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A modified version of the part period lot-sizing heuristic

Singa Wang Chiu(1), **Chia-Kuan Ting**(2) and **Yuan-Shyi Peter Chiu**(2)
(1)Department of Business Administration, Chaoyang University of Technology, 168 Jifong East Road, Wufong, Taichung 413, TAIWAN, e-mail: swang@cyut.edu.tw
(2)Department of Industrial Engineering and Management, Chaoyang University of Technology, 168 Jifong East Road, Wufong, Taichung 413, TAIWAN,
e-mail: djk@cyut.edu.tw, ypchiu@cyut.edu.tw

SUMMARY

This paper proposes a modified version of the part period lot-sizing heuristic for the case of deterministic time-varying demands. The Part Period Balancing heuristic (PPB) is to select the number of periods covered by the replenishment order such that the total holding costs are made as close as possible to the setup cost. This paper presents a modification of the PPB by adding a procedure to the end of the PPB to test whether the elimination of last replenishment order by combining it with the preceding order is cost-beneficial. If the condition holds, then it amalgamates the last two replenishment lot-sizes, and hence total inventory costs are reduced. Numerical examples are provided to demonstrate its practical usage and the proof of cost saving of the proposed mv-PPB heuristic is given in Appendix.

Key words: *part period balancing (PPB), cost control, heuristic, lot-sizing.*

1. INTRODUCTION

Wagner and Whitin [1] first proposed an optimal algorithm for solving the dynamic lot-sizing problem several decades ago. Although their decision procedure guarantees to provide the minimal overall costs, there are drawbacks for the algorithm and hence it has received limited acceptance in practice [2]. One major reason among others is that the Wagner-Whitin algorithm requires the complex calculations.

For a simpler approach that can provide a practical suboptimum solution, a considerable amount of research has been carried out. Various heuristic approaches and decision rules are proposed for solving the dynamic lot-sizing problem, examples among them are surveyed as follows. DeMatteis [3] proposed a Part-Period Algorithm (PPA). This algorithm chooses the number of periods covered by the replenishment order such that the total holding costs are made as close as possible to the setup cost. Gorham [4] presented a Least Unit Cost (LUC) heuristic for selecting replenishment quantities that minimizes the total relevant costs per unit of demand. Berry [5] examined several lot-sizing procedures including the EOQ, periodic order quantity, part period balancing, and the Wagner-Whitin algorithm, and presented a framework for analyzing such procedures with respect to inventory related costs and computing time. Silver and Meal [6] suggested a heuristic for selecting lot-size quantities that minimizes the total relevant costs per unit time, for the case of a deterministic time-varying demand rate and discrete opportunities for replenishment (this decision procedure has been called the Silver-Meal heuristic). Karni [7] proposed the lot-sizing procedures for unconstrained and constrained requirements planning systems. The procedures repeatedly scan the order quantities, shift an order by adding it to the nearest adjacent prior order, and select the one that makes the maximum part-period gain. Silver and Miltenburg [8] provided two modifications of the Silver-Meal (SM) heuristic to address the following situations: (1) when the demand pattern drops rapidly with time over several periods; and (2) when there are a large number of periods having no demand. Baker

[9] suggested additional heuristics and reconciled the differences among the prior literature. Bookbinder and Tan [10] proposed two lot-sizing heuristics. One simplifies the stopping rule of the SM heuristic and the other combines the merits of both the SM and LUC heuristic. Stadtler [11] suggested a modified single-level lot-sizing model with a principle idea of improving rolling schedules. His model considered only a portion of the fixed cost related to a decision with an impact on periods beyond the planning horizon and looked for a production plan that minimizes the sum of setup and holding cost over the planning horizon. Tibben-Lembke [12] restated the stopping conditions of the SM and LUC heuristic for the purpose of increasing intuitive interpretation to the user.

