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HIGHER–ORDER CONTRIBUTION TO THE ION–ACOUSTIC SOLITON IN A DRIFT NEGATIVE–ION PLASMA

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The effects of higher-order nonlinearity and dispersiveness on the ion-acoustic solitons in a multicomponent plasma, having warm electrons, positive ions and negative ions have been investigated using the pseudopotential method. It is observed that drifting negative ions considerably modify the potentials as well as widths of the solitary wave.

1. Introduction

Propagation of ion-acoustic solitons has been studied theoretically and experimentally by many authors. Washimi and Taniuti [1] first derived the Korteweg-de-Vries (KdV) equation for the study of ion-acoustic solitons in a simple plasma model, assuming, e.g., cold ions and isothermal electrons, collisionless and unmagnetised. Ikezi et al. [2] first experimentally observed ion-acoustic solitons in cold plasma. Subsequently, many authors incorporated other parameters of the plasma for the study of ion-acoustic solitons to obtain a clear understanding for the behaviour of solitary waves [3-7]. The presence of negative ions in the plasma has been found to give more interesting results than that of two-electron-temperature plasma, beam plasma etc. [8-16]. However, it has been found that theoretically predicted values of the amplitude, width and velocity of the solitary waves do not obey the experimental results. To remove the discrepency of the theoretical and experimental results, several authors considered the higher order contributions of non-linearity and dispersiveness to the formation of ion-acoustic solitary waves [17-18]. Tagare and Reddy [19] investigated the effect of higher order non-linearity on ion-acoustic solitary waves in a plasma consisting of negative ions and showed the peculiar nature of the solitary waves which is supported by the experimental results [20-22]. However, the effects of drifting ions have not yet been fully explored for the study of ion-acoustic solitary waves. Therefore, we have been motivated to study the higher order contributions of non-linearity and dispersiveness and also the effect of drifting ions on the existence and properties of ion-acoustic solitary waves. In Section 2, we derive the non-linear evolution equation for the first order and second order ion-acoustic solitary waves in a drift negative-ion plasma with isothermal electrons, using the pseudopotential methods. In Section 3, numerical estimates have been made for the soliton profiles and widths.

2. Formulation

We assume that the plasma is homogeneous, collisionless and unmagnetised. It consists of warm positive ions and warm negative ions. The ions are moving with drift velocities. The temperature of ions is much lower than the temperature of electrons, i.e., $T_i \ll T_e$. Therefore, Landau damping is neglected. For such a plasma, the governing equations in dimensionless form are [19],

$$\frac{\partial n_\alpha}{\partial t} + \frac{\partial}{\partial x}(n_\alpha u_\alpha) = 0 \tag{1}$$

$$\frac{\partial u_\alpha}{\partial t} + u_\alpha \frac{\partial u_\alpha}{\partial x} + \frac{\sigma_\alpha}{Q_\alpha n_\alpha} \frac{\partial p_\alpha}{\partial x} = -\frac{Z_\alpha}{Q_\alpha}\frac{\partial \phi}{\partial x} \tag{2}$$

$$\frac{\partial p_\alpha}{\partial t} + u_\alpha \frac{\partial p_\alpha}{\partial x} + 3 p_\alpha \frac{\partial u_\alpha}{\partial x} = 0 \tag{3}$$

$$\frac{\partial^2 \phi}{\partial x^2} = n_e + \sum_\alpha Z_\alpha n_\alpha \tag{4}$$

where $n_e = \exp(\phi)$, the subscript $\alpha = i$ is for positive ions and $\alpha = j$ for negative ions, n_α, u_α and p_α are the number density, velocity and pressure of the ions, respectively, $\sigma_\alpha = T_\alpha / T_e, Z_\alpha = 1$ for positive ions and $Z_\alpha = -Z$ for negative ions, $Q_\alpha = m_j/m_i$, i.e., $Q_\alpha = 1$ for positive ions and $Q_\alpha = Q$ for negative ions and ϕ denotes the electrostatic potential.

