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ABSTRACT
This article focuses on the cooperative innovation under oligopoly with game theory approach. The effects of cooperative innovation are addressed. First, we argue that the total innovation investment of cooperators is more than that of an individual firm without cooperation. Second, the number of cooperative partners has different effects on firms' innovative investment. Cooperative innovation investment decreases in both the number of cooperators and the total number of firms in the industry, while the innovation investment of non-cooperators increases. Finally, the optimal number of cooperators is initially highlighted in theory. The optimal number of firms in cooperation is achieved for cooperators, non-cooperators and social welfare, respectively.

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1. Introduction

Many factors affect firms' innovative investment and there exists extensive research on innovation in economics and cooperative innovation becomes more and more popular in recent years. For example, the operation of International Space Station (I.S.S.) is a $100 billion research outpost in low-Earth orbit. It was cooperatively built by 15 different countries and overseen by 5 space agencies, among them N.A.S.A., Russia's Roscosmos agency, the European Space Agency, the Canadian Space Agency and the Japan Aerospace Exploration Agency. Construction began in 1998 and rotating astronaut crews have lived on the I.S.S. continuously since 2000. Today, the I.S.S. is the largest human-built structure in space. See the latest news, photos and videos from I.S.S. missions here. In I.S.S., many cooperative innovations yield great achievements. (Planetminecraft, 2020, https://www.planetminecraft.com/project/international-space-station-iss-1781186/)Therefore, cooperative innovation plays crucial role in the society. Another typical example of cooperation is European Organization for Nuclear Research (C.E.R.N.), was founded by 12 European member states in 1954. C.E.R.N. now owns 21 members and is the largest and most powerful scientific instrument, providing data all over the world. (OpenAIRE, 2020, https://www.openaire.eu/european-organization-for-nuclear-research-cern).

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Although cooperative innovation appears popularly, many issues about cooperative innovation should be addressed. Does cooperative innovation improve innovative investment? How does the number of cooperative innovation partners affect the innovative investment? What is the optimal number of cooperators?

To answer the above questions, this article highlights the cooperative innovation and focuses on the effects of cooperative innovation on the total innovative investment with game theory approaches. Moreover, we also capture the optimal cooperators.

The rest of this article is organised as follows: Section 2 reviews the literature about innovation and cooperative innovation. Section 3 establishes a two stage game theory model. In the first stage, firms determine the innovative investment. In the second stage, firms compete in outputs. Section 4 analyses the model and main conclusions are achieved. Section 5 further discusses the model about the optimal number of cooperators.

Conclusions are remarked on in the final section.

2. Literature review

Innovation is extensively investigated in recent years in management (Akgün et al., 2014; Nie et al., 2021; Nie & Wang, 2019), industries (Chen et al., 2017; Nie et al., 2018; Wang & Nie,2016), agricultural sectors (Chen et al., 2017; Schwartz et al., 2012), cultural field (Bătrâncea, 2009; Bătrâncea & Nichita, 2012), environment fields (Acemoglu & Cao, 2015; Benigni et al.,2018; Chen et al., 2019; Yang et al., 2019; 2020) and so on. For example, Vives (2008) discovered that competitive market structure yields more innovation. Intellectual property protection also has impact on the R.&.D. investment. Bessen and Maskin (2009) argued that encouraging innovation is more useful than intellectual property protection in static case for some types of innovation. Moreover, opposite conclusions are achieved for dynamic situation by Bessen and Maskin (2009). Chen and Nie (2014) addressed the effects of product externality on the R.&.D. investments. Nie et al. (2018) captured the relationship between switching costs and innovative investment. Moreover, innovation is attached importance by many governments (Batrancea et al., 2019; Xiao et al., 2020; Yang et al., 2019). Nie and Yang (2020) captures the effects of mixed economy on R.&.D.

In the literature about innovation, cooperative innovation attracts extensive attention in recent years (Bausch, 2014; Nie, 2014; 2018; Nie et al., 2018; 2019; Wang et al., 2017; Wang & Nie, 2020; Yang et al., 2018; Yang & Nie, 2015). According to the view of Becker and Dietz (2004), the benefits of R.&.D. cooperation are jointly financing of R.&.D., reducing uncertainty, realising cost-saving and realising economies of scale and scope. The disadvantages of joint R.&.D. are transaction costs. On one hand, some literature highlights the effects of cooperative R.&.D. About cooperative R.&.D., the earlier paper of Katz (1986) discussed the relationship between the cooperative R.&.D. and the development. The interesting and significant papers of D'Aspremong and Jacquemin (1988, 1990) introduced cooperative R.&.D. under duopoly and compared cooperative R.&.D. with non-cooperative R.&.D.. Suzumura (1992) further extended cooperative R.&.D. to multiple firms. Steurs (1995) also addressed cooperative R.&.D. under oligopoly with spillover about inter-industry. On the other hand, some authors focus on some special industries to address cooperative

R.&.D.. For example, recently, Song (2011) considered the cooperative research of semiconductor industry with dynamic model.

There also exists some significant empirical evidence about cooperative R.&.D. With data on a large sample of Dutch innovating firms in two waves of the Community Innovation Survey, Belderbos et al. (2004) examined the impact of R.&.D. cooperation on firm performance differentiating between four types of R.&.D. partners (competitors, suppliers, customers, and universities and research institutes). Based on the German manufacturing industry, Becker and Dietz (2004) confirmed the relationship of cooperative R.&.D. and firms' innovation. Okamuro et al. (2011) identified that founder-specific characteristics such as educational background, prior innovation output, and affiliation to academic associations are fairly important in determining R.&.D. cooperation with academic institutes. Ghosh and Lim (2013) examined the relationship between innovative investment and trade costs. Poyago-Theotoky (1995) discussed the optimal size of joint venture in oligopoly. Based on a large data set of 406 subsidised R.&.D. cooperation projects, Schwartz et al. (2012) provided detailed insights into the relationship between project characteristics and innovation output. Niedergassel and Leker (2011) identified that the codification of knowledge plays an important role for the success of cooperative R.&.D. projects. van den Broek et al. (2018) checked cooperative innovation in health care industry and found that competition hinders cooperative innovation.

This article further focuses on firms' innovation and emphasises the cooperative innovation. The existing theoretical literature pays less attention to the relationship between the number of partners and the R.&.D. investment, while this article develops the theory about this relationship. We establish the theory about the relationship between the partner number and the innovation investment. Based on a two-stage model, this article addresses the cooperative innovation under the oligopoly. At the first stage, firms determine their innovative investment. In this stage, cooperative innovation is introduced and we assume that some firms cooperatively innovate while others do not cooperate. At the second stage, all firms compete in quantities with no cooperation. For cooperators, non-cooperators and social welfare, the optimal number of firms engaging in cooperative innovation is analysed.

Compared with the interesting paper of Suzumura (1992), this article characterises the cooperative innovation with multiple firms, while spillover is not addressed. Compared with Poyago-Theotoky (1995), this article presents the analytic solution of engaging the number of cooperators, which is proposed in the interesting paper of Poyago-Theotoky (1995). This article is organised as follows: A two-stage model is established in Section 2. In Section 3, the model is addressed and the relationship between innovation investment and capacity constraints is captured. Some further discussion is represented in Section 4. We give the optimal number of firms in the cooperative innovation. Conclusions are remarked in the final section.

3. The Model

A game theory model or Cournot competition model is established to analyse the cooperative innovation. This model covers both innovation and output competition.

Consider an industry of N firms. A two-stage model of the substitutability product under oligopoly innovation is established. Denote the set of$N$ firms to be $\mathrm{N} = \{1, 2, \ldots, N\}$. At the first stage, firms simultaneously choose whether to cooperatively innovate with innovative investment I_i for $i \in \{1, 2, \ldots, N\}$. Assume that there are K firms in cooperative innovation. For $i \in \{1, 2, \ldots, N\}$, launching innovative investment $I_i, \frac{1}{2}I_i^2$ represents the costs incurred by innovation, which is similar to that of Sacco and Schmutzer (2011) and Nie et al. (2019). At the second stage, firms simultaneously compete in quantity, which constitutes a Cournot model.

Demand. For $i \in \{1, 2, \ldots, N\}$, p_i is the price and q_i is the quantity of production. Denote$p = (p_1, p_2, \ldots, p_N)$ and $q = (q_1, q_2, \ldots, q_N)$. The utility function of a representative consumer is outlined by:

$$u(p, q) = \alpha \sum_{i=1}^{N} q_i - \frac{1}{2} \sum_{i=1}^{N} q_i^2 - \sum_{i=1}^{N} p_i q_i - \frac{\gamma}{N} \sum_{i=1}^{N} \sum_{i,j=1, j \neq i}^{N} q_i q_j, \tag{1}$$

where $\alpha > 0$ is a constant and $\gamma \in [0, 1]$. $\alpha > 0$ means the total market size and $\gamma \in [0, 1]$ indicates the degree of substitutability. $\gamma = 0$ means that goods are independent and $\gamma = 1$ manifests perfect substitutes (Liu et al., 2012; Nie & Chen, 2012). Actually, the quadratic utility function seems to simplify the problem. By this way, this model seems to be tractable. The above utility function is generally employed by Nie et al. (2020), Nie et al. (2019), Wang et al. (2019), Chen et al. (2020) and Yang & Nie (2020). The inverse demand function, which is the same as that in Liu et al. (2012), is given as follows:

$$p_i = \alpha - q_i - \frac{\gamma}{N} \sum_{j=1, j \neq i}^{N} q_j, i \in \{1, 2, \ldots, N\}. \tag{2}$$

Note that the inverse demand function is directly induced by the above utility function. We also note the above inverse demand function is similar to that in Deneckere and Davidson (1985).

