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ON THE FORMATION OF SOLITONS IN AN ISOTHERMAL RELATIVISTIC PLASMA WITH POSITIVE AND NEGATIVE IONS

SUDESHNA CHAKRABORTY, SAILENDRA N. PAUL and ASESH R. CHOWDHURY

High Energy Physics Division, Department of Physics, Jadavpur University, Calcutta 700032, India

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We have studied the phenomenon of solitary wave formation in a relativistic plasma having both positive and negative ions. The state of the plasma is assumed to be isothermal and ions are considered to be warm. The phase velocity, width and amplitude of the soliton are explicitly obtained as functions of σ_α, σ_β, $n_{\beta 0}/n_{\alpha 0}$, $u_{\alpha 0}/c$, $u_{\beta 0}/c$ and Q, where σ_α and σ_β are the temperatures of the two kinds of ions, $n_{\alpha 0}$ and $n_{\beta 0}$ are the respective equilibrium densities, $u_{\alpha 0}$ and $u_{\beta 0}$ are the corresponding streaming velocities and Q is the mass ratio of the negative and positive ions.

1. *Introduction*

Study of solitary waves in various types of plasma constitutes an important aspect of modern plasma theory. Washimi and Taniuti [1] were the first to investigate ion-acoustic solitary waves in a cold, collisionless, unmagnetised plasma through the derivation of Kortweg-deVries (K-dV) equation using the reductive perturbation method. Subsequently many authors incorporated different parameters, e.g., temperature of ions [2], non-isothermality [3], ion-beam [4], two-temperature electrons [5], density gradient [6], temperature gradient [7], Landau damping [8], magnetic field [9] etc. for theoretical analysis of solitary waves in plasmas. It is found that each of the above parameters has important role on the modification of the structure of the solitons. However, the solitary waves are found to have interesting characteristics in a plasma in the presence of negative ions together with positive ions [10-12]. If negative ions are present in a plasma as a contamination, rare-factive and negative potential solitons may exist there [13-15].

The works of the above authors on the ion-acoustic soliton are no doubt very important in different physical situations in plasmas. But in those cases the plasma was non-relativistic. In relativistic plasma, mass corrections of the plasma species due to relativistic effect are found to have significant contribution on the ion-acoustic solitons. Das and Paul [16] have shown that relativistic effect is important on the formation of solitons in a plasma only in the presence of streaming ions. Nejoh [17, 18] have introduced the temperature effect and Das et al. [19] assumed the non-isothermality in case of relativistic ion-acoustic solitary waves. Subsequently, Roy Chowdhury et al. [20] have derived higher order evolution equation for nonlinear waves in a relativistic plasma having cold ions and two-temperature electrons and have found periodic cnoidal wave like solution of it. Roy Chowdhury et al. [21] have also investigated the propagation of ion-acoustic soliton in an ion-beam relativistic plasma system. They showed that the width and phase velocity of the solitary wave are very much influenced by the relativistic effect as well as due to ion beam in the plasma. Later Salauddin [22] and other authors [23, 24] have studied the propagation of solitary waves in relativistic plasma in the presence of magnetic field collision between ions and electron etc. and have obtained interesting results. But recently, both the negative ions and the relativistic effect have been considered by Das et al. [25] and Chakraborty et al. [26, 27] for the study of ion-acoustic solitons in a plasma. The results of these authors are much more interesting than that of earlier authors [10-15]. However, the effect of temperature of ion in a relativistic plasma having negative ions has not yet been considered by any of the previous workers, altough the temperature effect is found to be very important for the study of ion-acoustic solitons in plasma. We are therefore here motivated to study the propagation of ion-acoustic solitary waves in a relativistic plasma with warm positive and negative ions having drift motions. In fact, we have investigated the role of ionic temperature together with negative ion-concentrations on the formation of ion-acoustic solitons in a relativistic plasma. In Section 2 we have derived the KdV equation using the reductive perturbation method. Then in Section 3, we have found out the width and amplitude of the solitons. The effects of negative ion-concentrations and temperature on the phase velocity, width and amplitude of the solitary wave have been shown in tabular form and also discussed graphically. In Section 4, some critical comments are given to analyse the results and study further on the present topic.

2. Formulation

Let us consider a plasma consisting of electrons and two types of ions, i.e. positive ions and negative ions. The plasma is collisionless, unmagnetized and isothermal. Moreover, the ions are hot and have uniform stream velocities of the relativistic order. So the basic equation governing the plasma dynamics in unidirectional propagation can therefore be written, using the usual hydrodynamic description of the plasma, as:

$$\begin{aligned} &\frac{\partial n_\alpha}{\partial t} + \frac{\partial}{\partial x}(n_\alpha u_\alpha) = 0 \\ &\frac{\partial \overline{u}_\alpha}{\partial t} + u_\alpha \frac{\partial \overline{u}_\alpha}{\partial x} + \frac{\sigma_\alpha}{n_\alpha}\frac{\partial p_\alpha}{\partial x} = -\frac{\partial \phi}{\partial x} \\ &\frac{\partial p_\alpha}{\partial t} + u_\alpha \frac{\partial p_\alpha}{\partial x} + 3p_\alpha \frac{\partial \overline{u}_\alpha}{\partial x} = 0 \end{aligned} \tag{1}$$

for the positive ions, and

$$\begin{aligned} &\frac{\partial n_\beta}{\partial t} + \frac{\partial}{\partial x}(n_\beta u_\beta) = 0 \\ &\frac{\partial \overline{u}_\beta}{\partial t} + u_\beta \frac{\partial \overline{u}_\beta}{\partial x} + \frac{\sigma_\beta}{n_\beta}\frac{\partial p_\beta}{\partial x} = \frac{Z}{Q}\frac{\partial \phi}{\partial x} \\ &\frac{\partial p_\beta}{\partial t} + u_\beta \frac{\partial p_\beta}{\partial x} + 3p_\beta \frac{\partial \overline{u}_\beta}{\partial x} = 0 \end{aligned} \tag{2}$$

for the negative ions, where Z is the charge, Q is the mass ratio of negative ions and positive ions, n_α and n_β are the densities of the two types of ions, u_α and u_β are their velocities, p_α and p_β denote corresponding pressure terms, σ_α and σ_β are temperatures of the ions and $\overline{u}_\alpha$ and $\overline{u}_\beta$ are given by

$$\overline{u}_\alpha \approx u_\alpha \left(1 + \frac{u_\alpha^2}{2c^2}\right)$$

and

$$\overline{u}_\beta \approx u_\beta \left(1 + \frac{u_\beta^2}{2c^2}\right)$$

where c is the velocity of light.