This paper presents a modified version of the Part Period Balancing (mv-PPB) heuristic. The logic employed by this study is similar to the reasoned thought used by Karin [7] and Silver and Miltenburg [8], with the difference that we intend to propose a simpler and more generalized mv-PPB heuristic.

2. THE PART PERIOD BALANCING HEURISTIC

The part period balancing heuristic is one of the most widely used methods for dynamic lot-sizing. The name part-period was first introduced by DeMatteis [3] back to several decades and it refers to the equivalent of a part held for one period. The logic of the PPB is to set the replenishment horizon equal to the number of periods that most closely matches the total inventory holding cost with the setup cost over that period. Exact equality of holding and setup costs is usually not possible because of the discrete nature of the replenishment periods. To illustrate, the following notations are used:

- r_i = requirement (demand) in period i, where $i=1$ to T,
- T = planning horizon, a finite number of periods,
- n = number of periods covered by the replenishment (i.e. ordering horizon),
- h = cost per part per period for holding inventory (\$/item/period),
- H_n = total holding costs accumulated from period 1 to period n,
- y^* = replenishment quantity,
- K = fixed setup cost (\$/order),
- m = m^{th} replenishment, where $m \le T$,
- i = i^{th} period, where $i=1$ to T,
- y_i = replenishment quantity in period i, where $i=1$ to T,
- n_l = number of periods covered by the preceding replenishment,
- $TC(PPB)_m$ = total costs summed up to the m^{th} replenishment, when the basic PPB is used,
- $TC(mv\text{-}PPB)_m$ = total costs summed up to the m^{th} replenishment, when the proposed mv-PPB is used.

The part period balancing heuristic has the following main decision steps in determining the replenishment quantity [2, 3, 9, 13]:

1. Let $n=1$, $H_n=0$
2. Let $n=n+1$, $H_n=H_{n-1}+h(n-1)r_n$
3. If $H_n<K$, go to step 2
4. If $(H_n-K)>(K-H_{n-1})$, then $n=n-1$
5. $y^* = \sum_{j=1}^{n} r_j$, go to step 1 and repeat until the end of the planning horizon.

From the above steps, one notices that the 'end of period' criterion is employed to compute the inventory holding cost. A similar procedure that uses an 'average carrying cost' criterion for calculating holding cost is given in Appendix A.

3. MODIFICATION OF THE PART PERIOD BALANCING HEURISTIC

The part period balancing heuristic is not aware of the possible cost saving by eliminating the last replenishment lot. In this paper, a modified version of the PPB (mv-PPB) heuristic is proposed by adding a procedure to the end of the PPB to test the following condition: whether the elimination of the last replenishment by combining it with the preceding lot is cost-beneficial. If the condition holds, then it amalgamates the last two replenishment lot-sizes, otherwise it leaves the replenishment decision derived by basic PPB unchanged. In other words, the logic of proposed mv-PPB heuristic, using the end of period criterion for computing carrying cost, is to perform the basic PPB heuristic first; then to check if the following condition is satisfied:

$$[K-h\, n_l\, \boldsymbol{y}_{T-(n-1)}] > 0 \qquad (1)$$

If it holds, then let $\boldsymbol{y}_{T-(n-1)-nl} = \boldsymbol{y}_{T-(n-1)-nl} + \boldsymbol{y}_{T-(n-1)}$ and let $\boldsymbol{y}_{T-(n-1)}=0$, otherwise leave all $\boldsymbol{y}_i$ unchanged. The complete decision procedures of mv-PPB, including the computation of total setup and holding costs and total number of replenishments, are listed below:

1. Let $i=0$, $m=0$, and $TC(PPB)_m=0$
2. Let $n=1$, $H_n=0$
3. Let $n=n+1$, if $(i+n)\le T$, then $H_n=H_{n-1}+h(n-1)\, r_{i+n}$ otherwise $n=n-1$ and go to step 6
4. If $H_n<K$, go to step 3
5. If $(H_n-K)>(K-H_{n-1})$, then $n=n-1$
6. $y_{i+1} = \sum_{j=1}^{n} r_{i+j}$, $m=m+1$, $i=i+n$

