#### LETTER TO THE EDITOR

# EFFECTIVE MICROWAVE CONDUCTIVITY AND MAGNETORESISTANCE IN THE MIXED STATE OF TYPE-II SUPERCONDUCTORS

ANTONIJE DULČIĆ\*, \*\* AND MIROSLAV POŽEK\*

\*Ruder Bošković Institute, University of Zagreb, POB 1016, 41001 Zagreb, Croatia \*\*Department of Physics, Faculty of Science, University of Zagreb, POB 162, 41001

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#### Original scientific paper

An effective microwave conductivity is derived for the case of type-II superconductors in the mixed state. The ensuing expression for the microwave magnetoresistance predicts a nonlinear behaviour in agreement with the experiments.

Measurements of the magnetoresistance at microwave frequencies can be very important for the study of type-II superconductors, because the effects of pinning, which usually obscure dc and low frequency measurements, become negligible in the response to microwaves, so that intrinsic properties can be revealed even in hard superconductors. For a theoretical formulation of the microwave absorption it is essential to identify the loss mechanisms, and derive an effective microwave conductivity, which, in its turn, can be used in the expression for the surface resistance

Both, classical and high- $T_c$  superconductors have been extensively studied by magnetic field dependent microwave absorption<sup>1-16</sup>). At low fields, one could observe features characteristic of weak links<sup>1-6</sup>). These phenomena were explained by the loss mechanism which occurs in Josephson junctions driven by the microwave current<sup>4-6</sup>). In order to study bulk superconducting properties in the mixed state, magnetoresistance measurements had to be extended to higher field values. However, controversial conclusions were reached by various authors about the microwave loss mechanism and its dependence on the applied magnetic field. Buluggiu et al.<sup>7</sup>) considered the phase slippage mechanism, originally proposed by Tinkham<sup>8</sup>) for

dc resistivity in an applied magnetic field. The predictions of their model could mimic the temperature dependence of modulated absorption signals at a given field. However, Zuo et al.<sup>9)</sup> showed that the field dependence at various temperatures was inconsistent with that mechanism. Fastampa et al.<sup>10)</sup> and Marcon et al.<sup>11)</sup> studied experimentally the microwave magnetoresistance in the mixed state of YBa<sub>2</sub>Cu<sub>3</sub>0<sub>7- $\delta$ </sub> and Bi<sub>2</sub>Sr<sub>2</sub>Ca<sub>2</sub>Cu<sub>3</sub>O<sub>10</sub>. Their results showed that, close to  $T_c$  microwave magnetoresistance was nonlinear. As the temperature was reduced below  $T_c$ , the initial slope of the experimental curves first increased and then decreased. These autors used the model of microwave driven viscous oscillations of vortices in a pinning potential, originally introduced by Gittleman and Rosenblum<sup>12,13)</sup> and later developed by Portis et al.<sup>2)</sup>, leading to a field dependent surface impedance. This model, however, could not explain the salient features of the experimental curves without recurring to additional assumptions, e.g. significant role of fluctuation effects. Nonlinear microwave magnetoresistance was observed also in classical superconductors<sup>14-16)</sup>, but not studied theoretically.

The aim of this paper is to review the microwave electrodynamics in the mixed state, and find an effective conductivity which yields correct field dependences of the surface resistance at various temperatures.

When an electromagnetic field impinges on the surface of a good conductor, or superconductor, curent is induced on the surface, within a penetration depth determined by the conductivity  $\sigma = \sigma_1 - i\sigma_2$ . The surface impedance is given by

$$Z_s = \sqrt{i \frac{\mu \omega}{\sigma}} \tag{1}$$

where  $\omega$  is the angular frequency of the electromagnetic wave, and  $\mu$  is the magnetic permeability. We are interested here in the surface resistance  $R_s = \text{Re}(Z_s)$ . When a superconductor is cooled in a vanishing dc external magnetic field, the microwave penetration depth is reduced from the skin depth  $\delta$  of the normal metal above  $T_c$ , to the London penetration depth  $\lambda_{\rm L}$ . Also, the surface resistance  $R_s(T)$  is reduced from its value in the normal state  $R_s(T_c) = R_{sn}$ , and the ratio  $R_s(T)/R_{sn}$  is usually plotted as an experimentally determined curve. If surface reactance  $X_s = \text{Im}(Z_s)$  is also measured, one can infer the temperature dependences of  $\sigma_1$  and  $\sigma_2$ . The peak in  $\sigma_1$ , observed below  $T_c^{-17,18}$  was interpreted to be due to the coherence factors, as predicted in the BCS theory. The above treatment and analysis involves the Meissner state.