The electrostatic potential ϕ satisfies the equation

$$\frac{\partial^2 \phi}{\partial x^2} = n_e + Z n_\beta - n_\alpha \tag{3}$$

where $n_e = \exp(\phi)$.

In our above equations we have normalised the velocities by the characteristic velocity $\sqrt{kT_e/m}$, all the densities by the equilibrium value n_0, and lenght by the Debye lenght $\sqrt{kT_e/4\pi e^2 n_0}$ whereas the potential is normalised to kT_e/e, so that the equations appear in totally dimensionless form. Now, to derive the KdV equation we use the reductive perturbative method following the works of Washimi and Taniuti [1]. Here x is scaled in such a way that it is the dispersion relation which gives the information about the scaling of the time. $\zeta = \epsilon^p(x - vt)$ and $\eta = \epsilon^{3p}t$ yields a natural scaling for x and t. The new variables ζ and η are long in the sense that it needs a large change in x and t in order to obtain appreciable change in ζ and η. The value of p will be different for different physical situations. In our case of isothermal plasma, $p = 1/2$, i.e. the new stretched coordinates are

$$\zeta = \epsilon^{1/2}(x - vt)$$

$$\eta = \epsilon^{3/2}t \tag{4}$$

where v is the unknown phase velocity to be determined later and ϵ is the expansion parameter.

We now assume that the physical parameters $n_{\alpha,\beta}$, $u_{\alpha,\beta}$, $p_{\alpha,\beta}$ and ϕ are in the following perturbation form,

$$n_\alpha = n_{\alpha 0} + \epsilon n_{\alpha 1} + \epsilon^2 n_{\alpha 2} + ...$$

$$n_\beta = n_{\beta 0} + \epsilon n_{\beta 1} + \epsilon^2 n_{\beta 2} + ...$$

$$u_\alpha = u_{\alpha 0} + \epsilon u_{\alpha 1} + \epsilon^2 u_{\alpha 2} + ...$$

$$u_\beta = u_{\beta 0} + \epsilon u_{\beta 1} + \epsilon^2 u_{\beta 2} + ...$$

$$p_\alpha = 1 + \epsilon p_{\alpha 1} + \epsilon^2 p_{\alpha 2} + ... \tag{5}$$

$$p_\beta = 1 + \epsilon p_{\beta 1} + \epsilon^2 p_{\beta 2} + ...$$

$$\phi = \phi_0 + \epsilon \phi_1 + \epsilon^2 \phi_2 + ...$$

$$(\phi_0 = 0)$$

where the first terms in the right hand side of (5) represent the equilibrium values of the respective parameters, second terms, third terms etc. represent the first order, second order etc. quantities, respectively.

Furthermore, we assume that the basic equations are supplemented by the following boundary conditions as $|x| \to \infty$

$$
\begin{aligned}
&(i) \quad n_e \to 1, \quad \phi \to 0 \\
&(ii) \quad n_\alpha \to n_{\alpha 0}, \quad n_\beta \to n_{\beta 0} \\
&(iii) \quad u_\alpha \to u_{\alpha 0}, \quad u_\beta \to u_{\beta 0} \\
&(iv) \quad p_\alpha \to 1, \quad p_\beta \to 1
\end{aligned}
\tag{6}
$$

and the charge neutrality condition of the plasma is

$$n_{\alpha 0} = 1 + Z n_{\beta 0}. \tag{7}$$

Now using Eqs. (4)-(6) in the basic equations (1)-(3) and then collecting the terms that are first order in ϵ, we obtain:

$$
\begin{aligned}
p_{\alpha 1} &= \frac{3\gamma_\alpha}{u_{\alpha 0} - v} u_{\alpha 1} \\
p_{\beta 1} &= \frac{3\gamma_\beta}{u_{\beta 0} - v} u_{\beta 1} \\
\phi_1 &= n_{\alpha 1} - n_{\beta 1} \\
n_{\alpha 1} &= \frac{n_{\alpha 0}}{v - u_{\alpha 0}} u_{\alpha 1} \\
n_{\beta 1} &= \frac{n_{\beta 0}}{v - u_{\beta 0}} u_{\beta 1} \\
\phi_{\alpha 1} &= (v - u_{\alpha 0})\gamma_\alpha u_{\alpha 1} - \frac{\sigma_\alpha}{n_{\alpha 0}} p_{\alpha 1} \\
\phi_{\beta 1} &= Q(v - u_{\beta 0})\gamma_\beta u_{\beta 1} - \frac{\sigma_\beta}{n_{\beta 0}} p_{\beta 1}
\end{aligned}
\tag{8}
$$

where $\gamma_\alpha = (1 + 3u_{\alpha 0}^2/2c^2)$, $\gamma_\beta = (1 + 3u_{\beta 0}^2/2c^2)$ and $Z = 1$.