Firms. There is a unique final good. Assume that there are $K(2 \leq K < N)$ firms engaging in the cooperative innovation. Without loss of generality, firm $i \in \{1, 2, \ldots, K\}$ cooperatively innovates, while firm $i \in \{K+1, K+2, \ldots, N\}$ innovates non-cooperatively.

For $i \in \{K+1, K+2, \ldots, N\}$, given innovative investment I_i under non-cooperative innovation, the profit functions of the firms are as follows:

$$\pi_i = p_i q_i - c(I_i) q_i - N I_i^2, \tag{3}$$

where $c(I_i) = c_0 - I_i$ denotes the costs incurred by innovative investment I_i in production. $c_0 > 0$ is a constant. The term $p_i q_i$ is the revenue of firm i. The cost function in this work is different from that in Eső et al. (2010). Apparently, $c(I_i)$ is continuously convex in I_i. $N I_i^2$ denotes the costs incurred by innovative investment with innovation investment I_i. We note that the innovation cost $N I_i^2$ is employed to avoid too much innovative investment. Actually, too much innovative investment may destroy the condition $c(I_i) = c_0 - I_i > 0$. This also guarantees the existence and the uniqueness of the solution to the profit maximisation problems of the firms. Equations (2) and (3) jointly imply that π_i is continuously concave in q_i for $i \in \{K+1, K+2, \ldots, N\}$.

For firm $i \in \{1, 2, \ldots, K\}$, the profit functions of the firms with cooperative innovation are listed as follows:

$$\pi_i = p_i q_i - c\left(\sum_{k=1}^{K} I_k\right) q_i - N I_i^2. \tag{4}$$

In (4), $c(\sum_{k=1}^{K} I_k) = c_0 - \sum_{k=1}^{K} I_k$. This implies the technology spillover is complete among the cooperative firms, while the technology spillover does not exist between firms who cooperate and who do not. Moreover, there does not exist technology spillover in firms without cooperation.

Compared with the paper of D'Aspremong and Jacquemin, (1988), this article models multiple firms and neglects technology spillover. Moreover, this work addresses the cases that firms cooperatively innovate and compete in quantities. This is similar to the hypothesis of Suzumura (1992). The achievement of innovation is shared by each firm in the cooperatively innovating group. Therefore, at the second stage, firms in cooperative innovation own the same marginal costs incurred by production and engage a Cournot competition. To guarantee the existence and the uniqueness of the solution, the following assumption is launched.

Assumption $\frac{\gamma^2 K^2 N^4}{(2N-\gamma)^4}[2N + \gamma(N-2)](N-K)[2N + \gamma(N-K-1)] <$

$$\{N(2N + \gamma N - \gamma)^2 - \frac{N^2 K}{(2N-\gamma)^2}[2N + \gamma(N-K-1)]^2\}\{N(2N + \gamma N - \gamma)^2 - \frac{N^2(2N + \gamma K - \gamma)}{(2N-\gamma)^2}[2N + \gamma(N-2)]\}.$$

For small γ or for not big enough K, the above inequality holds. As an extreme case, Assumption holds for $\gamma = 0$ or $K = 1$. We can show the exists solution $\gamma*(K^*)$ for the following equation for given K (γ).

$$\frac{\gamma^2 K^2 N^4}{(2N-\gamma)^4}[2N + \gamma(N-2)](N-K)[2N + \gamma(N-K-1)] =$$

$$\{N(2N + \gamma N - \gamma)^2 - \frac{N^2 K}{(2N-\gamma)^2}[2N + \gamma(N-K-1)]^2\}\{N(2N + \gamma N - \gamma)^2 - \frac{N^2(2N + \gamma K - \gamma)}{(2N-\gamma)^2}[2N + \gamma(N-2)]\}.$$

Thus, Assumption is satisfied for small γ or for not big enough K and the proof in detail is neglected because of page restriction.

The timing of this game is listed as follows: In the first stage, firms determinate to cooperate or not and determine the innovative investment. In the second stage, firms determine outputs to maximise the profits.

4. Analysis and primary results

The above model is solved by backward induction. We first analyse the second stage. Then, we consider the first stage.

4.1. The second stage

At the second stage, given the innovation investment, firms non-cooperatively compete in quantities. Firm $i \in \{1, 2, \ldots, K\}$ maximises its profits as follows:

$$\max_{q_i} \pi_i = \left(\alpha - q_i - \frac{\gamma}{N}\sum_{j=1, j\neq i}^{N} q_j\right) q_i - (c_0 - \sum_{k=1}^{K} I_k) q_i - N I_i^2. \tag{5}$$

Equation (5) is concave in q_i and there exists a unique solution to equation (5) for any $q_{-i} = (q_1, q_2, \ldots, q_{i-1}, q_{i+1}, \ldots, q_N)$. The corresponding first-order necessary conditions for optimality are given by:

$$\frac{\partial \pi_i}{\partial q_i} = \left(\alpha - \frac{\gamma}{N}\sum_{j=1, j\neq i}^{N} q_j\right) - 2q_i - (c_0 - \sum_{k=1}^{K} I_k) = 0. \tag{6}$$

Or $q_i = \frac{1}{2}\left(\alpha - \frac{\gamma}{N}\sum_{j=1, j\neq i}^{N} q_j\right) - \frac{1}{2}(c_0 - \sum_{k=1}^{K} I_k)$.

Firm $i \in \{K+1, K+2, \ldots, N\}$ maximises the following profit function:

$$\max_{q_i} \pi_i = \left(\alpha - q_i - \frac{\gamma}{N}\sum_{j=1, j\neq i}^{N} q_j\right) q_i - (c_0 - I_i) q_i - N I_i^2. \tag{7}$$

Equation (7) is concave in q_i and there exists a unique solution to equation (7) for any q_{-i}. The corresponding first-order necessary conditions of equation (7) are given by:

$$\frac{\partial \pi_i}{\partial q_i} = \left(\alpha - \frac{\gamma}{N}\sum_{j=1, j\neq i}^{N} q_j\right) - 2q_i - (c_0 - I_i) = 0. \tag{8}$$

Or $q_i = \frac{1}{2}\left(\alpha - \frac{\gamma}{N}\sum_{j=1, j\neq i}^{N} q_j\right) - \frac{1}{2}(c_0 - I_i)$.

Moreover, by symmetry, for $i \in \{1, 2, \ldots, K\}$, from equations (6) and (8), we have:

$$q_i = \frac{N}{2N-\gamma}\left[(\alpha - c_0)\frac{2N-\gamma}{2N+\gamma(N-1)} + \sum_{k=1}^{K} I_k - \frac{\gamma}{2N+\gamma(N-1)}\left(K\sum_{k=1}^{K} I_k + \sum_{k=K+1}^{N} I_k\right)\right]. \tag{9}$$

For $i \in \{K+1, K+2, \ldots, N\}$, equations (6) and (8) imply:

$$q_i = \frac{N}{2N-\gamma}\left[(\alpha - c_0)\frac{2N-\gamma}{2N+\gamma(N-1)} + I_i - \frac{\gamma}{2N+\gamma(N-1)}\left(K\sum_{k=1}^{K} I_k + \sum_{k=K+1}^{N} I_k\right)\right]. \tag{10}$$

4.2. The first stage

At the first stage, all firms determine their innovation. Firm $i \in \{1, 2, \ldots, K\}$ cooperatively innovates while firm $i \in \{K+1, K+2, \ldots, N\}$ non-cooperatively innovates. Firm $i \in \{1, 2, \ldots, K\}$ maximises its profits as follows:

$$\max_{I_i} \pi_i = \frac{N^2}{(2N-\gamma)^2}\left[(\alpha - c_0)\frac{2N-\gamma}{2N+\gamma(N-1)} + \sum_{k=1}^{K} I_k - \frac{\gamma}{2N+\gamma(N-1)}\left(K\sum_{k=1}^{K} I_k + \sum_{k=K+1}^{N} I_k\right)\right]^2 - NI_i^2. \tag{11}$$

Firm $i \in \{K+1, K+2, \ldots, N\}$ maximises the following profit function:

$$\max_{I_i} \pi_i = \frac{N^2}{(2N-\gamma)^2}\left[(\alpha - c_0)\frac{2N-\gamma}{2N+\gamma(N-1)} + I_i - \frac{\gamma}{2N+\gamma(N-1)}\left(K\sum_{k=1}^{K} I_k + \sum_{k=K+1}^{N} I_k\right)\right]^2 - NI_i^2. \tag{12}$$

By Assumption, equations (11) and (12) are both concave. From equations (11) and (12), the first order necessary conditions for optimality yield: For $i \in \{1, 2, \ldots, K\}$, we have:

$$\frac{\partial \pi_i}{\partial I_i} = \frac{2N^2}{(2N-\gamma)^2}\left[(\alpha - c_0)\frac{2N-\gamma}{2N+\gamma(N-1)} + \sum_{k=1}^{K} I_k - \frac{\gamma}{2N+\gamma(N-1)}\left(K\sum_{k=1}^{K} I_k + \sum_{k=K+1}^{N} I_k\right)\right] \frac{2N+\gamma(N-K-1)}{2N+\gamma(N-1)} - 2NI_i = 0. \tag{13}$$

