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Modelling the road bridge-actual traffic dynamic interaction

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SUMMARY

Dynamic response of the road bridge under actual traffic load continuously degrades serviceability of the bridge and thus its service life. Wheel loads contribute to fatigue surface wear and cracking of concrete which leads to corrosion. Stochastic nature of the actual traffic load, uncertainty in estimation of dynamic characteristic of real vehicles, dynamic bridge-vehicle interaction, etc., complicates calculation procedure of the dynamic response of the bridge in many senses. In this paper we present the approach based on a stochastic parameters of the actual traffic flow and numerical calculation of dynamic interaction system bridge-heavy vehicle. Vehicles are modelled as a group of axles (SDOF model) with the dynamic characteristics (stiffness and damping), and bridge is modelled as continuous elastic system. The proposed method is very useful for numerical simulation of actual traffic influence on the bridge and, at the same time for practical calculation because of its simplicity and acceptable accuracy.

***Key words**: actual traffic, bridge–vehicle dynamic interaction, road roughness, dynamic factor, bridge dynamic response, simulations of traffic flow.*

1. INTRODUCTION

The bridge dynamic is a subject of many researches over the last 150 years. The first known researches deal with analytical solution of the moving force over the bridge. Computers development and their applications introduce much more details in the bridge and vehicle systems.

In usual circumstances the bridge failures are not caused by the bridge-vehicle dynamics but wheel loads contribute to surface wear fatigue and cracking of concrete which leads to corrosion. Thus, dynamic loads continually degrade bridges increasing the necessity of regular maintenance. This fact is much more important because heavy vehicles have become larger and have increased in number while at the same time bridges are lighter and more flexible.

Better understanding of dynamic response of the interaction bridge-vehicle system is important from the aspect of designing bridges that are more resistant to deterioration effect caused by the actual traffic load. It is neccessary to design better vehicles to reduce bridge damage («road-friendly» vehicle) and to regulate vehicle load and suspension, etc. The increasing number of lorries and their weight, as well as lighter and more flexible bridges additionally emphasize this fact.

Deterioration effects occur not only under the extreme traffic load (or from load schemes considered in designing procedure), but under continuous traffic of actual heavy vehicle across the bridge. Thus, calculation procedure should be based on actual load on the bridge, which can be very difficult because of stochastic nature being a characteristic of actual traffic flow.

Determination of the dynamic response of the bridge resulting from the traffic of a heavy vehicle across the bridge is very complicated and depends on many parameters such as dynamic characteristics of the vehicle and the bridge and their relation, heavy vehicle geometry and speed, roughness of the surface profile of the bridge, ratio between bridge and vehicle natural frequencies, magnitude of initial vehicle oscillation, etc. The problem has become nonlinear if we do not neglect the vehicle mass in the mass of the bridge-vehicle system.

The dynamic behavior of the bridges in practical calculations usually is described by dynamic load factor defined as ratio between maximum dynamic and static stresses, which allow the use of the simplified calculation of the dynamic bridge response based on classical procedure with increased static load. It is worth mentioning that the estimated dynamic behavior of the bridges based on such dynamic factors is often conservative. It can be shown that static deflections always increase with the number of vehicles on the bridge while in a dynamic case this is not generally true (damping effects, possible resonance, etc.).

However, the traffic of the vehicle across the bridge can not be considered separately but as an interactive vehicle-bridge system («moving oscillator» problem) [1, 2, 3]. That means that initial vehicle oscillation and the road roughness are the input for the vehicle dynamics which generate dynamic type forces. These forces applied to the bridge cause larger dynamic displacement. This feedback mechanism of interaction force couples the dynamic response of the bridge to that of the vehicle. Especially dangerous situation may appear if the vehicle speed and road roughness is superimposed so that the resonance can occurr.

In this paper we present approach based on a stochastic characteristic of the representative lorry vehicles and numerical calculation of dynamic interaction system bridge-heavy vehicle. Vehicles are modelled as a group of axles (SDOF model) with the dynamic characteristics (stiffness and damping), and the main characteristic of vehicles (weight and distance between axles) are treated as stochastic variables. Dynamic bridge-vehicle system has been defined by the system of coupled partial differential equations.

