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Creep of timber structures

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SUMMARY

Apart from the criterion of load capacity, the criterion of deformability is very often a decisive one in designing timber structures. Displacements of the structure should stay within the limits so that its functionality could be assured during the entire service life in order that they do not cause damage to non-bearing elements of the structure and equipment and do not have negative psychological impact on the user. There, however, it should be kept in mind, the displacements of the structure include initial elastic displacements and displacements due to the creep of timber, normally emerging during a longer period of time. The creep of timber consists of three parts that appear simultaneously which can hardly be distinguished from one another. Those are time dependent viscoelastic creep, mechano-sorptive creep and the pseudo creep and recovery. The numerical analysis of the response of structure to long-term loading allows the time dependent increase in the displacements of structure due to the creep of timber to be determined.

In this article, the mathematical modelling of the timber creep by using mechanical models is presented. The creep parameters applied in the numerical simulation of this phenomenon have been obtained through long-lasting experimental research. The application of the developed method of numerical analysis of time dependent response of timber structures, taking into consideration the viscous creep and mechano-sorptive creep, is illustrated by calculation examples. Good correlation of the results obtained by calculation with those obtained experimentally verifies the suitability of both the calculation method and the developed software.

Key words: *timber structures, creep, four-point bending test.*

1. INTRODUCTION

The visco-elastic creep of timber depends on, to the largest extent, the stress-level, the temperature and moisture. The mechano-sorptive creep, on the other hand, depends mainly on the magnitude of change and, to a lesser extent, on the rate of change in the timber moisture [1]. Mechano-sorptive behaviour of timber is a consequence of slips or local failures of ordered bundles of cellulose molecules or micro fibrils, which are the basic constituents of the cellular walls [2]. The indicated timber damage is caused by the stresses, which may be caused by the external load or non-uniform timber swelling or shrinkage. Any change in timber moisture, irrespective of its increase or decrease, causes an increase in the strains of the loaded timber. The strain changes are bigger in the case of dehumidification than humidification of the specimen. In case of cyclic moisture changes, the increase in timber strains is even more distinctive. When the timber moisture is cycled within a set range, there is a gradual decrease in the creep rate; but any further increase in the moisture content causes the creep rate to increase up to the initial highest rate [1]. Pseudo creep and recovery result from different shrinkage and swelling of timber in the tension and compression zones due to moisture changes**.** Individual parts of timber creep have long been treated independently of one another. Hunt [1], however, found out by experimental research that all types of creep are interconnected, and consequently suggested a uniform approach to modelling the total creep.

The results of Raczkowski's experimental research [3] lead to the conclusion that rapid moisture changes in a loaded specimen promote the process of local timber decay. The increase in rate and extent of the creep of bending specimens, alternately exposed to humidification and dehumidification, is a consequence of different response of compressed and tensioned timber fibres to moisture changes. Accordingly, longitudinal swelling of timber fibres caused by humidification is less conspicuous in the compressive than in the tensile zone. During dehumidification, however, the longitudinal shrinkage of tensioned fibres is the same as the simultaneous shrinkage of compressed fibres. This means that moisture induced increase in timber creep is a result of a joint effect of external load and strains due to moisture changes causing additional stresses.

It is not as simple, though, to explain the fact that the rate of creep, which increases spontaneously with humidity changes, stabilizes later on to resume its initial value. It is a characteristic feature within the mechanism of the decay of polymers, one of which is timber as well, that applied stresses can change the molecular structure. High stresses at the root of a crack change the orientation of molecules, resulting in the strengthening of the polymer around the crack. Murphey [4] proved experimentally that the cellulose orientation in timber increases with increased loading. On the basis of these results, Raczkowski tried to explain the established fact that the spontaneous increase in creep rate, caused by an abrupt moisture change, can be stabilized again after some time. That is, when the rate of reorientation of molecules exceeds the rate of crack propagation, the strengthening of polymer around the area of the root of the crack inhibits further development of cracks. In the opposite case, when the rate of reorientation of molecules is lower than the rate of crack propagation, local damage and decay do occur.

Timber moisture as well as time stability of a permanently loaded structure (a continuously loaded condition of a structure) may have a crucial effect on the creep, even more in case of cyclic moisture changes. The creep increases with the increasing amplitude and period of humidity oscillation. Once the balance of timber moisture between individual moisture levels has been established, which depends on diffusion resistance and specimen size, any further timber creep only depends on stress at given moisture conditions [5].

In timber structures, the creep is of major concern because of the time dependent rise of deflection of bending elements or the risk of buckling of compressive loaded elements of the structure. Experiments prove that the creep of compressed fibres of a bending element differs from that of tensioned fibres. The creep is experimentally determined by measuring time increments of deflections in four-point bending test with time increments measured in the mid section of the beam with no lateral force being applied, while the bending moment is kept constant. The creep coefficient k_{def} is defined by the ratio between the time increment of deflection $u(t)–u(t_0)$ and the initial deflection $u(t_0)$:

$$k_{def}\left(t-t_0\right)=\frac{u\left(t\right)-u\left(t_0\right)}{u\left(t_0\right)} \tag{1}$$

Therefore, so defined coefficient k_{def} represents the substitutive total creep of the compression and tension zones. In the European pre-standard for timber structures Eurocode 5 (ENV 1995-1-1), the creep factor for deflection is given in relation to the load duration and the service class.

The results of experimental research carried out by Taylor and Pope [6] have led us to conclusion that there was no essential difference between the creep of solid timber and that of glued laminated timber made of the same wood. By increasing the cross-section of the specimens, the creep factor k_{def} decreases. Even more favourable effect on the decrease in mechano-sorptive creep of timber can be obtained by a surface coating that is especially efficient under conditions of variable environmental humidity.

