Optimal Design and Cogging Torque Minimization of a Permanent Magnet Motor for an Electric Vehicle

Hejra MSADDEK, Ali MANSOURI*, Hafedh TRABELSI

Abstract: With the increase of electric vehicle mobility, the integration of an electric motor inside the vehicle wheel is an interesting architecture for electric vehicles. This solution has the advantage of a great compactness of the motor and the elimination of mechanical transmission. This paper deals with the analytical modeling and the optimal dimension’s finding of an in-wheel motor. An optimization procedure, based on Sequential Quadratic Programming SQP, is performed. The main objectives are the minimization of the machine weight and the maximization of its efficiency. While respecting constraints, an optimal machine with a weight of 11.61 kg and an efficiency of 93% is reached. To verify the satisfaction of the design requirements and the motor performances, finite element analysis (FEA), was applied. The comparison of induction results shows a good accuracy with a maximum error of 14%. The output torque and the air gap flux density are assessed. A maximum output torque of 90 Nm is achieved, and the slotting effect is noticed. Then the cogging torque of the machine is investigated; different rotor structures were studied. It is concluded, from results, that the topology with circular segmented magnets has the lowest cogging torque which does not exceed 7 Nm.

Keywords: analytical model; cogging torque; finite element analysis; optimization problem permanent magnet motor; SQP algorithm

1 INTRODUCTION

The pollution and the problem of greenhouse gases are two major problems that threaten the life of the human being. These problems are mainly due to the use of internal combustion engines. In addition, these engines make extensive use of fossil fuel resources. For this reason, the vehicle industries are encouraging researchers to overcome these problems by the emergence of electric vehicles [1-6].

Because of the cost of magnet materials and the complexity of the machine’s structures, electric vehicles had not been very successful at the beginning [7]. With the technological progress of the last decade, the concept of electric cars has been again adopted. Great progress is being made in this domain [8-9]. In fact, an improvement in the electrical and magnetic performances of electric vehicles shows that these vehicles can substitute combustion vehicles [10-11].

The present work is situated in this context. We are looking to design an optimal permanent magnet motor for an electric traction application with a minimized cogging torque value.

A particular solution for electric vehicle design is adopted and studied, which is to independently drive every wheel by direct drive electric motor without any transmission or differential [12]. The motor is located inside the wheel. The weight of the motor is limited by the wheel dimensions. Therefore, the in-wheel motor design became a crucial optimization problem aiming to limit the motor weight without deteriorating other performances such as the motor efficiency. In the literature, few prototypes have been developed and many studies have been carried out in this context for electric and hybrid vehicle applications [13-17]. The comparison between different motors topologies done in [18] shows that permanent magnet machines are the best candidate for in wheel electric motor for electric vehicle application. Due to their numerous advantages, it has been mentioned in [19] that permanent magnet motors are widely applied for high performances applications.

Compared to other machines, it was shown in [20] that the machine with outer rotor presents numerous advantages: (i) the outer rotor machine is the most suitable for in-wheel vehicle applications, (ii) the large machine air gap diameter, enables the use of several poles, and (iii) the fact that the rotor is mounted on the outside makes the system more compact.

Depending on the shape of the induced electromotive force (emf) and their mode of feeding, permanent magnet machines can be classified into two families: 1) Permanent Magnet Synchronous Machines (PMSM) which are fed by sinusoidal currents and generally have sinusoidal emf and 2) Brushless Direct Current Motors (BDCM) which are fed by slot currents and generally have trapezoidal emf. The ease and the low cost of implementation as well as the simplicity of control of BDCM in comparison with PMSM [21] encouraged us to choose the BDCM for the present application. In [22], the design of a BLDC motor by means of FEA is proposed.

In this paper, the studied topology of the BDCM is described and the major geometric parameters are presented. An analytical model dealing with magnetic, electric, geometric, and thermal machine properties is presented. Then, for the design procedure of the considered machine according to the specifications, an optimization problem is formulated. In fact, this optimization problem is applied to minimize the motor weight and to maximize the efficiency. The optimization method is based on the Sequential Quadratic Programming (SQP). Once the optimization is done, a numerical modeling of the optimized topology, using the finite elements analysis, is defined. A comparison between analytical method and numerical ones is performed. The obtained results are discussed.

In the purpose of studying the machine performances, deep finite element analysis simulations were conducted. The air gap flux density and the output torque are investigated.

To minimize the cogging torque of the machine, the effect of the rotor structure studied. Therefore, the cogging torque is computed for different magnets shapes. In the literature, different techniques are used to minimize the cogging torque in permanent magnet motor [23-27]. In our work, we are interested in the method based on segmenting and spacing the magnets. According to the obtained results...
this technique has significantly reduced the cogging torque value.