   $TC(PPB)_m=TC(PPB)_{m-1}+K+H_n$
7. If $i<T$, then $n_l=n$ and go to step 2
8. If $[K-h\, n_l\, \boldsymbol{y}_{T-(n-1)}]>0$, then let

$$y_{T-(n-1)-n_l}=y_{T-(n-1)-n_l}+y_{T-(n-1)},\quad y_{T-(n-1)}=0,\quad \text{and}\quad TC(mv-PPB)_{m-1}=TC(PPB)_m-[K-h\ n_l\ y_{T-(n-1)}].$$

The aforementioned mv-PPB heuristic is simple from both conceptual and computation standpoints. Proof of cost saving of the mv-PPB in comparison with the basic PPB is given in Appendix B. Numerical examples in the following section demonstrate its practical usage.

4. NUMERICAL EXAMPLES AND DISCUSSION

4.1 Example 1

Consider the following demand pattern of an item with the setup cost $K=\$54$, holding cost $h=\$0.4$, and the end of period carrying cost criterion is used (data taken from Silver et al. [2]):

Period i	1	2	3	4	5	6	7	8	9	10	11	12
Demand r_i	10	62	12	130	154	129	88	52	124	160	238	41

First, by performing basic PPB heuristic one obtains the replenishments as presented in the third row of Table 1.

Table 1 Results of using the basic PPB heuristic on numerical example 1

Period i	1	2	3	4	5	6	7	8	9	10	11	12	Total
Requirements r_i	10	62	12	130	154	129	88	52	124	160	238	41	1200
Replenishment	**84**	**0**	**0**	**284**	**0**	**217**	**0**	**176**	**0**	**398**	**0**	**41**	**1200**

y=(84, 0, 0, 284, 0, 217, 0, 176, 0, 398, 0, 41); total number of replenishments m=6; $n_l=2$; n=1;
Total replenishment plus carrying costs = 6×\$54+690×0.4\$/\$/month = \$600.00

Secondly, testing the following condition of the mv-PPB heuristic:

$$[K-h\ n_l\ y_{T-(n-1)}]=[\$54-(\$0.4)(2)(41)]=\$21.2>0,$$

Since Eq. (1) holds, let $y_{T-(n-1)-nl} = y_{10}^{new} = y_{10}^{old} + y_{12}^{old} = 437$ and let $y_{T-(n-1)} = y_{12} = 0$. One obtains a better replenishment decision, a *3.53 %* cost reduction (see Table 2).

Table 2 Results of using the mv-PPB heuristic on numerical example 1

Period i	1	2	3	4	5	6	7	8	9	10	11	12	Total	Total costs
Replenishment (by the mv-PPB)	**84**	**0**	**0**	**284**	**0**	**217**	**0**	**176**	**0**	**439**	**0**	**0**	**1200**	**\$578.8**

4.2 Example 2

Consider the following demand pattern of a product with the setup cost $K=\$300$, holding cost $h=\$2$, and the average carrying cost criterion is used (data taken from Berry [5]):

Period i	1	2	3	4	5	6	7	8	9	10	11	12
Demand r_i	10	10	15	20	70	180	250	270	230	40	0	10

The replenishment solution by basic PPB heuristic is given in the third row of Table 3.

$$[K-h\ n_l\ y_{T-(n-1)}]=[\$300-(\$2)(3)(10)]=\$240>0,$$

Since Eq. (1) holds, let $y_9^{new} = y_9^{old} + y_{12}^{old} = 280$ and let $y_{12} = 0$. The resulting replenishment decision by the mv-PPB reduces total inventory costs by *6.89 %*. It is noticed that the mv-PPB heuristic happens to generate the same answer as the optimal solution derived by Wagner-Whitin algorithm [1] (see Table 3).

