LORENTZIAN COMPONENT OF THE FIFTH FORCE

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The possibility of detecting the fifth force through the hypermagnetic moments produced by rotating hypercharge is discussed. The effects are calculated for rotating bodies of the laboratory dimensions and also for the rotating Earth. In both cases the calculated effects are smaller than the Lense-Thirring effects.

The original suggestion of Fischbach et al.[1] that besides the known four fundamental interactions, a fifth force of the finite range might exist in the nature, has attracted considerable attention. The searches for the evidence were focused on a wide range of apparent anomalies in the existing experimental data on different systems[2]. Furthermore, several new experiments were also performed in order to find a signal of a non-Newtonian component of gravity. Modified Eötvös experiments only in some cases report the evidence for a composition dependence of the acceleration of objects on Earth.

A rather inconclusive situation is also with experiments which study the variation of the gravitational coupling constant as a manifestation of the fifth force[3]. Namely, the Coulomb-like force mediated by a massive photon corresponds to the potential energy of two masses m_1 and m_2, at a distance r from each other,

$$V(r) = -G\frac{m_1 m_2}{r}(1 + ae^{-r/\lambda}) \tag{1}$$

where a measures the strength of the fifth force relative to Newtonian gravity, while λ measures its range. Then for $r/\lambda \gg 1$ ($\lambda \approx 100\,\mathrm{m}$) the classical Newtonian potential is recovered, G being the usual gravitation constant. Also for $r/\lambda \ll 1$

one again has the same potential, now with the gravitational constant $G(1+\alpha)$. If $\alpha \neq 0$ is observed ($\alpha \approx 0.01$, according to estimates in Ref. 1), or if the gravitational coupling varies for intermediate values of r, this would indicate the presence of the second term in (1).

At the other side, theoretical investigations in kaon-decay physics[4] showed that there could also be room for such massive photons (hyperphotons) that transmit the hypothetical force. So far only the Coulomb-like interaction of such a hypercharge, attributed to the baryon has been searched for within the existing experimental data. Our aim here is to estimate the possibility of detecting the interaction of hypermagnetic moments. Of course, this Lorentz-like interaction is damped by the relativictic factor $(v/c)^2$, however one might wander if it can be compensated by other parameters in the experiment.

The simplest experiment one can think of is to let a sphere, homogeneously filled by the hypercharge, rotate at high speed. Then we place a hypermagnetic moment in the hypermagnetic field generated along the axis of rotation, and try to detect the interaction. The test body may be another sphere rotating at high speed. This will be discussed at a later point, and now we first evaluate the field of a rotating sphere.

Since we deal with massive photons, hyperphotons, we will start from the stationary Proca equation

$$[\Delta - m^2]\vec{A}(\vec{r}) = -4\pi/c\,\vec{j}(\vec{r}), \tag{2}$$

where m denotes the mass of the heavy photon, while $\vec{j}$ is the stationary hypercharge current. Formal integration of this equation by standard methods gives the vector potential

$$\vec{A}(\vec{r}) = \frac{1}{4\pi c}\int \mathrm{d}^3 r' \frac{\exp[-m|\vec{r}-\vec{r}'|]}{|\vec{r}-\vec{r}'|}\vec{j}(\vec{r}'). \tag{3}$$

As the equation (1) reflects the change in the Coulomb law, in the similar way we expect a modified Biot-Savart law in the case when the photon becomes massive.

In the particular case when the current flows along a wire, the integration in (3) may be performed. The situation further simplifies if we replace the current density $\vec{j}$ by the stationary current J as follows

$$\mathrm{d}^3 r'\vec{j} = \mathrm{d}f\mathrm{d}s\,\vec{j} = |\vec{j}|\,\mathrm{d}f\mathrm{d}\vec{s} = J\mathrm{d}\vec{s}.$$

The Biot-Savart law for massive photons then becomes

$$\vec{B}(\vec{r}) = \mathrm{rot}\,\vec{A} = \frac{J}{4\pi c}\int_C \frac{\mathrm{d}\vec{s}\times\vec{L}\,(1+mL)\exp(-mL)}{L^3} \tag{4}$$

where $\vec{L} = \vec{r} - \vec{r}'$ and C is the contour of the wire.

Choosing C to be a circle of radius z, we may evaluate the hypermagnetic field at a distance x from the centre, perpendicular to the plane of the circle. We find

$$B(x) = \frac{J}{2c}\,\frac{z^2\,(1 + mh)\,\mathrm{e}^{-mh}}{h^3}, \qquad h = \sqrt{z^2 + x^2}. \tag{5}$$

The direction of the field, which points along the axis of the circle, is unimportant for our purpose here.

After having derived the Biot-Savart law for the massive photon case, we consider now a massive sphere of radius R, rotating at a constant frequency ω. Let the hypercharge density ϱ in the sphere be constant. Through an arbitrary ring of radius z, rotating at frequency ω, the current amounts to

$$J = 2\pi\, z\omega\varrho\, \mathrm{d}z\, \mathrm{d}y. \tag{6}$$

Such rings are easily integrated into a disc, and discs into a sphere, so that we finally obtain,

$$B(r) = \frac{\pi\varrho\omega}{c}\int\limits_{-R}^{R}\mathrm{d}y\int\limits_{0}^{z_0}\mathrm{d}z\,\frac{z^3\,(1 + ml)\exp(-ml)}{l^3}, \qquad l = \sqrt{z^2 + (r + y)^2}, \quad z_0 = \sqrt{R^2 - y^2}. \tag{7}$$

The geometry and notations are obvious.