To study the solitary waves solution using the pseudopotential method [23,24], we assume that the variables depend on a single independent variable $\eta = x - Vt$, where V is the soliton velocity. We also use the following boundary conditions

$$u_\alpha = u_\alpha^{(0)}, \quad n_\alpha = n_\alpha^{(0)}, \quad p_\alpha = 1, \text{ and } \quad \phi = 0\,, \quad \text{as} \quad \eta = \pm\alpha \tag{5}$$

From the Eqs. (1) to (4), we get

$$-V \frac{\mathrm{d} n_\alpha}{\mathrm{d}\eta} + \frac{\mathrm{d}}{\mathrm{d}\eta}(n_\alpha u_\alpha) = 0, \tag{6}$$

$$-V\frac{\mathrm{d}u_\alpha}{\mathrm{d}\eta}+u_\alpha\,\frac{\mathrm{d}u_\alpha}{\mathrm{d}\eta}+\frac{\sigma_\alpha}{Q_\alpha n_\alpha}\,\frac{\mathrm{d}p_\alpha}{\mathrm{d}\eta}=-\frac{Z_\alpha}{Q_\alpha}\,\frac{\mathrm{d}\phi}{\mathrm{d}\eta},\tag{7}$$

$$-V\frac{\mathrm{d}p_\alpha}{\mathrm{d}\eta}+u_\alpha\,\frac{\mathrm{d}p_\alpha}{\mathrm{d}\eta}+3p_\alpha\frac{\mathrm{d}u_\alpha}{\mathrm{d}\eta}=0,\quad\text{and}\tag{8}$$

$$\frac{\mathrm{d}^2\phi}{\mathrm{d}\eta^2}=n_e+\sum_\alpha Z_\alpha n_\alpha.\tag{9}$$

Integrating (6) and using the boundary conditions (5), we get

$$n_\alpha=\frac{n_\alpha^{(0)}\,(u_\alpha^{(0)}-V)}{u_\alpha-V}\tag{10}$$

Also, from (8), we get

$$p_\alpha=\frac{(u_\alpha^{(0)}-V)^3}{(u_\alpha-V)^3}\tag{11}$$

Inserting Eq. (11) in Eq. (7), and integrating the latter using the boundary condition (5), we obtain

$$(u_\alpha^2-u_\alpha^{(0)\,2})-2V(u_\alpha-u_\alpha^{(0)})+\frac{3\sigma_\alpha}{Q_\alpha n_\alpha^{(0)}}\left[\frac{(u_\alpha^{(0)}-V)^2}{(u_\alpha-V)^2}-1\right]=-2\phi\,\frac{Z_\alpha}{Q_\alpha}.\tag{12}$$

From (10) and (12), the density of ions is

$$n_\alpha=\sqrt{\frac{n_\alpha^{(0)\,3}\,Q_\alpha}{6\sigma_\alpha}}\times\tag{13}$$

$$\sqrt{\left(V^2+a_{0\alpha}-\frac{2Z_\alpha}{Q_\alpha}\phi\right)\pm\sqrt{\left(V^2+a_{0\alpha}+\frac{2Z_\alpha}{Q_\alpha}\phi\right)^2-\frac{12\sigma_\alpha V^2}{Q_\alpha n_\alpha^{(0)}}\left(\frac{u_\alpha^{(0)}}{V}-1\right)^2}},$$

where $a_{0\alpha}=u_\alpha^{(0)\,2}-2Vu_\alpha^{(0)}+(3\sigma_\alpha)/(Q_\alpha n_\alpha^{(0)})$