2. NUMERICAL MODELLING OF MECHANICAL AND RHEOLOGICAL CHARACTERISTICS OF TIMBER

Different rheological models that consist of springs and dashpots can simulate the behaviour of viscose material. Even though such models cannot completely describe the time dependent behaviour of actual materials, they can, nevertheless, be used for modelling all basic characteristics of the material. These models are especially effective in describing the behaviour of visco-elastic material with rheological characteristics that do not depend on age.

Mechanical and rheological characteristics of timber can be simulated realistically enough by a generalised Kelvin model (Figure 1). This model consists of a serially linked Hook body and a chain of n Kelvin bodies. The strain $\varepsilon(t)$ is obtained as the sum of the basic elastic strain $\varepsilon_e(t)$, which is modelled by a spring, and the creep induced strain $\varepsilon_c(t)$ modelled by the chain of Kelvin bodies. Therein considered is the equilibrium condition, Eq. (2), according to which the stress in the spring and in all Kelvin bodies should equal the enforced stress $\sigma(t)$ of the system:

$$\varepsilon\left(t\right)=\varepsilon_e\left(t\right)+\varepsilon_c\left(t\right)=\varepsilon_e\left(t\right)+\sum_{\mu=1}^{n}\varepsilon_\mu\left(t\right);$$
$$\sigma_\mu\left(t\right)=\sigma\left(t\right),\quad \text{for } \mu = 1, 2, \ldots, n. \tag{2}$$

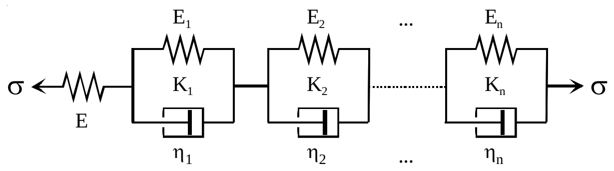

Fig. 1 Generalised Kelvin model

Time invariant characteristics of springs and dashpots (E = const.; E_μ = const., η_μ = const. for $\mu=1, 2\ldots, n$) are usually taken into account in modelling timber behaviour. For the stress values up to *30%* of short-term timber strength, the initial strain of timber $\varepsilon_e(t)$ is satisfactorily defined by the Hook law:

$$\varepsilon_e(t)=\frac{\sigma(t)}{E} \tag{3}$$

The creep induced strain $\varepsilon_c(t)$ is obtained as a sum of the strains of individual Kelvin bodies:

$$\varepsilon_c(t)=\sum_{\mu=1}^{n}\varepsilon_\mu(t) \tag{4}$$

When a well-known differential equation of a Kelvin rheological model $\sigma_\mu=E_\mu\varepsilon_\mu+\eta_\mu\dot{\varepsilon}_\mu$ is differentiated with respect to time t, and the retardation time $\tau_\mu=\eta_\mu/E_\mu$ is introduced, a differential equation is obtained:

$$\frac{\dot{\sigma}_\mu}{E_\mu\cdot\tau_\mu}=\frac{1}{\tau_\mu}\cdot\dot{\varepsilon}_\mu+\ddot{\varepsilon}_\mu \tag{5}$$

By solving the differential equation, Eq. (5), the strain rate $\dot{\varepsilon}_\mu$ of an individual Kelvin body is obtained:

$$\dot{\varepsilon}_\mu(\tau)=\dot{\varepsilon}_\mu(t_0)\cdot e^{-\frac{\tau-t_0}{\tau_\mu}}+\frac{1}{E_\mu\cdot\tau_\mu}\int_{t_0}^{\tau}e^{-\frac{\vartheta-t_0}{\tau_\mu}}\cdot\dot{\sigma}_\mu(\vartheta)\cdot d\vartheta \tag{6}$$

Additional integration over a period of time $(t-t_0)$ provides the strain ε_μ in time t:

$$\varepsilon_\mu(t)=\varepsilon_\mu(t_0)+\dot{\varepsilon}_\mu(t_0)\cdot\int_{t_0}^{t}e^{-\frac{\tau-t_0}{\tau_\mu}}\cdot d\tau+ + \frac{1}{E_\mu\cdot\tau_\mu}\int_{t_0}^{t}\int_{t_0}^{\tau}e^{-\frac{\vartheta-t_0}{\tau_\mu}}\cdot\dot{\sigma}_\mu(\vartheta)\cdot d\vartheta\cdot d\tau \tag{7}$$

Equations (6) and (7) can be applied to arbitrary course of stress within the time interval from t_0 to t. In case of a sectional linear course of stress (Figure 2) within the time interval from t_{i-1} to t_i:

$$\sigma(t)=\sigma(t_{i-1})+\frac{\Delta\sigma_i}{\Delta t_i}\cdot(t-t_{i-1}) \qquad \text{for:}\quad t_{i-1}\le t<t_i \tag{8}$$

the equations (9) and (10) for the strain rate $\dot{\varepsilon}_\mu$ and the strain ε_μ at the end of the interval $t=t_i$ respectively can be obtained from general Eqs. (6) and (7):

$$\dot{\varepsilon}_\mu(t_i)=\dot{\varepsilon}_\mu(t_{i-1})\cdot e^{-\frac{\Delta t_i}{\tau_\mu}}+\frac{\Delta\sigma_i\cdot\lambda_{\mu_i}}{E_\mu\cdot\tau_\mu} \tag{9}$$

$$\varepsilon_\mu(t_i)=\varepsilon_\mu(t_{i-1})+\left[\dot{\varepsilon}_\mu(t_{i-1})\cdot\Delta t_i\cdot\lambda_{\mu_i}+\frac{\Delta\sigma_i}{E_\mu}\left(1-\lambda_{\mu_i}\right)\right] \tag{10}$$

Here the factor λ_{μ_i}, representing a part of non-achieved strain $\Delta\sigma_i/E_\mu$, is defined by the next expression:

$$\lambda_{\mu_i}=\tau_\mu\left(1-e^{-\frac{\Delta t_i}{\tau_\mu}}\right)/\Delta t_i \tag{11}$$

