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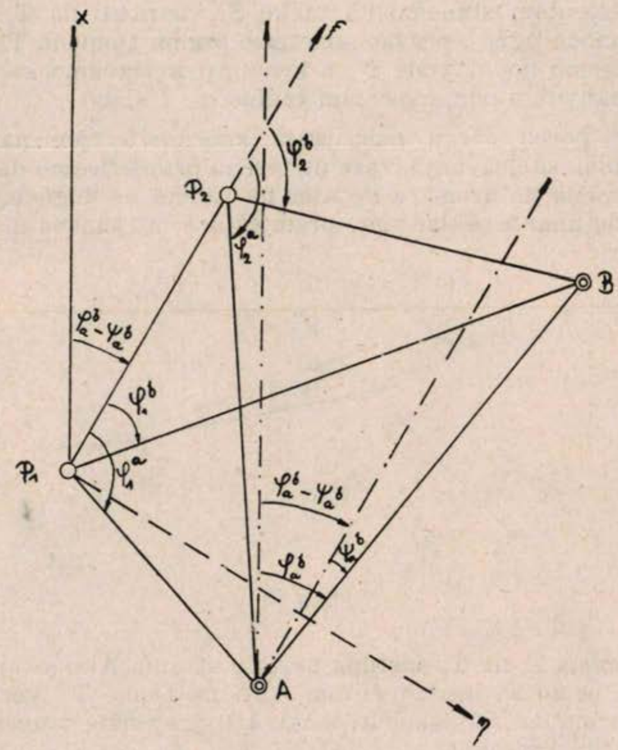
# Rješenje Hanzenovog zadatka računskim strojem

Hanzenov je zadatak moguće riješiti računskim strojem na dva načina. Prvi bi se način sastojao u slijedećem:

U slobodno odabranom koordinatnom sistemu izračunaju se koordinate zadanih točaka uzevši jednu od traženih točaka za ishodište koordinatnog sistema, udaljenost traženih točaka 1,100 ili sl., a spojnicu njihovu kao pozitivan smjer osi $\xi$.

Dobivši ovako koordinate zadanih točaka u slobodno izabranom sistemu, izvršimo transsormaciju koordinata iz slobodno izabranog sistema u (državni) zadani.




Rješenje tog zadatka je objavljeno god. 1942. u listu »Državna Izmjera broj 4.«

Drugo rješenje zadatka izgledalo bi ovako: Iz zadanih koordinata i opažanih pravaca izračunaćemo orjentacioni kut koji dodamo opažanim kutevima na traženim točkama i time dobivamo smjerne kuteve sa traženih točaka prema zadanim ili obrnuto. Presijecanjem napred lako odredimo koordinate traženih točaka. Razmotrimo ovaj drugi slučaj:

Zadane točke: A, B

Tražene točke: 1, 2;

Opažani kutevi: $\varphi^a_1, \varphi^b_1$
$\varphi^a_2, \varphi^b_2$

Smjerni kut trig. strane AB: $\varphi^b_a$ u zadanom sistemu.

Smjerni kut trig. strane AB: $\psi^b_a$ u slobodno izabranom sistemu.

Orjentacioni kut- koji treba dodati opažanim kutevima, da bi se dobili smjerni kutevi: $(\varphi^b_a - \psi^b_a)$.

Pretpostavimo da je tačka: $P_1$ ishodište koordinatnog sistema, pravac 12 os $\xi$, udaljenost 12 jednaka jedinici. Dakle, koordinate traženih točaka bile bi:

Točka 1: $\{\eta = 0,\ \xi = 0\}$ Točka 2: $\{\eta = 0,\ \xi = 1\}$.

