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Licensing under Cournot vs Bertrand competition

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ABSTRACT

In this paper we consider, on one hand, a differentiated Cournot model, and, on the other hand, a differentiated Bertrand model, when one of the firms engages in an R&D process that gives an endogenous cost-reducing innovation. The aim of the present paper is two-fold. The first is to study the licensing of the cost-reduction by a per-unit royalty and a fixed-fee in these Cournot and Bertrand models. The second is to do a direct comparison between Cournot model and Bertrand model. We analyse the implications of these types of licensing contracts over the R&D effort, the profits of the firms, the consumer surplus and the social welfare. We show that some previous results for two-part tariff licensing are not robust, in the sense that they can be not true for just either a per-unit royalty contract or a fixed-fee contract. Furthermore, by using comparative static analysis, we conclude that the degree of the differentiation of the goods assumes a great importance in the results. We also discuss the optimal licensing, meaning that which licensing method is preferred, in each of the duopoly models considered.

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1. Introduction

Licensing is one of the methods which allows the technology transfer between firms. This is one of the many reasons that makes the licensing an important phenomenon, because it is seen as a tool for managing the intellectual property of firms in high technology industries. Licensing can be defined as the granting of permission to use intellectual property rights (such as patents, trademarks or technology) under defined conditions. Among time, licensing activity has been the subject of much theoretical inquiry (e.g. Anand & Khanna, 2003;; Chang, Hwang, et al., 2013; Choi, 2001; Wang, 1998, 2002).

As can be found in the literature, technology licensing represents a major economic activity and plays an important role for growth of firms and economy. Getting a new technology by patent licensing is a low risk access to increase the profits.

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Although R&D is a good way to stimulate the growth of the firms' profit, it not only needs to invest a lot of money, but also to spend a lot of time. Many firms have not enough capital or time to engage in R&D activity, so they choose to adopt a new technology through a technology licensing.

There exists a vast literature focusing on the decision of the optimal licensing contract by the licensor (e.g. Cao & Kabiraj, 2018; Chang, Lin et al., 2013; Erkal, 2005; Ferreira, 2011; Fosfuri & Roca, 2004; Kamien et al., 1992; Kitagawa et al., 2018). Also, there exists a lot of studies that reveals two types of licensors, namely, the outsider licensor and the insider licensor. The licensor is an outsider when it is an independent R&D organization and not a competitor of the licensee in the product market. On the other hand, when the licensor competes with the licensee it becomes an insider licensor. Based on this, it has been discussed in the literature about the nature of licensing that should take place between the licensor and licensee(s). The studies of insider and outsider licensors have been done in different models. In the standard models, in a complete information framework, if the licensor happens to be an outsider, it can be said that fixed-fee licensing is optimal to the licensor (e.g. Banerjee & Poddar, 2019; Kamien, 1992; Katz & Shapiro, 1986). The reverse happens when the licensor is an insider that is a competitor, i.e. per-unit royalty licensing is optimal to the licensor (e.g. Kamien & Tauman, 2002; Marjit, 1990; Rockett, 1990; Wang & Yang, 1999).

Kabiraj and Kabiraj (2017) considered an international Cournot model competition, and they showed that a tariff on foreign products can influence the licensing strategy of the foreign firm. They also showed that a tariff can be chosen so as to induce fixed-fee licensing.

Poddar and Sinha (2004) opened up a new avenue of research related to patent licensing. By studying the optimal patent licensing strategy of an outsider licensor as well as of an insider licensor in a new environment, they contradict the existing results in the literature. They introduced the study of patent licensing in a spatial framework, and not in a standard framework of price and quantity competition as it was done before. In this way, two important research areas, patent licensing and competition in a spatial model, were bringed in one platform. So far, two main remarks must be mentioned: on one hand, no study has been done to reconcile the two results above and, on the other hand, in general a new technology is transferred from a firm who is at least as cost efficient as the recipient firm and in many cases it is the more efficient one.

Poddar and Sinha (2010) studied the optimal licensing contract when the new technology is transferred from a firm which is relatively cost-inefficient in the pre-innovation stage compared to the recipient firm and provided a framework to bridge the literature on external and internal licensor. Colombo and Filippini (2015) analysed an optimal two-part licensing scheme based on ad valorem royalties within a differentiated Bertrand duopoly where the innovator is also the downstream producer, and compares it with the optimal two-part per-unit royalty mechanism.

