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The Heterogeneity of Investors Based on Multi-fractal Features with Ultra-High Frequency Data

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Abstract: In the financial markets, the heterogeneity of investors is mostly focusing on very different underlying assets. However, there is one specific heterogeneity need to be discussed, that is the heterogeneity represented by investors who are investing in very similar underlying assets. In another word, whether there is a method to quantitatively describe the heterogeneity when investors have the same expectation in the future. In order to detect this kind of heterogeneity we introduced multi-fractal feature values and select SSE (Shanghai Security Exchange) 50 Index and its derivatives, SSE 50 Index ETF (Exchanged Tradable Fund) and SSE 50 Index Future to research on this topic, since these three underlying assets presented very similar fluctuation during the same period. With the static scenario analysis and dynamic analysis we successfully find that the multi-fractal feature values are not only able to detect this heterogeneity but also be able to describe it quantitatively. The false nearest point test had shown that the multi-fractal values are necessary and rational in this process. The other advantage by introducing multi-fractal values is that they are quantitative numbers which could be applied to models directly.

Keywords: multi-fractal analysis; R+realized volatility; scenario analysis; ultra-high frequency data

1 INTRODUCTION

In the research of heterogeneity of investors in Chinese financial markets, there are a lot of interesting results. However, there is still one specific heterogeneity that has not been discussed yet. That is if the investors are having the same expectation of future, whether they will behave differently during the same period, and whether there exists a quantitative method to show this heterogeneity. In order to solve the problem, we designed this study. In this study we need to solve the following problems, select a statistical factor to describe the investor features: find the quantitative values of this factor to represent the investor features and form up the environment to make sure the investors have the same expectations in the future.

To solve the first problem, we found that integrated realized volatility (*IRV*) is a rational choice. Merton (1980) firstly introduced monthly realized volatility by summing up the squared daily returns of underlying [1]. Andersen and Bollerslev (2003) finally proved the rationality and the universal suitability of realized volatility by high-frequency data [2]. Nowadays, it has been widely used in financial market researches such as, R. Vanaga etc. applied it in the commission financing research to identify the challenges [3]. Y. Gao applied it in financial risk analysis while the assets are still holding [4]. J. M. Lee etc. used several classification methods in financial market research and all the methods they applied used *IRV* as important factor to describe the investors' behaviour [5].

In order to detect the differences of *IRV* we also need to find the factors to evaluate the level of *IRV*. By comparing the past researches, it is interesting to find that multi-fractal features are detected and identified by many researchers; however, by comparing the results we found that during different time periods even the same underlying asset *IRV* would propose very different multi-fractal feature, and for the same time period the different underlying asset *IRV* would reflect different multi-fractal feature as well. For example, Z. Q. Zhang and W. X Zhou [6], L. Ureche-Rangau and Q. de Rorthays [7], X. Q. Liuand F. Y. Feietc. [8], researched on stocks, energy commodity, and indexes respectively during the year of 2008 in Chinese markets especially L. Zhang and his team [9] they applied the fractal features in return rate research in Chinese financial markets and proved that traditional method such as Power-law distribution is still effective in fractal features identification. However the multi-fractal features values show obvious differences. The same situation has been found in other researches, H. T. Chenand C. F. Wu [10], D. X. Mei, J. Liu [11] applied the multi-fractal features in forecasting of volatility in Chinese financial markets; however, they provided very different multi-fractal hypothesis. Similar results were found in the international financial markets such as, Casablanca markets by S. Lahmiri [12, 13], Hong Kong markets by Q. S. Ruan [14], Korea markets by K. Yim [15], and G. Oh [16]. During the year 2001 to 2019, researchers had indentified multi-fractal features of realized volatility in Chinese financial markets based on different frequency of data, such as D. Gu [17], L. Zhang [18], D. H. Wang [19] M. Ausloos [20]. With all these wonderful research results, researchers proposed the assumption that multi-fractal feature is able to reflect the heterogeneity of investors and some of the researches had provided relevant attempts. Z. P. Tang [21] and his team found the micro differences under different market situations based on multi-fractal and self-similarity features of high-frequency data of the SSE index. P. O´swie's team [22] and L. Zhang' team [23] both introduced multi-fractal into scenario analysis in financial markets in order to detect more information beyond the traditional statistical factors. O. Pont [24] had applied multi-fractal features in asset price forecasting since 2009 by considering the none-linear feature of the raw data, while C. N. Xing [25] introduced the multi-fractal into financial risk managements in order to increase the sensitivity of the risk manage model. Then we are able to go a step further: based on the multi-fractal features, researchers could apply the multi-fractal feature values of *IRV* to describe the investor feature quantitatively. This idea resulted from the research of X. Wang [26] who applied them in radar signal recognition. D. Stanujkic etc. [27] and A. Tkacenko [28] also provided very interesting multi-criteria method in financial markets analysis. It is important to point out that in the factor selection process the rationality test is very significant and in our research we applied false nearest point test since it is proved to be effective and easily applied.

