removal of large quantities of material. Abrasive machining is more precise than processes with large chip separation, known as creep feed grinding (CFG).

Disadvantages of machining with a grinding wheel in respect of a conventional cutting tool are high tool wear, significant heat generation and complexity of process modeling [2].

The latter disadvantage is especially important in a complex process of machining, i.e., in the modelling of regenerative chatter vibrations that are created in all machining cutting operations with chip removal. The self-excited vibrations between the workpiece and the cutting tool create unpleasant noise and, more important limit productivity and produce lowquality machined surface. For this reason, the prediction and avoidance of regenerative chatter vibrations are of crucial importance if stability, efficiency, and productivity of machining need to be ensured [3].

This topic has been researched since the beginning of the 20th century. Since abrasive processes are often the final machining processes, they are responsible for the quality of the surface and the dimensional accuracy of the workpiece. An abrasive process can destroy a product that has already undergone a series of expensive machining operations; therefore, abrasive processes are particularly important for industry. According to the literature, the first step in grinding research was made by Hahn, who proposed a relatively simple model of self-excited vibrations during grinding [4]. The model considered the surface regeneration of the workpiece and neglected that of the grinding wheel [4]. The model offers a possibility to predict a range of parameters in which chatter vibrations will not occur. Through experiments, Landberg confirmed that the grinding wheel does not wear evenly and that surface waves can be created both on the girding wheel and workpiece [5]. That revealed that surface regeneration can be created on the workpiece and on the grinding wheel simultaneously and can affect both in real time. The coupled waves on the grinding wheel and the workpiece were investigated by Snoeys and Brown, who were the first to confirm chatter vibrations with experiments [6]. The idea of regenerative stability was investigated by Thompson [7], who expanded scientific knowledge about grinding in his studies [8-11]. The review paper on grinding, written by Inasaki et al. [12] is fundamental, acknowledged, and most often cited.

Until 2006, almost all grinding theories were based on the following:

- Grinding wheel regeneration as a cause of instability is modelled as distributed radial wear around the circumference.
- Most grinding processes in practice are unstable due to workpiece regeneration.

Due to this, the growth rate of chatter vibrations is more important than absolute stability [11,12,13]. It has been observed that chatter instability develops much faster if it is caused by the workpiece [12] compared to chatter instability caused by the grinding wheel.

Li and Shin [13] presented another way of describing wheel regeneration in surface grinding. They combined distributed radial wear with distributed grain dullness (hence the distribution of cutting-edge dullness around the circumference of the wheel). They expressed the dullness of the grain with a specific energy that quantifies the grinding energy required to remove a unit volume of the workpiece material. The aforementioned energy is known as the equivalent grinding power. This means that a sharper (dull) grain corresponds to a small (large) specific energy. Obviously, Li and Shin included the physics of the surface workpiece waves and the specific energy waves on the grinding wheel. This is substantial progress from the general trend in the pre-2006 literature.

New grinding theories consider complex phenomena with self-excited vibrations, i.e., nonlinearities [14-16], workpiece unbalance [17] and parallel grinding [18]. There is a general consensus that grinding is an unstable process against grinding wheel regeneration and that the instability mechanisms are dominated by physical surface waves on the grinding wheel.

2. Stability model

A two-dimensional mechanical model of counter-directional, single-pass plane grinding with one degree of freedom is shown in Fig. 1. The grinding wheel with a radius R_g, can vibrate against the workpiece only in the direction of the cutting depth δ, which is marked with y. The grinding wheel modal mass, modal damping, and modal stiffness are marked by m, c, and k. The grinding wheel rotational speed is ω_g, i.e., the peripheral speed $v_g = \omega_g R_g$, and the feed is v_w. The grinding force is F, and the components are F_n (normal), F_t (tangential) and F_y.

Figure 1 shows the contact angle φ_c and the grinding force angle α, which defines the force angle in the cutting zone.