2. DYNAMIC RESPONSE OF THE BRIDGE UNDER ACTUAL TRAFFIC LOAD

2.1 Modelling of actual traffic flow

Actual traffic flow in this approach is modelled by a set of representative vehicles, whose characteristics can be obtained in different ways, such as: statistical publications, reports of on-site measurements, by using methods of weighing vehicles (for example, WIM – Weight In Motion), vehicle technical documentations, informations from particular road departments, etc. Regarding numerous kinds of lorries according to their carrying capacity and dimensions, it is necessary to select a few representative vehicles, taking into account the number of axles, particular frequency in actual traffic flow, type of vehicle, etc. The number of selected representative vehicles depends on required complexity of the model.

In this paper we use representative vehicles from Ref. [4], whose characteristics are obtained on the basis of collected and analyzed data about Croatian actual traffic flow. Total vehicle weight, taking into account its state (empty, full, overloaded, etc.), and the distance between axles are treated as random variables with an appropriate Beta probability density function of realistic vehicle weights and dimensions (Figure 1):

$$f(x)=\begin{cases} 0 & \text{for } x\le 0 \text{ and } x\ge 1 \\ \dfrac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)+\Gamma(\beta)}x^{\alpha-1}(1-x)^{\beta-1} & \text{for } 0<x<1 \end{cases} \tag{1}$$

In this way it was possible to set different representations of particular vehicle types, as well as traffic conditions on the bridge. Moreover, along with the data about vehicle representation in traffic, a simulation of vehicle movement across the bridge by the help of a computer is enabled.

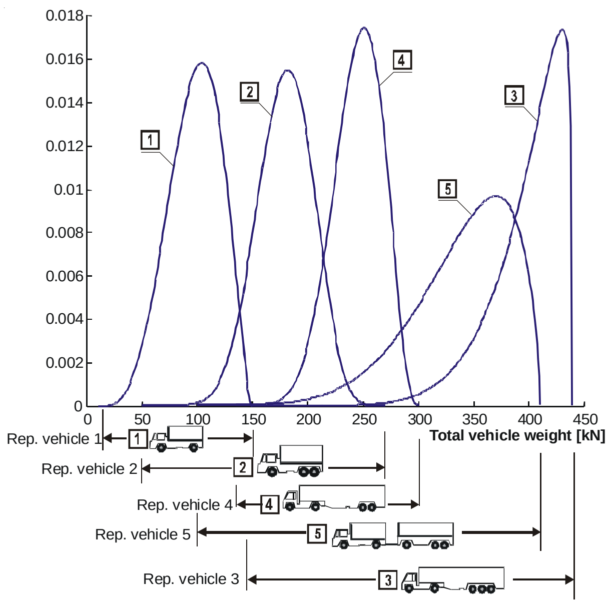

Fig. 1 Beta probability density function of weight for selected representative vehicles

2.2 Modelling of dynamic system bridge-vehicle

We consider the case of a moving set of vehicles across the bridge with the constant speed v, Figure 1.

Each vehicle has the mass m_i (that is weight $m_i g$), and dynamic characteristics defined by the stiffness k_i and damping c_i (quarter-car model with one degree of freedom). Roughness of the surface profile is generally modelled by the function $r(x)$, and we assume that vehicle and surface profile of the bridge are always in contact. Differential equation of motion can be written in the following form [3, 5, 6]:

$$\rho\frac{\partial^2 w_i(x,t)}{\partial t^2}+Kw_i(x,t)=-\left[m_i g+F_i(t)\right]\delta(x-vt) \tag{2}$$

where:

- ρ – the mass per unit length of the bridge [kg/m]
- $w_i(x,t)$ – the vertical deflection of the bridge under influence of *i-th* axle
- K – the stiffness of the bridge ($EI\frac{\partial^4}{\partial x^4}$)
- $\delta(x)$ – the Dirac delta function
- E – the Young's modulus of the material
- I – the second moment of area of the cross-section
- $F_i(t)$ – dynamic interaction force which is generated by relative displacement between *i-th* axle and the bridge, defined by equation:

$$F_i(t)=k_i\left[w_i(vt,t)+r(vt)-z_i(t)\right] \tag{3}$$

where:

- z_i – vertical displacement of the randomly generated weight of *i-th* axle $m_i g$

and $r(x)$ in this example is generally defined as:

$$r(x)=A\sin\left(\frac{2\pi}{b}x\right)$$

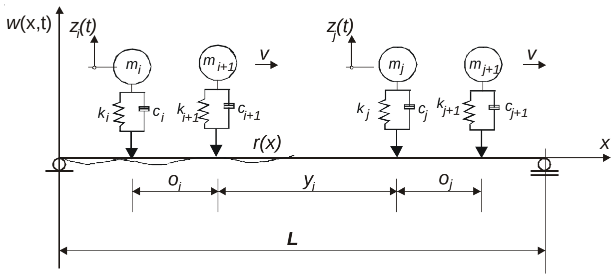

Fig. 2 Schematic model of the bridge-vehicles system

As we can see from the Figure 2, vehicle axles are excited by the interaction force, that is:

$$m_i\cdot\ddot{z}_i(t)=F_i(t) \tag{4}$$

In this paper we neglect damping of vehicle axles ($c_i=0$).

The solution of the dynamic system, Eqs. (2) and (4), can be written in the following form (method of separation of variables):

$$w_i(x,t)=\varphi_i(x)\cdot q_i(t) \tag{5}$$

where:

- $\varphi_i(x)$ – mass normalized mode shape,
- $q_i(t)$ – response of the *i-th* normal coordinate.

Thus, by substitution of the assumed solution in Eqs. (2) and (4) the dynamic bridge-vehicle system has been defined by the system of coupled partial differential equations:

$$\begin{aligned}&\rho\varphi_i(x)\ddot{q}_i(t)+EI\varphi_i^{IV}(x)q_i(t)=\\&=\left[-m_i g-k_i\varphi_i(vt)q_i(t)-k_i r(vt)+k_i z_i(t)\right]\delta(x-vt)\end{aligned} \tag{6}$$

$$m_i\ddot{z}_i(t)=k_i\varphi_i(vt)q_i(t)+k_i r(vt)-k_i z_i(t) \tag{7}$$

General solution of the equation above is defined as:

$$\begin{aligned}\varphi_i(x)=&C_1\cos\sqrt{\varpi}x+C_2\sin\sqrt{\varpi}x+\\&+C_3\cosh\sqrt{\varpi}x+C_4\sqrt{\varpi}x\end{aligned} \tag{8}$$

where constants C_1, C_2, C_3 and C_4 can be calculated from boundary conditions:

$$\begin{aligned}&\varphi_i(0)=0\ ;\ \varphi_i(L)=0\\&\varphi_i^{''}(0)=0\ ;\ \varphi_i^{''}(L)=0\end{aligned} \tag{9}$$

Assuming that the mode shape function $(\varphi_i)_n$ is orthonormal we obtain:

$$(\varphi_i)_n(x)=\sqrt{\frac{2}{\rho L}}\sin\left(\frac{n\pi}{L}x\right) \tag{10}$$

By substitution Eq. (10) into Eq. (6) we obtain:

$$\begin{aligned}&\rho(\varphi_i)_n(x)(\ddot{q}_i)_n(t)+EI\sqrt{\frac{2}{\rho L}}\left(\frac{n\pi}{L}\right)^4\cdot\\&\cdot\sin\left(\frac{n\pi}{L}x\right)(q_i)_n(t)=H_i(t)\delta(x-vt)\\&n=1,2,3,...\end{aligned} \tag{11}$$

where:

$$H_i(t)=-m_i g-k_i(\varphi_i)_n(vt)(q_i)(t)-k_i r(vt)+k_i z_i \tag{12}$$

Using substitution:

$$\omega_n^2=\frac{EI}{\rho}\frac{n^2\pi^2}{L^2} \tag{13}$$

Equation (11) has the following form:

$$\begin{aligned}&(\varphi_i)_n(x)(\ddot{q}_i)_n(t)+\omega_n^2(\varphi_i)_n(x)(q_i)_n(t)=\\&=\frac{1}{\rho}H_i(t)\delta(x-vt)\end{aligned} \tag{14}$$