The last problem that we need now is to find the underlying assets that will always have the same future that can make sure the investors have the exactly same expectations to be able to be solved by the establishment of multi-level capital markets. The derivatives markets of certain underlying developed considerably, and the trading of these derivatives became more and more frequent. This provided the possibility for researchers to improve the micro-structure analysis of financial markets. For example, the Shanghai Securities Exchange 50 Index (SSE 50 index) and its derivatives, the Shanghai Securities Exchange 50 Index Exchange Traded Funds (SSE 50 Index ETFs) and the Shanghai Securities Exchange 50 Index futures (SSE 50 Index futures), are now listed on the Shanghai Stock Exchange and the Chinese Financial Futures Exchange. The price of these three has strong similar fluctuation, and based on the mechanism of settlement, the price of them will be converged periodically. However, the basic investment requirements and their essence determined that their investors will be different in risk tolerances, trading strategies, as well as trading habits. Therefore, based on the identification of multi-fractal features of high-frequency realized volatility of the SSE 50 Index and its derivatives, we are able to analyze the heterogeneity of investors who are investing in the same underlying, and discover and comprehend the features of the micro-structure of Chinese financial markets.

In this paper, we designed a study path. With the first step, we captured the raw data of SSE 50 Index and its derivatives, and calculated the *IRV* as the criterion of investor feature. Then the multi-fractal feature values of *IRV* are calculated under different situations to prove that it is able to detect the differences between SSE 50 Index and its derivatives. At last the False Nearest Points method (FNP method) was applied to show that the traditional price and volume system is not enough in this specific heterogeneity research.

In Section 2, the methods that calculate realized volatility, multi-fractal values, and FNP are briefly introduced. In Section 3, the SSE 50 index and its derivatives are generally introduced and the empirical analysis is applied. Section 4 is the conclusion.

2 METHODOLOGY

2.1 Calculation of Integrated Realized Volatility (*IRV*)

Andersen and Bollerslev (2003) proved the rationality of *IRV* and determined that realized volatility based on tick data can be recognized as an effective and unbiased estimation of market fluctuation [2]. It can be calculated as Eq. (1) and Eq. (2)

$$IRV_t = \sum_{I=1}^{N}\left(r_i^2\right) \tag{1}$$

$$r_i = \log\left(P_i / P_{i-1}\right),\ i = 2, ..., n \tag{2}$$

where IRV_t is the realized volatility during time period t, N is the frequency of *IRV* (in this essay it is equal to 1 minute), P_i is the price at time i.

2.2 Multi-Fractal Identification by MF-DFA

There are many methods in multi-fractal identification, such as WF-WT, MF-WL, MF-DFA, MF-DMA, etc. In this research we are more focused on the multi-fractal features themselves rather than the accurate multi-fractal value, therefore the calculation convenience is the priority in the method selection and MF-DFA is chosen. This method is firstly applied by Kantelhardt based on DFA. With a time series $X(t)$, t =1, 2, 3, …, M with length M the detailed steps are as follows:

Step 1: Calculate the cumulative deviation $Y(t)$ of $X(t)$ by Eq. (3).

$$Y(t) = \sum_{t=1}^{t}\left(X_t - \overline{X}\right),\ t = 1,\ 2,\ 3,\ ...,\ M \tag{3}$$

where $\overline{X}$ is the mean value of $X(t)$.