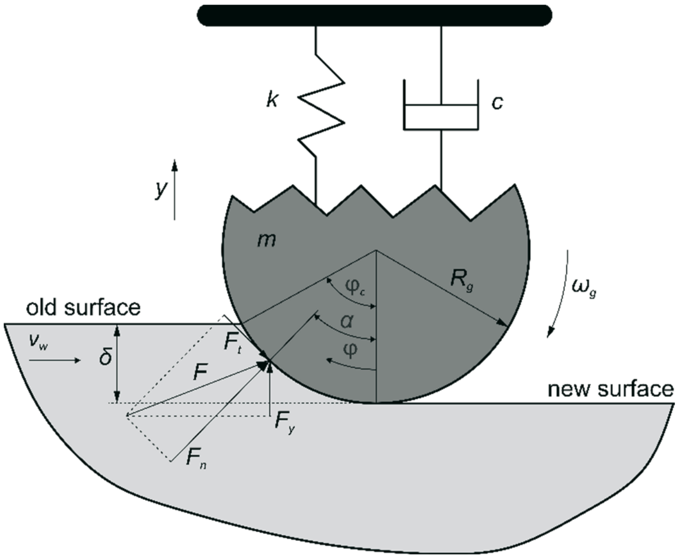

Fig. 1 Two-dimensional model of counter-directional plane grinding

Chatter vibration theory includes three component models: the grinding force model, the grinding wheel wear model, and the grinding wheel vibration model.

I) Grinding force model

One of the basic definitions of grinding force was given by Malkin and Guo [18] as:

$$F_t = w\,u\,\frac{v_w}{v_g}\,\delta, \tag{1}$$

where w is the grinding width and u is the specific energy. It is obvious that the three grinding mechanisms (sliding, plowing and chip formation) are not treated separately in that model; specific energy separates these mechanisms.

The component of the grinding force in the direction of grinding wheel vibration is:

$$F_y = \mu_y w\,u\frac{v_w}{v_g}\delta, \tag{2}$$

where μ_y is the ratio of the component in the y direction to the t direction of the grinding force.

II) Grinding wheel wear model

In case the only mechanism responsible for the regeneration effect is the dullness of grain, variation of the specific energy must be quantified both in space (around the circumference of the wheel) and in time (created during grinding). Thus, the two-variable description constitutes a complex mathematical problem, especially when coupled with differential equation stability analysis with a delay, which is typical of chatter vibrations [26-28]. The system can be simplified because the spatial and temporal variables of the specific energy function are not independent. Time is the only independent variable because angular coordinate depends on the time at a constant grinding wheel rotational speed. Thus, the problem is reduced to one variable as the spatial coordinate is removed; therefore, the specific energy u at time t corresponds to the point (or grain) on the grinding wheel that is just leaving the cutting zone. Thus, the specific energy u corresponds to the delay $\tau_g = \frac{T_g}{Z}$, i.e., the period of grain passing through the grip with proper grain distribution (T_g) divided by the number of cutting edges (Z) around the circumference of the grinding wheel.

It is obvious that certain assumptions have been made. First, due to the high density of grains (cutting edges) on the wheel and the high speed with which they exit the grip, it is assumed that the specific energy is a continuous function of time. Second, the specific energy is assumed to be constant across the width of the grinding wheel. Third, it is assumed that the specific energy increases in the grip zone due to wear, and is constant outside the grip. Thus, not a single grain becomes sharper during grinding, i.e., the self-sharpening property of the wheel is not taken into account.

III) Grinding wheel vibration model

It is assumed that the wheel vibrations affect the depth of cut but not the chip thickness. Therefore, grinding wheel vibrations are equivalent to workpiece vibrations and cause a change in the period of contact between the wheel and the workpiece. For reverse grinding, the input angle is changed, while for direct grinding, the output angle is changed.

Therefore, the regeneration effect in the model is produced as follows. Due to external excitation, the grinding wheel begins to vibrate against the workpiece. This changes the nominal depth of cut and the material removal rate over time. Since the material removal rate is not constant, different parts of the grinding wheel will remove different amounts of material. This causes different wear around the circumference of the grinding wheel, which is quantified by specific energy. As the grinding force depends on the specific energy, different amounts of specific energy cause different magnitudes of grinding force, and this leads to time-dependent displacement of the grinding wheel, i.e., vibrations of the grinding wheel. So, in a flowchart representation, this gives a closed loop: the initial excitation creates an initial displacement of the grinding wheel, and that displacement causes vibrations again. Depending on the magnitude and the phase difference between the vibrations of the grinding wheel and the grinding process, vibrations can be stable or unstable.