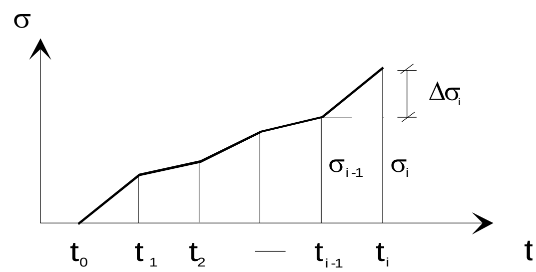

Fig. 2 Sectional linear time related course of stress

Using the Eqs. (3), (4) and (10), the constitutive law of the generalized Kelvin model, Eq. (2), can be written in an incremental form:

$$\Delta\varepsilon_i=\varepsilon(t_i)-\varepsilon(t_{i-1})= =\frac{\Delta\sigma_i}{E}+\sum_{\mu=1}^{n}\left[\dot{\varepsilon}_\mu(t_{i-1})\cdot\Delta t_i\cdot\lambda_{\mu_i}+\frac{\Delta\sigma_i}{E_\mu}\left(1-\lambda_{\mu_i}\right)\right] \tag{12}$$

By rearranging the Eq. (12), the final form of a linear relation between the increment of the total strain $\Delta\varepsilon_i$ and that of stress $\Delta\sigma_i$ within the observed time interval is obtained, Eq. (13). In its form, this incremental stress-strain relationship is similar to the relationship proposed by Bažant for modelling concrete behaviour [7]:

$$\Delta\varepsilon_i=\frac{\Delta\sigma_i}{E^*}+\Delta\varepsilon_i^*$$

$$\frac{1}{E^*}=\frac{1}{E}+\sum_{\mu=1}^{n}\frac{\left(1-\lambda_{\mu_i}\right)}{E_\mu} \tag{13}$$

$$\Delta\varepsilon_i^*=\sum_{\mu=1}^{n}\dot{\varepsilon}_\mu(t_{i-1})\cdot\Delta t_i\cdot\lambda_{\mu_i}$$

The benefit of the proposed constitutive law, Eq. (13), lies above all in the fact that the whole history of the previous loading is comprised in derivatives of strains of Kelvin bodies with respect to time ($\dot{\varepsilon}_\mu$) at the end of the previous time interval. Therefore, in any time interval the time derivatives of strains $\dot{\varepsilon}_\mu$ of all Kelvin bodies in the chain, which are needed for the calculation in the subsequent time interval, must additionally be calculated for the final time of interval t_i. The strains in individual Kelvin bodies ε_μ are the so-called inner unknowns of the model.

In a specific case where only one Kelvin body ($n=1$) is considered within the model, the constitutive law, Eq. (13), can serve to describe the behaviour, generally represented by a known linear differential equation, Eq. (14), of the basic rheological model (Figure 3a). This is the simplest model commonly used to analyze in practice the creep of timber structures:

$$\left(E\eta_1\right)\varepsilon+\left(EE_1\right)\dot{\varepsilon}=\left(E+E_1\right)\sigma+\eta_1\dot{\sigma} \qquad (14)$$

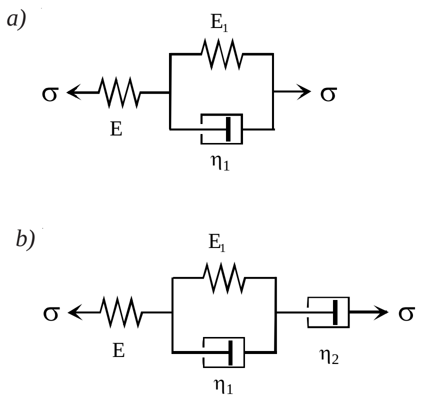

Fig. 3 Rheological models: (a) Basic model; (b) Burgers' model

In case of two Kelvin bodies ($n=2$) and the limit state $E_2 \to 0$ being considered in the generalized Kelvin model (Figure 1), the use of the constitutive law, Eq. (13), allows to describe the behaviour of Burgers' model too, (Figure 3b). In general, the behaviour of this model is described by linear differential equation of the second order:

$$\frac{\eta_1\,\eta_2}{E_1}\ddot{\varepsilon}+\eta_2\dot{\varepsilon}=\frac{\eta_1\,\eta_2}{E\,E_1}\ddot{\sigma}+\left(\frac{\eta_1}{E_1}+\frac{\eta_2}{E}+\frac{\eta_2}{E_1}\right)\dot{\sigma}+\sigma \qquad (15)$$

The elasticity modules E_μ of individual Kelvin bodies can be properly adjusted on the basis of the simulation of experimental results relative to the creep. The retardation times τ_μ are usually selected in advance, whereby it should be ensured that the whole spectrum of retardation times for the material concerned is as evenly represented as possible. The number n of Kelvin bodies is conditional on the required level of agreement between the calculated and experimental results.

The creep factor $k_{def}(t-t_0)$, Eq. (1), is most commonly determined by long-lasting experimental research of timber creep. It is defined as a ratio between the deflection increments, developed due to the creep under conditions of constant load in a given period of time $(t-t_0)$, and the initial elastic deflection. Experimentally determined development of the creep factor $k_{def}(t\text{-}t_0)$ can be well simulated by Dirichlet series, Eq. (16), where constants A_μ are defined by the least squares method:

$$k_{def}\left(t-t_0\right)=\sum_{\mu=1}^{n}A_\mu\left(1-e^{-\frac{t-t_0}{\tau_\mu}}\right) \qquad (16)$$