Sastavimo jednadžbe pravaca koje prolaze točkama: 1 i B, odnosno 2 i B uzevši u obzir gornje pretpostavke:

$$\left\{\begin{array}{l}\text{Jednadžba pravca u eksplicitnom obliku glasi:}\\ \eta = \operatorname{tg}\psi\cdot\xi + b\end{array}\right\}, \text{ dakle}$$

$$\begin{aligned}\eta_b - \eta_1 &= \operatorname{tg}\varphi^b_1(\xi_b - \xi_1)\\ \eta_b - \eta_1 &= \operatorname{tg}\varphi^b_2(\xi_b - \xi_1) + b'\end{aligned} \tag{I}$$

Drugu jednadžbu smo pisali u takovom obliku zato, jer su sve koordinate reducirane na točku 1 kao ishodište. Veličinu b′, odredićemo iz uvjeta:

$$\eta_2 - \eta_1 = \operatorname{tg}\varphi^b_2(\xi_2 - \xi_1) + b'$$

ili

$$b' = (\eta_2 - \eta_1) - \operatorname{tg}\varphi_2(\xi_2 - \xi_1) \tag{II}$$

Odbivši jednadžbe I dobivamo:

$$\xi_b - \xi_1 = -\frac{b'}{\operatorname{tg}\varphi^b_2 - \operatorname{tg}\varphi^b_1} \tag{III}$$

$$\left.\begin{aligned}\eta_b - \eta_1 &= -\operatorname{tg}\varphi^b_1\cdot\frac{b'}{\operatorname{tg}\varphi^b_2 - \operatorname{tg}\varphi^b_1}\\ &= b' - \operatorname{tg}\varphi^b_2\cdot\frac{b'}{\operatorname{tg}\varphi^b_2 - \operatorname{tg}\varphi^b_1}\end{aligned}\right\} \tag{IV}$$

Uvrstivši $\eta_1 = \eta_2 = 0$

$\xi_1 = 0, \xi_2 = 1$ u jednadžbu II dobivamo:

$$b' = -\operatorname{tg} \varphi_2^b \tag{V}$$

Ako uvrstimo V u III i IV uz gornje uvjete, dobivamo koordinate točke B u pomoćnom koord. sistemu:

$$\left.\begin{aligned} \eta_b &= -\operatorname{tg}\varphi_2^b + \frac{\operatorname{tg}^2\varphi_2^b}{\operatorname{tg}\varphi_2^b - \operatorname{tg}\varphi_2^b} \\ \xi_b &= \qquad\qquad + \frac{\operatorname{tg}\varphi_2^b}{\operatorname{tg}\varphi_2^b - \operatorname{tg}\varphi_1^b} \end{aligned}\right| \tag{VI}$$

Analogno dobijemo koordinate točke A :

$$\begin{aligned} \eta_a &= -\operatorname{tg}\varphi_2^a + \frac{\operatorname{tg}^2\varphi_1^a}{\operatorname{tg}\varphi_2^a - \operatorname{tg}\varphi_1^a} \\ \xi_a &= \qquad\qquad + \frac{\operatorname{tg}\varphi_2^a}{\operatorname{tg}\varphi_2^a - \operatorname{tg}\varphi_1^a} \end{aligned}$$

Izračunajmo smjerni kut stanice AB u pomoćnom koordinatnom sistemu:

$$\operatorname{tg}\psi_a^b = \frac{\eta_b - \eta_a}{\zeta_b - \xi_a} = \frac{+\operatorname{tg}\varphi_2^a - \operatorname{tg}\varphi_2^b + \dfrac{\operatorname{tg}^2\varphi_2^b}{\operatorname{tg}\varphi_2^b - \operatorname{tg}\varphi_2^b} - \dfrac{\operatorname{tg}^2\varphi_2^a}{\operatorname{tg}\varphi_2^a - \operatorname{tg}\varphi_1^a}}{\dfrac{\operatorname{tg}\varphi_2^b}{\operatorname{tg}\varphi_2^b - \operatorname{tg}\varphi_1^b} - \dfrac{\operatorname{tg}\varphi_2^a}{\operatorname{tg}\varphi_1^a - \operatorname{tg}\varphi_1^a}} =$$