2 MOTOR SIZING

2.1 Motor Geometry

As previously mentioned, the studied motor is a BDCM with trapezoidal emf fed by slots current. It has 36 slots, 24 poles with a surface mounted radial flux permanent magnet. Fig. 1 presents most of the geometrical parameters of the motor. Fig. 2 shows the geometry of the winding slots.

\[
N = \frac{pE}{D_{exs} \cdot L_{machine} \cdot B_c \cdot 2\pi f}
\]

where \( E \) is the emf, calculated from Lenz law.

\[
E = N \frac{d\phi}{dt} \cdot N \Omega \frac{d\phi}{d\theta}
\]

For a displacement of \( \pi/p \) with \( p \) as the number of pole pairs we have:

\[
\frac{d\phi}{d\theta} = \frac{2p\phi}{\pi}
\]

The expression of the emf is:

\[
E = N\Omega \frac{2p\phi}{\pi}
\]

The flux expression is obtained by assuming a linear variation as function of the rotor position:

\[
\phi = B_c \cdot S_{pole}
\]

The electromagnetic torque is:

\[
C\Omega = 2EI
\]

\[
C = 2NIB_c \frac{S_c}{\pi}
\]

2.2 Machine Design

The most important step in the machine design is the definition of the characteristics to be achieved. Also, in the design procedure, many types of models are coupled to compute the motor parameters and to obtain the desired by the results. Tab. 1 illustrates the motor specifications.

<table>
<thead>
<tr>
<th>Designation</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal voltage</td>
<td>V</td>
<td>48</td>
</tr>
<tr>
<td>Outer rotor diameter</td>
<td>mm</td>
<td>262</td>
</tr>
<tr>
<td>Axial length</td>
<td>mm</td>
<td>73</td>
</tr>
<tr>
<td>Weight</td>
<td>kg</td>
<td>11</td>
</tr>
<tr>
<td>Efficiency</td>
<td>%</td>
<td>93</td>
</tr>
</tbody>
</table>

2.2.1 Electromagnetic Equations

In the design procedure, the number of coils is a crucial parameter.

This parameter is calculated by applying the Lenz law:
2.2.3 Magnetic Equations

Some relations between geometric and magnetic equations are given by the flux conservation.

- Flux conservation between teeth and the airgap in the teeth’s vicinity:

\[ b_{ts} \cdot B_{ds} = \frac{D_{exs}}{2} \cdot \beta \cdot B_e \]  

(14)

which allows calculating \( b_{ts} \) which is the width of the tooth.

- Flux conservation between magnet and rotor yoke:

\[ B_{ry} \cdot h_{ry} \cdot K_{fud} = \frac{1}{2} B_a \cdot \alpha \cdot \left( e + \frac{D_{exs}}{2} \right) \]  

(15)

where \( k_{fud} \) is the leakage factor \( (k_{fud} < 1) \) [15].

- Flux conservation between magnet and stator yoke

Identically, for the stator:

\[ B_{sy} \cdot h_{sy} \cdot K_{fud} = \frac{1}{2} B_a \cdot \alpha \cdot \left( e + \frac{D_{exs}}{2} \right) \]  

(17)

2.2.4 Thermal Equations

In the design procedure, it is very important to calculate the motor temperature which is described by Eq. (18). We suppose that the temperature is equal in different parts of the motor (magnet, rotor, stator, teeth) because we suppose that the convection thermal resistances are always more important than the thermal conduction resistances.

\[ T = T_{ext} + \frac{P_j + P_{fer-s} + P_{fer-r} + P_m}{h_{conv} \cdot S_{ext}} \]  

(18)

where \( S_{ext} \) is the external surface of the motor.

\[ S_{ext} = \frac{\pi}{2} D^2_{exs} + \pi \cdot D_{exs} \cdot L_{machine} \]  

(19)

2.2.5 Motor Weight and Efficiency

The motor’s weight is computed from the sum of the weights of the different active parts: rotor, stator, magnet, and copper Eq. (20). The motor weight is the first objective to optimize. The second objective is the efficiency which is computed like in Eq. (21).

\[ m_{mat} = m_{cr} + m_{cs} + m_{pm} + m_{copper} \]  

(20)

\[ \eta = \frac{C_{em} \cdot \Omega \cdot P_m}{C_{em} \cdot \Omega + P_j + P_{fer-s} + P_{fer-r}} \]  

(21)

To calculate the copper losses, we must calculate the resistance per phase.

\[ R_{ph} = \rho_{civre} \cdot \frac{I_{cuvre}}{S_{civre}} \]  

(22)

hence the iron losses:

\[ P_j = 2 \cdot R_{ph} \cdot I^2 \]  

(23)

where \( \rho_{civre} \) is the resistivity of copper; \( L_{cuvre} \) is the length of the copper.

