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OSCILLATIONS IN A THREE-COMPONENT MIXED QUANTUM-CLASSICAL PLASMA

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A three-component plasma in which one component is higly degenerate and the other two are in classical regime is considered. The frequencies of acoustic and optical plasma modes are found using the RPA and expanding the response function in the low- and high-frequency limit.

1. Introduction

The problem of collective behaviour of charge carriers in solids was first addressed by Kronig and Korringa [1–2]. They introduced the concept of solid state plasma. The first rigorous treatment of electron plasma in metals was developed by Bohm and Pines [3–6] by applying the Random Phase Approximation (RPA). Subsequently, many authors studied plasma effects in metals. A rather complete list of references may be found, for instance, in the review article by Glicksman [7].

In contrast to normal metals in which only one kind of charge carriers exists (conduction electrons), semiconductors have more than one type of carriers. This naturally leads to the concept of multicomponent plasma. In a multicomponent plasma, in addition to high-frequency (optical) oscillations (as in a one-component plasma) there exist a low-frequency modes, the so-called acoustic branches. This name originates from the fact that, as in the case of lattice acoustic waves, the frequency of plasma acoustic waves tends to zero when the wavelength approaches infinity. Moreover, this mode can be appreciably Landau damped.

A problem of two-component plasma formed by electrons and holes in semiconductors was first considered by Pines and Schrieffer [8–9]. Later, Fröhlich [10–11] and Salustri [12] studied oscillations of a two-component plasma composed of s- and d-band electrons in metals.

The extension to an N-component plasma was natural, having its applications not only in solid state physics. Various properties of an N-component plasma have been studied by many authors [13–24].

In our earlier papers we studied the properties of a multicomponent classical [25] and a degenerate (quantum) [26] plasma. However, there could also exist systems in which one component is higly degenerate, and the other two are in the classical regime. In this paper we want to study plasma dispersion relations in such mixed systems.

2. *Calculation*

The dispersion relation of a three-component plasma may be expressed in the form [27]

$$1 + \frac{4\pi}{\varepsilon_0 k^2} \sum_{j=1}^{3} e_j^2 W_j(\mathbf{k}, \omega) = 0, \tag{1}$$

where ε_0 is the dielectric constant of the medium, $\mathbf{k}$ is the wave vector, ω is the eigenfrequency, e_j is the charge of the j^{th} species and $W_j(\mathbf{k}, \omega)$ is the corresponding response function. In the effective-mass approximation,

$$E(\mathbf{p}) = \frac{\hbar^2 p^2}{2m}. \tag{2}$$

After applying the RPA for the response function, one obtains [28]

$$W(\mathbf{k}, \omega) = \frac{2}{(2\pi)^3} \int \frac{f(\mathbf{p}+\mathbf{k}) - f(\mathbf{p})}{\hbar\omega - \frac{\hbar^2}{2m}(2\mathbf{pk} + k^2) + i\delta} d^3 p. \tag{3}$$

Here $f(\mathbf{p})$ is the Fermi-Dirac distribution function in the state of the wave vector $\mathbf{p}$ and δ is the positive infinitesimal.

An important role in the behaviour of a plasma is played by the screening wave vector k_s. For a rare plasma, when the motion of particles is described by the classical Maxwell-Boltzmann statistics, it is given by the Debye-Hückel wave vector

$$k_s = \omega_p \sqrt{\frac{m}{k_B T}}, \tag{4}$$

where k_B is the Boltzmann constant, T the temperature and ω_p the unperturbed long-wavelength plasma frequency

$$\omega_p^2 = \frac{4\pi N e^2}{\varepsilon_0 m}, \tag{5}$$

N being the concentration of particles. In the opposite case, namely for a dense plasma, the Debye-Hückel wave vector should be replaced by that calculated in the Thomas-Fermi approximation:

$$k_s = \omega_p \sqrt{\frac{3m}{2E_F}}, \tag{6}$$

where E_F is the Fermi energy.