Multiplying Eq. (14) by $(\varphi_i)_m(x)\rho$, integrating over the length of the bridge [0, L], and applying the orthogonality conditions, one can obtain the normal coordinate equations:

$$(\ddot{q}_i)_n(t)+\omega_n^2(q_i)_n(t)=H_i(t)(\varphi_i)_n(vt) \quad (15)$$

$$0 \le t \le \frac{L}{v}, \quad n=1,2,3,...$$

In this way we obtain system of N second order differential equations. The solution $(q_i)_n(t)$ of the system can be obtained using some of the numerical methods. Finally, the vertical deflection of the bridge under influence of *i-th* axle $w_i(x,t)$ is given as:

$$w_i(x,t)=-\sum_{n=1}^{N}(\varphi_i)_n(x)(q_i)_n(t) \quad (16)$$

In the case of simultaneous influence of more axles on the bridge, total deflection can be calculated as a linear combination of individual displacements caused by each axle (assumption of linear behavior of the bridge is valid for the considered problem, as we have pointed out in introduction of the paper).

Coupled system of Eqs. (6) and (7), as well as random generation of representative traffic flow in this paper have been solved by using MatLab software. The algorithm has been constructed by using numerical package based on an explicit Runge-Kutta formula given in Ref. [7]. It is one-step solver which in computing of the solution in the moment t_n, $(y(t_n))$, needs only the solution at the immediately preceding time point $(y(t_{n-1}))$.

2.3 Dynamic model of bridge-actual traffic load interaction

Since usual approaches are based on the models which consider separately actual traffic or dynamic response of the bridge in more details, we suggest a new approach which combines both mentioned models in paragraphs 2.1 and 2.2 in a simplified form. The flow chart of proposed method has been shown in the Figure 3. This method may be very useful for practical engineers' problems in this area since it essentially improves accuracy by using an acceptable mathematical model. In other words, by excluding dynamic and stochastic characteristics of bridge-vehicle system, we obtain basic moving force problem.

This method allows us calculation of dynamic response of the bridge under passage of stochastic generated sets of representative vehicles. For example we can calculate dynamic load factor in different traffic situations (which may be useful for conservative simple calculation of bridge dynamic behavior in particular cases) and design of moment range spectrum using rainflow moments cycle counting methods. The use of considered method will be illustrated in the numerical example in the next paragraph.

This approach extends previous researches [8, 9, 10], about investigation of Croatian actual traffic load characteristics and probabilistic estimation of safety of existing road bridges, which use classical procedure with increased static load.