Evaluation of the integral (7) is straightforward but very lengthy. As expected, an overall exponential damping factor $\exp(-mr)$ appears and guaranties the range property of the fifth force. The experiments should, therefore, be done at the north pole, $r = R$. For this particular point, in terms of a dimensionless variable $x = mR$, we obtain

$$B(R) = \frac{4R^2\pi\varrho\omega\exp(-x)}{x^5}\,f(x), \tag{8}$$

$$f(x) = (3 + 3x + x^2 + x^3)\,\mathrm{sh}\,x - (3x + 3x^2)\,\mathrm{ch}\,x. \tag{9}$$

The massless photon limit, $x = 0$, seems to be a singularity of the fifth order, however, lenghty straightforward manipulation shows that $f(x)$ at $x = 0$ also develops a zero of the fifth order. One therefore recovers the result for the ordinary magnetic field generated by the ordinary electric charge, namely,

$$B(R, m = 0) = \frac{4R^2\pi\varrho\omega}{15c}. \tag{10}$$

Let us now turn to the experimental aspect of the problem.

The basic idea to test experimentally the existence of the hypermagnetic field would be to place a test body, which possesses a hypermagnetic moment in the hypermagnetic field, produced by the massive rotating baryon body, and to observe the precession of the test body. If the strength of the interaction between two hypercharges Q and q is parametrized in the form

$$Qq = GMm\alpha \tag{11}$$

where M and m are baryon masses of the large and the test bodies, and α was estimated by Fischbach and collaborators to be of the order of 10^{-2}, then the precession of the test body would be

$$\Omega_y = \frac{B(R)}{2c}. \tag{12}$$

For $x \ll 1$ we obtain

$$\Omega_y \approx \frac{1}{10} \frac{GM\omega\alpha}{c^2 R}, \tag{13}$$

and for $x \gg 1$ we have

$$\Omega_y \approx \frac{3}{2} \frac{GM\omega}{c^2 R} \alpha \left(\frac{\lambda}{R}\right)^2. \tag{14}$$

Here λ is the range of the fifth force, and it was estimated by Fischbach to be of the order of 2×10^4 cm. We see that large $GM\omega/R$ factor is necessary if the effect is to be observable. For typical laboratory experiments $GM\omega/R$ is of the order of 10^{-1} which would produce precession of the test gyroscope of the order of 10^{-20} revolutions per sec. Even if we use a hypermagnetic field produced by proton which rotates with very high speed around its axis, the precession which it would produce on a nearby baryon would be of the order of 10^{-20} revolutions per sec.

On the other hand the precession of the test gyroscope in the hypermagnetic field caused by the rotating Earth would be reduced by the factor $(\lambda/R)^2$ where R is the radius of the Earth. In this case we would obtain the precession of the order of 10^{-27} revolutions per sec. One can expect the largest precession of the test gyroscope to be on the North pole of the neutron stars. For a typical neutron star $M = 10^{34}$ g, $R = 10^6$ cm, $\omega = 10^4$ s^{-1} and we obtain that the precession of the test body would be of the order of $\Omega_y \approx 10^{-1}$ revolutions per sec. This value is observable, however the experiments are not practical.

Here it should be mentioned that the precession of the test gyroscope would be produced also by the effect which is known in general relativity as the Lense-Thirring effect[5]. When split into space plus time the general relativistic gravitational field separates into three parts: (1) an electric-like part g^{00} which reduces to the Newtonian acceleration for weak fields, (2) a spatial metric g^{ij} related to the

»curvature of space«, and (3) g^{oj} whose curl for weak gravity is the gravitomagnetic field H_{GM}. This term represents the dragging of inertial frames by rotating massive bodies and was initially analysed by Lense and Thirring. These findings support the Mach's ideas of inertia, which were incorporated by Einstein into his theory of general relativity under the name »Mach's principle«. According to this principle the gravitational theory should be so constructed that the same results are obtained from assuming that a certain system is rotating within the fixed Universe or that a Universe is rotating around fixed system. In other words only the masses and relative motions of all the bodies in the Universe are important and no absolute entities enter any equation.

Thus one expects that the field near rotating massive body has Coriolis and centrifugal features, that is the plane of the Foucault pendutum is expected to be dragged around. It was shown by Shiff[6] that such a gravitomagnetic field would also produce a precession Ω_{GM} of a gyroscope. The precession would be

$$\Omega_{GM} = \frac{H_{GM}}{2c} = \frac{4GM\omega}{5c^2R}. \tag{15}$$

Therefore for $x \ll 1$ the ratio of the hypermagnetic and gravitomagnetic field would be

$$\frac{\Omega_y}{\Omega_{GM}} \approx \frac{a}{8} \approx 10^{-3}, \tag{16}$$

while for $x \gg 1$ the ratio would be

$$\frac{\Omega_y}{\Omega_{GM}} \approx \frac{15}{8}\, a\,(\lambda/R)^2 \approx 2 \cdot 10^{-11}. \tag{17}$$

We see that the effect of the hypermagnetic field in both cases is smaller than the effect of gravitomagnetic field. Therefore even if one finds a method to detect a very minute precession of the test gyroscope the precision should be 1 part in 10^3 for the $x \ll 1$ case or 1 part in 10^{11} for the $x \gg 1$ case. With present day techniques there is little hope that hypermagnetic field could be detected directly in the near future.

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Diskutirana je mogućnost detekcije pete sile preko hipermagnetskih momenata koje proizvodi rotirajući hipernaboj. Efekti su izračunati za slučaj rotirajućih tijela laboratorijskih dimenzija kao i za slučaj rotacije zemlje. U oba slučaja izračunati efekti su manji od efekata Lense-Thirringa.