We shall consider the case when the first component of a three-component plasma is in the quantum regime, whereas the second and the third one obey classical statistics. In other words, it will be supposed that the following equations hold:

$$E_{F1} \gg k_B T_1, \tag{7}$$

$$E_{Fj} \ll k_B T_j \quad j = 2, 3. \tag{8}$$

Next, our somewhat simplified model will be based on the assumption that the plasma frequencies of the coupled system are well separated. This assumption will be realized if both the frequencies ω_{pj} and the screening wave vectors k_{sj} satisfy the conditions

$$\omega_{p1}^2 \gg \omega_{p2}^2 \gg \omega_{p3}^2 \tag{9}$$

$$k_{s3}^2 \ll k_{s2}^2 \ll k_{s1}^2. \tag{10}$$

As is demonstrated later on, Eqs. (9) and (10) ensure that the damping of the waves is small in the collisionless plasma.

Confining oneself to the case $e_1 = e_2 = e_3$ and $N_1 \approx N_2 \approx N_3$, one can easily verify that Eqs. (9) and (10) will be satisfied if the species of the plasma have disparate masses and disparate characteristic energies:

$$m_1 \ll m_2 \ll m_3, \tag{11}$$

$$E_{f1} \gg k_B T_2 \gg k_B T_3. \tag{12}$$

Expanding the response function in the power series of the wave vector, one has in the classical limit:

$$\frac{4\pi e^2}{\varepsilon_0 k^2} W(\mathbf{k}, \omega) = \mathrm{i} \frac{\omega_p^2 \omega m}{k^3 k_B T} \sqrt{\frac{\pi m}{2 k_B T}} \exp\left(-\frac{m\omega^2}{2k^2 k_B T}\right) -$$

$$-\left(\frac{\omega_p}{\omega}\right)^2 \times \left[1 + 3\left(\frac{k^2 k_B}{m\omega^2}\right)^2 + 15\left(\frac{k^2 k_B T}{m\omega^2}\right)^2 + \ldots\right] \quad m\omega^2 \gg k^2 k_B T \quad (13)$$

and

$$\frac{4\pi e^2}{\varepsilon_0 k^2} W(\mathbf{k},\omega) = \frac{\omega_p^2 m}{k^2 k_B T}\left[i\frac{\omega}{k}\sqrt{\frac{\pi m}{2k_B T}}\exp\left(-\frac{m\omega^2}{2k^2 k_B T}\right) + 1 - \right.$$
$$\left. -\frac{m\omega^2}{k^2 k_B T} + \frac{1}{3}\left(\frac{m\omega^2}{k^2 k_B T}\right)^2 - \ldots\right] \quad m\omega^2 \ll k^2 k_B T, \quad (14)$$

whereas the corresponding high- and low-frequency expansions for the degenerate quantum plasma are, respectively,

$$\frac{4\pi e^2}{\varepsilon_0 k^2} W(\mathbf{k},\omega) = -\left(\frac{\omega_p}{\omega}\right)^2 \left[1 + \frac{6}{5}\frac{k^2 E_F}{m\omega^2} + \frac{12}{7}\left(\frac{k^2 E_F}{m\omega^2}\right)^2 + \right.$$
$$\left. + \frac{\hbar^2 k^4}{4m^2\omega^2} + \ldots\right] \quad m\omega^2 \gg k^2 E_F \quad (15)$$

and

$$\frac{4\pi e^2}{\varepsilon_0 k^2} W(\mathbf{k},\omega) = \frac{3\omega_p^2 m}{2k^2 E_F}\left[\mathrm{i}\frac{\pi\omega}{2k}\sqrt{\frac{m}{2E_F}} + 1 - \frac{m\omega^2}{2k^2 E_F} - \right.$$
$$\left. -\frac{\hbar^2\omega^2}{24E_F^2} - \frac{\hbar^2 k^2}{24 m E_F} - \ldots\right] \quad m\omega^2 \ll k^2 E_F. \quad (16)$$

The imaginary terms in Eqs. (13), (14) and (16) arise from the residue in the integral (3). They describe the famous Landau damping.