$$= \frac{(\operatorname{tg}\varphi_2^a - \operatorname{tg}\varphi_2^b)(\operatorname{tg}\varphi_2^b - \operatorname{tg}\varphi_1^b)(\operatorname{tg}\varphi_2^a - \operatorname{tg}\varphi_1^a) + \operatorname{tg}^2\varphi_2^b(\operatorname{tg}\varphi_2^a - \operatorname{tg}_1^a) - \operatorname{tg}^2\varphi_2^a(\operatorname{tg}_2^b - \operatorname{tg}\varphi_1^b)}{\operatorname{tg}\varphi_2^b(\operatorname{tg}\varphi_2^a - \operatorname{tg}\varphi_1^a) - \operatorname{tg}\varphi_2^a(\operatorname{tg}\varphi_2^b - \operatorname{tg}\varphi_1^b)}$$

Radi pojednostavljenja cijelog izraza razmotrimo posebno brojnik, a posebno nazivnik:

$$\begin{aligned} B = {} & (\operatorname{tg}\varphi_2^b - \operatorname{tg}\varphi_1^b)(\operatorname{tg}^2\varphi_2^a - \operatorname{tg}\varphi_2^a \cdot \operatorname{tg}\varphi_2^b - \operatorname{tg}\varphi_1^a \cdot \operatorname{tg}\varphi_2^a + \operatorname{tg}\varphi_1^a \cdot \operatorname{tg}\varphi_2^b) - \\ & - \operatorname{tg}^2\varphi_2^a(\operatorname{tg}\varphi_2^b - \operatorname{tg}\varphi_1^b) + \operatorname{tg}^2\varphi_2^b(\operatorname{tg}\varphi_2^a - \operatorname{tg}_1^a) = \\ = {} & (\operatorname{tg}\varphi_2^b - \operatorname{tg}\varphi_1^b)(\operatorname{tg}^2\varphi_2^a - \operatorname{tg}\varphi_2^a \cdot \operatorname{tg}_2^b - \operatorname{tg}\varphi_1^a \cdot \operatorname{tg}\varphi_2^a + \operatorname{tg}\varphi_1^a \cdot \operatorname{tg}\varphi_2^b - \operatorname{tg}^2\varphi_2^a) + \\ & + \operatorname{tg}^2\varphi_2^b(\operatorname{tg}\varphi_2^a - \operatorname{tg}\varphi_1^a) + \operatorname{tg}_2^b(\operatorname{tg}\varphi_2^a - \operatorname{tg}\varphi_1^a) = \\ = {} & -\operatorname{tg}^2\varphi_2^b \cdot \operatorname{tg}\varphi_2^a - \operatorname{tg}\varphi_2^b \cdot \operatorname{tg}\varphi_1^a \cdot \operatorname{tg}\varphi_2^a + \operatorname{tg}^2\varphi_2^b \cdot \operatorname{tg}\varphi_1^a + \operatorname{tg}\varphi_1^b \cdot \operatorname{tg}\varphi_2^a \cdot \operatorname{tg}\varphi_2^b + \\ & + \operatorname{tg}\varphi_1^a \cdot \operatorname{tg}\varphi_1^b \cdot \operatorname{tg}\varphi_2^b - \operatorname{tg}\varphi_1^a \cdot \operatorname{tg}\varphi_1^b \cdot \operatorname{tg}\varphi_2^b + \operatorname{tg}\varphi\varphi_2^b \cdot \operatorname{tg}\varphi_2^a - \operatorname{tg}\varphi_2^b \cdot \operatorname{tg}\varphi^a \end{aligned}$$