Eliminating $n_{\alpha 1}$, $n_{\beta 1}$, $u_{\alpha 1}$, $u_{\beta 1}$, $p_{\alpha 1}$, $p_{\beta 1}$ and ϕ_1 in Eq. (8) we obtain the following equation for the phase velocity v,

$$\frac{n_{\alpha 0}^2}{\gamma_\alpha}\left[(v-u_{\alpha 0})^2 n_{\alpha 0}-3\sigma_\alpha\right]^{-1}-$$

$$-\frac{1}{Q}\frac{n_{\beta 0}^2}{\gamma_\beta}\left[(v-u_{\beta 0})^2 n_{\beta 0}-3\sigma_\beta\right]^{-1}=1. \tag{9}$$

This phase velocity is completely different from that obtained by previous authors [25-27]. From Eq. (9) it is seen that concentration of negative ions together with their temperature changes the phase velocity of the ion-acoustic wave. Now, proceeding to the second order terms in ϵ, we obtain the KdV equation after the elimination of all the variables other than ϕ_1,

$$\frac{\mathrm{d}\phi_1}{\mathrm{d}\eta}+A_0\phi_1\frac{\mathrm{d}\phi_1}{\mathrm{d}\zeta}+B_0\frac{\mathrm{d}^3\phi_1}{\mathrm{d}\zeta^3}=0, \tag{10}$$

where the coefficients A_0, B_0 are given by:

$$\overline{A}=-1-\frac{6\sigma_\alpha}{b_\alpha M_{\alpha 0}\gamma_\alpha a_\alpha^2}-\frac{9\sigma_\alpha}{b_\alpha M_{\alpha 0}a_\alpha^2}+\frac{3vu_{\alpha 0}}{b_\alpha c^2\gamma_\alpha^2 a_\alpha^2}-$$

$$-\frac{9\sigma_\alpha u_{\alpha 0}(v-u_{\alpha 0})}{b_\alpha M_{\alpha 0}c^2\gamma_\alpha^2 a_\alpha^2}-\frac{2}{b_\alpha\gamma_\alpha a_\alpha(v-u_{\alpha 0})^2}-\left(1+\frac{9u_{\alpha 0}^2}{2c^2}\right)\frac{1}{\gamma_\alpha^2 a_\alpha^2 b_\alpha}+$$

$$+\frac{6\sigma_\beta}{Q^2 b_\beta M_{\beta 0}\gamma_\beta a_\beta^2}+\frac{9\sigma_\beta}{Q^2 b_\beta M_{\beta 0}a_\beta^2}-\frac{3vu_{\beta 0}}{Q^2 b_\beta c^2\gamma_\beta^2 a_\beta^2}+$$

$$+\frac{9\sigma_\beta u_{\beta 0}(v-u_{\beta 0})}{Q^2 b_\beta M_{\beta 0}c^2\gamma_\beta^2 a_\beta^2}+\frac{2}{Q^2 b_\beta\gamma_\beta a_\beta(v-u_{\beta 0})^2}+\left(1+\frac{9u_{\beta 0}^2}{2c^2}\right)\frac{1}{Q^2\gamma_\beta^2 a_\beta^2 b_\beta} \tag{11}$$

$$\overline{B}=\frac{3\sigma_\beta}{Qb_\beta M_{\beta 0}a_\beta}+\frac{1}{Qb_\beta a_\beta}+\frac{1}{Qb_\beta(v-u_{\beta 0})}-$$

$$-\frac{3\sigma_\alpha}{b_\alpha M_{\alpha 0}a_\alpha}-\frac{1}{b_\alpha a_\alpha}-\frac{1}{b_\alpha(v-u_{\alpha 0})} \tag{12}$$

$$M_{\alpha 0}=n_{\alpha 0}(v-u_{\alpha 0})^2$$

$$M_{\beta 0}=n_{\beta 0}(v-u_{\beta 0})^2$$

$$b_\alpha = \frac{1}{n_{\alpha 0}^2}\left[n_{\alpha 0}(v-u_{\alpha 0})^2 - 3\sigma_\alpha\right]\gamma_\alpha$$

$$b_\beta = \frac{1}{n_{\beta 0}^2}\left[n_{\beta 0}(v-u_{\beta 0})^2 - 3\sigma_\beta\right]\gamma_\beta$$

$$a_\alpha = \frac{b_\alpha n_{\alpha 0}}{(v-u_{\alpha 0})\gamma_\alpha}; \quad a_\beta = \frac{b_\beta n_{\beta 0}}{(v-u_{\beta 0})\gamma_\beta} \tag{13}$$

$$A_0 = \frac{\overline{A}}{\overline{\overline{B}}}; \quad B_0 = \frac{1}{\overline{\overline{B}}}.$$

3. Results and discussion

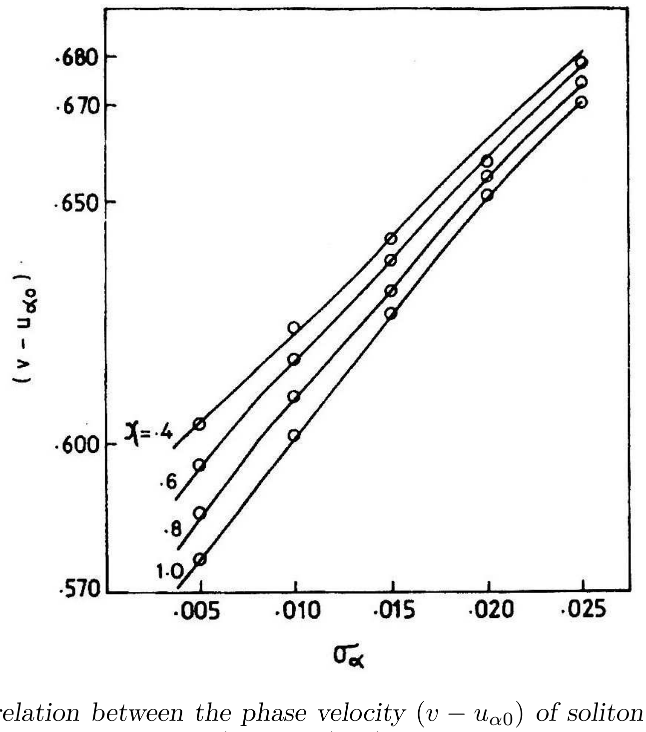

Fig. 1. The relation between the phase velocity $(v-u_{\alpha 0})$ of soliton against ionic temperature for different ratios $(\chi = n_{\beta 0}/n_{\alpha 0})$ of ion concentration: a) $\chi = 0.2/0.5$, b) $\chi = 0.3/0.5$, c) $\chi = 0.4/0.5$, d) $\chi = 0.5/0.5$ using formula (9). Other parameters being fixed at $u_{\alpha 0}/c = 0.5$, $Q = 0.6$.