IV) Equation of motion

Five mathematical equations are established: for grinding wheel vibration and cut depth (1), cut depth and material removed (2), material removed and specific energy (3), specific energy and grinding force (4), and grinding force and grinding wheel vibration (5).

The dependence of the grinding wheel displacement (y) on the chip thickness variation (δ) is:

$$\delta(t) = \delta_0 - y(t), \tag{3}$$

where δ_0 is the nominal (default) cutting depth. Obviously, the origin of coordinate y is determined by $y(t) \equiv 0$ and corresponds to stationary grinding.

The relationship between the cut depth and the material removed by one grain (edge) is

$$V_w'(t) = v_w \tau_g \delta(t), \tag{4}$$

where V_w' is the specific amount of removed material (the volume of the material per unit of grinding width).

The relationship between the removed material and the specific energy that quantifies the wear of grain at the moment in time (t) during one period of grinding wheel revolution $(t - T_g)$ can be defined with the specific amount of removed material, which causes grain wear, as

$$u(t) = u(t - T_g) + C_d V_w'(t), \tag{5}$$

where C_d is the dullness coefficient, which converts the removed material into specific energy with the concept of grain wear.

The relationship between the specific energy and the grinding force component in the y direction is determined by averaging the specific energy distribution in the cutting zone and the grinding force represented by equation (2), i.e.

$$F_y(t) = \frac{\mu_y\, w\, v_w\, \delta_0}{v_g\, \tau_{c,0}} \int_0^{\tau_{c,0}} u\big(t - T_g + \tau_{c,0} - \tau\big)\mathrm{d}\tau + C, \tag{6}$$

where τ is the local time coordinate related to the grip zone ($\varphi = \omega_g \tau$), $\tau_{c,0}$ is the contact period of one grain in stationary cutting conditions, and C is the time-independent part of the grinding force.

The relationship between the grinding force and the wheel vibrations is determined by Newton's second law

$$\ddot{y}(t) + 2\zeta\omega_n\, \dot{y}(t) + \omega_n^2\, y(t) = \frac{1}{m} F_y(t), \tag{7}$$

where $\zeta = c/(2m\omega_n)$ and $\omega_n = \sqrt{k/m}$ are the dimensionless damping and the natural circular wheel frequency.

The equation of motion follows from equations (3) to (7) by removing the variables y, δ, V_w' and F_y:

$$\ddot{u}(t) + 2\zeta\omega_n \dot{u}(t) + \omega_n^2 u(t) = \ddot{u}\big(t - T_g\big) + 2\zeta\omega_n\, \dot{u}\big(t - T_g\big) + \omega_n^2\, u\big(t - T_g\big)$$
$$- \frac{\mu_y\, C_d\, w\, v_w^2\, \tau_g\, \delta_0}{m} \int_0^{\tau_{c,0}} u\big(t - T_g + \tau_{c,0} - \tau\big)\mathrm{d}\tau\ , \tag{8}$$

where the force constant C is removed since it is compensated for and does not change the dynamics (or the stability) of the linear system. It is obvious that the grinding process equation of motion has two delays: T_g which corresponds to one wheel rotation and $\tau_{c,0}$ which corresponds to the period required for one grain to pass the cutting zone. It is important to point out that the delay $T_g > \tau_{c,0}$.

3. Chatter stability

The stability assessment is carried out using the Nyquist criterion [19-23], which starts from the system equation in the frequency domain. The Laplace transforms of the five state variables are marked by $\mathcal{L}\{y\}(s) = Y(s)$, $\mathcal{L}\{\delta\}(s) = D(s)$, $\mathcal{L}\{V_w'\}(s) = W_w'(s)$, $\mathcal{L}\{u\}(s) = U(s)$ and $\mathcal{L}\{F_y\}(s) = \Phi_y(s)$, where s is the complex Laplace frequency.