For $i \in \{K+1, K+2, \ldots, N\}$

$$\frac{\partial \pi_i}{\partial I_i} = \frac{2N^2}{(2N-\gamma)^2}\left[(\alpha - c_0)\frac{2N-\gamma}{2N+\gamma(N-1)} + I_i - \frac{\gamma}{2N+\gamma(N-1)}\left(K\sum_{k=1}^{K} I_k + \sum_{k=K+1}^{N} I_k\right)\right] \frac{2N+\gamma(N-2)}{2N+\gamma(N-1)} - 2NI_i = 0. \tag{14}$$

Therefore, the symmetry manifests $I_1 = I_2 = \cdots = I_K = I^c$ and $I_{K+1} = I_{K+2} = \cdots = I_N = I^{nc}$. Thus: $NI^c - \frac{N^2}{(2N-\gamma)^2}[(\alpha - c_0)\frac{2N-\gamma}{2N+\gamma(N-1)} + KI^c - \frac{\gamma}{2N+\gamma(N-1)}(K^2I^c + (N-K)I^{nc})] \frac{2N+\gamma(N-K-1)}{2N+\gamma(N-1)} = 0.$ $NI^{nc} - \frac{N^2}{(2N-\gamma)^2}[(\alpha - c_0)\frac{2N-\gamma}{2N+\gamma(N-1)} + I^{nc} - \frac{\gamma}{2N+\gamma(N-1)}(K^2I^c + (N-K)I^{nc})] \frac{2N+\gamma(N-2)}{2N+\gamma(N-1)} = 0.$

Based on the above two equations or (13)–(14), we hence have:

$$I^c = \frac{\det\begin{bmatrix} \frac{N^2}{2N-\gamma}[2N+\gamma(N-K-1)](\alpha-c_0) & \frac{\gamma N^2}{(2N-\gamma)^2}(N-K)[2N+\gamma(N-K-1)] \\ \frac{N^2}{2N-\gamma}[2N+\gamma(N-2)](\alpha-c_0) & N(2N+\gamma N-\gamma)^2 - \frac{N^2(2N+\gamma K-\gamma)}{(2N-\gamma)^2}[2N+\gamma(N-2)] \end{bmatrix}}{\det\begin{bmatrix} N(2N+\gamma N-\gamma)^2 - \frac{N^2K}{(2N-\gamma)^2}[2N+\gamma(N-K-1)]^2 & \frac{\gamma N^2}{(2N-\gamma)^2}(N-K)[2N+\gamma(N-K-1)] \\ \frac{\gamma K^2N^2}{(2N-\gamma)^2}[2N+\gamma(N-2)] & N(2N+\gamma N-\gamma)^2 - \frac{N^2(2N+\gamma K-\gamma)}{(2N-\gamma)^2}[2N+\gamma(N-2)] \end{bmatrix}} \tag{15}$$

and

$$I^{nc} = \frac{\det\begin{bmatrix} N(2N+\gamma N-\gamma)^2 - \frac{N^2K}{(2N-\gamma)^2}[2N+\gamma(N-K-1)]^2 & \frac{N^2}{2N-\gamma}[2N+\gamma(N-K-1)](\alpha-c_0) \\ \frac{\gamma K^2N^2}{(2N-\gamma)^2}[2N+\gamma(N-2)] & \frac{N^2}{2N-\gamma}[2N+\gamma(N-2)](\alpha-c_0) \end{bmatrix}}{\det\begin{bmatrix} N(2N+\gamma N-\gamma)^2 - \frac{N^2K}{(2N-\gamma)^2}[2N+\gamma(N-K-1)]^2 & \frac{\gamma N^2}{(2N-\gamma)^2}(N-K)[2N+\gamma(N-K-1)] \\ \frac{\gamma K^2N^2}{(2N-\gamma)^2}[2N+\gamma(N-2)] & N(2N+\gamma N-\gamma)^2 - \frac{N^2(2N+\gamma K-\gamma)}{(2N-\gamma)^2}[2N+\gamma(N-2)] \end{bmatrix}}. \tag{16}$$

Moreover, the symmetry manifests $q_1 = q_2 = \cdots = q_K = q^c$ and $q_{K+1} = q_{K+2} = \cdots = q_N = q^{nc}$. By equations (9)–(10) and equations (15)–(16), we obtain q^c and q^{nc}.

$$
\begin{aligned}
q^{c} &= \frac{N}{2N-\gamma}\left[(\alpha-c_0)\frac{2N-\gamma}{2N+\gamma(N-1)}+\sum_{k=1}^{K}I_k-\frac{\gamma}{2N+\gamma(N-1)}\left(K\sum_{k=1}^{K}I_k+\sum_{k=K+1}^{N}I_k\right)\right] \\
&= (\alpha-c_0)\frac{N}{2N+\gamma(N-1)}+ \\
&\Bigg[\frac{\frac{N^3}{(2N-\gamma)^2}(\alpha-c_0)\det\begin{bmatrix}[2N+\gamma(N-K-1)] & \frac{\gamma(N-K)}{[2N+\gamma(N-1)]}N(2N+\gamma N-\gamma)^2 \\ [2N+\gamma(N-2)] & \begin{array}{c}\frac{K[2N+\gamma(N-K-1)]}{[2N+\gamma(N-1)]}\{N(2N+\gamma N-\gamma)^2 \\ -\frac{N^2(2N+\gamma K-\gamma)}{(2N-\gamma)^2}[2N+\gamma(N-2)]\}+ \\ \frac{\gamma(N-K)}{[2N+\gamma(N-1)]}\frac{\gamma K^2N^2}{(2N-\gamma)^2}[2N+\gamma(N-2)]\end{array}\end{bmatrix}}{\det\begin{bmatrix}\begin{array}{c}N(2N+\gamma N-\gamma)^2- \\ \frac{N^2K}{(2N-\gamma)^2}[2N+\gamma(N-K-1)]^2\end{array} & \frac{\gamma N^2}{(2N-\gamma)^2}(N-K)[2N+\gamma(N-K-1)] \\ \frac{\gamma K^2N^2}{(2N-\gamma)^2}[2N+\gamma(N-2)] & \begin{array}{c}N(2N+\gamma N-\gamma)^2- \\ \frac{N^2(2N+\gamma K-\gamma)}{(2N-\gamma)^2}[2N+\gamma(N-2)]\end{array}\end{bmatrix}}\Bigg],
\end{aligned}
\tag{17}
$$

$$
\begin{aligned}
q^{nc} &= \frac{N}{2N-\gamma}\left[(\alpha-c_0)\frac{2N-\gamma}{2N+\gamma(N-1)}+I_i-\frac{\gamma}{2N+\gamma(N-1)}\left(K\sum_{k=1}^{K}I_k+\sum_{k=K+1}^{N}I_k\right)\right] \\
&= (\alpha-c_0)\frac{N}{2N+\gamma(N-1)}+ \\
&\Bigg[\frac{\frac{N^3}{(2N-\gamma)^2}(\alpha-c_0)\det\begin{bmatrix}\begin{array}{c}\frac{[2N+\gamma(K-1)]}{2N+\gamma(N-1)}\{N(2N+\gamma N-\gamma)^2- \\ \frac{N^2K}{(2N-\gamma)^2}[2N+\gamma(N-K-1)]^2\}+ \\ \frac{\gamma K^2}{2N+\gamma(N-1)}\frac{\gamma N^2}{(2N-\gamma)^2}(N-K)[2N+\gamma(N-K-1)]\end{array} & [2N+\gamma(N-K-1)] \\ \frac{\gamma K^2}{2N+\gamma(N-1)}N(2N+\gamma N-\gamma)^2 & [2N+\gamma(N-2)]\end{bmatrix}}{\det\begin{bmatrix}\begin{array}{c}N(2N+\gamma N-\gamma)^2- \\ \frac{N^2K}{(2N-\gamma)^2}[2N+\gamma(N-K-1)]^2\end{array} & \frac{\gamma N^2}{(2N-\gamma)^2}(N-K)[2N+\gamma(N-K-1)] \\ \frac{\gamma K^2N^2}{(2N-\gamma)^2}[2N+\gamma(N-2)] & \begin{array}{c}N(2N+\gamma N-\gamma)^2- \\ \frac{N^2(2N+\gamma K-\gamma)}{(2N-\gamma)^2}[2N+\gamma(N-2)]\end{array}\end{bmatrix}}\Bigg]
\end{aligned}
\tag{18}
$$

Moreover, we have the following conclusions

Proposition 1. *(13)–(18) implies* $I^c < I^{nc}$, $q^c > q^{nc}$ *and* $KI^c > I^{nc}$. *Moreover,* $\frac{\partial I^c}{\partial \gamma} < 0$ *and* $\frac{\partial I^{nc}}{\partial \gamma} > 0$.

Proof. See Appendix.

Remarks: Because of cooperative innovation, non-cooperative innovation firms play disadvantageous positions and share less market than every firm with cooperative innovation. Larger product substitutability yields more competitive innovation. Therefore, the innovation investments of non-cooperatively innovation firms are no more than those of the total innovation of cooperators.