Step 2: Cut the time series from the beginning and the tail by time scale s, and receive $2N$ spaces with length s. The spaces can be illustrated as v, v =1, 2, 3, …, $2N$, while $N = int\left(\frac{M}{s}\right)$.

Step 3: Fit the elements of each space v by Eq. (4).

$$\tilde{Y}_V(i) = \tilde{a}_0 + \tilde{a}_1 i + \tilde{a}_2 i^2 + ... + \tilde{a}_k i^k \tag{4}$$

In this research $k = 2$.

Step 4: Calculate the *RMSE* between the original data and the fitted data. The *RMSE* of spaces that are cut from the beginning is calculated by Eq. (5) while the spaces that are cut from the tail are calculated by Eq. (6).

$$F^2(v, s) = \frac{1}{s}\sum_{i=1}^{s}\left\{Y\left(\left[v-1\right]s + i\right) - \widetilde{Y}_v(i)\right\}^2 \tag{5}$$

$$F^2(v, s) = \frac{1}{s}\sum_{i=1}^{s}\left\{Y\left(M - \left[v-N\right]s + i\right) - \widetilde{Y}_v(i)\right\}^2 \tag{6}$$

Step 5: Calculate the qth order fluctuation function value $F_q(s)$ by Eq. (7) and Eq. (8) with different q values.

When $q = 0$:

$$F_q(s) = \exp\left\{\frac{1}{4N}\sum_{v=1}^{2N}\log\left[F^2(v, s)\right]\right\} \tag{7}$$

When $q \neq 0$.

$$F_q(s) = \left\{\frac{1}{2N}\sum_{v=1}^{2N}\left[F^2(v, s)\right]^{q/2}\right\}^{1/q} \tag{8}$$

Step 6: Change the scale s and repeat step 2 to step 5 and calculate the fluctuation function value $F_q(s)$ under different time scales.

Step 7: Fit $F_q(s)$ with time scale s by Eq. (9).

$$F_q(s) \sim s^{H(q)} \tag{9}$$

where $H(q)$is the generalized Hurst exponent.

2.3 Multi-Fractal Feature Values

$H(2)$ is the classical Hurst exponent to describe the long-term correlation of the raw data. When $H(2) > 0.5$, the data is long-term correlated, when $H(2) < 0.5$ the data is mean value reversion, and when $H(2) = 0.5$, the data is random.

Based on different q, the multi-fractal strength value could be represented by ΔH. The calculation is as follows (Eq. 10).

$$\Delta H = H(q_{\min}) - H(q_{\max}) \tag{10}$$

According to $H(q)$ and q, we are able to calculate the multi-fractal feature value by the following equations from Eq. (11) to Eq. (15). Especially, these values are capable of reflecting the implied information of the raw data. For instance, $\Delta\alpha$ represents the singularity level. The greater the $\Delta\alpha$, the more heterogeneous the level of *IRV*. $\Delta f(\alpha)$ is applied to reflect the skewness of the singular spectrum, where $\Delta f(\alpha) > 0$ means that *IRV* has a greater probability to stay at the local maximum.

$$\tau\left(q\right) = q \times H\left(q\right) - 1 \tag{11}$$

$$\alpha = \frac{\mathrm{d}\tau\left(q\right)}{\mathrm{d}q} \tag{12}$$

$$f\left(\alpha\right) = q\alpha - \tau\left(q\right) \tag{13}$$

$$\Delta\alpha = \alpha_{\max} - \alpha_{\min} \tag{14}$$

$$\Delta \mathrm{f}\left(\alpha\right) = f\left(\alpha_{\min}\right) - f\left(\alpha_{\max}\right) \tag{15}$$

2.4 The False Nearest Points (FNP) Test

The FNP method is a test to show whether it makes sense to include the potential factors in the analysis. It operates as follows:

Step1. With a set $\{A\}$ under the original dimension that $A_i = (a_1, a_2, a_3, \ldots, a_n)$, find the nearest two point pairs A_i and A_j based on the Euclidean distances between every two points.