Inserting the Laplace transforms into equations (3) to (7) and arranging them results in the following functions in the frequency domain;

$$J(s) = \frac{U(s)}{D(s)} = \frac{C_d\, w\, \tau_g}{1 - \mathrm{e}^{-T_g s}}, \tag{9}$$

$$H(s) = \frac{\Phi_y(s) - \frac{C}{s}}{U(s)} = \frac{\mu_y w v_w\, \delta_0\, \mathrm{e}^{-T_g s}\left(\mathrm{e}^{\tau_{c,0} s} - 1\right)}{v_g \tau_{c,0} s}, \tag{10}$$

$$G(s) = \frac{Y(s)}{\Phi_y(s)} = \frac{1}{m\left(s^2 + 2\zeta\omega_n s + \omega_n^2\right)}. \tag{11}$$

Transfer functions *J*, *H* and *G* define three ratios in the frequency domain between the specific energy and cutting depth (*J*), the grinding force and the specific energy (*H*) and the grinding wheel vibrations and grinding force (*G*). To determine stability, the Nyquist criterion uses the transfer function of the system's open loop negative feedback (12) [24,25] (which is a product of the previous three transfer functions).

$$T_0(s) = \frac{\mu_y \, C_d \, w \, \tau_g \, v_w^2 \, \delta_0 \, \mathrm{e}^{-T_g s}(\mathrm{e}^{\tau_{c,0} s} - 1)}{m \, v_g \tau_{c,0} \, s(1 - \mathrm{e}^{-T_g s})(s^2 + 2\zeta\omega_n s + \omega_n^2)}. \quad (12)$$

4. Stability graphs

Fig. 2 shows a chatter stability graph during grinding a steel workpiece, with rotational speeds ranging from 0 to 90,000 min^{-1}. It was planned to perform the experiment on the KELLENBERGER cylindrical grinder machine, but due to the lack of measuring equipment, the experiment was not performed. The parameters used in the model are shown in Fig. 3. Unstable areas are marked with “-“, and stable areas are marked with “+”.

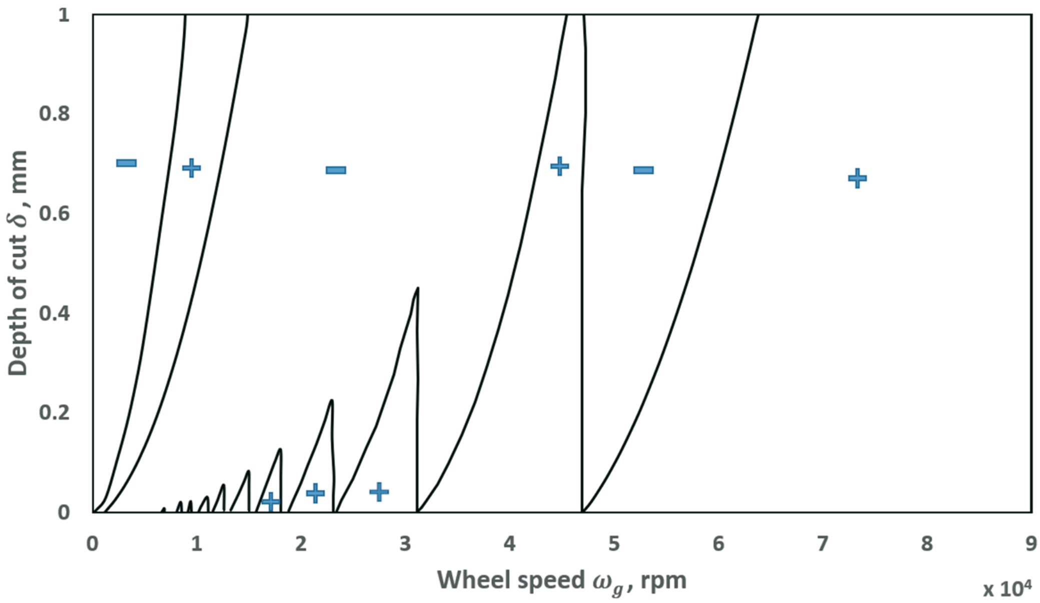

Fig. 2 Counter-directional surface grinding stability graph

In grinding operations, grinding wheels heat up the metal a lot, causing chips to be embedded in the grinding wheel. This effect, called clogging is not taken into account in this study.

