

THE TRANSFORMATION OF COORDINATES BASED ON TWO CONTROL POINTS

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In the issue for October—November—December 1971. Marko KACANSKI solved this problem in a matrix form. A similar, but possibly somewhat simpler approach is given here.

The problem of coordinate transformation usually arises when it is required to put a survey computed on a local grid on to a national system. In effecting such a change there are three basic factors to be considered:—

- a — a change of scale and possibly a change of unit.
- b — a change of azimuth to make the directions agree on both systems.
- c — a change of origin to make the axes coincide.

In order to be able to fulfil these conditions the minimum data required is the coordinates of two points on both systems. Let lower case letters refer to the old grid and upper case to the new grid then the necessary given data must be: — (e_a, n_a) ; (e_b, n_b) and (E_A, N_A) ; (E_B, N_B) .

a — Change of scale

By virtue of the national system being on a projection allowing for the spheroidal nature of the earth whereas a local survey may have been computed as if the earth were flat there is every possibility of a line computed on one system being a different length when computed on the other system. Similarly the old system may have been in one linear unit (say feet) and the new system in metres. This situation can be taken care of by the use of a distance correction M derived as: —

$$M = \frac{\text{Final length AB}}{\text{Initial length ab}}$$

Hence any expression involving a length on the old grid must be multiplied by M to get the equivalent length on the new grid.

TRASFORMACIJA KOORDINATA OSLANJAJUĆI SE NA DVIJE IDENTIČNE TOČKE

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U broju za Oktobar-Novembar-Decembar 1971. godine Marko Kačanski riješio je ovaj problem u formi matrice. Ovdje se daje jedan sličan, ali možda nešto jednostavniji pristup tom problemu.

Problem transformacije koordinata pojavljuje se obično kada treba preračunati neki premjer, izračunat u nekoj lokalnoj mreži, u državni koordinatni sistem. Pri takvoj transformaciji treba uzeti u obzir tri osnovna faktora:

- a — promjenu mjerila, a možda i promjenu jedinice za dužinu
- b — promjenu azimuta
- c — promjenu ishodišta koordinatnih sistema.

Da se mogu ispuniti ovi uvjeti, potreban je minimalni broj podataka a to su koordinate dviju identičnih točaka u oba koordinatna sistema.

Neka se označavanje malim slovima odnosi na staru mrežu, a označavanje velikim na novu mrežu, pa će onda neophodni podaci biti:

(e_a, n_a) , (e_b, n_b) i (E_A, N_A) , (E_B, N_B)

a — Promjena mjerila

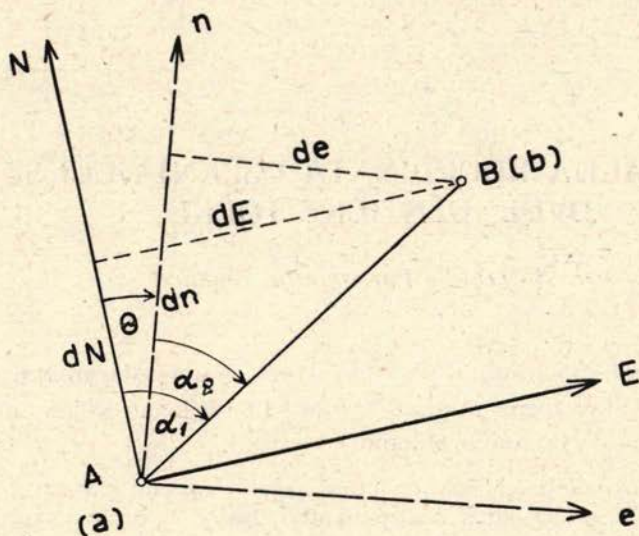
Na temelju toga, što je u državnom koordinatnom sistemu uzeto u obzir da je Zemlja sferoid, a budući da je neki lokalni premjer mogao biti izračunat na Zemlji kao ravnini, postoji svakako mogućnost, da dužina sračunata u jednom sistemu bude različita od one, izračunate u drugom. Isto tako stari sistem može imati jednu jedinicu za dužine (recimo stopu) a da je novi sistem metarski. Ova okolnost može se uzeti u obzir tako, da se upotrebi korekcija dužine M , izvedenu kao:

$$M = \frac{\text{konačna dužina AB}}{\text{početna dužina ab}}$$

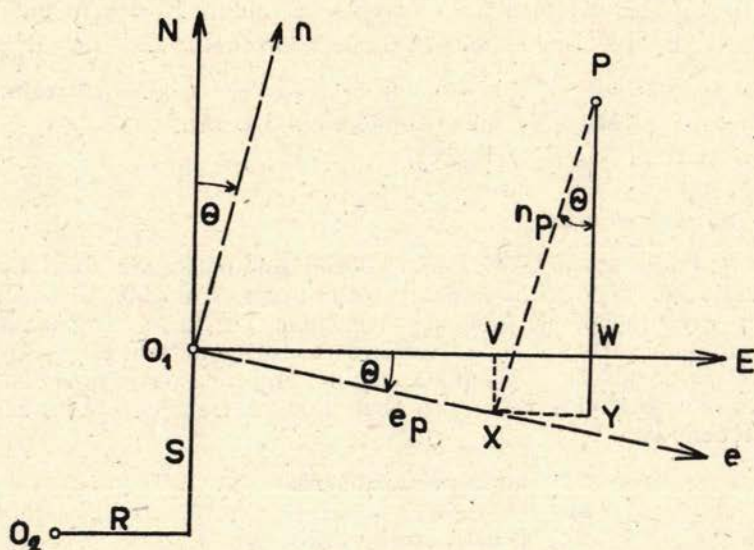
Radi toga, svaki izraz ili formula, koja sadrži dužinu u staroj mreži treba da se pomnoži sa M , kako bi se dobila dužina u novoj.

b — Change of azimuth

If Θ = rotation angle between the grids,
 = Bearing AB — bearing ab = $\alpha_1 - \alpha_2$



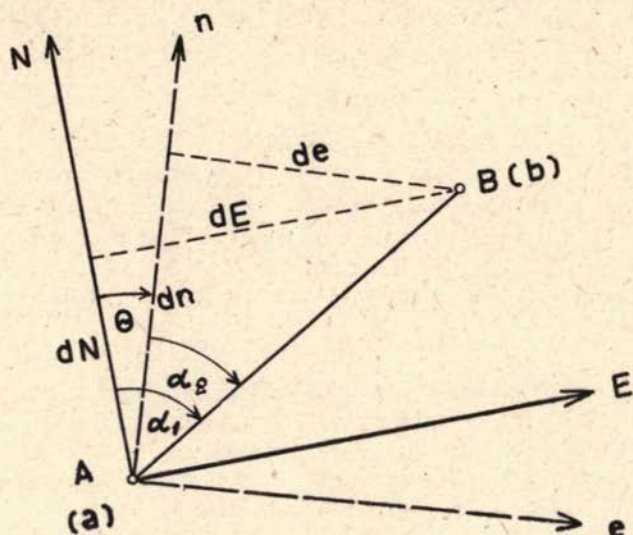
c — Change of origin



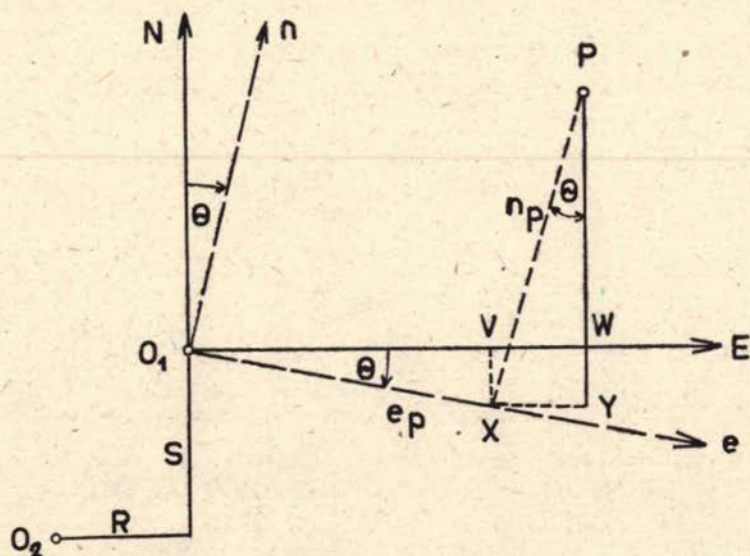
P has the initial coordinates e_p, n_p measured from origin O_1 on the original grid coordinates of O_1 in relation to the new origin O_2 are (R, S).

b — Promjena azimuta

Ako je: θ = kut rotacije između dviju mreža,
Smjerni kut AB — smjerni kut $ab = \alpha_1 - \alpha_2$



c — Promjena ishodišta



Točka P ima početne koordinate: e_p , n_p , mjerene od ishodišta O_1 u prvotnom koordinatnom sistemu. Neka su koordinate O_1 , u odnosu na novo ishodište $O_2 - R, S$.