Under equilibrium, firms' profits are outlined as follows. For $i \in \{1, 2, \ldots, K\}$,

$$\pi_i = q_i^2 - N(I^c)^2 = (I^c)^2 \left\{ \left[\frac{(2N-\gamma)(2N+\gamma N-\gamma)}{2N+\gamma N-\gamma-\gamma K} \right]^2 - N \right\}. \tag{19}$$

For $i \in \{K+1, K+2, \ldots \cdots, N\}$,

$$\pi_i = q_i^2 - N(I^{nc})^2 = (I^{nc})^2 \left\{ \left[\frac{(2N-\gamma)(2N+\gamma N-\gamma)}{2N+\gamma N-\gamma} \right]^2 - N \right\}. \tag{20}$$

In (19) and (20), I^c and I^{nc} are outlined by (15)–(16). Moreover, the profits of each cooperator are more than those of one non-cooperator because of $I^c < I^{nc}$ and $q^c > q^{nc}$.

Here we address the social optimal innovation investment. The social welfare is the value of consumer surplus and producer surplus, which is outlined as follows:

$$SW = \alpha \sum_{i=1}^{N} q_i - \frac{1}{2} \sum_{i=1}^{N} q_i^2 - \frac{\gamma}{N} \sum_{i=1}^{N} \sum_{j=1,\, i \neq j}^{N} q_i q_j - \sum_{i=1}^{K} c_i \left(\sum_{k=1}^{K} I_k \right) q_i - \sum_{i=K+1}^{N} c_i(I_i) q_i - N \sum_{i=1}^{N} I_i^2. \tag{21}$$

Denote the optimal output under (21) to be $q_1 = q_2 = \cdots = q_K = q^{c,sw}$ and $q_{K+1} = q_{K+2} = \cdots = q_N = q^{nc,sw}$ along with the optimal innovation investment $I_1 = I_2 = \cdots = I_K = I^{c,sw}$ and $I_{K+1} = I_{K+2} = \cdots = I_N = I^{nc,sw}$. We also discuss (21) by backward induction. (21) is concave in q^c and q^{nc}, and manifests the following relationship. For $i \in \{1, 2, \ldots, K\}$,

$$\frac{\partial SW}{\partial q_i} = \alpha - q_i - \frac{\gamma}{N} \sum_{j=1,\, j \neq i}^{N} q_j - \left(c_0 - \sum_{k=1}^{K} I_k \right) = 0.$$

For $i \in \{K+1, K+2, \ldots, N\}$,

$$\frac{\partial SW}{\partial q_i} = \alpha - q_i - \frac{\gamma}{N} \sum_{j=1,\, j \neq i}^{N} q_j - (c_0 - I_i) = 0.$$

Similar to the above analysis, we have: For $i \in \{1, 2, \ldots, K\}$,

$$q_i = \frac{N}{N-\gamma}\left[(\alpha - c_0)\frac{N-\gamma}{N+(N-1)\gamma} + \sum_{k=1}^{K} I_k - \frac{\gamma}{N+(N-1)\gamma}\left(K\sum_{k=1}^{K} I_k + \sum_{k=K+1}^{N} I_k\right)\right]. \tag{22}$$

For $i \in \{K+1, K+2, \ldots \cdots, N\}$, we have:

$$q_i = \frac{N}{N-\gamma}\left[(\alpha - c_0)\frac{N-\gamma}{N+(N-1)\gamma} + I_i - \frac{\gamma}{N+(N-1)\gamma}\left(K\sum_{k=1}^{K} I_k + \sum_{k=K+1}^{N} I_k\right)\right]. \tag{23}$$

Moreover, we have:

$$SW = \frac{1}{2}\sum_{k=1}^{N} q_i^2 + \frac{\gamma}{N}\sum_{i=1}^{N}\sum_{j=1, j\neq i}^{N} q_i q_j - N\sum_{i=1}^{N} I_i^2. \tag{24}$$

(24) is concave in I_i for all i. We therefore achieve the optimal innovation by the first order optimal conditions:

$$\frac{\partial SW}{\partial I_i} = \sum_{k=1}^{N} q_i \frac{\partial q_i}{\partial I_i} + \frac{\gamma}{N}\left(\frac{\partial q_i}{\partial I_i}\sum_{j=1, j\neq i}^{N} q_j + q_i \sum_{j=1, j\neq i}^{N} \frac{\partial q_j}{\partial I_i}\right) - 2NI_i = 0.$$

For $i \in \{1, 2, \ldots, K\}$,

$$\frac{\partial SW}{\partial I_i} = \frac{N^2}{(N-\gamma)^2}\left\{K\left[(\alpha - c_0)\frac{N-\gamma}{N+(N-1)\gamma} + \sum_{k=1}^{K} I_k - \frac{\gamma}{N+(N-1)\gamma}\right.\right.$$

$$\left.\left(K\sum_{k=1}^{K} I_k + \sum_{k=K+1}^{N} I_k\right)\right]\frac{N+(N-K-1)\gamma}{N+(N-1)\gamma}$$

$$-\sum_{i=K+1}^{N}\left[(\alpha - c_0)\frac{N-\gamma}{N+(N-1)\gamma} + I_i - \frac{\gamma}{N+(N-1)\gamma}\left(K\sum_{k=1}^{K} I_k + \sum_{k=K+1}^{N} I_k\right)\right]$$

$$\frac{K\gamma}{N+(N-1)\gamma} + \frac{N+(N-K-1)\gamma}{N+(N-1)\gamma}(K-1)$$

$$\left[(\alpha - c_0)\frac{N-\gamma}{N+(N-1)\gamma} + \sum_{k=1}^{K} I_k - \frac{\gamma}{N+(N-1)\gamma}\left(K\sum_{k=1}^{K} I_k + \sum_{k=K+1}^{N} I_k\right)\right]$$

$$+\frac{N+(N-K-1)\gamma}{N+(N-1)\gamma}\sum_{i=K+1}^{N}\left[(\alpha - c_0)\frac{N-\gamma}{N+(N-1)\gamma}\right.$$

$$+I_i-\frac{\gamma}{N+(N-1)\gamma}\left(K\sum_{k=1}^{K}I_k+\sum_{k=K+1}^{N}I_k\right)\Bigg]$$

$$+\left[(\alpha-c_0)\frac{N-\gamma}{N+(N-1)\gamma}+\sum_{k=1}^{K}I_k-\frac{\gamma}{N+(N-1)\gamma}\left(K\sum_{k=1}^{K}I_k+\sum_{k=K+1}^{N}I_k\right)\right]$$

$$\left[\frac{NK+K(N-K-1)\gamma}{N+(N-1)\gamma}-\frac{(N-K)K\gamma}{N+(N-1)\gamma}\right]\Bigg\}-2NI_i=0. \tag{25}$$

Or by symmetry, equation (25) is restated as:

$$f_3=\frac{N}{(N-\gamma)^2}\Bigg\{\left[(\alpha-c_0)\frac{N-\gamma}{N+(N-1)\gamma}+KI^c-\frac{\gamma}{N+(N-1)\gamma}(K^2I^c+NI^{nc}-KI^{nc})\right]$$

$$\left[\frac{N+(N-K-1)\gamma}{N+(N-1)\gamma}(2K-1)\right]+(N-K)\Bigg[(\alpha-c_0)\frac{N-\gamma}{N+(N-1)\gamma}$$

$$+I^{nc}-\frac{\gamma}{N+(N-1)\gamma}(K^2I^c+NI^{nc}-KI^{nc})\Bigg]\frac{N+(N-K-1)\gamma-K\gamma}{N+(N-1)\gamma}+$$

$$\left[(\alpha-c_0)\frac{N-\gamma}{N+(N-1)\gamma}+KI^c-\frac{\gamma}{N+(N-1)\gamma}(K^2I^c+NI^{nc}-KI^{nc})\right]$$

$$\frac{NK-K\gamma}{N+(N-1)\gamma}\Bigg\}-2I_i=0. \tag{25a}$$

For $i\in\{K+1,K+2,\ldots,N\}$,

$$\frac{\partial SW}{\partial I_i}=\frac{N^2}{(N-\gamma)^2}\Bigg\{\Bigg[(\alpha-c_0)\frac{N-\gamma}{N+(N-1)\gamma}+I_i-\frac{\gamma}{N+(N-1)\gamma}$$

$$(K\sum_{k=1}^{K}I_k+\sum_{k=K+1}^{N}I_k)\Bigg]\frac{N+(N-2)\gamma}{N+(N-1)\gamma}-K\Bigg[(\alpha-c_0)\frac{N-\gamma}{N+(N-1)\gamma}$$

$$+\sum_{k=1}^{K}I_k-\frac{\gamma}{N+(N-1)\gamma}(K\sum_{k=1}^{K}I_k+\sum_{k=K+1}^{N}I_k)\Bigg]\frac{\gamma}{N+(N-1)\gamma}-\sum_{j=K+1,j\neq i}^{N}$$

$$\left[(\alpha-c_0)\frac{N-\gamma}{N+(N-1)\gamma}+I_j-\frac{\gamma}{N+(N-1)\gamma}(K\sum_{k=1}^{K}I_k+\sum_{k=K+1}^{N}I_k)\right]\frac{\gamma}{N+(N-1)\gamma}$$

$$+\frac{N+(N-2)\gamma}{N+(N-1)\gamma}K\left[(\alpha-c_0)\frac{N-\gamma}{N+(N-1)\gamma}+\sum_{k=1}^{K}I_k-\frac{\gamma}{N+(N-1)\gamma}\left(K\sum_{k=1}^{K}I_k+\sum_{k=K+1}^{N}I_k\right)\right]$$

$$+\frac{N+(N-2)\gamma}{N+(N-1)\gamma}\sum_{j=K+1,j\neq i}^{N}\left[(\alpha-c_0)\frac{N-\gamma}{N+(N-1)\gamma}+I_j\right.$$

$$\left.-\frac{\gamma}{N+(N-1)\gamma}\left(K\sum_{k=1}^{K}I_k+\sum_{k=K+1}^{N}I_k\right)\right]-\left[(\alpha-c_0)\frac{N-\gamma}{N+(N-1)\gamma}\right.$$

$$\left.+I_i-\frac{\gamma}{N+(N-1)\gamma}\left(K\sum_{k=1}^{K}I_k+\sum_{k=K+1}^{N}I_k\right)\right]\left[\frac{K\gamma}{N+(N-1)\gamma}+\frac{(N-K-1)\gamma}{N+(N-1)\gamma}\right]\Bigg\}-2NI_i=0. \tag{26}$$