Step 2. Extend the original dimension to $A_i = (a_1, a_2, a_3, \ldots, a_n, \ldots, a_{n+m})$ and find the nearest two point pairs A_i and A_v based on the Euclidean distances between every two points.

Step3. Compare j and v, if $j \neq v$, then A_i and A_j are false nearest points. If the number of false nearest points has reached a certain level it is rational to say that the extended dimension of $(a_{n+1}, \ldots, a_{n+m})$ is necessary.

3 DATA AND RESULTS

3.1 Research Data

The SSE 50 Index is one of the most important comprehensive indexes in Chinese financial markets. It contains 50 extremely large companies that also have great liquidity. The SSE 50 Index ETF and the SSE 50 Index future are the derivatives dependent on the SSE 50 Index. The SSE 50 Index ETF historically was the first ETF in China. The SSE 50 Index future will be settled by cash every third Friday each month. If any one of the three experiences irrational fluctuation, the arbitrage strategies may be applied by longing the price that is underestimated and shorting the price that is overestimated. Therefore even though the price change of the three underlying may differ for some time, their final trend will definitely be convergent. Investors are able to invest in this index via all these three underlying since they will ultimately have the same return. This could be seen from Fig. 1.

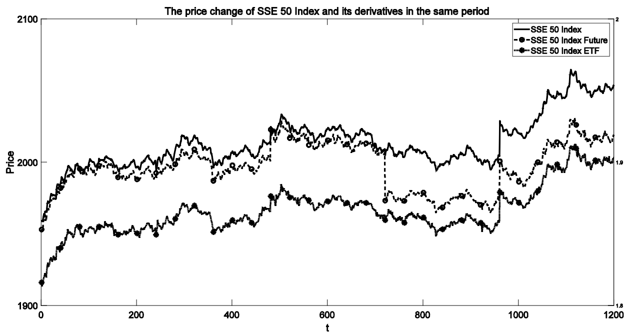

Figure 1 The price change of SSE 500 Index and its derivatives in the same period

However, when the price change is highly similar, the three underlying have different trading regulations and requirements, which are listed in Tab. 1.

Table 1 SSE50 Index and its derivatives

	SSE 50 Index	SSE 50 indexETF	SSE 50 Index future
Trading amount	huge	About ¥ 300.00	About ¥ 1 million
Trading cost	2 – 3 %/lot	2 - 3 %/lot	¥ 60/lot
Entry requirements	none	none	high
Trading method	Indirect trade	Trade, subscribe, redeem	trade
Trading direction	Long and limited short	Long and limited short	Long and short
Trading model	Full amount	Full amount	Marginal amount
Clearing frequency	$T + 1$	$T + 1$	$T + 0$
IRV frequency	1 min	1 min	1 min
N	12	20	120
Tick frequency	5seconds	3 seconds	0.5second
Investor type	Mainly individual	Mainly institutions	Mainly institutions
Risk preference	Comprehensive	Risk averse	Risk seeking
Acknowledge level	Comprehensive	Professional	Strong professional

3.2 Empirical Analysis
3.2.1 Static Scenario Analysis

In order to observe whether the multi-fractal features are able to reflect the investor differences under the exact same economic environment, two scenarios are selected in this research: similar fluctuation with different levels of market vitality and different market quotations with "maintain, rise and drop".

Tab. 2 demonstrates the values of each multi-fractal feature of the SSE 50 Index and its derivatives in two different weeks with different market vitality. During the two weeks, the trading frequency of the SSE 50 Index ETF increased by 4.63% from 20106 to 21037 and the SSE 50 Index future increased by 15.37% from 109343 to 126149. Meanwhile, the SSE 50 Index experienced fluctuations of 2.39% and 2.26% respectively.