Table 3 Replenishment decisions on numerical example 2

Period i	1	2	3	4	5	6	7	8	9	10	11	12	Total	Total costs
Requirements r_i	10	10	15	20	70	180	250	270	230	40	0	10	1105	
Replenishment (by PPB)	55	0	0	0	70	180	250	270	270	0	0	10	1105	$ 3,485
Replenishment (by mv-PPB)	**55**	**0**	**0**	**0**	**70**	**180**	**250**	**270**	**280**	**0**	**0**	**0**	**1105**	**$ 3,245**
Replenishment (Wagner-Whitin)	55	0	0	0	70	180	250	270	280	0	0	0	1105	$ 3,245

4.3 Discussion

Recall the testing condition of the mv-PPB from Section 3, if we multiply both sides of Eq. (1) by (*1/h*) then we have the following:

$$[K/h - n_l\, \boldsymbol{y}_{T-(n-1)}]>0, \text{ or:}$$

$$K/h > n_l\, \boldsymbol{y}_{T-(n-1)} \qquad (2)$$

Equation (2) implies that under the following situations the chance of cost saving is higher: (1) when the *K/h* value is larger; or (2) when the last replenishment lot $\boldsymbol{y}_{T-(n-1)}$ is smaller. These presumptions are confirmed by the result of a further analysis (see Table 4) using Berry's examples [2]. Among five sets of different *K* values (ranging from *200* to *400*, i.e. $100 \le K/h \le 200$) analyzed by the mv-PPB, four sets are cost-beneficial obtaining an average of *4.59 %* cost reduction. One also notices that four out of five resulting replenishments by the mv-PPB happen to be identical to the optimal solutions derived by Wagner-Whitin algorithm (see Table 5, Appendix C).

Although these analytical results appear to be attractive, they are contributed by the satisfaction of $K/h > n_l\, \boldsymbol{y}_{T-(n-1)}$ and they are also partly caused by the assumed termination of demand pattern at the end of planning horizon. Hence, the proposed mv-PPB heuristic may not be beneficial in a rolling schedule environment [8, 11].

5. CONCLUDING REMARKS

The part period balancing heuristic is one of the most widely used techniques for dynamic lot-sizing because of its simplicity. The modified version of PPB heuristic presented here is aimed a simple to apply and easy to solve by hand. Although an extra testing condition is added to basic PPB, it does not deteriorate the simplicity of the original heuristic. Hence, it is worthwhile to append the testing condition $K/h > n_l\, \boldsymbol{y}_{T-(n-1)}$ to any heuristic that deals with deterministic time-varying demands in a finite planning horizon environment.

Table 4 Results of using the basic PPB and the mv-PPB for different K/h ratios

Period	Requirements		Replenishments by the **PPB**					Replenishments by the **mv-PPB**				
		K/h=	200	175	150	125	100	200	175	150	125	100
1	10		55	55	55	55	55	55	55	55	55	55
2	10		0	0	0	0	0	0	0	0	0	0
3	15		0	0	0	0	0	0	0	0	0	0
4	20		0	0	0	0	0	0	0	0	0	0
5	70		250	250	70	70	70	250	250	70	70	70
6	180		0	0	180	180	180	0	0	180	180	180
7	250		250	250	250	250	250	250	250	250	250	250
8	270		270	270	270	270	270	270	270	270	270	270
9	230		280	270	270	230	230	280	280	280	280	280
10	40		0	0	0	50	50	0	0	0	0	0
11	0		0	0	0	0	0	0	0	0	0	0
12	10		0	10	10	0	0	0	0	0	0	0
		Total Cost =	3805	3845	3485	3095	2745	3805	3555	3245	2945	2645
							Cost saving=	0.00%	7.54%	6.89%	4.85%	3.64%

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APPENDIX A

The main decision steps of basic PPB heuristic using the average carrying cost criterion are listed as follows:

1. Let $n=1$, $H_n=h(n-0.5)r_n$
2. Let $n=n+1$, $H_n=H_{n-1}+h(n-0.5)r_n$
3. If $H_n<K$, go to step 2
4. If $(H_n-K)>(K-H_{n-1})$, then $n=n-1$
5. $y^*=\sum_{j=1}^{n} r_j$, go to step 1 and repeat until the end of the planning horizon.