At the unit stress ($\sigma=1$) effective from the time t_0 onwards, the creep induced strain $\varepsilon_c(t)$ is defined by the expression, Eq. (17), using the creep factor k_{def}, Eq. (16), and the initial elastic strain $\varepsilon_e(t)$, Eq. (3):

$$\varepsilon_c\left(t\right)=k_{def}\left(t\right)\cdot\varepsilon_e\left(t\right)=\frac{k_{def}\left(t\right)}{E}=\sum_{\mu}\frac{A_\mu}{E}\left(1-e^{-\frac{t-t_0}{\tau_\mu}}\right)$$

$$\text{for } \sigma=1 \qquad (17)$$

In order to get the response of Kelvin bodies, generally defined by the Eqs. (4), (10) and (11), at the constant stress of $\sigma=1$ being suddenly introduced at the time t_0, the conditions described by Eq. (18) should be given due to the consideration in these equations:

$$\varepsilon_\mu(t_0)=0;\ \dot{\varepsilon}_\mu\left(t_0\right)=\sigma\left(t_0\right)/\eta_\mu=1/\eta_\mu \ \text{ and } \ \dot{\sigma}=0 \qquad (18)$$

This allowed us to retrieve the time development of the creep induced strain, Eq. (19), as described by a chain of Kelvin bodies, at the constant stress of $\sigma=1$ and from the time t_0 onwards:

$$\varepsilon_c\left(t\right)=\sum_{\mu=1}^{n}\dot{\varepsilon}_\mu\left(t_0\right)\tau_\mu\left(1-e^{-\frac{t-t_0}{\tau_\mu}}\right)=\sum_{\mu=1}^{n}\frac{1}{E_\mu}\left(1-e^{-\frac{t-t_0}{\tau_\mu}}\right) \qquad (19)$$

The comparison of Eqs. (19) and (17) demonstrates that the proposed calculation model can serve to approximate the experimentally determined timber creep whenever the next equation is applied to each of the Kelvin bodies ($\mu=1, 2,\ldots, n$):

$$E_\mu=\frac{E}{A_\mu} \quad \text{for } \mu=1,2,\ldots,n \qquad (20)$$

Figure 4 represents the numerical simulation of experimentally determined creep factors k_{def} using the generalized Kelvin model with a chain of three Kelvin bodies (curves marked $n=3$) and a basic rheological model (curves marked $n=1$). The constants of Kelvin bodies A_μ ($\mu=1, 2, 3$) for the chosen retardation periods of *10*, *100* and *1000* days, as given in Table 1, were defined by the least squares method. In order to ensure an adequate effect of longer periods, deviations of the computed values from the measured ones were corrected by linear time weights. The creep factor k_{def} was experimentally determined by Taylor and Pope [6]. The specimens of the same size were tested at the same stress level in three different types of environmental conditions. The specimens marked CSGSU were kept in a covered and sheltered area under constant conditions. For the specimens marked VSGSU, the cyclical changes in the relative humidity of the surrounding air were controlled between *40%* and *80%* at a constant temperature of $T=20°C$. While roofed, the specimens marked ESGSU were exposed to external environment conditions.

The parameters of the basic rheological model had already been defined by Taylor and Pope [6] using the least squares method. According to them, an appropriate retardation time for the three different types of ambient conditions is $\tau_1 = 312.5$ days. The parameters of the single Kelvin body in the basic rheological model are $A_1=0.67$ for the specimens marked CSGSU, $A_1=1.21$ for the specimens marked VSGSU and $A_1=2.15$ for those marked ESGSU. Being common to the three typical ambient conditions, the retardation time τ_1 means that time development of the creep induced strain is the same. The parameters A_1 of the basic rheological model already represent themselves as the final values of the creep factor $k_{def,\infty}= k_{def}(t\rightarrow\infty)$.

The comparison of the numerical simulation of the creep factors k_{def} under all environmental conditions with the experimental results shows that by taking into consideration two (for the specimens marked CSGSU and VSGSU) or three Kelvin bodies alone (for the specimens marked ESGSU) in a generalized Kelvin model, a much better matching was achieved than with the basic rheological model.

Figure 5 represents the comparison of the numerical simulation with the experimentally determined creep factor for displacements k_{def} using a generalized Kelvin model with three Kelvin bodies ($n=3$). The appropriate parameters of Kelvin bodies A_μ ($\mu=1,2,3$) were defined by the least squares method with retardation times of *10*, *100* and *1000* days chosen in advance. These parameters are given in Table 2. Long-term measurements of deflections of glued laminated free-span timber beams under constant load were performed by Srpčič [8]. The boundary stress level in the middle of the span was $k_\sigma\approx 0.22$. Two series of specimens were tested. The first was exposed to environmental conditions of low relative humidity $RH_1=62\%$, whereas the second was tested under conditions of high relative humidity $RH_2=93\%$.