Table 2 Multi-fractal values of *IRV* under different trading frequency

	SSE 50 Index		SSE 50 Index ETF		SSE 50 Index future	
	Lower-vitality	Higher-vitality	Lower-vitality	Higher-vitality	Lower-vitality	Higher-vitality
$H(2)$	0.713	0.758	0.647	0.825	0.891	0.771
ΔH	0.416	0.360	0.320	0.417	0.369	0.856
$\Delta\alpha$	0.580	0.413	0.485	0.563	0.471	1.024
$\Delta f(\alpha)$	−0.246	−0.433	−0.318	−0.384	−0.244	−0.379

Tab. 2 shows that:
1. $H(2)$ shows the most obvious difference. In the lower-vitality trading week, the SSE 50 Index ETF shows only 0.647 while the SSE 50 Index future has a much greater value of 0.891. Furthermore, when vitality is increased, the $H(2)$ value of the SSE 50 Index ETF increases a lot from 0.647 to 0.825 while the SSE 50 Index future experiences a drop from 0.891 to 0.771. The SSE 50 index is only slightly affected with a rise from 0.713 to 0.758.
2. The ΔH of the SSE 50 Index and the SSE 50 Index ETF do not change too much, while the ΔH of the SSE 50 Index future sky rockets from 0.369 to 0.856. It is worth mentioning that the ΔH of the SSE 50 Index decreases by 0.056 while that of the SSE 50 Index ETF increases by 0.097, which means that when market vitality is changing, not only does the influence strength change but also the influence type.

Table 3 Multi-fractal values of *IRV* under different trading frequency

	SSE 50 Index		SSE 50 Index ETF		SSE 50 Index future	
	Lower-vitality	Higher-vitality	Lower-vitality	Higher-vitality	Lower-vitality	Higher-vitality
$H(2)$	0.713	0.758	0.647	0.825	0.891	0.771
ΔH	0.416	0.360	0.320	0.417	0.369	0.856
$\Delta\alpha$	0.580	0.413	0.485	0.563	0.471	1.024
$\Delta f(\alpha)$	−0.246	−0.433	−0.318	−0.384	−0.244	−0.379

Tab. 3 shows that:
1. $H(2)$ experiences obvious differences. The SSE 50 Index has the greatest value in drop quotation at 0.809 at the same point when the SSE 50 Index ETF and the SSE 50 Index future peak. The $H(2)$ value maintains its quotation level at 0.769 and 0.802, respectively. The influence level of market quotation is different, the SSE 50 Index and the SSE 50 Index future are affected strongly by the market quotation while the SSE 50 Index ETF is affected less.
2. ΔH provides very different results. Regarding the SSE 50 Index, ΔH is 0.591 in the rising quotation which is much greater than the quotation of maintaining and drop of 0.311 and 0.375 respectively. According to the SSE 50 Index future, its ΔH in quotations of rising and drop are 0.879 and 0.834 respectively, which are strongly greater than the maintained quotation of 0.661. The ΔH of the SSE 50 Index ETF reaches the peak of 0.543 when the market heavily drops, reaching a level greater than the rising quotation of 0.409 and the maintenance quotation of 0.333. It can be seen that ΔH affects the market quotation in a very complex way, as different market quotations of the SSE 50 Index and its derivatives changed differently not only in value level but also in direction.
3. The skewness of the singular spectrum changes differently. $\Delta f(\alpha)$ of the SSE 50 Index is $0.4356 > 0$ in maintenance quotation while that of the SSE 50 Index future is $0.0278 > 0$ in the rising quotation.

3.2.2 Dynamic Circumstances Analysis

In this Section, we intend to apply the continuously dynamic fluctuation of multi-fractal values to show the change of multi-fractal features continuously. The test length of the series was kept at 1200 and each candidate series was moved 1 min forward to observe the sensitivity of multi-fractal values in dynamic environments.