APPENDIX B

Proof: $TC(mv-PPB)_{m-1} \leq TC(PPB)_m$

(1) If condition: $[K-n_l\, \boldsymbol{y}_{T-(n-1)}] > 0$ does not hold,

then: $TC(\text{mv-PPB})_{m-1}=TC(\text{PPB})_m$

(2) If condition: $[K-n_l\, \boldsymbol{y}_{T-(n-1)}] > 0$ holds,

then: $\boldsymbol{y}_{T-(n-1)-nl}=\boldsymbol{y}_{T-(n-1)}-n_l+\boldsymbol{y}_{T-(n-1)}$, and

then: $\boldsymbol{y}_{T-(n-1)}=0$; and:

$$TC(mv-PPB)_{m-1}=TC(PPB)_m-K+h\ n_l\ \boldsymbol{y}_{T-(n-1)}$$

$$K-h\ n_l\ \boldsymbol{y}_{T-(n-1)}>0$$

$$TC(mv-PPB)_{m-1}<TC(PPB)_m$$

APPENDIX C

Table 5 Results of using the mv-PPB and the Wagner-Whitin algorithm for different K/h ratios

Period	Requirements		Replenishments by the **mv-PPB**					by the **Wagner-Whitin** algorithm				
		K/h=	200	175	150	125	100	200	175	150	125	100
1	10		55	55	55	55	55	55	55	55	55	55
2	10		0	0	0	0	0	0	0	0	0	0
3	15		0	0	0	0	0	0	0	0	0	0
4	20		0	0	0	0	0	0	0	0	0	0
5	70		250	250	70	70	70	250	70	70	70	70
6	180		0	0	180	180	180	0	180	180	180	180
7	250		250	250	250	250	250	250	250	250	250	250
8	270		270	270	270	270	270	270	270	270	270	270
9	230		280	280	280	280	280	280	280	280	280	280
10	40		0	0	0	0	0	0	0	0	0	0
11	0		0	0	0	0	0	0	0	0	0	0
12	10		0	0	0	0	0	0	0	0	0	0
		Total Cost =	3805	3555	3245	2945	2645	3805	3545	3245	2945	2645
		Cost penalty of the mv-PPB= (in comparison with the Wagner-Whitin algorithm)	0%	0.28%	0%	0%	0%					

MODIFICIRANI HEURISTIČKI POSTUPAK ZA ODREĐIVANJE VELIČINE SERIJE U OGRANIČENOM VREMENU

SAŽETAK

Ovaj rad predlaže modificirani heuristički postupak za određivanje veličine serije u ograničenom vremenu u slučaju određenih vremenski promjenjivih zahtjeva. Postupak uravnoteženja u ograničenom vremenu (PPB) predstavlja odabir broja perioda pokrivenih nadopunjavanjem narudžbi tako da holding troškovi budu što je moguće bliži troškovima pripreme. Ovaj rad iznosi modifikaciju PPB-a dodavanjem postupka na kraju PPB-a radi provjere je li opravdano eliminiranje posljednje izmjene narudžbe u kombinaciji s prethodnom narudžbom. Ako je uvjet opravdan, on spaja dva posljednja nadopunjavanja serija te smanjuje sveukupne troškove zaliha. Numerički primjeri pokazuju njegovu praktičnu primjenu, a dokaz o uštedi predloženog heurističkog mv-PPB-a nalazi se u dodatku.

Ključne riječi: *postupak uravnoteženja (PPB), kontrola troškova, ograničeno vrijeme, veličina serije.*