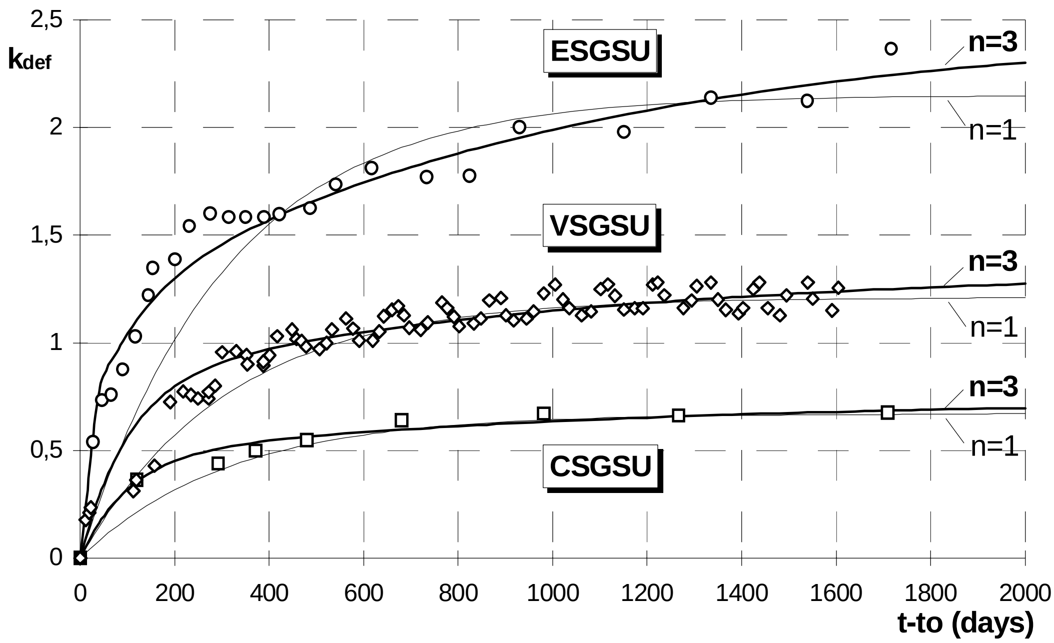

Fig. 4 Numerical simulation of experimentally determined creep factor k_{def} [6] under various environmental conditions

Table 1 Constants of Kelvin bodies A_μ determined on the basis of experimental results of Taylor and Pope [6]

	Constants of Kelvin bodies A_μ for generalized Kelvin model ($n=3$)		
	Retardation time τ_μ (days)		
Environmental conditions	10	100	1000
Normal external conditions (ESGSU)	0.5319	0.6058	1.3467
Variable conditions (VSGSU)	0	0.8093	0.5366
Constant conditions (CSGSU)	0	0.4644	0.2697

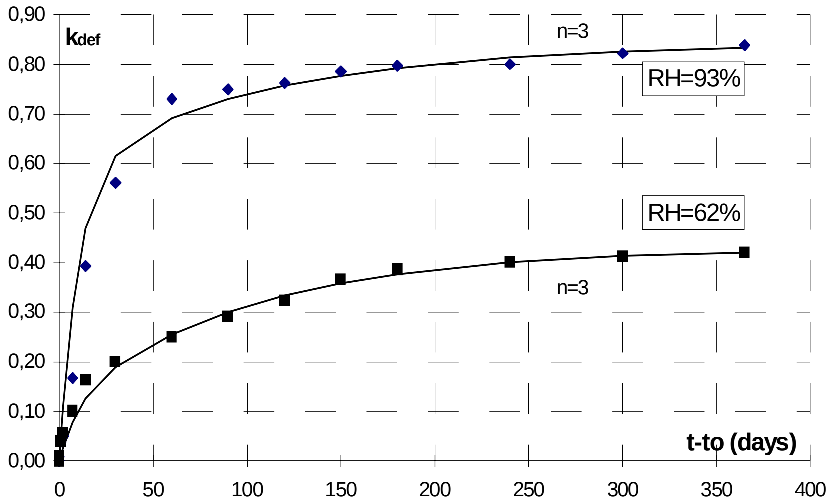

Fig. 5 Numerical simulation of the measured creep factor k_{def} from experiment [8]

Table 2 Appropriate constants of Kelvin bodies A_μ for the numerical simulation of experimental results of Srpčič [8]

	***Constants of Kelvin bodies A_μ for generalized Kelvin model** (n=3)*		
	***Retardation time** τ_μ (days)*		
Environmental conditions	*10*	*100*	*1000*
Low relative humidity, RH_1=62%	*0.1126*	*0.3167*	*0*
High relative humidity, RH_2=93%	*0.5792*	*0.2467*	*0.0474*

3. NUMERICAL ANALYSIS OF TIMBER PLANE FRAME STRUCTURES

Geometrical non-linearity of structures and timber creep are normally allowed for in the analysis of time dependent response of structures. The developed calculation method is based on the finite element method. It is only the influence of normal stresses on displacements that is taken into consideration, whereas the influence of shear stresses is neglected. The calculation of displacement and inner forces takes into account large displacements and moderate strains. The total period considered consisting of discrete times t_0, t_1, t_2, ... t_{i-1}, t_i is divided into intervals of arbitrary length. The analysis of the structure is performed step by step within the discrete times given. Environmental conditions and timber moisture are kept constant all the time while linear changes of the stresses may occur inside individual time intervals. $\Delta t_i=t_i-t_{i-1}$ is the length of the i-th time interval.

The geometrical non-linearity of the structure is included by appropriate kinematical equations of a finite element [9]. The elongation e_0 and the local declination β of the direction of axis of the deformed element from the connecting line between the nodes of the finite element can be expressed as (see Fig. 6):

$$\begin{aligned}(1+e_0)\cos\beta &= u_{,x}+\cos\psi \\ -(1+e_0)\sin\beta &= w_{,x}+\sin\psi\end{aligned} \tag{21}$$

Accordingly, $u_{,x}$ and $w_{,x}$ are the derivatives of displacements u and w in the direction of x or z-axis with respect to the coordinate x, while ψ is the angle encompassed by the connecting line between the nodes of the deformed finite element and the initial direction of the element.