The time selection is between February 17, 2020, to March 6, 2020, during which period the price increased by 2.26% at first, then decreased by 4.47% in the following week, and increased by 4.38% in the last week. In this period, the market initially experienced a rise, after which there was a drop and eventually a rise again. Therefore, it is rational to observe the change of multi-fractal features with the strong fluctuation in this period.

The total length of the *IRV* that is calculated is 3600. $\{X_i\}, i = 1, 2, ..., 2400$. Each identification will select the elements of X_j, X_{j+1}, …, X_{j+1199}, j = 1, 2, …, 2400, and identify 2400 times in order to see the multi-fractal features change every single minute.

The continuously dynamic fluctuation of the multi-fractal value is demonstrated in Fig. 2, Fig. 3, and Fig. 4.

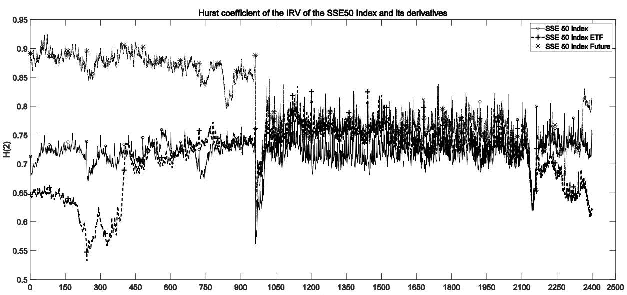

Figure 2 Hurst coefficient of the *IRV* of the SSE 50 Index and its derivatives

It is obvious that nearly half the time the $H(2)$ of the three underlying experienced extremely large different changes during the exact same period. A similar result has been found in ΔH and $\Delta f(\alpha)$ as well. The results are shown in Fig. 3 and Fig. 4.

The multi-fractal strength change is shown in Fig. 3.

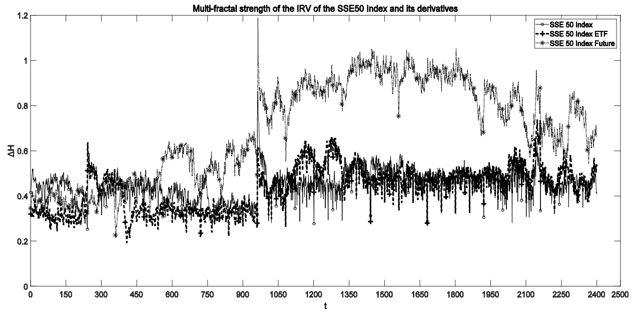

Figure 3 Multi-fractal strength of the *IRV* of the SSE 50 Index and its derivatives

The skewness of the singular spectrum is represented in Fig. 4.

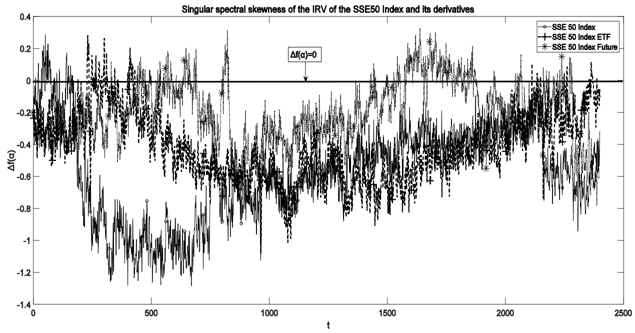

Figure 4 Singular spectral skewness of the *IRV* of the SSE 50 Index and its derivatives

Concluding the static and dynamic empirical analysis, it can be found that even with the exact same macro eco-environments, the investors who invest in the SSE 50 Index by different derivatives still represent very different behavior features and all these differences are able to be identified by multi-fractal feature values. This indicates that the multiple-level capital market estimation achieved its goal and is able to lead the investors to the markets that are most suitable for them.

3.2.3 False Nearest Points Test

The previous sections have proved that multi-fractal feature values are able to detect and describe the investor features under the high-frequency micro-structure. However, whether multi-fractal tools are essential or whether the traditional price-volume system is enough needs to be tested. The candidate data were selected for the full trading week in the year 2019. The traditional factors are selected as volume (V), return rate (R), greatest fluctuation in the week (F), and trading frequency (Fr). The multi-fractal factors were selected as $H(2)$, ΔH, $\Delta\alpha$, $\Delta f(\alpha)$ and will be added to the traditional factors to observe whether the result will be different.