Equation (10) is the KdV equation governing the motion of the soliton in the relativistic plasma having negative ions. We now search for a solitary wave solution in the form

$$\phi = A\,\mathrm{sech}^2(K\zeta - \omega\eta) \tag{14}$$

whence we get that the amplitude (A) of the wave is $3\omega/KA_0$ and the width (δ) is $2\sqrt{-B_0K/\omega}$ where $\omega/K = 1$. From the solution of the KdV equation it is found that amplitude, width and phase velocity of the solitary wave depend on the negative ion concentration, relativistic effect, the mass ratio and the temperature of the ions. To understand the actual behaviour of these parameters on the ion-acoustic solitons, we have numerically estimated the values of A, δ and $v^{'} = (v - u_{\alpha 0})$ and the results are plotted graphically. Numerical results are furnished in Tables 1 − 4. All the graphical representations are in accordance with the tabular data, presented in this paper.

TABLE 1.

Calculation of phase velocity ($v^{'} = v - u_{\alpha 0}$), width (δ) and amplitude (A) of soliton, respectively, for different ionic temperatures (σ_α) and negative ion concentration ($n_{\beta 0}$). The constant parameters are $u_{\alpha 0}/c = 0.5$, $Q = 6$, $n_{\alpha 0} = 0.5$.

Ionic Temperature (σ_α)	$n_{\beta 0} = 0.2$	$n_{\beta 0} = 0.3$	$n_{\beta 0} = 0.4$	$n_{\beta 0} = 0.5$
0.005	$v^{'} = 0.6047$ $\delta = 1.0605$ $A = 0.9965$	0.5957 1.0485 0.9447	0.5865 1.0367 0.8938	0.5771 1.0284 0.8436
0.01	$v^{'} = 0.6248$ $\delta = 1.0483$ $A = 0.9143$	0.6182 1.0303 0.883	0.6105 1.0163 0.884	0.6025 1.0029 0.8036
0.015	$v^{'} = 0.6418$ $\delta = 1.052$ $A = 0.8145$	0.6394 1.0153 0.822	0.6335 0.998 0.797	0.627 0.9832 0.767
0.02	$v^{'} = 0.6514$ $\delta = 1.1411$ $A = 0.6412$	0.6595 1.0041 0.7612	0.6557 0.9815 0.7547	0.6504 0.9653 0.734

In Figs. 1 − 4 we see that with the increase of the temperature of the ions, the phase velocity of the solitary wave increases. Moreover, with the increase of the mass ratio of negative and positive ions (i.e. Q), the phase velocity of the solitons also increases. Further, we see that the presence of negative ions decreases the phase velocity of the solitary wave. Also the relativistic effect has significant contribution on the phase velocity of the soliton. Here and in all subsequent computations, all quantities are dimensionless due to our normalisation as given before Eq. (4).

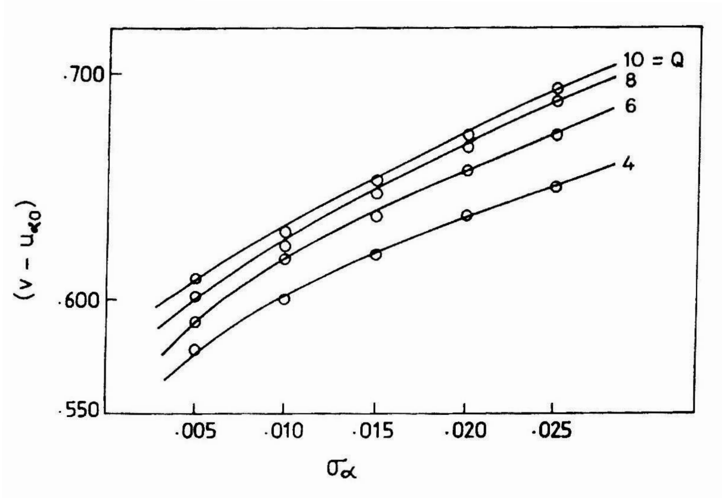

Fig. 2. Ion-temperature-dependence of phase velocity ($v - u_{\alpha 0}$) of soliton for different mass ratio (Q) of negative and positive ions; $u_{\alpha 0}/c$ and χ remaining constant ($u_{\alpha 0}/c = 0.5$ and $\chi = 0.6$).

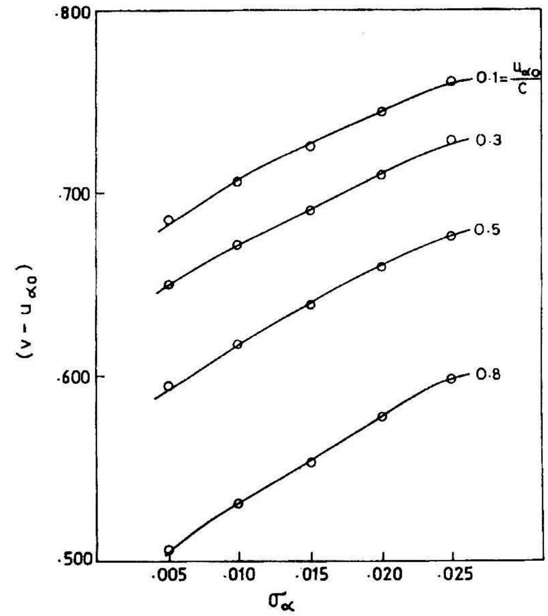

Fig. 3. Phase velocity of solitons as a function of ionic temperature for different streaming velocities:
a) $u_{\alpha 0}/c = 0.1$, b) $u_{\alpha 0}/c = 0.3$, c) $u_{\alpha 0}/c = 0.5$, d) $u_{\alpha 0}/c = 0.8$, where the remaining two parameters are kept fixed, i.e., $Q = 6$, $\chi = 0.6$.