Then on the new grid:

$$E_p = R + O_1W = R + O_1V + XY \\ = R + e_p \cdot M \cdot \cos\theta + n_p \cdot M \cdot \sin\theta$$

and

$$N_p = S + WP = S + (YP - XV) \\ = S + n_p \cdot M \cdot \cos\theta - e_p \cdot M \cdot \sin\theta$$

Let $M \cdot \cos\theta = P$ and $M \cdot \sin\theta = Q$

Then

$$E_p = R + P \cdot e_p + Q \cdot n_p$$

and

$$N_p = S + P \cdot n_p - Q \cdot e_p$$

As 2 sets of values are known for the two points A and B then R and S can be found as:

$$R = E_A - P \cdot e_a - Q \cdot n_a = E_B - P \cdot e_b - Q \cdot n_b$$

and

$$S = N_A - P \cdot n_a + Q \cdot e_a = N_B - P \cdot n_b + Q \cdot e_b$$

With the modern small programmable desk calculators such as the Hewlett-Packard 9100 or 9810, the need to physically evaluate $\sin\theta$ and $\cos\theta$ as functions of the differences in coordinates disappears as the keys give the facility to convert rectangular coordinates directly into polar form and the subsequent use of automatic trigonometric function keys does all these operations within a few instructions steps. The print-out shown is from the Hewlett-Packard 9810 and shows the same example as that in the original article. In effect, the two equations being evaluated for each new point are:

$$E_p = 353\,229.920 - 1.895\,044\,49e_p - 0.046\,044\,88n_p$$

and

$$N_p = 262\,066.818 - 1.895\,044\,49n_p + 0.046\,044\,88e_p$$

CHANGE OF GRID

ENTER DATA

COORDINATES OF

POINT A

EA → Y

NA → X

429355.600

38780.400

COORDINATES OF

POINT B

EB → Y

NB → X

431167.900

38491.040

COORDINATES OF

POINT A*

EA* → Y

NA* → X

COORDINATES OF

POINT B*

EB* → Y

NB* → X

-43008.420

116781.480

-43967.900

116910.860

COORDINATES OF

POINT ON THE

OLD GRID WHOSE

COORDINATES ARE

REQUIRED ON THE

NEW GRID

EP* → Y

NP* → X

-43171.680

116778.210

Tada će u novom sistemu koordinate točke P biti:

$$\begin{aligned} E_p &= R + O_1W = R + O_1V + XY \\ &= R + e_p \cdot M \cdot \cos\theta + n_p \cdot M \cdot \sin\theta \end{aligned}$$

i

$$\begin{aligned} N_p &= S + WP = S + (YP - XV) \\ &= S + n_p \cdot M \cdot \cos\theta - e_p \cdot M \cdot \sin\theta \end{aligned}$$

Uzmimo da je $M \cos \theta = P$, a $M \sin \theta = Q$, pa će biti

$$\begin{aligned} E_p &= R + P \cdot e_p + Q \cdot n_p \\ N_p &= S + P \cdot n_p - Q \cdot e_p \end{aligned}$$

Budući da su poznate dvije vrijednosti za dvije točke A i B, može se naći vrijednost za R i S kao:

$$R = E_A - P \cdot e_a - Q \cdot n_a = E_B - P \cdot e_b - Q \cdot n_b$$

i

$$S = N_A - P \cdot n_a + Q \cdot e_a = N_B - P \cdot n_b + Q \cdot e_b$$

Pri radu s malim stolnim kalkulatorima kao npr. sa Hewlett-Packard 9100 ili 9810 otpada potreba računanja stvarnih vrijednosti $\sin \theta$ i $\cos \theta$ kao funkcija koordinatnih razlika, jer posebna tipka na kalkulatoru daje mogućnost pretvaranja pravokutnih koordinata neposredno u polarne, pa se narednom upotrebom tipaka za automatske trigonometrijske funkcije, u nekoliko stupnjeva prema uputi, obavljaju sve ove operacije. Prikazani primjer izrađen je na kalkulatoru Hewlett-Packard 9810 i prikazuje isti primjer, kao u originalnom članku. U stvari, jednadžbe, koje su određene za svaku novu točku jesu:

$$E_p = 353\,229.920 - 1.895\,044\,49e_p - 0.046\,044\,88n_p$$

i

$$N_p = 262\,066.818 - 1.895\,044\,49n_p + 0.046\,044\,88e_p$$

Slobodni prijevod: S. Macarol

CHANGE OF GRID

ENTER DATA

COORDINATES OF

POINT A

EA → Y

NA → X

429355.600

38780.400

COORDINATES OF

POINT B

EB → Y

NB → X

431167.900

38491.040

COORDINATES OF

POINT A*

EA* → Y

NA* → X

COORDINATES OF

POINT B*

EB* → Y

NB* → X

—43008.420

116781.480

—43967.900

116910.860

COORDINATES OF

POINT ON THE

OLD GRID WHOSE

COORDINATES ARE

REQUIRED ON THE

NEW GRID

EP* → Y

NP* → X

—43171.680

116778.210

COORDINATES OF
POINT P ON THE
NEW GRID

429665.136
38779.080

IF FURTHER POINT
TO BE CONVERTED
ENTER OLD COORDS

-43361.030
116711.380

COORDINATES OF
POINT P ON THE
NEW GRID

430027.039
38897.007

-43604.090
116792.260

COORDINATES OF
POINT P ON THE
NEW GRID

430483.925
38732.544
-43824.520
116883.610

COORDINATES OF
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430897.443
38549.282

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