$$B = \operatorname{tg}\varphi_1^b \cdot \operatorname{tg}\varphi_2^a(\operatorname{tg}\varphi_2^b + \operatorname{tg}\varphi_1^a) - \operatorname{tg}\varphi_1^a \cdot \operatorname{tg}\varphi_2^b(\operatorname{tg}\varphi_2^a + \operatorname{tg}\varphi_1^b)$$

$$N = \mathrm{tg}\,\varphi_2^b \cdot \mathrm{tg}\,\varphi_2^a - \mathrm{tg}\,\varphi_2^b \cdot \mathrm{tg}\,\varphi_1^a - \mathrm{tg}\,\varphi_2^a \cdot \mathrm{tg}\,\varphi_2^b + \mathrm{tg}\,\varphi_1^b \cdot \mathrm{tg}\,\varphi_2^a =$$

$$= \mathrm{tg}\,\varphi_2^a \cdot \mathrm{tg}\,\varphi_1^b - \mathrm{tg}\,\varphi_2^b \cdot \mathrm{tg}\,\varphi_1^a$$

$$\mathrm{tg}\,\psi_a^b = \frac{\mathrm{tg}\,\varphi_1^b \cdot \mathrm{tg}\,\varphi_2^a (\mathrm{tg}\,\varphi_2^b + \mathrm{tg}\,\varphi_1^a) - \mathrm{tg}\,\varphi_1^a \cdot \mathrm{tg}\,\varphi_2^b (\mathrm{tg}\,\varphi_2^a + \mathrm{tg}\,\varphi_1^b)}{\mathrm{tg}\,\varphi_2^a \cdot \mathrm{tg}\,\varphi_1^b - \mathrm{tg}\,\varphi_2^b \cdot \mathrm{tg}\,\varphi_1^a} \quad \text{ili}$$

$$\mathrm{tg}\,\psi_a^b = \frac{\dfrac{1}{\mathrm{ctg}\,\varphi_1^a \cdot \mathrm{ctg}\,\varphi_2^a}\left(\dfrac{1}{\mathrm{ctg}\,\varphi_2^b} + \dfrac{1}{\mathrm{ctg}\,\varphi_1^a}\right) - \dfrac{1}{\mathrm{ctg}\,\varphi_1^a \cdot \mathrm{ctg}\,\varphi_2^b}\left(\dfrac{1}{\mathrm{ctg}\,\varphi_2^a} + \dfrac{1}{\mathrm{ctg}\,\varphi_1^b}\right)}{\dfrac{1}{\mathrm{ctg}\,\varphi_2^a \cdot \mathrm{ctg}\,\varphi_1^b} - \dfrac{1}{\mathrm{ctg}\,\varphi_2^b \cdot \mathrm{ctg}\,\varphi_1^a}}$$

ako taj izraz skratimo dobivamo konačno:

$$\mathrm{tg}\,\psi_a^b = \frac{\mathrm{ctg}\,\varphi_1^a + \mathrm{ctg}\,\varphi_2^b) - (\mathrm{ctg}\,\varphi_2^a + \mathrm{ctg}\,\varphi_1^b)}{\mathrm{ctg}\,\varphi_1^a \cdot \mathrm{ctg}\,\varphi_2^b - \mathrm{ctg}\,\varphi_2^a \cdot \mathrm{csg}\,\varphi_1^b}$$

Konačne smjerne kuteve (sa traženih točaka prema zadanim dobivamo na slijedeći način:

$$\alpha_2^a = (\varphi_a^b - \psi_a^b) + \varphi_2^a$$
$$\alpha_2^b = (\varphi_a^b - \psi_a^b) + \varphi_2^b$$
$$\alpha_1^a = (\varphi_a^b - \varphi_a^q) + \varphi_1^a$$
$$\alpha_1^b = (\varphi_a^b - \psi_a^b) + \varphi_1^b$$

Smjerni kut među zadanih točaka računamo po poznatoj formuli.

$$\mathrm{tg}\,{}_a^b = \frac{Y_b - Y_a}{X_b - X_a}$$

Kontrolu računa koordinata točaka 1, 2 možemo provesti tako da iz koordinata novih točaka računamo opet koordinate zadanih točaka, ili da pomoću razlike smjernih kuteva na novim točkama provjerimo opažane kuteve (pravce).