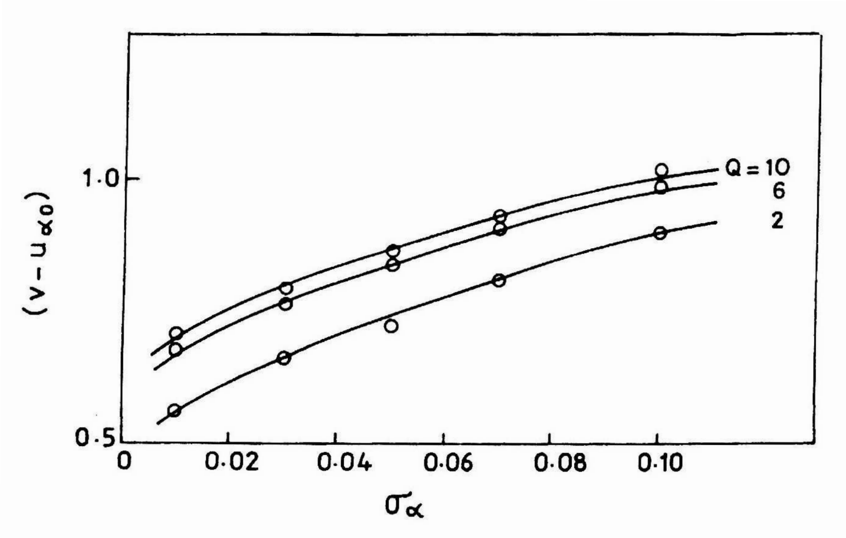

Fig. 4. Effect of ionic temperature (σ_α) on the phase velocity ($v' = v - u_{\alpha 0}$) of soliton for different mass ratios where the velocity of drift motion is kept unaltered ($u_{\alpha 0}/c = 0.1$). In this case $n_{\alpha 0} = n_{\beta 0}$.

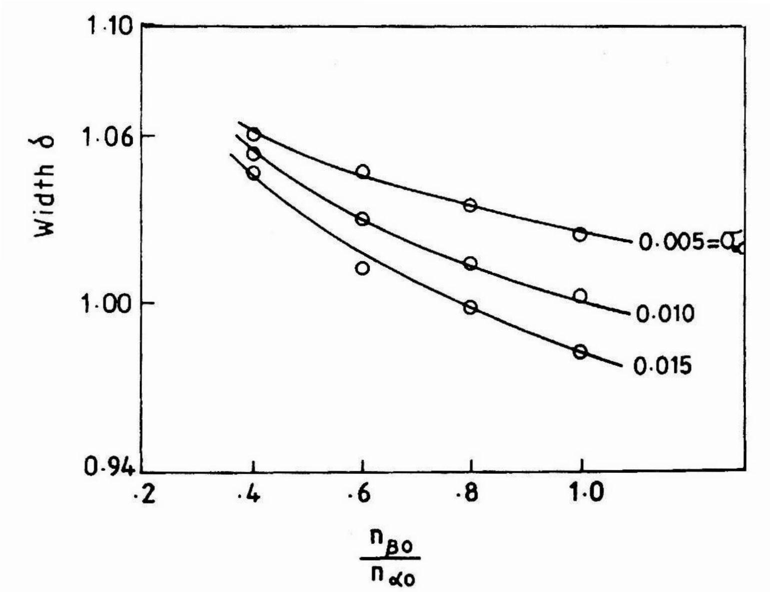

Fig. 5. Ion-temperature dependence of width of soliton for different negative ion concentrations. Other parameters $Q = 6$ and $u_{\alpha 0}/c = 0.5$ being maintained constant.

TABLE 2.
Temperature dependence of phase velocity (v'), width (δ) and amplitude (A) of soliton with the variation of mass ratio (Q). Other parameters are $u_{\alpha 0}/c = 0.5$, $n_{\beta 0} = 0.3$, $n_{\alpha 0} = 0.5$.

Ionic Temperature (σ_α)	$Q = 4$	$Q = 6$	$Q = 8$	$Q = 10$
0.005	$v' = 0.579$ $\delta = 1.0338$ $A = 0.861$	0.5957 1.0485 0.945	0.6038 1.0557 0.99	0.6087 1.06 1.018
0.01	$v' = 0.6006$ $\delta = 1.0172$ $A = 0.794$	0.6182 1.0303 0.883	0.6266 1.037 0.9306	0.6316 1.0410 0.960
0.015	$v' = 0.6206$ $\delta = 1.0055$ $A = 0.726$	0.6394 1.0153 0.822	0.6483 1.0209 0.873	0.6535 1.0245 0.905
0.02	$v' = 0.6388$ $\delta = 1.0015$ $A = 0.654$	0.6595 1.0041 0.761	0.669 1.0077 0.817	0.6745 1.0103 0.852

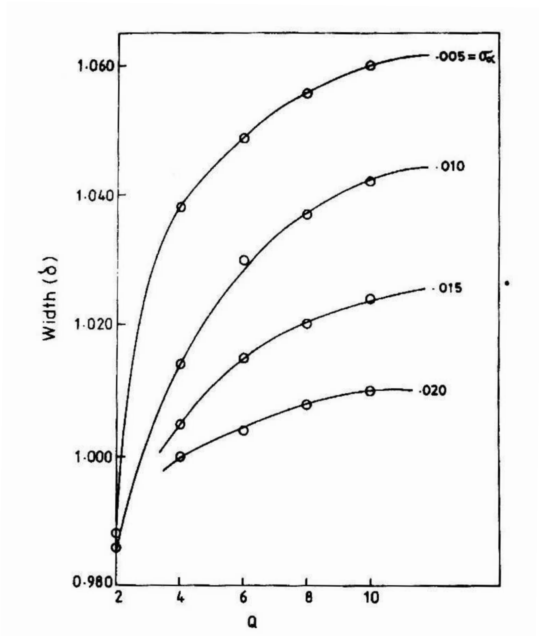

Fig. 6. Variation of width of soliton with the increase of mass ratio and ionic temperature. Fixed values of other parameters are $n_{\alpha 0}/n_{\beta 0} = 0.6$ and $u_{\alpha 0}/c = 0.5$, respectively.