Grinding wheel model parameters	
m=0.3 kg	modal mass
zeta=0.02	dimensionless damping
ω_n=1580 Hz	natural frequency

Grinding wheel and workpiece parameters:	
Rg=12 mm	grinding wheel radius
w=6 mm	grinding width
z=10000	number of grinding wheel blades
Cd=4e21 J/m^3/m^2	plowing coefficient
μ_u=0.4	sliding friction coefficient (= Ft / Fn)
v_w=38 mm/min	workpiece speed

Limits and resolution of the stability graph	
wg_min = 1000 rpm	minimum circuit speed
wg_max = 90000 rpm	maximum circuit speed
wg_n = 1500	circuit speed resolution
doc_min = 0.02 mm	minimum cutting thickness
doc_max = 1 mm	maximum cutting thickness
doc_n = 500	cutting thickness resolution

Fig. 3 Model parameters

5. Conclusion

Grinding is typically an unstable process considering the regeneration of the grinding wheel, but even at high grinding speeds, there are stable areas with huge cutting depths, which is especially important for industry. The presented model provides relevant information on how to avoid chatter vibrations, i.e., which changes in the model parameters stabilize the system. This is particularly significant because, depending on the nature of the instability, certain grinding parameters do not have a stabilizing effect on the process. For industry, this has special importance because it will be possible to determine the parameters of the grinding process that will ensure surface quality and dimensional accuracy. This will reduce production costs and workpiece processing time.

In conclusion, the parameter field in which the process is stable and its improvement are currently our research areas; results of the research may pave the way for increased efficiency and productivity in industry.

REFERENCES

[1] A.J. Lynch, C.A. Rowland, The history of grinding, Society for Mining, Metallurgy & Exploration (SME), Littleton, Colorado, 2005.

[2] B. Stone, Chatter and Machine Tools, Springer, Switzerland, 2014. https://doi.org/10.1007/978-3-319-05236-6

[3] J. Munoa, X. Beudaert, Z. Dombóvári, Y. Altintas, E. Budak, C. Brecher, G. Stépán, Chatter suppression techniques in metal cutting, CIRP Ann. 65 (2) (2016) 785-808. https://doi.org/10.1016/j.cirp.2016.06.004

[4] R.S. Hahn, On the theory of regenerative chatter in precision-grinding operations, Trans. ASME (1954) 593-597.h ttps://doi.org/10.1115/1.4014908

[5] P. Landberg, Experiments on grinding, Microtecnic 11 (1) (1957) 18-26.

[6] R. Snoeys, D. Brown, Dominating parameters in grinding wheel-and workpiece regenerative chatter, in: Proceedings of the 10th International Conference on Machine Tool Design and Research, 1969, pp. 325-348. https://doi.org/10.1016/B978-0-08-015661-3.50025-4

[7] R.A. Thompson, On the doubly regenerative stability of a grinder, J. Eng. Ind. 96 (1) (1974) 275-280. https://doi.org/10.1115/1.3438310

[8] R.A. Thompson, On the doubly regenerative stability of a grinder: the combined effect of wheel and workpiece speed, J. Eng. Ind. 99 (1) (1977) 237-241. https://doi.org/10.1115/1.3439144

[9] R.A. Thompson, On the doubly regenerative stability of a grinder: the theory of chatter growth, J. Eng. Ind. 108 (2) (1986) 75-82. https://doi.org/10.1115/1.3187054

[10] R.A. Thompson, On the doubly regenerative stability of a grinder: the mathematical analysis of chatter growth, J. Eng. Ind. 108 (2) (1986) 83-92. https://doi.org/10.1115/1.3187055

[11] R.A. Thompson, On the doubly regenerative stability of a grinder: the effect of contact stiffness and wave filtering, J. Eng. Ind. 114 (1) (1992) 53-60. https://doi.org/10.1115/1.2899758