It is convenient to separate the plasma frequency into the real and the imaginary part:

$$\omega_j = \Omega_j - \mathrm{i}\gamma_j \quad j = 1, 2, 3, \quad (17)$$

where we assume that in the long-wavelength limit the damping term γ_j is negligibly small,

$$\gamma_j \ll \Omega_j. \quad (18)$$

Taking the high-frequency expansions (13) and (15) for all the three components of the plasma we calculate the frequency of the optical plasma mode by virtue of the conditions (9) and (10):

$$\Omega_1 = \omega_{p1} \left\{ 1 + \frac{6k^2 E_{F1}}{5m_1\omega_{p1}^2} + \left(\frac{2k^2 E_{F1}}{3m_1\omega_{p1}^2}\right)^2 \left[\frac{108}{175} + \frac{9}{16}\left(\frac{\hbar\omega_{p1}}{E_{F1}}\right)^2\right] + \dots \right\}^{1/2}, \tag{19}$$

$$\gamma_1 = 0. \tag{20}$$

Similarly to acoustic phonons, the frequencies of the second and the third modes go to zero as $k \to 0$. They are called acoustic plasma modes. The upper acoustic frequency is obtained by applying the low-frequency expansion (16) to the first component and the high-frequency expansion (13) to $j = 2, 3$. Under the assumption that Ω_2 is much larger than γ_2, we arrive at

$$\Omega_2 = \frac{k\omega_{p2}}{\sqrt{\frac{3m_1}{2E_{F1}}\left[\omega_{p1}^2 + \frac{2k^2 E_{F1}}{3m_1}\left(1 - \frac{\hbar^2\omega_{p1}^2}{16E_{F1}^2}\right)\right]}}, \tag{21}$$

$$\gamma_2 = \frac{3\pi m_1 \omega_{p1}^2 \Omega_2^4}{8E_{F1}\omega_{p2}^2 k^3}\sqrt{\frac{m_1}{2E_{F1}}}. \tag{22}$$

It is interesting to note that the frequency of the second component does not depend on temperature although this component is described by classical statistical physics. This is because the collective motion of the second species is screened by the oscillations of the first species and they are distributed according to quantum statistics.

To calculate the lower acoustic frequency, we apply the low-frequency expansions (16) and (14) to the first and to the second component, respectively, and the high-frequency expansion (13) to the third component. Assuming again that the damping term is small, after performing some algebra we obtain

$$\Omega_3 = k\omega_{p3}\sqrt{\frac{k_B T_2}{m_2\omega_{p2}^2 + k^2 k_B T_2}}, \tag{23}$$

$$\gamma_3 = \frac{m_2\omega_{p2}^2\Omega_3^4}{2k_B T_2 k^3 \omega_{p3}^2}\sqrt{\frac{\pi m_2}{2k_B T_2}}. \tag{24}$$

Contrary to the frequencies Ω_1 and Ω_2, the frequency Ω_3 increases with increasing temperature. It should be further emphasized that, similarly to the upper acoustic frequency Ω_2 which depends on m_1 and E_{F1} but not on T_2, the lower acoustic frequency Ω_3 depends on m_2 and T_2 but not on the temperature of the third species T_3.

Starting from Eqs. (21) and (23), in the limit $k \to 0$ one obtains for the ratio of the upper and the lower acoustic frequency

$$\frac{\Omega_2}{\Omega_3} = \frac{\omega_{p2} k_{s2}}{\omega_{p3} k_{s1}}, \tag{25}$$

which is large by virtue of Eqs. (9) and (10). Hence, at small wave vectors the slope of the curve describing the upper acoustic frequency Ω_2 as a function of k is much larger than that of the curve describing the lower acoustic frequency Ω_3.

Comparing Eq. (21) with Eq. (22) and Eq. (23) with Eq. (24), we conclude that up to the unimportant numerical factor, at long wavelengths the ratio of the imaginary to the real part of the acoustic frequency is

$$\frac{\gamma_j}{\Omega_j} \approx \frac{\omega_{pj}}{\omega_{pj-1}} \ll 1, \quad j = 2, 3 \;, \tag{26}$$

where in the last step we have used Eq. (9). This verifies Eq. (18), which we have applied in the derivation of dispersion relations (21), (22), (23) and (24).