Therefore, from Eq. (9), putting the values of n_α and n_e, we get

$$\frac{\mathrm{d}^2\phi}{\mathrm{d}\eta^2}=\mathrm{e}^\phi+\sum_\alpha Z_\alpha\sqrt{\frac{n_\alpha^{(0)\,3}\,Q_\alpha}{6\sigma_\alpha}}\sqrt{\left(b_{1\alpha}+\frac{2Z_\alpha}{Q_\alpha}\phi\right)\pm\sqrt{\left(b_{1\alpha}+\frac{2Z_\alpha}{Q_\alpha}\phi\right)^2-b_{2\alpha}^2}},\tag{14}$$

where

$$b_{1\alpha}=V^2+a_{0\alpha},\quad\text{and}$$

$$b_{2\alpha}^2 = \frac{12\sigma_\alpha V^2}{Q_\alpha n_\alpha^{(0)}} \left(\frac{u_\alpha^{(0)}}{V} - 1 \right)^2 .$$

After expansion of the right-hand-side, Eq. (14) can be written in the form

$$\frac{\mathrm{d}^2\phi}{\mathrm{d}\eta^2} = A\phi - B\phi^2 + C\phi^3, \qquad (15)$$

where

$$A = 1 + \sum_\alpha Z_\alpha^2 \sqrt{\frac{n_\alpha^{(0)\,3}}{12 Q_\alpha \sigma_\alpha}} \left(d_{1\alpha}^{-1/2} - d_{2\alpha}^{-1/2} \right),$$

$$B = -\frac{1}{2} \left[1 + \sum_\alpha Z_\alpha^2 \sqrt{\frac{n_\alpha^{(0)\,3}}{12 Q_\alpha^3 \sigma_\alpha}} \left(d_{1\alpha}^{-3/2} - d_{2\alpha}^{-3/2} \right) \right],$$

$$C = \frac{1}{2} \left[\frac{1}{3} + \sum_\alpha Z_\alpha^4 \sqrt{\frac{n_\alpha^{(0)\,3}}{12 Q_\alpha^5 \sigma_\alpha}} \left(d_{2\alpha}^{-5/2} - d_{1\alpha}^{-5/2} \right) \right],$$

$$d_{1\alpha} = b_{1\alpha} + b_{2\alpha}, \qquad \text{and}$$

$$d_{2\alpha} = b_{1\alpha} - b_{2\alpha}.$$

Taking the terms up to ϕ^2 in (15), we obtain

$$\frac{\mathrm{d}^2\phi}{\mathrm{d}\eta^2} = A\phi - B\phi^2. \qquad (16)$$

Equation (16) gives the solitary wave solution like the first order KdV equation,

$$\phi^{(1)} = \frac{3A}{2B}\,\mathrm{sech}^2\theta, \qquad (17)$$

where

$$\theta = \sqrt{\frac{A}{4}}\eta.$$

The width of the soliton is

$$D_1 = \frac{2}{\sqrt{A}}. \qquad (18)$$

Taking terms up to ϕ^3, after integration, we get from (15)

$$\left(\frac{d\phi}{d\eta}\right)^2 = \alpha_1\phi^2 - \alpha_2\phi^3 + \alpha_3\phi^4, \tag{19}$$

where

$$\alpha_1 = A\,, \quad \alpha_2 = \frac{2B}{3}, \quad \text{and} \quad \alpha_3 = \frac{C}{2}.$$

Integrating again, we finally obtain the soliton solution like the second order KdV equation

$$\phi^{(2)} = \frac{2\alpha_1}{\sqrt{\left(\alpha_2^2 - 4\alpha_1\alpha_3\right)}\,\left(2\cosh^2\psi - 1\right) + \alpha_2}, \tag{20}$$

where $\psi = \eta\sqrt{\alpha_1}/2 \equiv \theta$. The width of the soliton is

$$D_2 = \frac{2}{\sqrt{\alpha_1}}\cosh^{-1}\sqrt{\frac{0.6905\alpha_2}{\sqrt{\alpha_2^2 - 4\alpha_1\alpha_3}} + 1.6905}\,. \tag{21}$$

It is to be noted that in the limit $\alpha_3 = 0$, D_2 of Eq. (21) becomes equal to D_1 of Eq. (18), if negative value of $\sqrt{\alpha_2^2}$ is taken.