Yang and Nie (2015) investigated the effects of different R&D subsidy strategies under asymmetric competition. They showed that, in the asymmetric duopoly market, subsidising the small firm instead of the large one does help to maximise social welfare in most cases, and it is conducive to enlarge the profits of the industry. Furthermore, if the government intends to stimulate social R&D investment and total outputs, the optimal strategy depends on the cost gap of the asymmetric duopoly. Offering R&D subsidies to the large firm becomes the optimal choice for the authority if the cost gap is large enough.

Fan et al. (2018) showed that per-unit royalty licensing is more profitable if the licensor is more efficient in using the innovation, whereas ad valorem licensing is more profitable if the licensee is more efficient. Hsu et al. (2019) compared, in a Cournot duopoly model, two licensing forms between competitors of different productivity, ad valorem and per-unit royalty licensing. They found that ad valorem royalty licensing is superior to per-unit royalty licensing for the patent-holding firm when the cost-reducing innovation is non-drastic. Yan and Yang (2018) investigated the licensing behavior in a differentiated Bertrand model by considering uncertain R&D outcomes and technology spillover. They showed that, in the case of a non-drastic innovation, fixed-fee licensing is better than royalty licensing when product substitution and technology spillover are both small, while it is royalty licensing otherwise. Furthermore, allowing a two-part tariff licensing, this is superior (equivalent) to royalty licensing when technology spillover is small (large), but always better than fixed-fee licensing for any degree of product substitution and technology spillover. Zou and Chen (2020) examined product innovation licensing in both exclusive and non-exclusive schemes each under unit/revenue royalty and fixed fee in a vertically differentiated Cournot oligopoly, where a quality-leading firm is an internal licensor. They found that, under a non-exclusive licensing, royalty licensing is the optimal policy choice for the licensor if quality difference within firms is small, regardless of whether a unit or revenue royalty scheme is offered. In the case of exclusive licensing, a two-part tariff is optimal. Wang et al. (2020) studied the relationship between privatization and licensing (by public or private firms) with the consideration of either a domestic or a foreign private firm. They showed that, in the case of a domestic private firm, public licensing facilitates privatization, but private licensing hinders privatization. Furthermore, in the case of a foreign private firm, both public and private licensing facilitate privatization.

As can be found in the literature, three types of licensing contract can occur: (i) (per-unit) royalty licensing; (ii) fixed-fee licensing; and (iii) two-part tariff licensing (fixed-fee plus royalty). In the present paper, we analyze the cases of licensing by means of a per-unit royalty and licensing by means of a fixed-fee in a differentiated-good Cournot duopoly, on one hand, and in a differentiated-good Bertrand duopoly, on the other hand, when one of the firms engages in an R&D process that gives an endogenous cost-reducing innovation. So, in our case the licensor is an insider. Also, we analyse, in each case, the social welfare implications. Then, we do a direct comparison of the most used cases of the licensing contracts for these two differentiated-good duopoly models.

Li and Ji (2010) develop a duopoly model where one of the firms engages in an endogenous cost-reducing innovation and licenses its innovation to its rival firm. But, the authors consider only the licensing by a two-part tariff. Our work differs by considering the licensing by means of a per-unit royalty and the licensing by means of a fixed-fee, in the same Cournot and Bertrand models. Furthermore, we do a comparative static analyses, showing how the results depend on the degree of the differentiation of the goods, a parameter that plays an important role in this paper. We also compare the results obtained in this paper and by Li and Ji (2010), for these two cases of duopoly.

Ferreira and Bode (2013) considered a differentiated Stackelberg model, when the leader firm engages itself in an *R&D* process that gives an endogenous cost-reducing innovation. The aim was to study the licensing of the cost-reduction by a two-part tariff. A direct comparison between the Stackelberg duopoly model and the Cournot duopoly model for the case of the two-part tariff licensing was done. By considering the same differentiated Stackelberg duopoly model, when the leader firm engages itself in an *R&D* process that gives an endogenous cost-reducing innovation, Ferreira and Tuns (2012) studied the licensing of the cost-reduction by a per-unit royalty and a fixed-fee licensing (see also Bode et al., 2014).

The remained of the paper is organized as follows. Section 2 lays down the basic framework and derives our main purposes. Section 3 deals with the case of licensing by means of a per-unit royalty and licensing by means of a fixed-fee, in a duopoly market modeled as a Cournot competition. Section 4 deals with the case of licensing by means of a per-unit royalty and licensing by means of a fixed-fee, in a duopoly market modeled as a Bertrand competition. Section 5 yields the main results gained by a direct comparison between the Cournot duopoly model and the Bertrand duopoly model studied in the present paper and by Li and Ji (2010). Conclusions are drawn in Section 6.