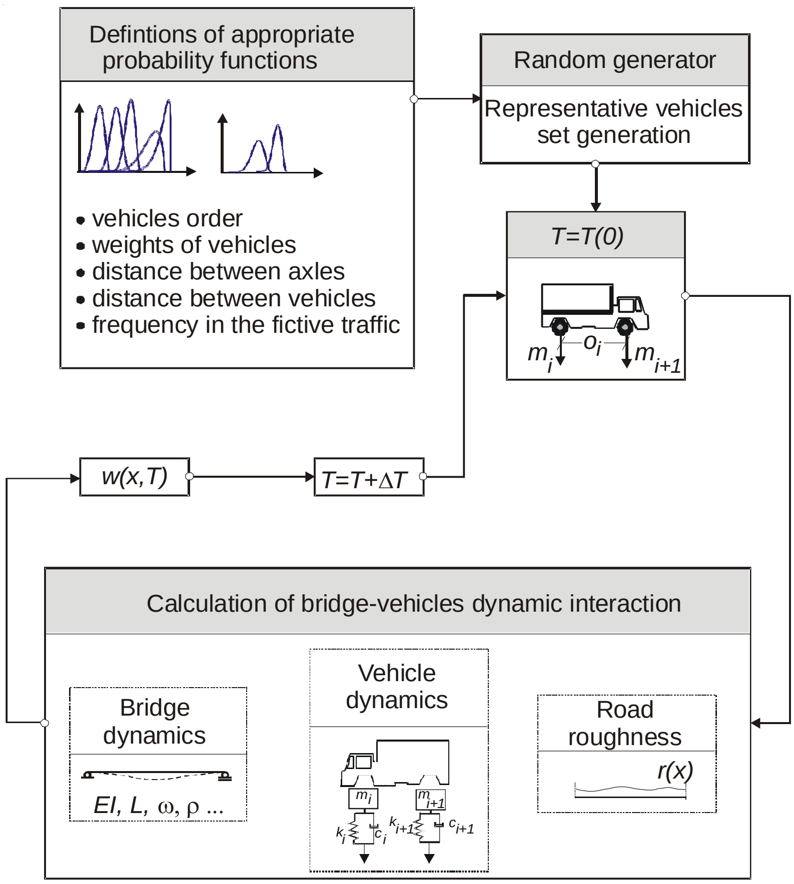

Fig. 3 Flow chart of proposed method

3. NUMERICAL EXAMPLE OF THE PROGRAM USE

In this example we consider a simple supported beam bridge with the span *L=20 m*, with the following material characteristic:

$\rho=1.65\cdot10^4$ kg/m, $I=0.059$ m^4, $E=2.1\cdot10^{11}$ N/m^2

The caracteristics of representative vehicle in this example are taken from Ref. [4] as mentioned in Section 2.1. For the stiffness of the vehicle the value *k=2.0 MN/m* is taken, which is a usual value in normal conditions [3], while we neglect damping in this example. Distance between vehicles is also modelled by beta probability function with coefficients (*1.7839*, *4.994*). The road roughness has been modelled with sinusoidal function:

$r(x) = A sin\left(\frac{2\pi x}{b}\right)$, where *A* has value of *0.02 m.*

Numerical simulations of moving representative sets of vehicles across the bridge were performed, where are some important characteristics of bridge-vehicle dynamic system: velocity of vehicle, roughness of bridge surface, frequency of particular representative vehicle in fictive traffic flow, etc. Some of the obtained results for the vertical deflections of the bridge at the midspan under different traffic conditions are presented in the Figures 4, 5, 6 and 7.

Post processing of obtained data (moments and displacement in characteristic point of the bridge) is also possible. For example, the Figure 8 shows calculated dynamic load factors and moment range spectrum obtained using rainflow moments cycle counting methods.

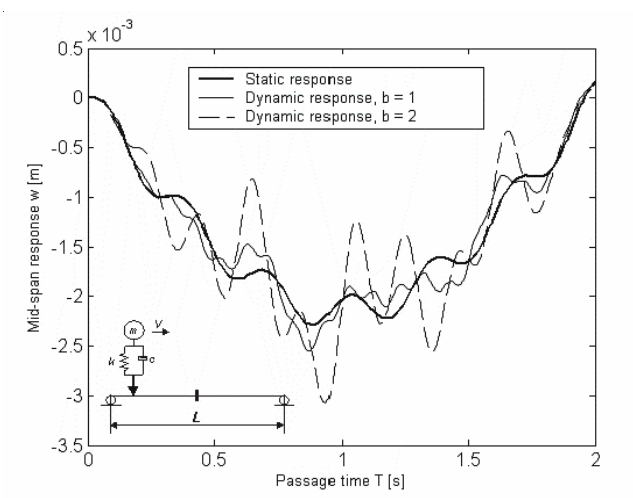

Fig. 4 Mid-span response of the bridge under passage of one axle with m=12 t and velocity v=10 m/s