3. Results and discussion

From the expressions (17), (18), (20) and (21), it is observed that negative ions and the drift velocity of the ions affect the formation of ion-acoustic solitons in the plasma. It is to be noted that the results of previous authors [19] can be recovered for the non-drifting plasma from our present results. To investigate the effects of the drift velocity and of negative ions, we consider the plasmas having $H^+ + O^-$, $He^+ + Cl^-$ and $Ar^+ + SF_5^-$ ions with different ionic temperature. Using the solitary wave solutions (17) and (20), potentials $\phi^{(1)}$ and $\phi^{(2)}$ have been calculated for different negative ion concentrations, ionic temperatures and mass ratios of negative ion and positive ion and are shown in Table 1 and Table 2. The potentials $\phi_1 (= \phi^{(1)})$ and ϕ_2 $(= \phi^{(1)} + \phi^{(2)})$ are plotted in Fig. 1. and Fig. 2. Figure 1 shows that an increase of the mass ratio of negative and positive ions decreases the soliton amplitude. But from Fig. 2, we observe that the soliton amplitudes for the first-order solution for both $H^+ + O^-$ and $He^+ + Cl^-$ plasma ϕ_1 is negative, while ϕ_2 is positive for $H^+ + O^-$ and negative for $H^+ + Cl^-$ plasmas. The widths of solitons are numerically calculated and shown in Table 3 and Table 4. Figures 3 and 4 illustrate the variation of the widths of the solitons for different negative ion concentrations. It is observed that widths of the solitons increase for higher concentations of negative ions. It is also observed that $D_2 < D_1$, i.e., the higher-order contribution of the non-linearity and dispersiveness decreases the width of the solitons.

TABLE 1. Potentials of the solitary waves in negative-ion plasmas. The value of $n_j^{(0)}$ was assumed equal 0.1.

$He^+ + O^-$ ions, $\sigma = 0.06$

θ	$\phi^{(1)}$	$\phi^{(2)}$
0	0.39081	0.15893
0.4	0.33438	0.12525
0.6	0.27809	0.09656
1.0	0.16412	0.04965
1.2	0.11920	0.03432
1.6	0.05882	0.01590
2.0	0.02761	0.00723

$He^+ + Cl^-$ ions, $\sigma = 0.13$

θ	$\phi^{(1)}$	$\phi^{(2)}$
0	-22.63323	0.98575
0.4	-19.36567	0.73302
0.6	-16.10512	0.53916
1.0	-9.505227	0.25789
1.2	-6.90336	0.17427
1.6	-3.40675	0.07865
2.0	-1.59903	0.03535

$H^+ + O^-$ ions, $\sigma = 0.24$

θ	$\phi^{(1)}$	$\phi^{(2)}$
0	-3.54221	9.66045
0.4	-3.03082	5.37433
0.6	-2.52053	3.31291
1.0	-1.48762	1.28316
1.2	-1.08041	0.82072
1.6	-0.53317	0.34904
2.0	-0.25025	0.15290

$Ar^+ + SF_5^-$ ions, $\sigma = 0.047$

θ	$\phi^{(1)}$	$\phi^{(2)}$
0	0.1499	0.0802
0.4	0.1283	0.0643
0.6	0.1067	0.0503
1.0	0.0629	0.0265
1.2	0.0457	0.0185
1.6	0.0225	0.0086
2.0	0.0105	0.0039