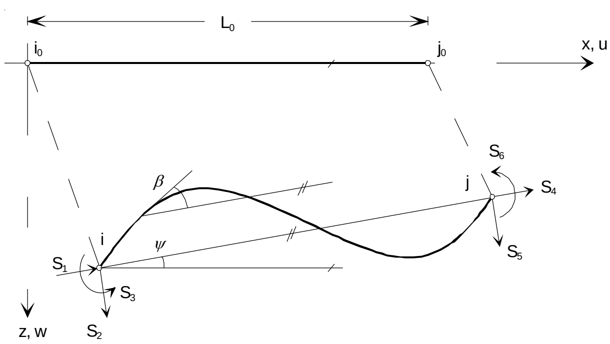

Fig. 6 Kinematics of a finite element

Bernoulli's hypothesis is adopted for the course of normal strains across the cross-section. In this way, the strains $\varepsilon(z, t)$ in time t can be expressed by the next equation where $\varepsilon_0(t)$ represents the elongation, and $\beta_{,x}(t)$ the curvature of the axis of the finite element:

$$\varepsilon(z,t)=e_0(t)+z\beta_{,x}(t) \tag{22}$$

If the time at the end of i-th interval t_i is allowed for the time t and if the strain $\varepsilon_{i-1}(z)$ in the initial time t_{i-1} is furthermore considered to be known, the increment of the geometrical strain $\Delta\varepsilon_i(z)$ in the time interval $\Delta t_i = t_i - t_{i-1}$ can be written where z is the coordinate of an arbitrary point of the cross-section:

$$\Delta\varepsilon_i(z) = \varepsilon_i(z) - \varepsilon_{i-1}(z) = \left[e_0(t_i) + z\beta_{,x}(t_i)\right] - \varepsilon_{i-1}(z) \tag{23}$$

Once the increment of the geometrical strain $\Delta\varepsilon_i(z)$, Eq. (23), in the i-th time interval is known, the stress increment $\Delta\sigma_i(z)$ in an arbitrary point of the cross-section with the coordinate z can be calculated using the constitutive law, Eq. (13). Also, the effect of timber creep is included in this way. The required parameters of the generalized Kelvin model are defined on the basis of simulation of experimentally determined creep factors k_{def} by means of the Eqs. (16) and (19). The least squares method is applied therein:

$$\Delta\sigma_i(z) = \Delta\sigma\left(z, \Delta\varepsilon_i, \Delta t_i, E, k_{def}\right) \tag{24}$$

The stress in an arbitrary point of the cross-section with the coordinate z at the end of the time interval $\sigma_i(z)$ concerned is defined by the equation (25). There, $\sigma_{i-1}(z)$ represents the stress at the beginning of the time interval concerned:

$$\sigma_i(z) = \sigma(t_i, z) = \sigma_{i-1}(z) + \Delta\sigma_i(z) \tag{25}$$

The axial force N and the bending moment M of the cross-section are obtained by integrating the stresses σ over the cross-section, which are defined by the Eqs. (25) and (24):

$$\begin{aligned} N(t_i) &= \int \sigma\left(z, e_0(t_i), \beta_{,x}(t_i), E, k_{def}\right) dA \\ M(t_i) &= \int z \cdot \sigma\left(z, e_0(t_i), \beta_{,x}(t_i), E, k_{def}\right) dA \end{aligned} \tag{26}$$

When joining the elements into the structure, the equilibrium and kinematic conditions of individual nodes are taken into consideration. Since the equations of the structure are non-linear because of the geometric non-linearity and the timber creep considered, they are solved iteratively step by step using the Newton-Raphson method. The finite element used is of a "mixed" type [9]. With this element, the curvature and axial force are approximated along its reference axis. On the assumption of the equilibrium as obtained at the beginning of the time interval t_{i-1}, the analysis of the stress-strain status of the structure at the end of the interval t_i concerned is performed in the following way. At any instance of iteration, the curvature $\beta_{,x}(t_i)$ and the angle ψ are determined by means of kinematic equations of the element. The elongation ε_0 and the corresponding bending moment M for any integration node of the finite element, on the other hand, are determined on the basis of the constitutive equations of the cross-section, Eq. (26). This is followed by a calculation of generalized forces and generalized elongations as well as a calculation of the tangential stiffness matrix of the finite element. The tangential stiffness matrix of the structure is obtained by appropriately joining the matrices of individual finite elements. The equations of the structure in their incremental form are expressed for the *k-th* iteration in time t_i by the matrix equation:

$$\left[\bar{K}(t_i)\right]_{(k)} \left\{\Delta v_r\right\}_{(k+1)} = \left\{F(t_i)\right\} - \left\{\bar{S}_r(t_i)\right\}_{(k)} \tag{27}$$

where $\left[\bar{K}(t_i)\right]_{(k)}$ is the tangential stiffness matrix of the structure, $\left\{\Delta v_r\right\}_{(k+1)}$ is a vector of the unknown node displacements, $\left\{F(t_i)\right\}$ is the vector of a given node load and $\left\{\bar{S}_r(t_i)\right\}_{(k)}$ is the vector of the condensed boundary forces of the finite element. The iterations are repeated until the degree of precision required to meet the equilibrium conditions is obtained.

4. COMPUTER PROGRAMME

The relevant software was developed to analyze the time dependent response of timber structures in consideration of the creep. A highly effective finite element P_4 [9, 10, 11, 12] was used. The curvature of the element along its axis was approximated with a polynomial of the *4-th* degree. Lobbato's quadrature formula with five integration nodes was applied for numerical integration of quantities along the axis of the finite element. Non-linear equations of the structure in individual time intervals were solved numerically by Newton-Raphson's method.

The effect of creep on the time dependent response of the structure is included in the relevant constitutive law of timber and the corresponding constitutive equations of the cross-section. In general, any timber creep function can be taken into account.

External load can consist of concentrated and distributed loads on one hand, or enforced displacements and rotations on the other. Within any time interval, the load of the element and node forces consist of a changeable and unchangeable part that are both linear with respect to time. The deformed geometry and stiffness of the structure at the end of a given time interval represent the starting geometry and starting stiffness of the structure in the next time interval. The effect of timber creep alone or the simultaneous effect of timber creep and the change of external load on the time dependent response of the structure can be taken into account within a given time interval.

5. CALCULATION EXAMPLES

The applicability of the developed calculation method and software for the analysis of the time dependent response of timber structures is demonstrated by calculation examples.

5.1 Free span beam

The time dependent behaviour of the glued laminated timber beams, experimentally tested by Srpčič [8], was analyzed by using the developed software. Figure 7 shows a structural system, the load and the geometry of the beams. For the parameters A_i of the Kelvin bodies in the generalized Kelvin model the values from Table 2 were considered. The development of the measured and calculated deflections w in the middle of the span as compared for two different values of the relative humidity of the ambient air, are indicated in Figure 8. A fairly high degree of matching between measured and calculated values can be noticed.