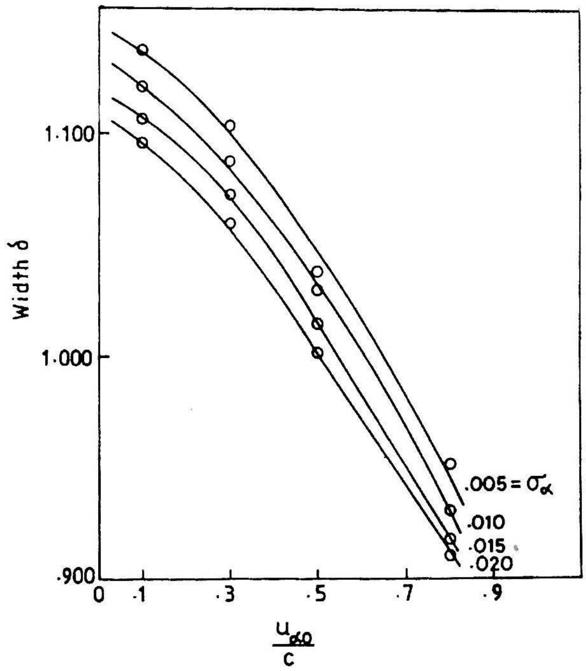

Fig. 7. Width of soliton as a function of ionic temperature for the increase of relativistic effect. In the above variation $Q = 6$ and $\chi = 0.6$.

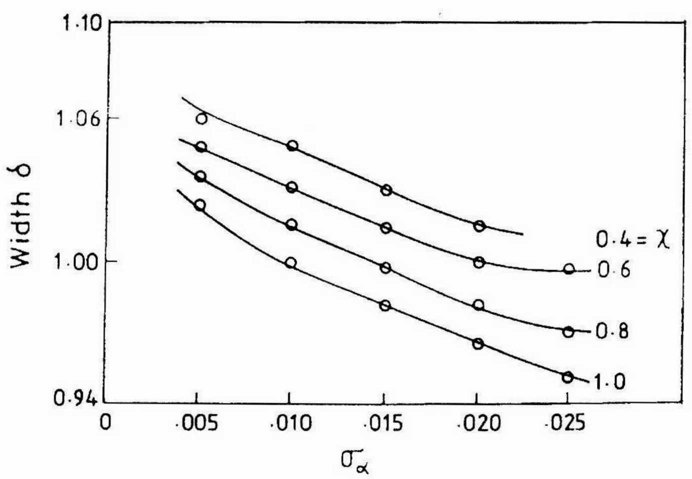

Fig. 8. Effect of ionic temperature on the width of soliton for different negative ion concentration. Mass ratio (Q) and $u_{\alpha 0}/c$ being kept fixed at 6 and 0.5, respectively.

TABLE 3.
Variation of phase velocity, width and amplitude of soliton with respect to ionic temperatures and streaming velocities. Other parameters remaining the same (i.e. $Q = 6$, $n_{\beta 0} = 0.3$, $n_{\alpha 0} = 0.5$).

Ionic Temperature (σ_α)	$u_{\alpha 0}/c = 0.1$	$u_{\alpha 0}/c = 0.3$	$u_{\alpha 0}/c = 0.5$	$u_{\alpha 0}/c = 0.8$
0.005	$v' = 0.6863$ $\delta = 1.1368$ $A = 0.8405$	0.6512 1.1036 0.9125	0.5957 1.0485 0.9447	0.507 0.952 0.9235
0.01	$v' = 0.706$ $\delta = 1.1214$ $A = 0.8018$	0.672 1.087 0.866	0.618 1.0303 0.883	0.533 0.9311 0.825
0.015	$v' = 0.725$ $\delta = 1.1078$ $A = 0.7635$	0.6917 1.073 0.8196	0.6394 1.0153 0.822	0.5568 0.9168 0.7307
0.02	$v' = 0.743$ $\delta = 1.096$ $A = 0.7254$	0.7106 1.0612 0.773	0.659 1.004 0.761	0.578 0.9143 0.632
0.025	$v' = 0.7605$ $\delta = 1.0865$ $A = 0.687$	0.7286 1.0522 0.726	0.678 0.998 0.697	0.596 0.946 0.509

TABLE 4.
Numerical estimation of phase velocity and width of solitary wave for different ionic temperature and mass ratio where the concentrations of both type (positive and negative) of ions are the same, $u_{\alpha 0}/c$ being kept fixed at 0.1.

Ionic Temperature (σ_α)	$Q = 2$	$Q = 4$	$Q = 6$	$Q = 10$
0.01	$v' = 0.5534$ $\delta = 0.9433$	0.6553 1.0619	0.6859 1.0937	0.7095 1.118
0.03	$v' = 0.6520$ $\delta = 0.8683$	0.7412 0.998	0.7684 1.0335	0.7895 1.0596
0.05	$v' = 0.7391$ $\delta = 0.8237$	0.8182 0.9491	0.8428 0.98647	0.8621 1.0138
0.07	$v' = 0.8163$ $\delta = 0.777$	0.8885 0.91204	0.9113 0.94904	0.9291 0.9887
0.1	$v' = 0.9199$ $\delta = 0.7315$	0.9846 0.866	1.005 0.8771	1.0214 0.9319

From Figs. $5 - 7$, the variations of width of the solitary wave with negative ion concentration, temperature, mass ratio etc. are well understood. It is seen that the width of solitons decreases with the increase of negative ions in the plasma. The effect of mass ratio is significant for the width of the soliton. If Q increases, the width δ also increases. But for higher ionic temperatures, the width decreases.