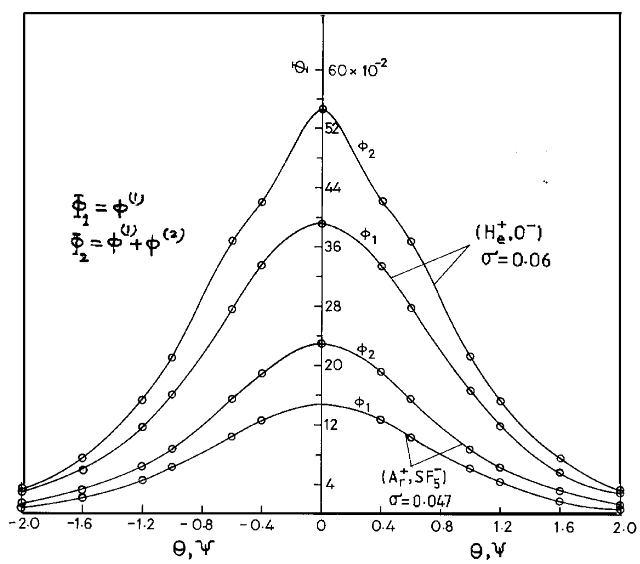

Fig. 1. The structure of the solitary waves for the ions $He^+ + O^-$ *and* $Ar^+ + SF_5^-$ *for different ion temperatures* (ϕ_1 *– first order,* ϕ_2 *– second order).*

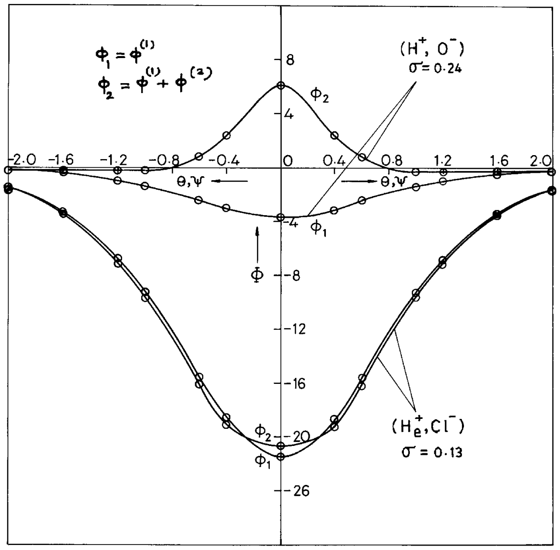

Fig. 2. The structure of the solitary waves (first order and second order) for the ions $H^+ + O^-$ *and* $He^+ + Cl^-$ *for different ion temperatures (right).*

TABLE 2. Width of the solitons in negative-ion plasmas

$He^+ + O^-$ ions, $\sigma = 0.06$

$n_j^{(0)}$	D_1	D_2
0.10	2.4506	2.0382
0.15	2.5419	2.1343
0.20	2.6497	2.2453
0.25	2.8007	2.3985
0.30	3.0090	2.6077
0.35	3.3008	2.8983
0.40	3.7615	3.3577
0.45	4.5558	4.1545
0.50	6.8657	6.5028

$He^+ + Cl^-$ ions, $\sigma = 0.13$

$n_j^{(0)}$	D_1	D_2
0.10	2.0446	1.0573
0.15	2.0532	1.0912
0.20	2.0645	1.2673
0.25	2.0796	1.3154
0.30	2.0985	1.3616
0.35	2.1233	1.4291
0.40	2.1882	1.4466
0.45	2.1919	1.5109
0.50	2.2405	1.5526

$H^+ + O^-$ ions, $\sigma = 0.24$

$n_j^{(0)}$	D_1	D_2
0.10	2.1157	1.5864
0.15	2.1350	1.6179
0.20	2.1606	1.6530
0.25	2.1914	1.6895
0.30	2.2313	1.7318
0.35	2.2797	1.7785
0.40	2.3415	1.8373

$Ar^+ + SF_5^-$ ions, $\sigma = 0.047$

$n_j^{(0)}$	D_1	D_2
0.10	2.7840	2.3961
0.15	2.9753	2.5941
0.20	3.2367	2.8620
0.25	3.6436	3.2774
0.30	4.3411	3.9898
0.35	6.0331	5.7230
0.40	43.1331	43.0714