The optical mode is the same as in a degenerate quantum plasma [26], and the behaviour of the two acoustic modes is also qualitatively the same (regarding their dependence on k and their relative intensities). A new feature is the temperature dependence of the third mode. This is illustrated in Fig. 1. Choosing $\omega_{p1}/\omega_{p2} = 10$ and $k_{s1}/k_{s2} = 1$, we have plotted the third (acoustic) frequency for four temperatures defined by

$$\frac{E_{F1}}{k_B T_2} = 5,\ 20,\ 50,\ 100. \tag{27}$$

As can be seen from the figure, this frequency increases with temperature, but we should keep in mind that this is the temperature of the second species. We also note that the frequency saturates earlier (as a function of k) at higher temperatures.

We also note here that in contrast to the frequency, the damping of the third mode decreases with T_2.

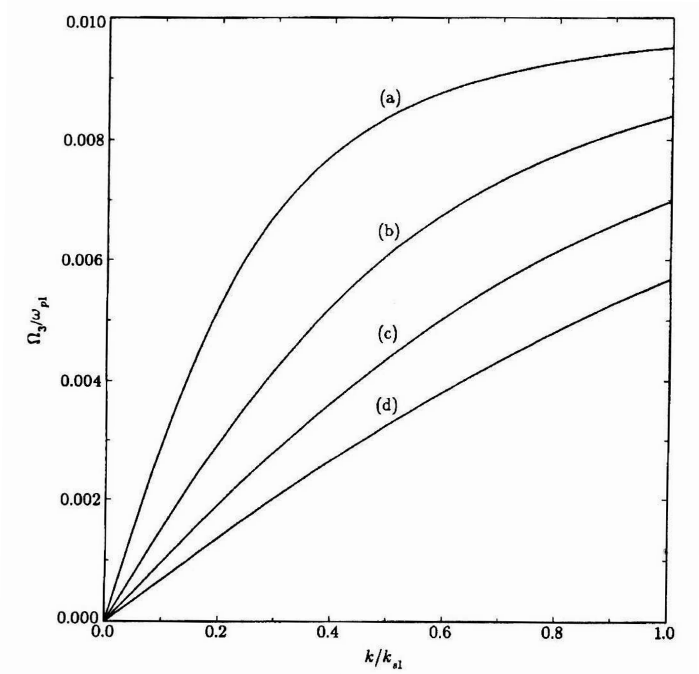

Fig. 1. Acoustic plasma frequency Ω_3 as a function of the wave vector k (in relative units) for four different temperatures: (a) $E_{F1}/k_BT_2 = 5$, (b) $E_{F1}/k_BT_2 = 20$, (c) $E_{F1}/k_BT_2 = 50$ and (d) $E_{F1}/k_BT_2 = 100$. Other parameters used in the calculations are $k_{s1}/k_{F1} = 1$, $\omega_{p1}/\omega_{p2} = 10$.

3. Conclusion

We have studied the collective motion of a three-component plasma in which one component is degenerate and the other two are in classical regime. Assuming that each component is formed of free charged particles with isotropic effective masses, calculating the response function in the RPA and confining our consideration to the case where eigenfrequencies and screening wave vectors of the components are quite different, we have calculated the real and the imaginary parts of plasma frequencies by expanding the response function in the low- and high-frequency limits.

Although the behaviour of the frequencies is qualitatively the same as in a degenerate plasma, the frequency of the third species becomes temperature dependent—but on the temperature of the second species.

Although our considerations are based on an extremely simplified model, we believe that the results obtained describe the main features of a mixed three-component plasma and that they can be useful in the understanding of the collective behaviour of such systems.

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OSCILACIJE U TRO-KOMPONENTNOJ MJEŠANOJ KVANTNO-KLASIČNOJ PLAZMI

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