The variations of the width of the solitary wave with the ionic temperatures, negative ion concentration, mass variation of two types of ions as well as relativistic effect may also be understood from Figs. 8 − 11.

The characteristics of the amplitude of the soliton for different mass ratio and negative ion concentration, particulary the effect of ionic temperatures including the relativistic effect, are shown in Figs. 12 − 14. It is found that ionic temperature have significant contribution on the amplitude of solitary wave. It is very important to note that the trends of above figures for phase velocity width, amplitude of the solitons are quite usual and agree with those obtained by earlier authors [26] when both the positive and negative ions are cold i.e. $\sigma_\alpha = \sigma_\beta = 0$ and the situation is isothermal.

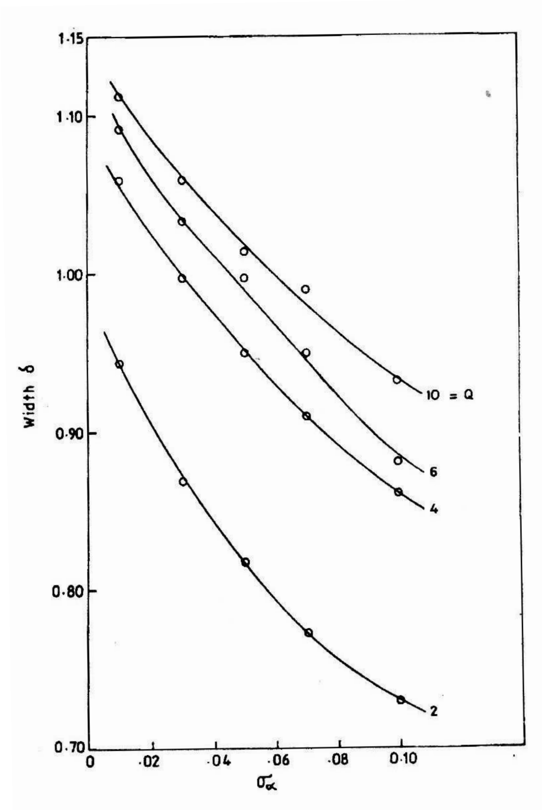

Fig. 9. The ratio between the width (δ) of soliton against ionic temperature (σ_α) for different mass ratio:
a) $Q = 2$, b) $Q = 4$, c) $Q = 6$, d) $Q = 10$. Other parameters being kept unaltered (i.e. $u_{\alpha 0}/c = 0.1$ and $\chi = 1$).

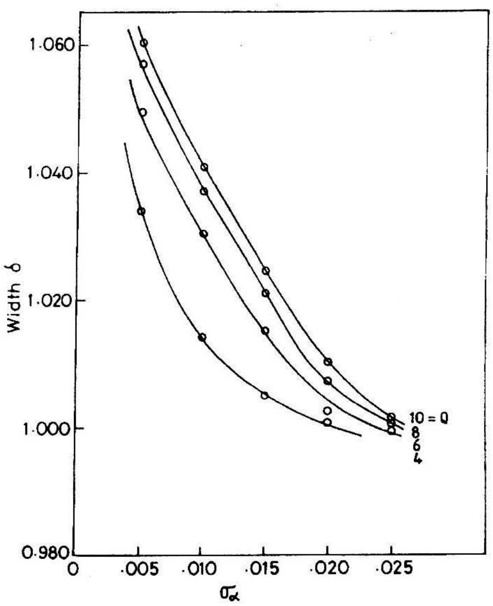

Fig. 10. Width (δ) of soliton as a function of ionic temperature (σ_α) for four different mass ratios (Q) ranging from 4 to 10. Other parameters being kept unaltered (i.e. $u_{\alpha 0}/c = 0.5$ and $\chi = 0.6$).

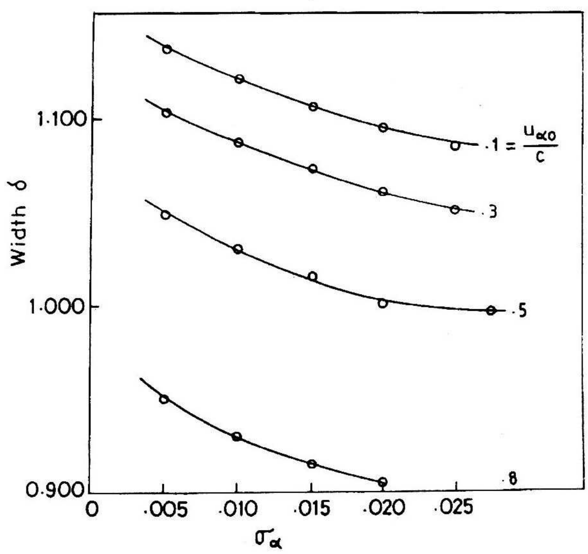

Fig. 11. Effect of ionic temperature (σ_α) on the width (δ) of soliton for different relativistic velocities where mass ratio (Q) and ratio of ion-concentration ($\chi = n_{\beta 0}/n_{\alpha 0}$) are kept fixed at 6 and 0.6, respectively.

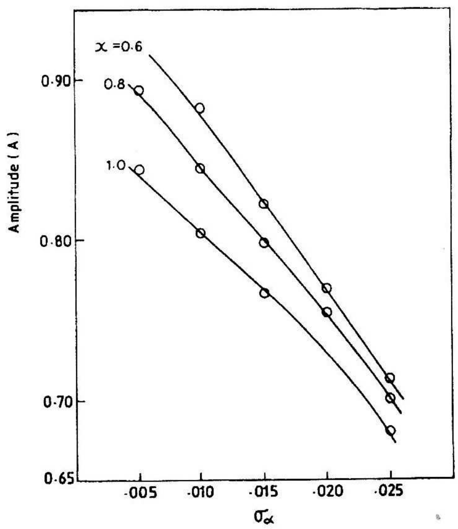

Fig. 12. Variation of amplitude (A) of soliton for the increase of ionic temperature (σ_α) for three different negative ion concentrations: a) $n_{\beta 0} = 0.3$, b) $n_{\beta 0} = 0.4$, c) $n_{\beta 0} = 0.5$. Other parameters being fixed at $Q = 6$ and $u_{\alpha 0}/c = 0.5$.

