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Calculating the constants of the second-order process using grey model GM(3/t,1)

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SUMMARY

Grey model GM(3/t,1) and its application in the identification of the static sensitivity, the frequency of natural oscillations and the damping coefficient of the second-order process are introduced. Second-order differential equation is substituted by discrete-time grey model GM(3/t,1). Grey model GM(3/t,1) can be built by state variable measured data.

Key words: *discrete-time series, grey model, second-order process.*

1. INTRODUCTION

The subject of our investigation is the possibility to influence grey system's performance by control of deterministic processes with grey system. Some authors presented the application of grey system theory in control problems, especially in the grey prediction technology. For example, C.C. Wong (1998) proposed a grey prediction controller design and C.H. Chou (2001) introduced a variable structure controller based on the grey prediction technology.

Grey system theory was initiated by D. Julong in 1982 [1]. Grey models GM(1,1) and GM(1,N) are introduced by D. Julong in control problems [2]. Grey models can be used when mathematical process model is unknown. For example, Cheng proposed a grey prediction controller to control an industrial process without knowing the system model in 1986. We can say that grey models substitute the unknown mathematical process models in control problems. After D. Julong had introduced grey models GM(1,1) and GM(1,N) some authors proposed new grey models. For instance, Z. Zhen proposed a new grey model GM(3/t,1) in thermodynamic process control [3]. In that paper, differential equation of the second-order thermodynamic process is substituted by discrete-time grey model GM(3/t,1). Grey model GM(3/t,1) can be built only by state variable measured data. In this paper, grey model GM(3/t,1) and its application in calculating the static sensitivity, the frequency of natural oscillations and the damping coefficient of the second-order process are presented.

The paper is organized as follows: in Section 2 the second-order process is described, in Section 3 grey model GM(3/t,1) is introduced, in Section 4 the application of grey model GM(3/t,1) is presented and Section 5 is conclusion.

2. THE SECOND-ORDER PROCESS

The second-order process can be expressed by the following differential equation:

$$\frac{1}{\omega_n^2}\ddot{y}+\frac{2\zeta}{\omega_n}\dot{y}+y=k_s x(t) \tag{1}$$

where:

$y(t)$ is a function of the process output,

$x(t)$ is a function of the process input,

and the constants of the second-order process are as follows:

k_S is the static sensitivity,

ω_n is the frequency of natural oscillations, and

ζ is the damping coefficient.

The following substitution can be introduced:

$$y(t)=x^{(1)}(t) \tag{2}$$

Differential Eq. (1) with substitution (2) and $x=a$ can be written as:

$$\frac{d^2x^{(1)}}{dt^2}+a_1\frac{dx^{(1)}}{dt}+a_2x^{(1)}=b \tag{3}$$

where the constants are as follows:

$$\begin{aligned} a_1 &= 2\zeta\omega_n \\ a_2 &= \omega_n^2 \\ b &= ak_s\omega_n^2 \end{aligned} \tag{4}$$

3. GREY MODEL GM(3/T,1)

Let $x^{(1)}$ be discrete-time sequence of state variable measured data:

$$x^{(1)}=\left(x^{(1)}(1),...,x^{(1)}(n)\right) \tag{5}$$

and let $x^{(0)}$ be IAGO series of $x^{(1)}$:

IAGO:

$$\begin{aligned} &x^{(0)}(k)=\alpha^{(1)}\left(x^{(1)}(k)\right)=x^{(1)}(k)-x^{(1)}(k-1),\\ &x^{(1)}(0)=0, \quad k=1,2,...,n \end{aligned} \tag{6}$$

$$x^{(0)}=\left(x^{(0)}(1),...,x^{(0)}(n)\right) \tag{7}$$

Equation (3) can be substituted by grey model GM(3/t,1) [3]:

$$\alpha^{(2)}\left(x^{(1)}(k)\right)+a_1\alpha^{(1)}\left(x^{(1)}(k)\right)+a_2z^{(1)}(k)=b \tag{8}$$

where:

$$\alpha^{(2)}\left(x^{(1)}(k)\right)=\alpha^{(1)}\left(x^{(1)}(k)\right)-\alpha^{(1)}\left(x^{(1)}(k-1)\right) \tag{9}$$

$$\alpha^{(1)}\left(x^{(1)}(k)\right)=x^{(1)}(k)-x^{(1)}(k-1) \tag{10}$$

$$z^{(1)}(k)=0{,}5x^{(1)}(k)+0{,}5x^{(1)}(k-1) \tag{11}$$

$$x^{(1)}(0)=0,\ k=2,3,...,n$$

The matrix equation for Eq. (8) is as follows:

$$Y=B\theta \tag{12}$$

$$\boldsymbol{Y}=\left[\alpha^{(2)}\left(x^{(1)}(2)\right)\quad \alpha^{(2)}\left(x^{(1)}(3)\right)\quad ...\quad \alpha^{(2)}\left(x^{(1)}(n)\right)\right]^T \tag{13}$$

$$\boldsymbol{B}=\begin{bmatrix} -x^{(0)}(2) & -x^{(0)}(3) & ... & -x^{(0)}(n) \\ -z^{(1)}(2) & -z^{(1)}(3) & ... & -z^{(1)}(n) \\ 1 & 1 & ... & 1 \end{bmatrix}^T \tag{14}$$

$$\boldsymbol{\theta}=\left[a_1\quad a_2\quad b\right]^T \tag{15}$$

Afterwards, by means of the least square method, we have:

$$\boldsymbol{\theta}=\left[\boldsymbol{B}^T\boldsymbol{B}\right]^{-1}\boldsymbol{B}^T\boldsymbol{Y} \tag{16}$$