5.2 Buckling of the column

The time dependent response of eccentrically loaded columns is analyzed in the calculation example. The eccentricity e of the force P amounts to *1.5 cm*, i.e. 1/10 of the dimension of the cross-section in the direction of the eccentricity.

The vertical force $P=160\ kN$ is applied abruptly and then kept constant throughout the test.

The rectangular cross-section has dimensions *15/20 cm*. The structural system and the cross-section are shown in Figure 9.

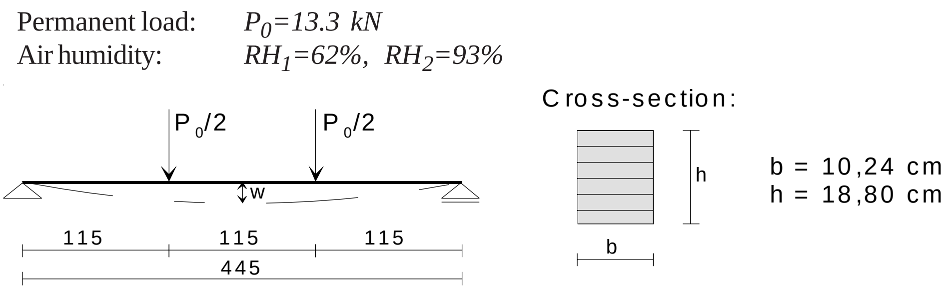

Fig. 7 The geometry and the load of the beam

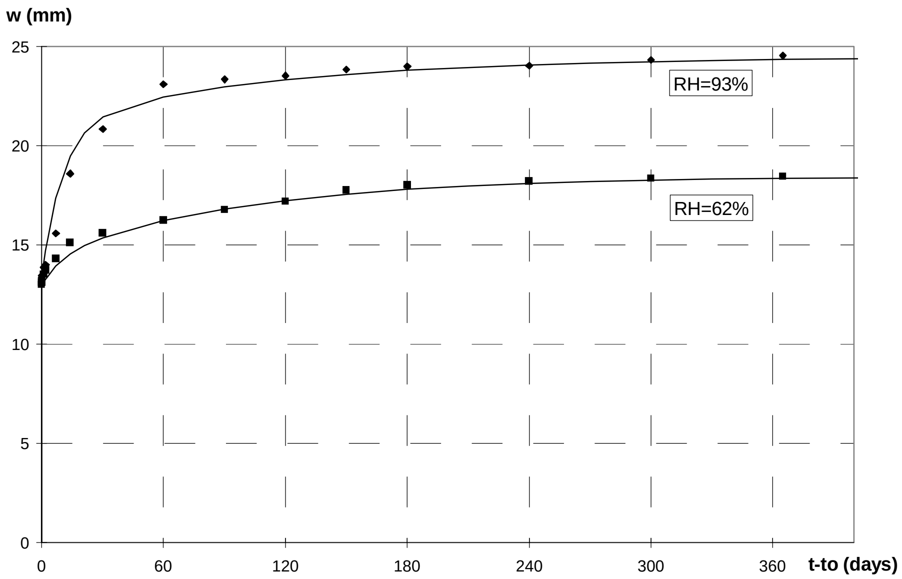

Fig. 8 Diagram of calculated and measured time developments of beam deflections

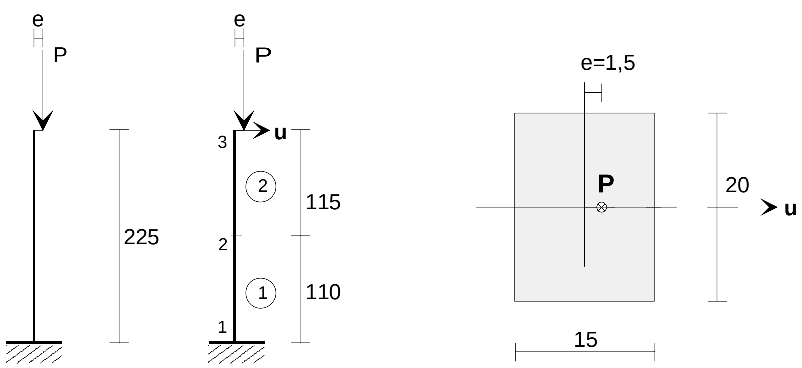

Fig. 9 Structural system and the cross-section of the column

The elasticity module *E=12500 MPa* and the compressive strength of timber f_c*=40 MPa* were adopted in the calculation. The values from Table 1 were used for the parameters A_i of the Kelvin bodies in the generalized Kelvin model to approximate the creep factors as determined experimentally by Taylor and Pope [6]. Three columns of the same size were analyzed. They were exposed to the same loading mode yet under different ambient conditions. The creep factor considered was determined in a covered and sheltered area with constant conditions for the column marked CSGSU, under conditions of relative air humidity cycling between *40%* and *80%* for the column marked VSGSU, and under ambient conditions for the column marked ESGSU.

Figure 10 indicates the calculated horizontal displacements of the top of the column at a constant eccentric load of *P=160 kN* for three different types of ambient conditions. The initial deflection just after applying the load is *1.6 cm* in all three cases. The column is stable only under constant conditions at low relative humidity of the ambient air (CSGSU). In this case the calculated horizontal displacement after 100 years amounts to *7.8 cm*. In the columns for which the creep factors under variable and ambient conditions (VSGSU and ESGSU) are considered, the boundary stresses exceed the compressive strength at the time of approximately *120* days and *700* days respectively. The horizontal displacement amounts to *14.75 cm* when the boundary stress achieves the value of the compressive strength $\sigma = f_c$.