We want to consider in this paper what happens when an external magnetic field is applied, and the superconductor brought into the mixed state. The response of the superconductor to the electromagnetic field should be determined by an effective conductivity which relates the induced microwave current and electric field. In order to find this conductivity, we shall use the well known concepts of flux flow<sup>19)</sup> and adapt them to the microwave case. Let us first consider for a moment that the vortices are fixed by a pinning potential. The current density J and the field E in the superconductor are related through the expression

$$J = [(1-b)\sigma + b\sigma_n]E \tag{2}$$

where  $\sigma$  is still the conductivity which holds for the Meissner state, and  $b = B/B_{c2}$  is the reduced field which stands for the fraction of the sample volume taken by the vortex cores containing normal electrons.  $\sigma_n$  is the normal state conductivity at a given temperature. In the case of a dc current, the field E vanishes since the imaginary part of  $\sigma$  becomes infinite, i.e. the whole current is carried by the superconducting electrons outside of the vortex cores. For ac currents, however, the inertia of the superconducting electrons causes a finite  $\sigma$  and a nonzero electric field E. The electric current is partly carried by both, the superconducting and the normal electrons, as if they acted in parallel.

Next, we can treat the case of vortices which are allowed to move due to the Lorentz force  $\vec{f}_{\rm L} = \vec{J} \times \vec{F}$  per unit length of a flux line. The effective electric field in the superconductor becomes

$$E_{\text{eff}} = E + b(E_c - E) \frac{v}{v_{\text{max}}}$$
(3)

where E is the electric field throughout the sample given by Eq. (2), and  $(E_c - E)(v/v_{\rm max})$  is an additional field which develops in the vortex core when the vortex is in motion at velocity v. One may remark that Eq. (3) holds for both, dc and ac cases. In the familiar case of dc flux flow, the field E vanishes, and  $v = v_{\rm max}$ , so that the electric field in the core is  $E_c = J/\sigma_n$ , and the effective field in the sample becomes the well known flux flow field  $E_{\varphi} = bE_c = (b/\sigma_n)J^{19}$ . For the case of microwaves, the ratio  $v/v_{\rm max}$  can be determined by solving the differential equation of motion for flux lines<sup>12,13</sup>)

$$\frac{v}{v_{\text{max}}} = \frac{1 + i\left(\omega_0/\omega\right)}{1 + (\omega_0/\omega)^2} \tag{4}$$

where  $\omega_0$  is the depinning frequency. The effective conductivity  $\sigma_{\text{eff}}$  is obtained from Eqs. (2) – (4)

$$\frac{1}{\sigma_{\text{eff}}} = \frac{1 - b(v/v_{\text{max}})}{(1 - b)\sigma + b\sigma_n} + \frac{b}{\sigma_n} \frac{v}{v_{\text{max}}}$$
 (5)

This effective conductivity determines the penetration of the microwave field and current into the superconductor, and the losses which occur.

At this point one has to consider the relative geometry of vortices and current. If the vortices are parallel to the microwave current, the Lorentz force vanishes, and no motion is driven. As already explained above, the effective field in Eq. (3) is then just the field E, and the effective conductivity is given by Eq. (2). Next, we may consider the perpendicular cases. Vortices can be perpendicular to the microwave current in two different configurations, parallel and perpendicular to the sample surface. In the former case, the motion of vortices can be envisaged as compressional waves from the surface into the bulk<sup>20</sup>, while the latter case yields vortex tilt waves within the effective penetration depth<sup>21</sup>. In the flux flow regime, achieved at microwave frequencies ( $\omega \gg \omega_0$ ), there can be no difference between

these two cases. Namely, the motion of an infinitesimal segment of a vortex line depends only on the local current density and viscosity<sup>21)</sup>.