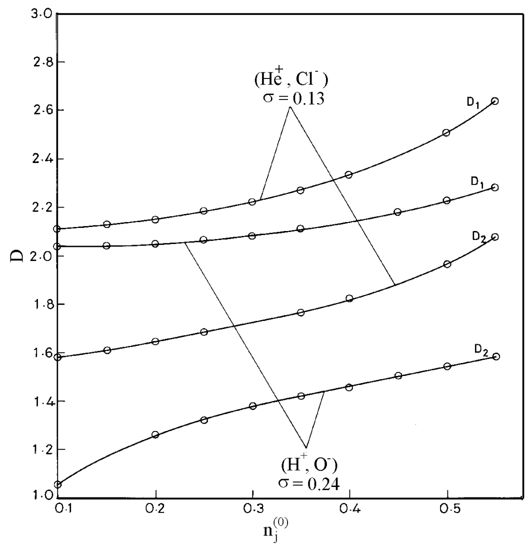

Fig. 3. The variation of the width of solitary vaves with the concentration of negative ions for the ions $He^{+} + Cl^{-}$ *and* $H^{+} + O^{-}$ *(D_1 - first, D_2 - second order).*

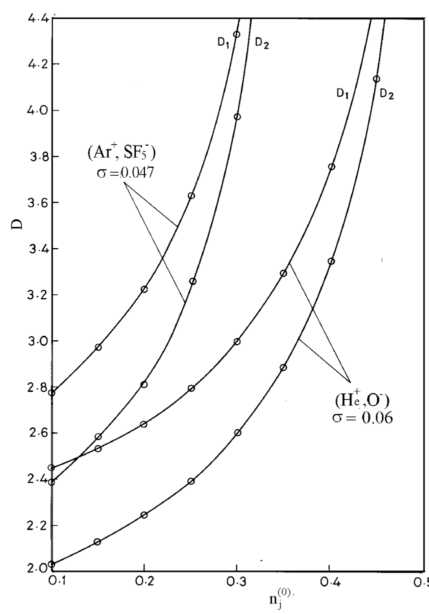

Fig. 4. The variation of the width of solitary waves with the concentration of negative ions for the ions $Ar^{+}SF_5^{-}$ *and* $He^{+} + O^{-}$.

4. Conclusion

A theoretical investigation of the ion-acoustic solitons in multi-component plasmas, consisting of warm electrons, positive ions and negative ions, has been made. Pseudopotentioal method has been applied, taking into consideration the effects of higher-order non-linearity and dispersiveness. Numerical estimations of the potentials and widths have been made for plasmas of (He^{+}+O^{-}), (He^{+}+Cl^{-}), (H^{+}+O^{-}) and (Ar^{+}+SF_5^{-}) ions with different ionic temperatures. It is observed that for (He^{+}+O^{-}) and (Ar^{+}+SF_5^{-}) plasma, both potentials ϕ_1 and ϕ_2 are positive, while for plasma (H^{+}+O^{-}), ϕ_1 is negative and ϕ_2 is positive. It is very interesting to observe that for (He^{+}+Cl^{-}) plasma, both ϕ_1 and ϕ_2 are negative. Tagare [19] and other authors obtained the same type of negative and positive potentials in a negative ion plasma at the critical density of negative ions, considering the modified KdV equation and using the reductive perturbation method. The first- and second-order KdV equation have not been studied using the pseudopotential method after the non-linear term vanished at the critical density. We plan to make an exhaustive study of the ion-acoustic solitary waves at critical density of the negative ions using the pseudopotential method, and also to try to obtain the double-layer solution.

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DOPRINOSI VIŠEG REDA IONSKO–AKUSTIČNIM SOLITONIMA U PLAZMI S VUČENIM NEGATIVNIM IONIMA

Učinci viših redova nelinearnosti i disperzije na ionsko-akustične solitone u više-komponentnoj plazmi s vrućim elektronima i pozitivnim i negativnim ionima se istražuju primjenom pseudopotencijalne metode. Pokazuje se da vučeni negativni ioni znatno mijenjaju potencijale i širine solitonskih valova.