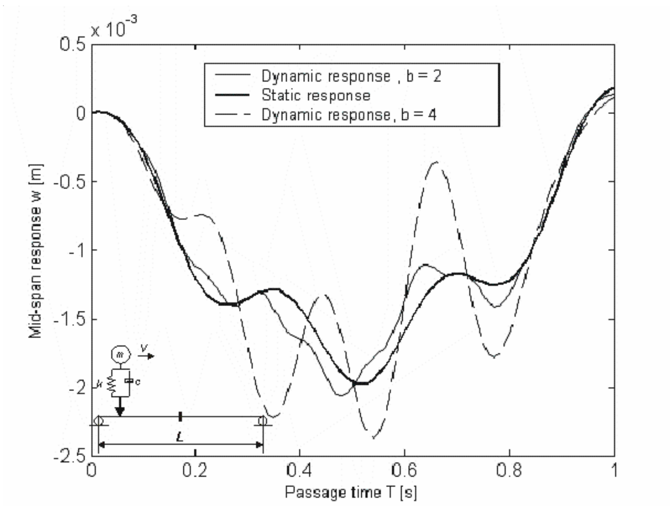

Fig. 5 Mid-span response of the bridge under passage of one axle with m=12 t and velocity v=20 m/s

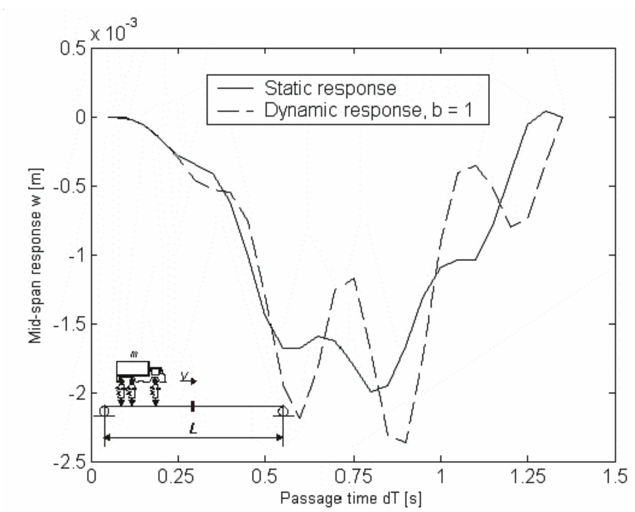

Fig. 6 Mid-span response of the bridge under passage of representative vehicle 2 (with mass of axles 30.8 t, 54.6 t and 54.6 t) and velocity v = 20 m/s

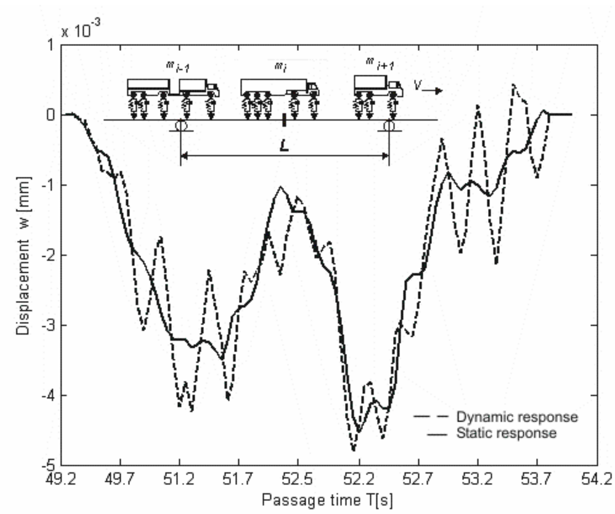

Fig. 7 One segment of mid-span response of the bridge under passage of a set of randomly generated vehicles with velocity v = 20 m/s and b = 1