[12] I. Inasaki, B.A. Karpuschewski, H.S. Lee, Grinding chatter - origin and suppression, CIRP Ann. 50 (2) (2001) 515-534. https://doi.org/10.1016/S0007-8506(07)62992-8

[13] H. Li, Y.C. Shin, Wheel regenerative chatter of surface grinding, J. Manuf. Sci. Eng. 128 (2) (2006) 393-403. https://doi.org/10.1115/1.2137752

[14] R.A. Thompson, The dynamic behavior of surface grinding: Part 2 - some surface grinding tests, J. Eng. Ind. 93 (2) (1971) 492-497. https://doi.org/10.1115/1.3427956

[15] M. Weck, J. Alldieck, The originating mechanisms of wheel regenerative grinding vibration, CIRP Ann. 38 (1) (1989) 381-384. https://doi.org/10.1016/S0007-8506(07)62728-0

[16] F. Hashimoto, A. Kanai, M. Miyashita, K. Okamura, Growing mechanism of chatter vibrations in grinding processes and chatter stabilization index of grinding wheel, CIRP Ann. 33 (1) (1984) 259-263. https://doi.org/10.1016/S0007-8506(07)61421-8

[17] K.W. Chung, Z. Liu, Nonlinear analysis of chatter vibration in a cylindrical transverse grinding process with two time delays using a nonlinear time transformation method, Nonlinear Dyn. 66 (4) (2011) 441-456. https://doi.org/10.1007/s11071-010-9924-y

[18] S. Malkin, C. Guo, Grinding Technology: Theory and Applications of Machining with Abrasives, second ed., Industrial Press, New York, 2008.

[19] H. Nyquist, Regeneration theory, Bell Syst. Tech. J. 11 (1) (1932) 126-147. https://doi.org/10.1002/j.1538-7305.1932.tb02344.x

[20] R.C. Dorf, R.H. Bishop, Modern Control Systems, thirteenth ed., Pearson, Harlow, 2011.

[21] C. Desoer, Y.T. Wang, On the generalized Nyquist stability criterion, IEEE Trans. Autom. Control 25 (2) (1980) 187-196. https://doi.org/10.1109/TAC.1980.1102280

[22] R.A. Thompson, The dynamic behavior of surface grinding: Part 1 - a mathematical treatment of surface grinding, J. Eng. Ind. 93 (2) (1971) 485-491. https://doi.org/10.1115/1.3427955

[23] T. Insperger, G. Stépán, Semi-discretization for Time-delay Systems, Springer, New York, 2011. https://doi.org/10.1007/978-1-4614-0335-7

[24] M. Tóth, N. Sims, D. Curtis, An analytical study of wheel regeneration in surface grinding, Procedia CIRP (2019) 214-219. https://doi.org/10.1016/j.procir.2019.04.046

[25] M. Tóth, N. Sims, D. Curtis, An alternative wheel regenerative mechanism in surface grinding: distributed grit dullness captured by specific energy waves, Mechanical Systems and Signal Processing 1 (2022) 1-18. https://doi.org/10.1016/j.ymssp.2021.107964

[26] Jenn-Yih Chen, Yi-Ling Lin, Bean-Yin Lee, Prediction Model of End Mill Cutting Edge Based on Material Properties and Cutting Conditions 69-84, Transactions of FAMENA, Vol. 46 No. 2, 2022. https://doi.org/10.21278/TOF.462033821

[27] Lyubomir Gruyitch, Zoran Bučevac, Radiša Jovanović, Vladimir Zarić, Discrete-Time System Conditional Optimisation in the Parameter Space via the Full Transfer Function Matrix 45-62, Transactions of FAMENA, Vol. 45 No. 3, 2021. https://doi.org/10.21278/TOF.453014220

[28] Shengyuan Li, Longxi Zheng, Vibration Attenuation Mechanism of the Rotor System with Anisotropic Support Stiffness 129-144, Transactions of FAMENA, Vol. 45 No. 1, 2021. https://doi.org/10.21278/TOF.451017220

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