The effective conductivity in Eq. (5) satisfies three limiting conditions. For b=0, one obtains  $\sigma_{\rm eff}=\sigma$ , as required for a superconductor in the Meissner state. For full flux penetration b=1, the sample becomes normal, and one finds  $\sigma_{\rm eff}=\sigma_n$ . Also for  $T=T_c$ , one has to set  $\sigma=\sigma_n$ , so that  $\sigma_{\rm eff}=\sigma_n$ , as expected. Note that the conductivity in Eq. (2) also satisfies these limiting conditions.

The temperature dependence of  $\sigma_{\rm eff}$  is determined by the temperature dependence of  $\sigma$ , while the field dependence is given explicitly in Eq. (5). One should note that b is determined through the flux density B in the superconductor, and not by the external field H. Only for  $H \gg H_{c1}$  one may take that  $B \approx \mu_0 H$ . The effective microwave conductivity given by Eq. (5) is the central result of this paper.

In what follows, we shall provide some experimental evidence for the correctness of Eq. (5). It will be tested through the field dependence of the surface resistance  $R_s$ , which results when  $\sigma_{\text{eff}}$  is substituted for  $\sigma$  in Eq. (1), and the real part evaluated.

We have carried out microwave absorption measurements on a sintered ceramic sample of YBa<sub>2</sub>Cu<sub>3</sub>0<sub>7- $\delta$ </sub>. The sample was placed in the center of a reflection type microwave cavity operating in  $TE_{102}$  mode. Operating frequency was 9.4 GHz. The coupling of the cavity was tuned so that the reflected microwave power was zero when the sample was just above  $T_c$ . When the sample was cooled below  $T_c$ , we could detect a reflected power due to the change of the surface resistance of the sample. Similarly, application of an external dc field changes the surface resistance, and consequently changes the reflected microwave power. We used Bruker microwave bridge model ER 046 MRP, which allows pulsing of the microwave power, so that an external lock-in amplifier could be used to measure the output current of the microwave detector. This method improves significantly the signal to noise ratio of the measured reflected microwave power. Figure 1. shows the set of experimental curves. Both, temperature and field dependences are given in the same scale. At very low fields (less than 2 mT), there occurs a pronounced field dependence due to intergranular weak links<sup>10,22)</sup> but we have omitted it in Fig. 1, since we are interested here only in the intragranular bulk behaviour. Moreover, we wish to retain only the region where  $H \gg H_{c1}$ , so that the approximation  $B \approx \mu_0 H$  can be used. Therefore, the curves in Fig. 1. are cut off below 0.04 T. One can notice the nonlinearity of the magnetoresistance curves near  $T_c$ , and an almost linear behaviour at lower temperatures. Also, the peculiar variation of the initial slopes with temperature is found as previously  $^{11}$ .

For the purpose of testing the field dependences of our theoretical expression for  $\sigma_{\rm eff}$ , one needs the values of  $\sigma_1$  and  $\sigma_2$  at various temperatures. These quantities can be uniquely determined from the zero field values of  $R_s$  and  $X_s$ .

We have extrapolated the curves in Fig. 1 to zero field. Also  $X_s$  was measured

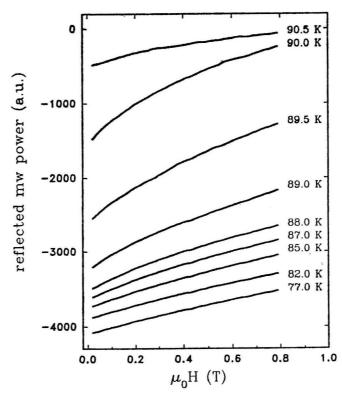


Fig. 1. Experimentally obtained temperature and field dependences of the microwave absorption in ceramic superconductor  $YBa_2Cu_3O_{7-\delta}$ .

by the frequency shift method<sup>17)</sup>. Detailed analysis of these quantities will be the subject of another paper. With the zero field parameters fixed, and assuming the flux flow limit  $(\omega \gg \omega_0)^{13}$ , we could proceed to fit the theoretical expression for  $R_s(T,b)/R_{sn}$  to each of the magnetoresistance curves in Fig. 1. The only fitting parameter was then  $B_{c2}$  at a given temperature. The resulting theoretical curves are shown in Fig. 2. All features of the experimental curves are remarkably well reproduced. The theoretical and experimental curves overlap at all temperatures.