6. CONCLUSION

The stress level and the timber moisture represent two essential parameters of the creep. Modifying the timber moisture causes internal stresses and local structural damage to the timber. The consequence is a time related increase in displacements. In some cases the indicated structural damage can provoke failure of constructional elements even at a load considerably lower than the nominal short-term failure load. Based on the results of experimental research it can be concluded that the creep of the laminate is approximately the same as that of the massive timber.

Mechanical and rheological characteristics of timber can be simulated fairly well with a generalized Kelvin model that consists of a serially connected Hook model and a chain of *n* Kelvin bodies. The experimentally measured creep factor can normally be approximated well enough by considering a chain of three Kelvin bodies.

The finite element method was used for the numerical analysis of the time dependent response of timber structures. Relevant software was developed. The computational results of the time dependent response of the free span beam during the period of one year were compared with the measured values. The matching between the measured and the computational deflections is relatively good. The example of the computational simulation of the buckling of timber column shows that changes in timber moisture can greatly affect the structural stability.

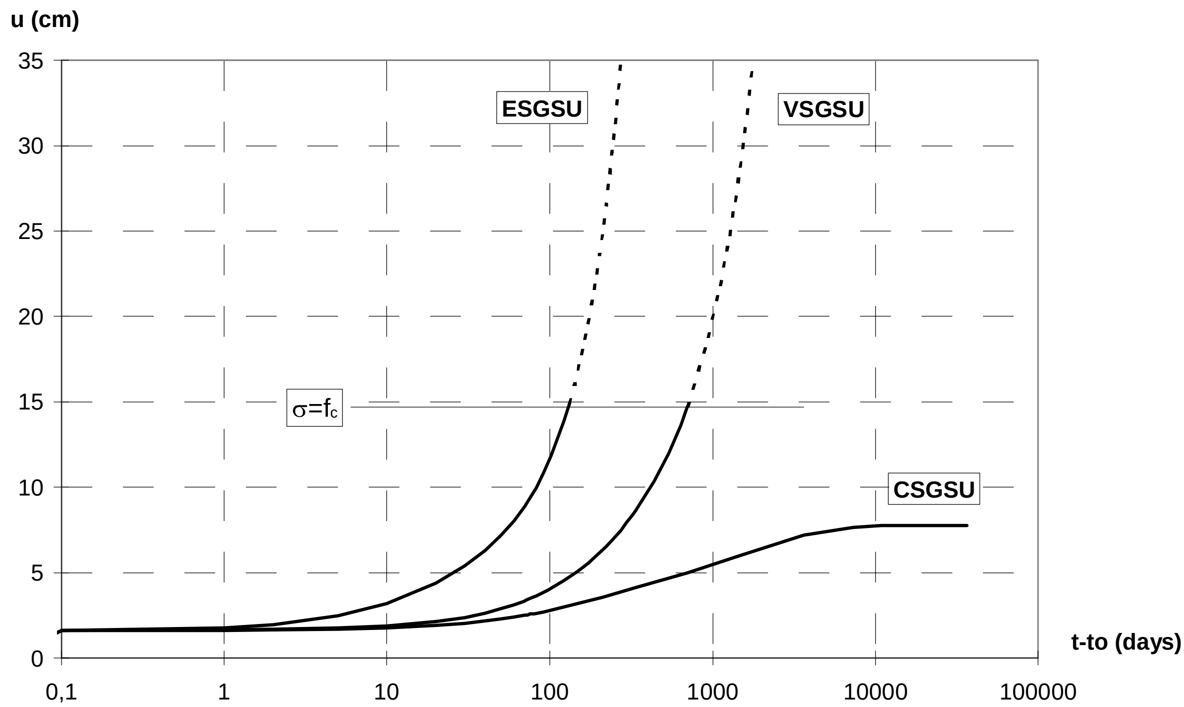

Fig. 10 Calculated time development of horizontal displacements of the node 3 under various ambient conditions

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PUZANJE DRVENIH KONSTRUKCIJA

SAŽETAK

Za projektiranje drvenih konstrukcija od velike su važnosti dva kriterija, kriterij nosivosti i kriterij deformacija. Naime, pomaci konstrukcije moraju ostati unutar granica tako da se osigura njezina funkcionalnost tijekom vijeka trajanja da ne dođe do oštećenja na nenosivim elementima konstrukcije kao i opremi, te da se izbjegne negativni psihološki utjecaj na korisnike. Pri tome treba imati na umu da pomaci konstrukcije uključuju početne elastične pomake kao i pomake uslijed puzanja drvenih konstrukcija koji se uobičajeno pojavljuju tijekom dužeg vremenskog perioda. Puzanje drvenih konstrukcija sastoji se od tri dijela koja se pojavljuju istovremeno i koja je teško međusobno razlikovati. To su viskoelastično puzanje ovisno o vremenu, mehano-sorptivno puzanje i lažno puzanje te oporavljanje. Numerička analiza odgovora konstrukcije pri dugotrajnom opterećenju dozvoljava povećanje pomaka konstrukcije ovisno o proteklom vremenu uslijed puzanja drvene građe koje treba odrediti.

Ovaj članak opisuje matematičko modeliranje puzanja drveta, koristeći mehaničke modele. Parametri puzanja primijenjeni u numeričkoj simulaciji ovog fenomena dobili su se dugotrajnim eksperimentalnim istraživanjem. Primjena razvijene metode numeričke analize vremenski ovisnog odgovora drvenih konstrukcija, uzimajući u obzir viskozno puzanje i mehano-sorptivno puzanje, prikazano je numeričkim primjerima. Dobro slaganje rezultata dobivenih proračunima i onih dobivenih pomoću eksperimenata potvrđuje primjerenost jedne i druge metode kao i razvijenog software-a.

Ključne riječi: *drvene konstrukcije, puzanje, test savijanja s dvije sile.*