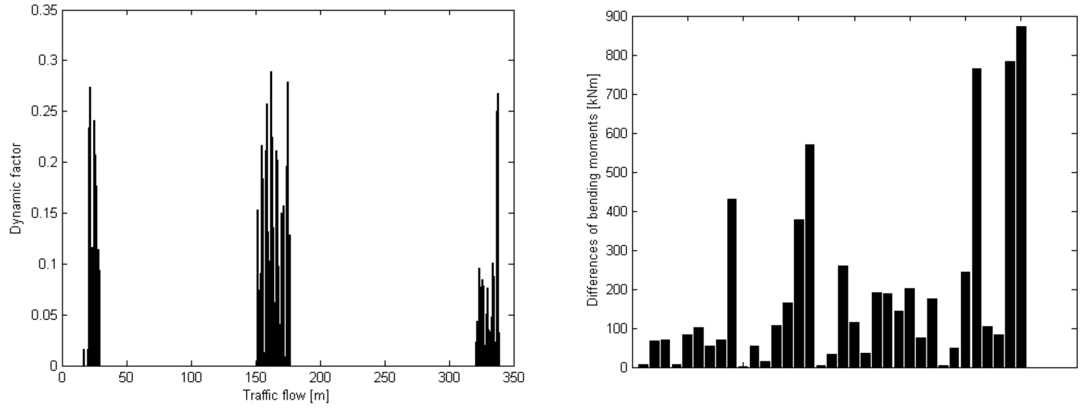

Fig. 8 Dynamic load factors and moment range spectrum

4. CONCLUSION

In this paper we present the approach to dynamic analysis of interaction system bridge-vehicle under the influence of actual traffic load. The influence of the actual traffic load should be considered in the following cases: estimation of the bridge reliability and durability, fatigue analysis, maintenance cost optimisation, development of the more effective bridge design, etc.

The presented method combines probabilistic approach for modelling actual traffic load and direct numerical procedure for the dynamic analysis of bridge-vehicle interaction system. The actual traffic flow is modelled by the set of representative vehicles with main characteristics (weight and distance between axles) treated as random variables. Advantages of these methods can be summarized in three points as follows:

– method may be very interesting for engineers because of its simplicity – in a simple manner actual traffic flow can be modelled by a few types of representative vehicles whose weights, dimensions and frequencies can be obtained on the basis of appropriate experimental or statistical data; SDOF vehicle model is described by the minimum number of the parameters, which in many cases adequately describes the interaction of the bridge–vehicle system; the main characteristic of the vehicle can be also chosen as deterministic value;
– dynamic response of the coupled system may be significantly larger than the static response (see the numerical example) - the considered method uses direct numerical procedure (instead of iterative method) for calculation of dynamic response of coupled system which saves processor time and accelerates calculation;
– the considered method is very useful for running parametric analysis and numerical simulations with varying different characteristics of the bridge, vehicle and traffic flow.

Many aspects of the interaction between the heavy vehicle and the bridge (roads) are now well understood, but still it remains a considerable further work to be performed with the purpose of minimizing the damage caused by actual traffic flow. Complex models of bridge-vehicle system are appropriate for the purpose of research, but they are not an adequate tool for practical analysis since engineers should use simpler methods as much accurate as possible. In this sense the proposed method can be very effective and useful.

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MODELIRANJE INTERAKTIVNOG DJELOVANJA CESTOVNOG MOSTA I STVARNOG PROMETNOG OPTEREĆENJA

SAŽETAK

Dinamičko ponašanje mostova uslijed utjecaja od realnog prometnog opterećenja je izvor sustavnog narušavanja općeg stanja mosta, smanjenja njegove uporabljivosti i trajnosti. Osovinska opterećenja vozila uzrokuju oštećenja kolovozne plohe te na taj način dovode do pojave korozije i ugrožavanja konstrukcije mosta. Stohastičnost realnog prometnog opterećenja, nepouzdanosti u procjeni stvarnih dinamičkih parametara vozila, postojanje dinamičke međuovisnosti most-vozilo i slično, samo su neke od teškoća pri provedbi dinamičkog proračuna mosta na utjecaje od realnog opterećenja. U ovom je radu izložena metoda proračuna koja kombinira probabilistički pristup za modeliranje stvarnog prometnog opterećenja i metodu direktnog numeričkog proračuna interaktivnog sustava most-vozilo. Vozila su modelirana kao skup određenog broja osovina s dinamičkim karakteristikama (krutost i prigušenje), dok je most modeliran kao kontinuirani elastični sustav. Predložena metoda je vrlo efikasna za provedbu numeričkih simulacija utjecaja stvarnog opterećenja na most, te istodobno i vrlo prikladna za praktične proračune zbog svoje jednostavnosti i zadovoljavajuće točnosti.

Ključne riječi: *stvarni promet, dinamička međuovisnost most-vozilo, oštećenja ceste, dinamički faktor, dinamički odgovor mosta, simulacija prometa.*