The iron losses in the rotor yoke \( P_{fer-r} \) and in the stator yoke \( P_{fer-s} \) are calculated as follows [28]:

\[ P_{fer-r} = P_{specific} \left( \frac{f}{f_{const}} \right)^{1.5} \cdot m_r \cdot \left( \frac{B_{ry}}{B_{const}} \right)^2 \]  

(24)

\[ P_{fer-s} = P_{specific} \left( \frac{f}{f_{const}} \right)^{1.5} \cdot m_s \cdot \left( \frac{B_{sy}}{B_{const}} \right)^2 \]  

(25)

3 OPTIMIZATION PROCEDURE

The machine geometry is completely defined through 9 parameters. To identify these unknown parameters, an optimization procedure must be done. In the literature several objective functions were used: minimization of the permanent magnet material weight, minimization of the total losses or maximization of the efficiency [20, 29, 30]. In our work, because the motor will be inside the wheel, the chosen objective function is the minimization of the motor weight. But we must not deteriorate the efficiency, so the second objective is the maximization of machine efficiency. An optimization design procedure is generally realized throughout three major steps. It begins by identifying the unknown machine design variables. Secondly the machine constraints and the objective function are defined and finally an appropriate solver is applied to find the optimum geometry satisfying all the requirements [31].

3.1 Optimization Variables

For the design, we have 9 optimization variables which are cited in the next table.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Unit</th>
<th>Designation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( B_x )</td>
<td>T</td>
<td>Induction in the rotor yoke</td>
</tr>
<tr>
<td>( B_y )</td>
<td>T</td>
<td>Induction in the stator yoke</td>
</tr>
<tr>
<td>( B_z )</td>
<td>T</td>
<td>Induction in the teeth</td>
</tr>
<tr>
<td>( \delta )</td>
<td>mm</td>
<td>Thickness of the airgap</td>
</tr>
<tr>
<td>( D_{exs} )</td>
<td>mm</td>
<td>Outer diameter of the stator</td>
</tr>
<tr>
<td>( D_{incs} )</td>
<td>mm</td>
<td>Inner diameter of the stator</td>
</tr>
<tr>
<td>( h_{sw} )</td>
<td>mm</td>
<td>Slot wedge height</td>
</tr>
</tbody>
</table>
3.2 Constraints

Depending on the motor application, wheel-motor, many other constraints related to the wheel dimension have to be respected:
- The outer diameter of the rotor is fixed by the wheel diameter: $D_{exr} = 262$ mm.
- The axial length of the motor: $L_{machine} = 73$ mm.

3.3 Specifications

According to the given specifications we have:
- The motor weight: approximately 11 kg.
- The efficiency: 93%

3.4 Optimization Problem

After selecting the objective functions, the optimization variables and the constraints, the optimization problem is expressed as follows:

$$\begin{align*}
\min_X & \left( m_{mot} \right) \\
\min_X & \left( 1 - \eta \right) \\
\text{under constraints} & \\
1 \leq B_{xg} \leq 1,7T \\
1 \leq B_{yg} \leq 1,7T \\
1 \leq B_{zg} \leq 1,7T \\
0,85 \leq B_s \leq 0,95T \\
0,8 \leq \delta \leq 2 \text{ mm} \\
130 \leq D_{incs} \leq 200 \text{ mm} \\
3 \leq h_{pm} \leq 5 \text{ mm} \\
200 \leq D_{exs} \leq 250 \text{ mm} \\
D_{exr} = 262 \text{ mm}
\end{align*}$$

4 RESULTS

At first the optimization problem is solved by means of the SQP algorithm. The variations of the two objective functions, weight and efficiency, are illustrated in Fig. 3 and 4. According to these figures, we can clearly notice that after 140 iterations the objectives tend towards specifications.

Fig. 5 shows the variation of the weight of the motor according to its efficiency. From this figure, two regions can be distinguished. The first region is located on the left of the point 1 in which the motor is unfeasible. In fact, we notice that in this region the weight of the engine decreases while the efficiency increases which is not logical, because if the weight decreases, then the weight of all the materials decreases and as a result the efficiency decreases. So, the second region of the curve, in which the motor is feasible, is located on the right of the point 1.

![Figure 3 Weight variation according to iterations](image)

![Figure 4 Efficiency variation according to iterations](image)

![Figure 5 Front of Pareto for weight and efficiency](image)

The final point of the optimization is meeting the specification (weight = 11.61 Kg, efficiency = 92.24% which is close to 93%). The outer diameter constraint of the rotor is respected ($D_{exr} = 262$ mm) with an active length of the machine $L_{machine} = 73$ mm.

The different machine regions weights are illustrated in Tab. 3.