Formula za sračunavanje kuta $\psi_a^b$ je veoma jednostavna i prikladna za mašinsko računanje; kut $\varphi_a^b$ računa se također jednostavno, i prema tome se cijeli problem svodi na rješenje zadatka »presijecanje naprijed«, a u tom baš i leži prednost ovog drugog načina rješenja pred prvim.

Upotrebljena literatura:

S. Horvat: Geodetski instrumenti.

H. Wittke: Die Rechenmaschine und ihre Rechentechnik.

**Primjer:**

**Zadano:** $\overset{\triangle}{\circ}$ 53  $y = 6\ 376\ 186.61_8$  $x = 4\ 800\ 468.35_5$
$\overset{\triangle}{\circ}$ 51  $y = 6\ 381\ 866.21_5$  $x = 4\ 798\ 264.24_1$

| | Vizura | Opažani pravac | Smjerni kut |
|---|---|---|---|
| Stajalište 4 | $\overset{\triangle}{\circ}$ 3 | 0° 00′ 00.″0 | 100° 28′ 22.″0 |
| | $\overset{\triangle}{\circ}$ 51 | 33 50 30.″7 | 134° 18′ 52.″7 |
| | $\overset{\triangle}{\circ}$ 53 | 102 45 55.″0 | 203° 14′ 17.″0 |

| | Vizura | Opažani pravac | Smjerni kut |
|---|---|---|---|
| Stajalište 3 | $\overset{\triangle}{\circ}$ 4 | 0° 00° 00.″0 | 280° 28′ 22.″0 |
| | $\overset{\triangle}{\circ}$ 53 | (332)<br>152° 38′ 17.″1 | 253° 06′ 39.″1 |
| | $\overset{\triangle}{\circ}$ 51 | (252)<br>72° 20′, 09.″2 | 172 48′ 81.″2 |

| | | | | | |
|---|---|---|---|---|---|
| $\operatorname{ctg}\varphi_4^{53}$ | — 0.226 | 557 | $\operatorname{ctg}\varphi_4^{53}\operatorname{ctg}\varphi_3^{51}$ | — 0.072 | 147 |
| $\operatorname{ctg}\varphi_3^{51}$ | + 0'318 | 450 | $\operatorname{ctg}\varphi_3^{53}\operatorname{ctg}\varphi_4^{51}$ | + 3.881 | 936 |
| $\Sigma$ | + 0.091 | 893 | $\Sigma_2$ | + 2.809 | 789 |
| $\Sigma_1$ | — 0,440 | 915 | $\Sigma\text{-}\Sigma_1 = \Sigma_3$ | + 0.537 | 808 |
| $\operatorname{ctg}\varphi_8^{53}$ | — 1'932 | 340 | $\operatorname{tg}\psi = \dfrac{\Sigma_3}{\Sigma_2}$ | + 0.189 | 629 |
| $\operatorname{ctg}\varphi_4^{51}$ | + 1.491 | 425 | $\psi$ | 10° 44′ 14.″9 | |

$$\varphi_{53}^{51} = 111^\circ\ 12'\ 36''.9$$
$$\psi_{53}^{51} = \quad 10\ \ 44'\ 14.9$$
$$\varphi_4^3 = \varphi_{53}^{51} - \psi_{53}^{51} = 100^\circ\ 28'\ 22''.0$$

Ako ovaj kut dodamo opažanim pravcima na stajalištima 3 i 4, dobivamo smjerne kuteve prema zadanim točkama. Sada lako izračunamo koordinate traženih točaka presijecanjem napred, prethodno promijenivši smjerne kuteve za veličinu ± 180°.