Or:

$$f_4=\frac{N}{(N-\gamma)^2}\left\{\left[(\alpha-c_0)\frac{N-\gamma}{N+(N-1)\gamma}+I^{nc}-\frac{\gamma}{N+(N-1)\gamma}(K^2I^c+NI^{nc}-KI^{nc})\right]\right.$$

$$\frac{N+(K-1)\gamma}{N+(N-1)\gamma}-K\left[(\alpha-c_0)\frac{N-\gamma}{N+(N-1)\gamma}+KI^c-\frac{\gamma}{N+(N-1)\gamma}(K^2I^c+NI^{nc}-KI^{nc})\right]$$

$$\frac{\gamma}{N+(N-1)\gamma}+\frac{N+(N-2)\gamma}{N+(N-1)\gamma}K\left[(\alpha-c_0)\frac{N-\gamma}{N+(N-1)\gamma}+KI^c-\frac{\gamma}{N+(N-1)\gamma}\right.$$

$$\left.(K^2I^c+NI^{nc}-KI^{nc})\right]+\frac{N+(N-2)\gamma}{N+(N-1)\gamma}\left[(\alpha-c_0)\frac{N-\gamma}{N+(N-1)\gamma}+I^{nc}\right.$$

$$\left.-\frac{\gamma}{N+(N-1)\gamma}(K^2I^c+NI^{nc}-KI^{nc})\right](N-K-1)-\left[(\alpha-c_0)\frac{N-\gamma}{N+(N-1)\gamma}\right.$$

$$\left.+I^{nc}-\frac{\gamma}{N+(N-1)\gamma}(K^2I^c+NI^{nc}-KI^{nc})\right]\left[\frac{K\gamma}{N+(N-1)\gamma}+\frac{(N-K-1)\gamma}{N+(N-1)\gamma}\right]\Bigg\}-2I_i=0. \tag{26a}$$

Denote $A_{11}=\frac{\partial f_3}{\partial I^c}$, $A_{12}=\frac{\partial f_3}{\partial I^{nc}}$, $A_{22}=\frac{\partial f_4}{\partial I^{nc}}$, $A_{21}=\frac{\partial f_4}{\partial I^c}$, $B_1=f_3-I^c\frac{\partial f_3}{\partial I^c}-I^{nc}\frac{\partial f_3}{\partial I^{nc}}$ and $B_2=f_4-I^c\frac{\partial f_4}{\partial I^c}-I^{nc}\frac{\partial f_4}{\partial I^{nc}}$. By symmetry, denote the solution to the above equations (25)–(26) to be $(I^{c,sw},I^{nc,sw})$. The above two equations manifest:

$$I^{c,sw}=-\frac{\det\begin{bmatrix}B_1 & A_{21}\\ B_2 & A_{22}\end{bmatrix}}{\det\begin{bmatrix}A_{11} & A_{21}\\ A_{12} & A_{22}\end{bmatrix}}, \tag{27}$$

$$I^{nc,sw} = -\frac{\det\begin{bmatrix} A_{11} & B_1 \\ A_{12} & B_2 \end{bmatrix}}{\det\begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix}}. \tag{28}$$

For $(I^{c,sw}, I^{nc,sw})$, we have the similar conclusions to Proposition 1. The cooperators benefit from cooperation. Comparing (27)–(28) with (15)–(16), we immediately have the following conclusions:

Proposition 2. *The innovation investment is lower than the social optimum.*

Remarks: Under competition, there always exists under-investment of innovation. Because of the market power under Cournot, firms invest less than the optimal innovation. This conclusion is consistent with the existed results of Bessen and Maskin (2009), and Sacco and Schmutzer (2011).

5. Further discussion

This section addresses the number of firms in cooperative innovation and some related empirical evidence is addressed. Firms in cooperative innovation benefit from this cooperation and the profit for each firm is outlined by (19)–(20). In this section, we restrict $K \geq 2$. The equilibrium innovation is determined by (15)–(16).

5.1. The effects of the number of firms with cooperative innovation

Firstly, we address the effects of the number of cooperating firms about R.&D. Then, the effects of both the cooperating number on firms' profits and the total number of firms in this industry are considered. From the above formulations, we have:

Proposition 3. *The innovative investment of cooperative firms decreases in K, while the innovative investment of firms without cooperative innovation increases in K. The total innovative investment of cooperative partners increases in K. The innovative investment of cooperative firms decreases in N, while the innovative investment of firms without cooperative innovation increases in N.*

Proof. See Appendix.

Remarks: The above conclusions manifest that the number of cooperative firms yields different trend of the innovative investment. The large number of cooperative firms stimulates non-cooperative firms to improve the innovation, while reduces the innovative investment of cooperative firms. More competition yields more innovation for non-cooperators while less for cooperators. For non-cooperators, this conclusion is consistent with Vives' result (2008), while for cooperators, this is contrary to Vives' (2008). This conclusion seems interesting.

Cooperative innovation has two-edge effects. On one hand, firms in cooperation reduce the production cost. On the other hand, the competition of firms in cooperative innovation becomes fiercer because firms compete both in outputs and in innovational investment. About the optimal number of cooperatively innovating firms, by envelope theorem, we have the following conclusion

Proposition 4. *For firms in cooperation, the optimal number of cooperatively innovating firms is* $K^{c,*} = 2$ *and* $K^{nc,*} = N-1$.*The socially optimal number of cooperatively innovating firms is* $K^{sc,*} = N$

Proof. See Appendix.

Remarks: For cooperators, the optimal number of cooperators is 2. For cooperators, when there are more cooperators, firms in cooperation face more fierce competition at the second stage. Moreover, there are free-rider phenomena when there are more cooperators. It seems rational that the optimal number of cooperators is 2 for cooperators.

For non-cooperators, the optimal number of cooperators is $N-1$. When there are more cooperators, non-cooperators launch more innovation investment while the cooperators innovate less. The optimal number of cooperators is $N-1$.

The social welfare is maximised if $K^{sc,*} = N$ firms cooperatively innovate. It is consistent with social phenomenon. The number of cooperators under competition is lower than social optimality. Government always encourages cooperative innovation to improve the social welfare because $2<K^{sc,*}$. This conclusion of social welfare is consistent with that of Poyago-Theotoky (1995).

This conclusion seems interesting because there exists no such conclusion in the existing literature. This conclusion neglects technology factor. When complicated technology is considered, the conclusions may depend on the technology factor. Moreover, free-riders are not introduced in the above model. Taking free-riders into account, the number of the social optimality of the cooperators should be less than N.

5.2. Related empirical evidence

This article analytically the cooperative innovative and some interesting conclusions are achieved. There exist some empirical evidences supporting the above conclusions. For example, Becker and Dietz (2004) employed the data of the German manufacturing industry to identify the positive relationship of R.&D. investment and the number of cooperative firms.

In Proposition 1, we showed that $KI^c>I^{nc}$. This conclusion is also supported by Becker and Dietz (2004). Becker and Dietz (2004) examined that 'firms invest more in the innovation process when they are engaged in R.&D. cooperation' (p. 218). Based on the German service sector, Kaiser (2002) supported empirical evidence that cooperating firms launch more R.&D. investment in research than non-cooperating firms.

Becker and Dietz (2004) confirmed the positive output effects of cooperative innovation, which is consistent with the above theoretic conclusion $q^c>q^{nc}$. In summary, Proposition 1 rationally explains the empirical results of Becker and Dietz (2004) about both input effects and output effects.

This article argues that the total innovative investment of cooperative partners increases in the number of cooperative partners$K$. This theoretic conclusion is supported by the empirical evidence of Becker and Dietz (2004).

6. Conclusion

This article addresses the cooperative innovation in some industry and achieves some interesting conclusions. This article argues that the number of cooperative partners has different effects on the innovative investment to firms. The innovation investment of cooperators decreases while the innovation investment of non-cooperators increases in the number of cooperators. The optimal number of firms in cooperation is achieved for cooperators, non-cooperators and social welfare, respectively. For cooperators, two cooperators are optimal. For non-cooperators, all other firms' cooperative innovation is optimal. The optimal number of firms to cooperate under social welfare optimality is obtained.