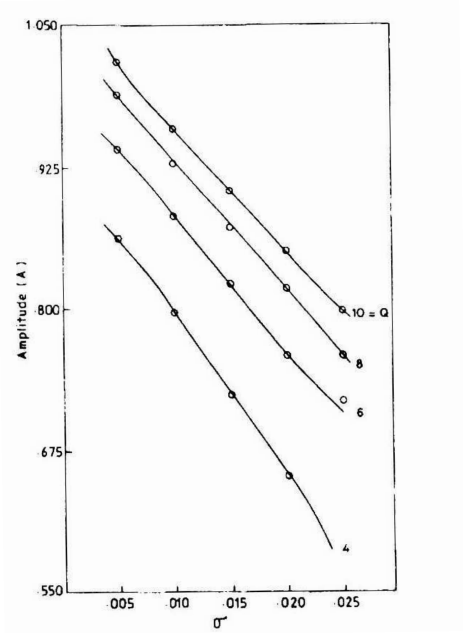

Fig. 13. The relation between the amplitude (A) of soliton against ionic temperature (σ_α) for different mass ratios (Q), where $u_{\alpha 0}/c = 0.5$ and $n_{\beta 0}/n_{\alpha 0} = 0.6$.

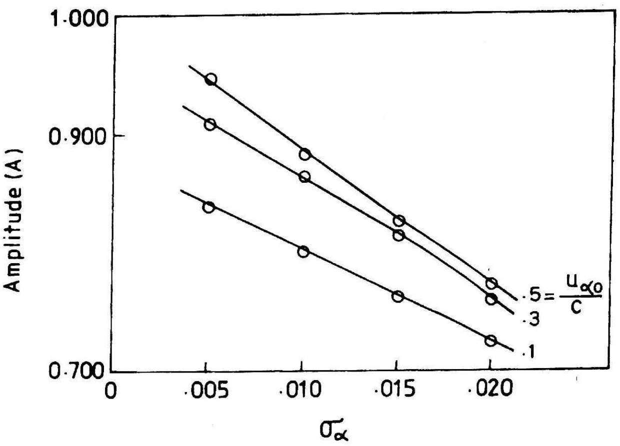

Fig. 14. Effect of ion-temperature (σ_α) on the amplitude (A) of soliton for different relativistic cases:
a) $u_{\alpha 0}/c = 0.1$, b) $u_{\alpha 0}/c = 0.3$, c) $u_{\alpha 0}/c = 0.5$ where other parameters ($Q = 6$, $\chi = 0.6$) remain constant.

4. *Concluding remarks*

In our present investigation we have shown the effect of ionic temperature on the ion acoustic soliton in a relativistic plasma having both positive ions and negative ions. It is to be mentioned that negative ions are produced in the laboratory by electron attachment of neutral particles with an electronegative gas (gas with high electron attachment cross section, e.g. halogens, hexa flouride (SF_6) and oxygen) when admitted into electric gas discharges [28]. In ionosphere, negative ions are of the type O^- and O_2^- [29]. In the F-region of the ionosphere, negative ions can be produced artifically by injecting SF_6 vapours [30]. During solar bursts, evolution of stars and pulsar radiation, relativistic ions and electrons are ejected and come into the space and also enter the ionosphere [31]. In laboratory relativistic plasma can be produced using high power laser radiation of the order 10^{10} to 10^{18} W/cm^2 [32]. So in astrophysical plasma and in laser-induced laboratory plasma our present investigation of soliton would be relevant, from which we may get some new information about the propagation of waves in such plasma. In the laboratory, various authors [33-35] studied the behaviour of ion acoustic soliton in presence of negative ion in the plasma and obtained interesting results. However, our theoretical results can not be compared with the experimental values since we have not yet seen any experimental report for the ion-acoustic soliton in relativistic plasma. It is expected that scientists belonging to the experimental groups would be able to make more sophisticated instruments in near future for the observation of ion-acoustic solitons in relativistic plasma having both positive and negative ions. Anyway, our present theoretical investigation gives some new ideas on the solitary waves which will help to further develop the infra-structure for the study of solitons in a warm relativistic plasma having multicomponent negative ions (e.g. O^-, O_2^-, SF_6^-) and that would be of interest for the laboratory and space plasmas. It is known that contributions of higher order nonlinearity and dispersiveness yield the theoretical values of width and phase velocity of solitons which are very close to the experimentally observed values [36, 37]. Higher order effects on ion-acoustic soliton in a relativistic plasma with positive and negative ions may be theoretically investigated following our previous works [38]. In fact, we have recently solved this problem which will be communicated elsewhere in near future.

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TVORBA SOLITONA U IZOTERMNOJ RELATIVISTIČKOJ PLAZMI S POZITIVNIM I NEGATIVNIM IONIMA

SUDESHNA CHAKRABORTY, SAILENDRA N. PAUL i ASESH R. CHOWDHURY

High Energy Physics Division, Department of Physics, Jadavpur University, Calcutta 700032, India

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Proučavana je tvorba solitonskih valova u relativističkoj plazmi koja sadrži pozitivne i negativne ione. Pretpostavljeno je da je plazma izotermna i da su ioni vrući. Određena je fazna brzina, širina i amplituda solitona u ovisnosti o σ_α, σ_β, $n_{\beta 0}/n_{\alpha 0}$, $u_{\alpha 0}/c$, $u_{\beta 0}/c$ i Q, gdje su σ_α i σ_β temperature iona, $n_{\alpha 0}$ i $n_{\beta 0}$ pripadne ravnotežne gustoće, $u_{\alpha 0}$ i $n_{\beta 0}$ odgovarajuće brzine strujanja, a Q je omjer mase negativnih i pozitivnih iona.