Finaly, one may find it interesting to compare the field dependence of  $R_s$  predicted by our effective conductivity with those found by previous authors. The effective resistivity in the work of Portis et al.<sup>2)</sup>, and other authors<sup>11,20,21)</sup> contains two terms, one of which is the same as the second term on the right-hand side of our Eq. (5) in the flux flow limit ( $\omega \gg \omega_0$ ). However, for the other term in the effective resistivity, those authors use an expression which is proportional to the square of the London penetration depth  $\lambda_{\rm L}$ , and contains no field dependence. In our notation, they would have  $(1/\sigma_{\rm eff})_p = \left[1/(-{\rm i}\sigma_2) + b/\sigma_n\right]$ . This expression fails at

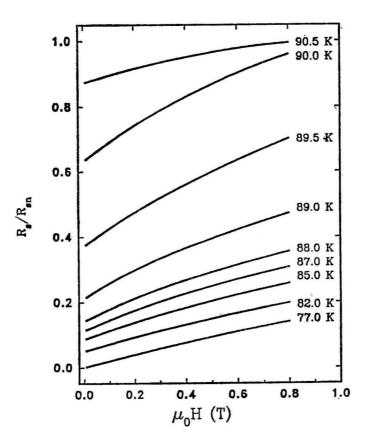


Fig. 2. Theoretical curves for the normalized surface resistance calculated using the effective conductivity of Eq. (5) with variable  $B_{c2}$  in order to find the best fit to the corresponding experimental curves in Fig. 1. The fitting parameters  $B_{c2}$  for the curves from top to bottom are (in Tesla): 1.0, 1.1, 2.2, 4.4, 7.2, 8.9, 11.4, 14.9 and 18.5.

b=0 where the effective conductivity should reduce to the Meissner conductivity  $\sigma=\sigma_1-\mathrm{i}\sigma_2$ . It fails also for b1 where the effective conductivity should become equal to  $\sigma_n$ . One may also consider the revised treatment of Coffey and Clem<sup>23</sup>, which takes into account the effects of the normal electrons through the two fluid concept. For the flux flow regime, the effective conductivity of Coffey and Clem can be expressed, in our notation, as  $(1/\sigma_{\mathrm{eff}})_{CC} = \left[1-\mathrm{i}\sigma_2(1-b)b/\sigma_n\right]/\left[(1-b)\sigma+b\sigma_n\right]$ . The ensuing expression for  $R_s(T,b)/R_{sn}$  is compared in Fig. 3 to the experimental magnetoresistance curve at T=89.0 K.

In conclusion, we have derived an effective conductivity for the response of type-II superconductors in the mixed state to microwave irradiation. The ensuing surface resistance shows nonlinear field dependence, in agreement with the experiments.

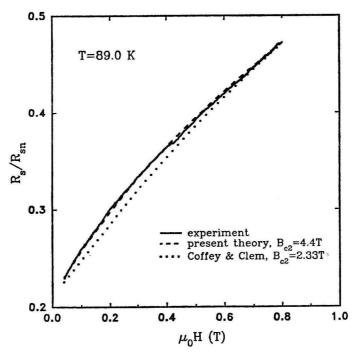


Fig. 3. Comparison of the experimental curve of  $R_s/R_{sn}$  at T=89.0 K with the theoretical forms of  $R_s/R_{sn}$  using effective conductivities: a)  $1/\sigma_{\rm eff}$  of Eq. (5) with  $\omega\gg\omega_0$  (dashed line,  $B_{c2}=4.4$  T), b)  $(1/\sigma_{\rm eff})_{CC}$  as explained in text (dotted line,  $B_{c2}=2.33$  T).

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## EFEKTIVNA MIKROVALNA VODLJIVOST I MAGNETOOTPOR U MIJEŠANOM STANJU SUPRAVODIČA DRUGE VRSTE

#### ANTONIJE DULČIĆ\*, \*\* i MIROSLAV POZEK\*

\*Institut Ruđer Bošković, Sveučilište u Zagrebu, POB 1016, 41001 Zagreb, Hrvatska

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Izveden je izraz za efektivnu mikrovalnu vodljivost u slučaju miješanog stanja supravodiča druge vrste. Dobiveni izrazi za mikrovalni magnetootpor predviđaju nelinearno ponašanje u skladu s eksperimentalnim opažanjima.

<sup>\*\*</sup> Department of Physics, Faculty of Science, University of Zagreb, POB 162, 41001, Hrvatska