Based on the above conclusions, some policy implications are presented. On one hand, government should encourage cooperative innovation to increase the total innovative investment. When technologies own complementary effect, cooperative innovation seems more efficient. On the other hand, the optimal number of cooperators is important for government to make decisions.

The limit of this article is lack of data to testify our conclusions. Because of time and data restriction, no empirical research is launched. This is our future research. The uncertain situation is neglected, which is another limit of this article.

Some interesting topics arise in this work. Firstly, we assume that there exists a unique group in cooperative innovation in this industry. If there are multiple groups to cooperatively innovate, the innovative investment and the number of the groups are all important. Secondly, we assume the linear cost function to simplify the problem. It is important to extend to general cases. Finally, governmental subsides are not included in this work and the government subsides have crucial effects on the cooperative innovation, although R.&D. projects' funding is an important predictor of the innovative success of R.&D. cooperation projects. This is our further researching topics.

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Appendix

Proof of Proposition 1. From (15) and (16), we have:

$$I^c - I^{nc} = \frac{\dfrac{N^2}{2N-\gamma}(\alpha - c_0)\det\begin{bmatrix} [2N+\gamma(N-K-1)] & \begin{array}{c}\dfrac{\gamma N^2}{(2N-\gamma)^2}(N-K)[2N+\gamma(N-K-1)]+\\ N(2N+\gamma N-\gamma)^2 - \dfrac{N^2K}{(2N-\gamma)^2}[2N+\gamma(N-K-1)]^2\end{array} \\ [2N+\gamma(N-2)] & \begin{array}{c}\dfrac{\gamma K^2N^2}{(2N-\gamma)^2}[2N+\gamma(N-2)] + N(2N+\gamma N-\gamma)^2 - \\ \dfrac{N^2(2N+\gamma K-\gamma)}{(2N-\gamma)^2}[2N+\gamma(N-2)]\end{array} \end{bmatrix}}{\det\begin{bmatrix} \begin{array}{c} N(2N+\gamma N-\gamma)^2 - \\ \dfrac{N^2K}{(2N-\gamma)^2}[2N+\gamma(N-K-1)]^2\end{array} & \dfrac{\gamma N^2}{(2N-\gamma)^2}(N-K)[2N+\gamma(N-K-1)] \\ \dfrac{\gamma K^2N^2}{(2N-\gamma)^2}[2N+\gamma(N-2)] & N(2N+\gamma N-\gamma)^2 - \dfrac{N^2(2N+\gamma K-\gamma)}{(2N-\gamma)^2}[2N+\gamma(N-2)] \end{bmatrix}}.$$

$$\det\begin{bmatrix} [2N+\gamma(N-K-1)] & \begin{array}{c}\dfrac{\gamma N^2}{(2N-\gamma)^2}(N-K)[2N+\gamma(N-K-1)]+\\ N(2N+\gamma N-\gamma)^2 - \dfrac{N^2K}{(2N-\gamma)^2}[2N+\gamma(N-K-1)]^2\end{array} \\ [2N+\gamma(N-2)] & \begin{array}{c}\dfrac{\gamma K^2N^2}{(2N-\gamma)^2}[2N+\gamma(N-2)] + N(2N+\gamma N-\gamma)^2 - \\ \dfrac{N^2(2N+\gamma K-\gamma)}{(2N-\gamma)^2}[2N+\gamma(N-2)]\end{array} \end{bmatrix}$$

$$= [2N+\gamma(N-K-1)]\{\frac{\gamma K^2N^2}{(2N-\gamma)^2}[2N+\gamma(N-2)] + N(2N+\gamma N-\gamma)^2$$

$$- \frac{N^2(2N+\gamma K-\gamma)}{(2N-\gamma)^2}[2N+\gamma(N-2)]\}$$

$$-[2N+\gamma(N-2)]\{\frac{\gamma N^2}{(2N-\gamma)^2}(N-K)[2N+\gamma(N-K-1)]$$

$$+N(2N+\gamma N-\gamma)^2 - \frac{N^2K}{(2N-\gamma)^2}[2N+\gamma(N-K-1)]^2\}$$

$$= -N(2N+\gamma N-\gamma)^2(K-1) + \frac{N^2}{(2N-\gamma)^2}[2N+\gamma(N-K-1)]$$

$$[2N+\gamma(N-2)](K-1)(2N+\gamma N-\gamma)$$

$$= -\frac{N(2N+\gamma N-\gamma)(K-1)}{(2N-\gamma)^2}\{(2N+\gamma N-\gamma)(2N-\gamma)^2$$

$$-N[2N+\gamma(N-K-1)][2N+\gamma(N-2)]\} = -\frac{N(2N+\gamma N-\gamma)(K-1)}{(2N-\gamma)^2}$$

$$\{(2N+\gamma N-\gamma)(2N^2-\gamma N^2+\gamma NK+N\gamma-4N\gamma+\gamma^2)+N\gamma[2N+\gamma(N-K-1)]\}<0.$$

The above inequality holds because of $N \geq 2$ and $K \geq 2$. Therefore, $I^c - I^{nc} < 0$. By a similar way, here we show $KI^c > I^{nc}$.

$$KI^c - I^{nc} = \frac{\frac{N^2}{2N-\gamma}(\alpha-c_0)\det\begin{bmatrix} [2N+\gamma(N-K-1)] & \begin{matrix}\frac{\gamma N^2K}{(2N-\gamma)^2}(N-K)[2N+\gamma(N-K-1)]+\\ N(2N+\gamma N-\gamma)^2-\frac{N^2K}{(2N-\gamma)^2}[2N+\gamma(N-K-1)]^2\end{matrix} \\ [2N+\gamma(N-2)] & \begin{matrix}\frac{\gamma K^2N^2}{(2N-\gamma)^2}[2N+\gamma(N-2)]+NK(2N+\gamma N-\gamma)^2-\\ \frac{N^2K(2N+\gamma K-\gamma)}{(2N-\gamma)^2}[2N+\gamma(N-2)]\end{matrix}\end{bmatrix}}{\det\begin{bmatrix} N(2N+\gamma N-\gamma)^2-\frac{N^2K}{(2N-\gamma)^2}[2N+\gamma(N-K-1)]^2 & \frac{\gamma N^2}{(2N-\gamma)^2}(N-K)[2N+\gamma(N-K-1)] \\ \frac{\gamma K^2N^2}{(2N-\gamma)^2}[2N+\gamma(N-2)] & N(2N+\gamma N-\gamma)^2-\frac{N^2(2N+\gamma K-\gamma)}{(2N-\gamma)^2}[2N+\gamma(N-2)]\end{bmatrix}}.$$

$$\det\begin{bmatrix} [2N+\gamma(N-K-1)] & \begin{matrix}\frac{\gamma N^2K}{(2N-\gamma)^2}(N-K)[2N+\gamma(N-K-1)]+\\ N(2N+\gamma N-\gamma)^2-\frac{N^2K}{(2N-\gamma)^2}[2N+\gamma(N-K-1)]^2\end{matrix} \\ [2N+\gamma(N-2)] & \begin{matrix}\frac{\gamma K^2N^2}{(2N-\gamma)^2}[2N+\gamma(N-2)]+NK(2N+\gamma N-\gamma)^2-\\ \frac{N^2K(2N+\gamma K-\gamma)}{(2N-\gamma)^2}[2N+\gamma(N-2)]\end{matrix}\end{bmatrix}$$

$$=\det\begin{bmatrix} [2N+\gamma(N-K-1)] & N(2N+\gamma N-\gamma)^2-\frac{N^2K}{(2N-\gamma)}[2N+\gamma(N-K-1)] \\ [2N+\gamma(N-2)] & NK(2N+\gamma N-\gamma)^2-\frac{N^2K}{(2N-\gamma)}[2N+\gamma(N-2)]\end{bmatrix}$$

$$= N(2N+\gamma N-\gamma)^2\{K[2N+\gamma(N-K-1)]-[2N+\gamma(N-2)]\}>0.$$

We thus achieve $KI^c > I^{nc}$. Combined (9)–(10) and $KI^c > I^{nc}$, we have $q^c - q^{nc} > 0$. Although one cooperator innovates less than an individual non-cooperator, the cooperators totally launch more innovation investment than a single non-cooperator.