4. APPLICATION AND EXAMPLE

According to the Eq. (16) (using Mathematica Wolfram software), we get:

$$\boldsymbol{\theta}=\left[a_1\quad a_2\quad b\right]^T=\left[-4700{,}6\quad 876{,}422\quad 360{,}715\right]^T$$

From Eqs. (4) with $a=0{,}925\ mol/m^3$, we obtain:

$$\begin{aligned} k_s &= 0{,}445 \\ \omega_n &= 29{,}6\ min^{-1} \\ \zeta &= 79{,}4 \end{aligned}$$

Because of $\omega_n=\pm\sqrt{a_2}$, for calculating ζ the solution $\omega_n=-\sqrt{a_2}$ is used, but defacto $\omega_n=\sqrt{a_2}$ (the constants k_S, ω_n and ζ must be positive).

Equation (8) with substitute $x^{(1)}(k)=\hat{x}^{(1)}(k)$ can be rewritten as:

$$\begin{aligned} \hat{x}^{(1)}(k)=\hat{x}^{(1)}(k-1)&\frac{2+a_1-0{,}5a_2}{1+a_1+0{,}5a_2}- \\ -\hat{x}^{(1)}(k-2)&\frac{1}{1+a_1+0{,}5a_2}+ \\ +&\frac{b}{1+a_1+0{,}5a_2} \end{aligned} \tag{17}$$

$$k = 3,\ 4,\ ...,\ 8$$

where:

$$\hat{x}^{(1)}(1)=x^{(1)}(1) \text{ and } \hat{x}^{(1)}(2)=x^{(1)}(2)$$

The whitening time-response of Eq. (1) is as follows:

$$y(t)=c_1e^{\lambda_1 t}+c_2e^{\lambda_2 t}+ak_s \tag{18}$$

From the characteristic equation of Eq. (1) we get:

$$\lambda_{1,2}=\omega_n\left(-\zeta\pm\sqrt{\zeta^2-1}\right)$$

$$\begin{aligned} \lambda_1 &= -0{,}186 \\ \lambda_2 &= -4700{,}3 \end{aligned} \tag{19}$$

Because $e^{-4700{,}3t}\to 0$ we obtain:

$$y(t)=ce^{-0{,}186t}+0{,}4116 \tag{20}$$

Table 1. A series of state variable measured data obtained from the second-order process (two reactors connected in series)

k	1	2	3	4	5	6	7	8
$x^{(1)}(k)$ (mol/m^3)	0,4123	0,4124	0,4126	0,4127	0,413	0,4133	0,4137	0,4141

Table 2. The results

t (min)	10,3	10,4	10,5	10,6	10,7	10,8
k	3	4	5	6	7	8
$x^{(1)}(k)$ (mol/m^3)	0,4126	0,4127	0,413	0,4133	0,4137	0,4141
$\hat{x}^{(1)}(k)$ (mol/m^3)	0,4125	0,4126	0,4127	0,4128	0,413	0,4132
$y(t)$ (mol/m^3)	0,41228	0,41226	0,41225	0,41224	0,41223	0,41222

From $y(t_1)=x^{(1)}(1)$ as the boundary condition, $t_1=10,1$ min (the moment when from value zero reaches value a), it follows:

$$c = 0,005$$

Results are presented in Table 2 where $x^{(1)}(k)$ are state variable measured data, $\hat{x}^{(1)}(k)$ are state variable values obtained by grey model GM(3/t,1), Eq. (17), and are $y(t)$ state variable values obtained by mathematical process model Eq. (18).

There are small differences between values $x^{(1)}(k), \hat{x}^{(1)}(k)$ and $y(t)$ in Table 2. It means that the static sensitivity, the frequency of natural oscillations and the damping coefficient are successfully calculated. Values of $y(t)$ in Table 2 are in monotonic decrease. It is because of small error in mathematical method mentioned above (grey model cannot completely substitute the differential equation of the second-order process). If k_s will be a little greater values of $y(t)$ it will be in monotonic increase.

5. CONCLUSION

1. The constant item in the whitened response describes the equilibrium feature of the second-order process.
2. The real roots of the characteristic equation reflect the inertia and delay property of the second-order process.
3. The high precision of grey differential model GM(3/t,1) features the excellent adaptability of grey model to the second-order process.

The second-order process can be described by discrete-time grey model GM(3/t,1). Mathematical procedure mentioned in this paper can be used when mathematical process model is unknown and only state variable measured data are available. Easy way to calculating the static sensitivity, the frequency of natural oscillations and the damping coefficient of the second-order process is presented.

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7. REFERENCES

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IZRAČUNAVANJE KONSTANTI PROCESA DRUGOG REDA KORISTEĆI SIVI MODEL GM(3/t,1)

SAŽETAK

U ovom radu prikazani su sivi model GM (3/t,1) i njegova primjena u identifikaciji statičke osjetljivosti, frekvencije prirodnog titranja i koeficijenta prigušenja procesa drugog reda. Diferencijalna jednadžba drugog reda zamijenjena je diskretnim vremenskim sivim modelom GM(3/t,1). Sivi model GM(3/t,1) može biti formiran pomoću izmjerenih podataka.

Ključne riječi: *vremenski diskretni podaci, sivi model, proces drugog reda.*