H_0: Introducing multi-fractal factors will provide more useful information in historical data analysis.

H_1: Introducing multi-fractal factors will not provide more useful information in historical data analysis.

The test result is shown in Tab. 4.

Table 4 Test results of the SSE50 Index and its derivatives by false nearest points

	SSE 50 Index	SSE 50 Index ETF	SSE 50 Index future
Proportion of false nearest points	69.77%	55.81%	74.42%

Since the proportion of false nearest points is much greater than 5%, the null hypothesis, that the introduction of multi-fractal factors is rational and essential, should be accepted. Furthermore, it is demonstrated by this test that, in the high-frequency micro-structure, even if the investors are investing in the same underlying such as the SSE 50 Index, the heterogeneity of their investing behaviors is still able to be detected by multi-fractal tools.

3.3 Implications

This research had proved that even the investors may have the same expectation in the future; they still will behave very differently due to the heterogeneity of the investors such as risk tolerance, trading strategies and trading habits. All these differences are able to be presented by their multi-fractal feature values. These values may describe in detail how the investors will repeat their investing behaviors through the Hurst coefficient and other derived parameters. For instances, $H(2)$ is the classical Hurst exponent to describe the long-term correlation of the raw data, while the multi-fractal strength value could be represented by ΔH. Importantly, the multi-fractal feature values are quantitatively numbers; these numbers are capable to be directly applied into models such as, classification and cluster. This may improve the ability in modeling of financial markets researches.

In the real life, the variety of investors may make the markets more complex, such as unexpected fluctuation, liquidity risks and arbitrage opportunities. With the new factors that were applied by heterogeneity researches, the participants of the markets are able to recognize and track the diversified investors. Then, they may adjust their strategies to fit the investing environments or make the adjustments of policies to stabilize the markets. Furthermore, with the progress of calculation power, it is better to introduce ultra-high frequent data in the research since it could contain more information.

4 CONCLUSION

There is a hypothesis that even the underlying assets are affected by very similar factors, the investors may behave differently due to the heterogeneity of the investors such as risk tolerance, trading strategies and trading habits. However, due to the research condition in the past periods, it is difficult to detect these differences. This results from the fact that the fluctuation of these underlying assets such as SSE 50 Index and its derivatives is strongly similar, it is impossible to detect the differences based on the traditional volume and price system. Therefore, in this research we firstly selected the strongly relevant underlying assets, SSE 50 Index and its derivatives; they are affected by the same factors and there will be convergence due to the settlement mechanism. Secondly, as the traditional factors of volume and price are not enough, we introduced multi-fractal feature values into this process. Based on the static and dynamic scenario analysis, it is proved that multi-fractal feature values are able to reflect the heterogeneity of the investor even in the exactly same period. At last we proposed a false nearest points test to show the rationality of the introducing the multi-fractal features in the heterogeneity analysis. This research is only an initiation, actually investors are diversified strongly, and therefore, even though they share the same expectation during the same time period, they still would behave differently. These differences may provide unknown volatility, risk, liquidity, as well as arbitrage opportunities. Thus the heterogeneity research on investors is a significant topic for both the participants and the supervisors of capital markets. However, with the fast development of the capital markets, this study still needs to be improved. For instances, in this research an eight dimension vector was applied to describe the investors behaviour features. However, the trading frequency is getting faster and faster and the complexity of investors is getting stronger as well. This requires stronger calculation ability of the models to satisfy the high-frequent data need. Secondly, the accuracy of the factors needs to be discussed deeper to find less factors to include more information. To be concluded, it is proved that the heterogeneity among investors who have the same expectation in the future exists, and could be represented quantitatively by multi-fractal feature values. It is rational to introduce traditional nonlinear complex research methods into financial markets research.

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