(14) and 15) are restated as:

$$f_1 = [2N+\gamma(N-1)]^2NI^c - \frac{N^2}{(2N-\gamma)^2}[(\alpha-c_0)(2N-\gamma)+K(2N+\gamma N-\gamma)I^c$$
$$-\gamma(K^2I^c+(N-K)I^{nc})][2N+\gamma(N-K-1)] = 0. \tag{A1}$$

$$f_2 = [2N + \gamma(N-1)]^2 N I^{nc} - \frac{N^2}{(2N-\gamma)^2}\left[(\alpha-c_0)(2N-\gamma) + (2N+\gamma N-\gamma)I^{nc} - \gamma(K^2 I^c + (N-K)I^{nc})\right][2N+\gamma(N-2)] = 0. \tag{A2}$$

Apparently, the concavity of profit functions implies $\det\begin{bmatrix} \frac{\partial f_1}{\partial I^c} & \frac{\partial f_1}{\partial I^{nc}} \\ \frac{\partial f_2}{\partial I^c} & \frac{\partial f_2}{\partial I^{nc}} \end{bmatrix} > 0$. Moreover,

$$\frac{\partial f_1}{\partial \gamma} = 2[2N+\gamma(N-1)](N-1)NI^c - \frac{2N^2}{(2N-\gamma)^3}[(\alpha-c_0)(2N-\gamma) + K(2N+\gamma N-\gamma)I^c$$

$$-\gamma(K^2 I^c + (N-K)I^{nc})][2N+\gamma(N-K-1)] - \frac{N^2}{(2N-\gamma)^2}[(\alpha-c_0)(2N-\gamma)$$

$$+K(2N+\gamma N-\gamma)I^c - \gamma(K^2 I^c + (N-K)I^{nc})](N-K-1) - \frac{N^2}{(2N-\gamma)^2}$$

$$\left[-(\alpha-c_0) + K(N-1)I^c - (K^2 I^c + (N-K)I^{nc})\right][2N+\gamma(N-K-1)]$$

$$= 2[2N+\gamma(N-1)](N-1)NI^c - \frac{2}{(2N-\gamma)}[2N+\gamma(N-1)]^2 NI^c$$

$$-\frac{N-K-1}{[2N+\gamma(N-K-1)]}[2N+\gamma(N-1)]^2 NI^c - \frac{N^2}{(2N-\gamma)^2}[-(\alpha-c_0) + K(N-1)I^c$$

$$-(K^2 I^c + (N-K)I^{nc})][2N+\gamma(N-K-1)] = [2N+\gamma(N-1)]NI^c$$

$$\left\{N+K-3-\frac{2\gamma N}{(2N-\gamma)} - \frac{(N-K-1)\gamma K}{[2N+\gamma(N-K-1)]}\right\} - \frac{N^2}{(2N-\gamma)^2}$$

$$\left[-(\alpha-c_0) + K(N-1)I^c - (K^2 I^c + (N-K)I^{nc})\right][2N+\gamma(N-K-1)].$$

$$\frac{\partial f_2}{\partial \gamma} = 2[2N+\gamma(N-1)](N-1)NI^{nc} - \frac{2}{(2N-\gamma)}[2N+\gamma(N-1)]^2 NI^{nc}$$

$$-\frac{N-2}{[2N+\gamma(N-2)]}[2N+\gamma(N-1)]^2 NI^{nc} - \frac{N^2}{(2N-\gamma)^2}$$

$$\left[-(\alpha-c_0) + (N-1)I^{nc} - (K^2 I^c + (N-K)I^{nc})\right][2N+\gamma(N-2)]$$

$$= [2N+\gamma(N-1)]NI^{nc}\left\{N-2-\frac{2\gamma N}{(2N-\gamma)} - \frac{(N-2)\gamma}{[2N+\gamma(N-2)]}\right\}$$

$$-\frac{N^2}{(2N-\gamma)^2}\left[-(\alpha-c_0) + (N-1)I^{nc} - (K^2 I^c + (N-K)I^{nc})\right][2N+\gamma(N-2)].$$

By the implicit function theorem to (A1)–(A2), we immediately have:

$$\frac{\partial I^c}{\partial \gamma} = -\frac{\det\begin{bmatrix}\frac{\partial f_1}{\partial \gamma} & \frac{\partial f_1}{\partial I^{nc}} \\ \frac{\partial f_2}{\partial \gamma} & \frac{\partial f_2}{\partial I^{nc}}\end{bmatrix}}{\det\begin{bmatrix}\frac{\partial f_1}{\partial I^c} & \frac{\partial f_1}{\partial I^{nc}} \\ \frac{\partial f_2}{\partial I^c} & \frac{\partial f_2}{\partial I^{nc}}\end{bmatrix}}$$

$$= -\frac{\det\begin{bmatrix}\frac{\partial f_1}{\partial \gamma} & \frac{\gamma N^2}{(2N-\gamma)^2}(N-K)[2N+\gamma(N-K-1)] \\ \frac{\partial f_2}{\partial \gamma} & \begin{matrix}N(2N+\gamma N-\gamma)^2- \\ \frac{N^2(2N+\gamma K-\gamma)}{(2N-\gamma)^2}[2N+\gamma(N-2)]\end{matrix}\end{bmatrix}}{\det\begin{bmatrix}\frac{\partial f_1}{\partial I^c} & \frac{\partial f_1}{\partial I^{nc}} \\ \frac{\partial f_2}{\partial I^c} & \frac{\partial f_2}{\partial I^{nc}}\end{bmatrix}} < 0.$$

The inequality holds because of:

$$\det\begin{bmatrix}\frac{\partial f_1}{\partial \gamma} & \frac{\gamma N^2}{(2N-\gamma)^2}(N-K)[2N+\gamma(N-K-1)] \\ \frac{\partial f_2}{\partial \gamma} & \begin{matrix}N(2N+\gamma N-\gamma)^2- \\ \frac{N^2(2N+\gamma K-\gamma)}{(2N-\gamma)^2}[2N+\gamma(N-2)]\end{matrix}\end{bmatrix} =$$

$$N^2\{(2N+\gamma N-\gamma)^2-\frac{N(2N+\gamma K-\gamma)}{(2N-\gamma)^2}[2N+\gamma(N-2)]\}[2N+\gamma(N-1)]I^c\times$$
$$\{N+K-3-\frac{2\gamma N}{(2N-\gamma)}-\frac{(N-K-1)\gamma K}{[2N+\gamma(N-K-1)]}\}$$
$$-\{N(2N+\gamma N-\gamma)^2-\frac{N^2(2N+\gamma K-\gamma)}{(2N-\gamma)^2}[2N+\gamma(N-2)]\}\times$$
$$\frac{N^2}{(2N-\gamma)^2}\left[-(\alpha-c_0)+K(N-1)I^c-(K^2I^c+(N-K)I^{nc})\right][2N+\gamma(N-K-1)]$$
$$-\frac{\gamma N^2}{(2N-\gamma)^2}(N-K)[2N+\gamma(N-K-1)][2N+\gamma(N-1)]$$

$$NI^{nc}\{N-2-\frac{2\gamma N}{(2N-\gamma)}-\frac{(N-2)\gamma}{[2N+\gamma(N-2)]}\}+\frac{\gamma N^2}{(2N-\gamma)^2}(N-K)[2N+\gamma(N-K-1)]$$

$$\frac{N^2}{(2N-\gamma)^2}\left[-(\alpha-c_0)+(N-1)I^{nc}-(K^2I^c+(N-K)I^{nc})\right][2N+\gamma(N-2)]>0.$$

The above inequality holds for large N. Similarly, we achieve the relationship $\frac{\partial I^{nc}}{\partial \gamma}>0$. The proof is complete

Proof of Proposition 3. Actually, since K is an integer, (A1) and (A2) are not continuous in$K$. This proposition focuses on the monotonic property of the innovation investment and we regard K as a continuous variable. From (A1) and (A2), by implicit function theorem, we have:

$$\frac{\partial I^c}{\partial K} = -\frac{\det\begin{bmatrix}\frac{\partial f_1}{\partial K} & \frac{\partial f_1}{\partial I^{nc}} \\ \frac{\partial f_2}{\partial K} & \frac{\partial f_2}{\partial I^{nc}}\end{bmatrix}}{\det\begin{bmatrix}\frac{\partial f_1}{\partial I^c} & \frac{\partial f_1}{\partial I^{nc}} \\ \frac{\partial f_2}{\partial I^c} & \frac{\partial f_2}{\partial I^{nc}}\end{bmatrix}} = -\frac{N^2}{(2N-\gamma)^2}\times$$

$$\frac{\det\begin{bmatrix}\begin{array}{c}\gamma\left[(\alpha-c_0)(2N-\gamma)+K(2N+\gamma N-\gamma)I^c-\gamma(K^2I^c+(N-K)I^{nc})\right] \\ -[(2N+\gamma N-\gamma)I^c-\gamma(2KI^c-I^{nc})][2N+\gamma(N-K-1)]\end{array} & \frac{\gamma N^2}{(2N-\gamma)^2}(N-K)[2N+\gamma(N-K-1)] \\ [\gamma(2KI^c-I^{nc})][2N+\gamma(N-2)] & \begin{array}{c}N(2N+\gamma N-\gamma)^2- \\ \frac{N^2(2N+\gamma K-\gamma)}{(2N-\gamma)^2}[2N+\gamma(N-2)]\end{array}\end{bmatrix}}{\det\begin{bmatrix}\frac{\partial f_1}{\partial I^c} & \frac{\partial f_1}{\partial I^{nc}} \\ \frac{\partial f_2}{\partial I^c} & \frac{\partial f_2}{\partial I^{nc}}\end{bmatrix}} < 0.$$

The above inequality comes from sufficiently large N. The total innovative investment of cooperative partners increases in K.

Similarly, we have the relationship:

$$\frac{\partial I^{nc}}{\partial K} = -\frac{\det\begin{bmatrix}\frac{\partial f_1}{\partial I^c} & \frac{\partial f_1}{\partial K} \\ \frac{\partial f_2}{\partial I^c} & \frac{\partial f_2}{\partial K}\end{bmatrix}}{\det\begin{bmatrix}\frac{\partial f_1}{\partial I^c} & \frac{\partial f_1}{\partial I^{nc}} \\ \frac{\partial f_2}{\partial I^c} & \frac{\partial f_2}{\partial I^{nc}}\end{bmatrix}} > 0.$$

Similarly, we have $\frac{\partial I^{c,sw}}{\partial K} < 0$ and $\frac{\partial I^{nc,sw}}{\partial K} > 0$. Moreover, $KI^c > I^{nc}$ implies $K\frac{\partial I^c}{\partial K} + I^c > 0$. By a similar way, we have that the innovative investment of cooperative firms decreases in N, while the innovative investment of firms without cooperative innovation increases in N.