<table>
<thead>
<tr>
<th>Entities</th>
<th>Before optimization</th>
<th>After optimization</th>
</tr>
</thead>
<tbody>
<tr>
<td>Copper weight / kg</td>
<td>1,15</td>
<td>0,45</td>
</tr>
<tr>
<td>Rotor weight / kg</td>
<td>3,03</td>
<td>3,67</td>
</tr>
<tr>
<td>Stator weight / kg</td>
<td>9,48</td>
<td>6,35</td>
</tr>
</tbody>
</table>

![Table 3 Weight of different parts of the motor](image)

![Figure 6 Initial and final geometries](image)
Fig. 6 presents the geometry of the motor before and after optimization.

The obtained design variables, subject of the optimization, are presented in tab. 4.

<table>
<thead>
<tr>
<th>Table 4 Motor geometric parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variables</td>
</tr>
<tr>
<td>(b_1) / mm</td>
</tr>
<tr>
<td>(b_d) / mm</td>
</tr>
<tr>
<td>(b_{ts}) / mm</td>
</tr>
<tr>
<td>(B_e) / T</td>
</tr>
<tr>
<td>(b_{sa}) / mm</td>
</tr>
<tr>
<td>(b_{sa}^s) / mm</td>
</tr>
<tr>
<td>(D_{ex}) / mm</td>
</tr>
<tr>
<td>(h_{sy}) / mm</td>
</tr>
</tbody>
</table>

5 VERIFICATION WITH FINITE ELEMENT ANALYSIS

At performing our optimization procedure, we have obtained the optimal geometric dimensions of the motor. Then, these parameters will be used to model the machine by means of the finite element software. This step is carried out to validate the obtained optimization results. FEA is a powerful tool used by researchers to analyse electromagnetic devices [32-33]. In our work, the magnetic field problems solution is obtained using the magnetic vector potential \(A\).

For reasons of periodicity and symmetry in the machine [34], the study is limited to 2-D and to 1/12 of the machine. To assure the FEA model continuity, additional boundary conditions are applied. A particular attention is paid to the meshing method to ensure the rotor movement. Indeed, a sliding line is constructed in the middle of the air gap and periodicity boundary conditions were applied.

The previous figure (Fig. 7) illustrates the study domain of the investigated machine in which are reported the boundary conditions.

The magnetic flux densities through the magnetic circuit are summarized in Tab. 5.

<table>
<thead>
<tr>
<th>Table 5 Weight of different parts of the motor</th>
</tr>
</thead>
<tbody>
<tr>
<td>(B_y)</td>
</tr>
<tr>
<td>Analytical value / T</td>
</tr>
<tr>
<td>Numerical value / T</td>
</tr>
<tr>
<td>Error / %</td>
</tr>
</tbody>
</table>

Referring to tab. 5, we note that the difference between the FEA results and the analytical values is slight, which validates our optimization procedure.

According to these previous figures, we can clearly notice that the airgap flux density waveform turns to be trapezoidal, the slotting effect is also remarkable. Concerning the output torque, we can notice that a maximum value of 90 Nm is reached, and a high ripple torque is also detected. This torque can induce vibration and noise in the permanent magnet hub. Therefore, it must be considered when developing the motor control strategy.

In a brushless permanent magnet machine, cogging torque is due to the magnetic attraction between the permanent magnets mounted in the rotor and the stator teeth. In this section, the effect of different permanent magnets shapes: conventional magnets (a), segmented
circular magnets (b) and segmented circular spaced magnets (c) on the cogging torque is studied. The obtained results are illustrated in Fig. 10. According to these results, we can clearly notice that:
- The machine with circular permanent magnets presents the highest cogging torque reaching a maximum value equal to 35 Nm.
- The topology with circular segmented magnets has the lowest value of the cogging torque which does not exceed 7 Nm. Therefore, a decrease of about 28 Nm is obtained compared to the first machine.

6 CONCLUSION

In this work, an analytical model, required to optimize a brushless direct current machine, has been developed. This motor will be used for the application of in-wheel motor electric vehicle. This application makes many constraints for motor design. In order to find optimal motor geometry, an optimization procedure, based on Sequential Quadratic Programming (SQP) has been presented. The purpose of this optimization is the minimisation of motor weight and the maximisation of its efficiency.

To achieve this goal, many simulations have been carried out to find the optimum design variables. Finally, the optimization is finished, and optimal motor parameters are given. The comparison of the obtained results with those based on FEA, shows the effectiveness of the proposed optimization procedure. An investigation of the machine performances, by means of FEA, is carried out. The existence of a high ripple torque and cogging torque was shown. To minimize the machine cogging torque, deep FEA was carried out. Different machine rotor structures were investigated. A decrease of the cogging torque value of about 80% is achieved.

Despite the considerable improvement of the machine performances, several other works remain essential such as the analytical model improvements, the investigation of different stator and rotor structures and the development of the appropriate control strategy of the studied motor.

7 REFERENCES