Conclusion is achieved and the proof is complete.

Proof of Proposition 4. By (20) and (21), for $i \in \{1, 2, \ldots, K\}$, the optimal number of cooperatively innovating firms is determined by the following equation.

$$\frac{\partial \pi_i}{\partial K} = 2I^c\left\{\left[\frac{(2N-\gamma)(2N+\gamma N-\gamma)}{2N+\gamma N-\gamma-\gamma K}\right]^2 - N\right\}\frac{\partial I^c}{\partial K} + 2\gamma(I^c)^2\frac{[(2N-\gamma)(2N+\gamma N-\gamma)]^2}{(2N+\gamma N-\gamma-\gamma K)^3}.$$

From Proposition 3, we have:

$$2I^c\{\left[\frac{(2N-\gamma)(2N+\gamma N-\gamma)}{2N+\gamma N-\gamma-\gamma K}\right]^2-N\}\frac{\partial I^c}{\partial K}+2\gamma(I^c)^2\frac{[(2N-\gamma)(2N+\gamma N-\gamma)]^2}{(2N+\gamma N-\gamma-\gamma K)^3}<0.$$

The above inequality holds for large enough N. Because of the hypothesis $K\geq 2,$ the solution is $K^{c,*}=2.$ Actually, the profit function of cooperator is not continuously at $K=1.$ Compared the profit function of $K=1$ with that of $K=2,$ we obtain $K^{c,*}=2.$

For $i\in\{K+1,K+2,\ldots\cdots,N\},$ the optimal number of cooperatively innovating firms is determined by $\frac{\partial\pi_i}{\partial K}=2I^{nc}\{[\frac{(2N-\gamma)(2N+\gamma N-\gamma)}{2N+\gamma N-\gamma}]^2-N\}\frac{\partial I^{nc}}{\partial K}>0.$ The hypothesis $K\geq 2$ implies the solution to be $K^{nc,*}=N-1.$

For the social welfare, we have:

$$\begin{aligned}SW=&\frac{1}{2}\frac{NK}{N-\gamma}\left[(\alpha-c_0)\frac{N-\gamma}{N+(N-1)\gamma}+KI^{c,sw}-\frac{\gamma}{N+(N-1)\gamma}(K^2I^{c,sw}+NI^{nc,sw}-KI^{nc,sw})\right]^2\\&+\frac{1}{2}\frac{N(N-K)}{N-\gamma}\left[(\alpha-c_0)\frac{N-\gamma}{N+(N-1)\gamma}+I^{nc,sw}-\frac{\gamma}{N+(N-1)\gamma}(K^2I^{c,sw}+NI^{nc,sw}-KI^{nc,sw})\right]^2\\&+\frac{\gamma N}{(N-\gamma)^2}\{\frac{K(K-1)}{2}\left[(\alpha-c_0)\frac{N-\gamma}{N+(N-1)\gamma}+KI^{c,sw}-\frac{\gamma}{N+(N-1)\gamma}(K^2I^{c,sw}+NI^{nc,sw}-KI^{nc,sw})\right]^2\\&+\frac{(N-K)(N-K-1)}{2}\left[(\alpha-c_0)\frac{N-\gamma}{N+(N-1)\gamma}+I^{nc,sw}-\frac{\gamma}{N+(N-1)\gamma}(K^2I^{c,sw}+NI^{nc,sw}-KI^{nc,sw})\right]^2\\&K(N-K)\left[(\alpha-c_0)\frac{N-\gamma}{N+(N-1)\gamma}+KI^{c,sw}-\frac{\gamma}{N+(N-1)\gamma}(K^2I^{c,sw}+NI^{nc,sw}-KI^{nc,sw})\right]\times\\&\left[(\alpha-c_0)\frac{N-\gamma}{N+(N-1)\gamma}+I^{nc,sw}-\frac{\gamma}{N+(N-1)\gamma}(K^2I^{c,sw}+NI^{nc,sw}-KI^{nc,sw})\right]\}\\&-NK(I^{c,sw})^2-N(N-K)(I^{nc,sw})^2.\end{aligned}$$

Therefore:

$$\begin{aligned}\frac{\partial SW}{\partial K}=&\frac{1}{2}\frac{N}{N-\gamma}\Big[(\alpha-c_0)\frac{N-\gamma}{N+(N-1)\gamma}+KI^{c,sw}\\&-\frac{\gamma}{N+(N-1)\gamma}(K^2I^{c,sw}+NI^{nc,sw}-KI^{nc,sw})]^2\frac{NK}{N-\gamma}\\&\left[(\alpha-c_0)\frac{N-\gamma}{N+(N-1)\gamma}+KI^{c,sw}-\frac{\gamma}{N+(N-1)\gamma}(K^2I^{c,sw}+NI^{nc,sw}-KI^{nc,sw})\right]\\&\left[I^{c,sw}-\frac{\gamma}{N+(N-1)\gamma}(2KI^{c,sw}-I^{nc,sw})\right]-\frac{1}{2}\frac{N}{N-\gamma}\Big[(\alpha-c_0)\frac{N-\gamma}{N+(N-1)\gamma}+I^{nc,sw}\\&-\frac{\gamma}{N+(N-1)\gamma}(K^2I^{c,sw}+NI^{nc,sw}-KI^{nc,sw})]^2-\frac{N(N-K)}{N-\gamma}\Big[(\alpha-c_0)\frac{N-\gamma}{N+(N-1)\gamma}\\&+I^{nc,sw}-\frac{\gamma}{N+(N-1)\gamma}(K^2I^{c,sw}+NI^{nc,sw}-KI^{nc,sw})]\left[\frac{\gamma}{N+(N-1)\gamma}(2KI^{c,sw}-I^{nc,sw})\right]\\&+\frac{2\gamma N}{(N-\gamma)^2}\{\frac{2K-1}{2}\Big[(\alpha-c_0)\frac{N-\gamma}{N+(N-1)\gamma}+KI^{c,sw}\\&-\frac{\gamma}{N+(N-1)\gamma}(K^2I^{c,sw}+NI^{nc,sw}-KI^{nc,sw})]^2+K(K-1)\Big[(\alpha-c_0)\frac{N-\gamma}{N+(N-1)\gamma}\end{aligned}$$

$$+KI^{c,sw}-\frac{\gamma}{N+(N-1)\gamma}(K^2I^{c,sw}+NI^{nc,sw}-KI^{nc,sw})]\left[I^{c,sw}-\frac{\gamma(2KI^{c,sw}-I^{nc,sw})}{N+(N-1)\gamma}\right]$$

$$-\frac{2(N-K)-1}{2}\left[(\alpha-c_0)\frac{N-\gamma}{N+(N-1)\gamma}+I^{nc,sw}-\frac{\gamma}{N+(N-1)\gamma}(K^2I^{c,sw}+NI^{nc,sw}-KI^{nc,sw})\right]^2$$

$$-(N-K)(N-K-1)\left[(\alpha-c_0)\frac{N-\gamma}{N+(N-1)\gamma}+I^{nc,sw}-\frac{\gamma}{N+(N-1)\gamma}(K^2I^{c,sw}+NI^{nc,sw}-KI^{nc,sw})\right]$$

$$\frac{\gamma(2KI^{c,sw}-I^{nc,sw})}{N+(N-1)\gamma}+(N-2K)\left[(\alpha-c_0)\frac{N-\gamma}{N+(N-1)\gamma}+KI^{c,sw}-\frac{\gamma}{N+(N-1)\gamma}\right.$$

$$(K^2I^{c,sw}+NI^{nc,sw}-KI^{nc,sw})]\times\left[(\alpha-c_0)\frac{N-\gamma}{N+(N-1)\gamma}+I^{nc,sw}-\frac{\gamma}{N+(N-1)\gamma}\right.$$

$$(K^2I^{c,sw}+NI^{nc,sw}-KI^{nc,sw})]-K(N-K)\left[(\alpha-c_0)\frac{N-\gamma}{N+(N-1)\gamma}+KI^{c,sw}\right.$$

$$-\frac{\gamma}{N+(N-1)\gamma}(K^2I^{c,sw}+NI^{nc,sw}-KI^{nc,sw})]\frac{\gamma(2KI^{c,sw}-I^{nc,sw})}{N+(N-1)\gamma}+K(N-K)$$

$$\left[(\alpha-c_0)\frac{N-\gamma}{N+(N-1)\gamma}+I^{nc,sw}-\frac{\gamma}{N+(N-1)\gamma}(K^2I^{c,sw}+NI^{nc,sw}-KI^{nc,sw})\right]$$

$$\left[I^{c,sw}-\frac{\gamma(2KI^{c,sw}-I^{nc,sw})}{N+(N-1)\gamma}\right]\}-NK(I^{c,sw})^2+N(I^{nc,sw})^2.$$

Apparently, $\frac{\partial SW}{\partial K}>0$. We achieve that the socially optimal number of cooperators is $K^{sc,*}$ such that $K^{sc,*}=N$.