2. The basic framework

We consider a duopoly model where two firms, denoted by F_1 and F_2, produce a differentiated good.

The inverse demand functions are given by $p_i = 1-q_i-dq_j$, where:

- p_i represents the price of the good produced by firm F_i, $i=1, 2$;
- q_i and q_j represent, respectively, the outputs of firms F_i and F_j, $i,j = 1,2, i \neq j$;
- d represents the degree of product substitutability, $d \in (0,1)$.

The duopoly market is modeled either as a Cournot or as a Bertrand competition: the firms decide simultaneously the level of their decision variables (respectively, either output quantities or prices).

We recall the basic model of a licensing contract. Initially, both firms have identical unit production cost $c_i = c$, with $i=1, 2$ and $0<c<1$. We consider that one of the two firms can engage in an R&D process in order to improve its technology. This allows a reduction of its production costs by an amount that we call *innovation size*. The cost-reducing innovation creates a new technology that reduces innovating firm's unit cost by the amount of k, while the amount invested in R&D is $k^2/2$. So, the innovation size is endogenous. There are many papers that use this approach to model process innovations (e.g. Li & Ji, 2010; Lin & Saggi, 2002; Qiu, 1997). However, in other papers the innovation size is exogenous (e.g. Filippini, 2005;

Kabiraj, 2005). Furthermore, we assume that only the firm F_1 can engage in process innovation. So, firm F_1 is the licensor and, in case the technology transfer occurs, firm F_2 is the licensee.

We consider the following three stage licensing game. In the first stage, the innovator (firm F_1) decides whether to license the technology, because licensing reduces the marginal cost of its rival firm (firm F_2). If firm F_1 decides to license it, then it charges a payment from the licensee (a royalty rate or a fixed licensing fee). In the second stage, the firm F_2 decides whether to accept or reject the offer made by firm F_1. Then, both firms represent the players either of a Cournot or a Bertrand game. So, in the third stage both firms simultaneously decide their outputs or prices and compete against each other. The game will be solved by using the backward induction.

We will also analyze, in each licensing contract, the consumer surplus CS and the social welfare W, that are, respectively, defined by

$$CS = \frac{q_1^2 + 2dq_1q_2 + q_2^2}{2} \quad \text{and} \quad W = \pi_1 + \pi_2 + CS.$$

3. Cournot competition

In this section we will study the situation when there can exist a technology transfer from firm F_1 (the innovator) to firm F_2, based on a per-unit royalty or a fixed-fee licensing contract, in a differentiated Cournot duopoly model.[1]

We recall that Li and Ji (2010) studied the pre-licensing and licensing by means of a two-part tariff. From their paper we know that, in the pre-licensing equilibrium, two cases appear: non-drastic innovation (for $d \in (0, d_1)$)[2] and drastic innovation (for $d \in (d_1, 1)$),[3] where $d_1 \in (0, 1)$ is such that $d_1^3 - 2d_1^2 - 4d_1 + 4 = 0$. Throughout the paper, all the results for the pre-licensing and licensing by means of a two-part tariff are considered the ones obtained by Li and Ji (2010).

3.1. Per-unit royalty licensing

In case of the per-unit royalty licensing[4], the unitary production costs of firm F_1 and firm F_2 are, respectively, given by $c - k$ and $c - k + r$, where r denotes the per-unit royalty. It is obvious that if $r \geq k$ it is not convenient for firm F_2 to accept the licensing, so the following restriction is imposed: $r < k$. In this situation, the profits of the firms F_1 and F_2 are, respectively, given by

$$\pi_{1,r}^C = (1 - q_{1,r}^C - dq_{2,r}^C - c + k_r^C)q_{1,r}^C - (k_r^C)^2/2 + r^C q_{2,r}^C$$

and

$$\pi_{2,r}^C = (1 - q_{2,r}^C - dq_{1,r}^C - c + k_r^C - r^C)q_{2,r}^C.$$

Standard computations yield the optimal cost reduction, optimal royalty and optimal outputs given, respectively, by

$$k_r^C = \frac{(c-1)(d-2)(d-6)}{7d^2-8d-4}, \qquad r^C = \frac{(c-1)(d-2)(d^2-2d-4)}{7d^2-8d-4},$$
$$q_{1,r}^C = \frac{(c-1)(2-d)(d+4)}{7d^2-8d-4} \quad \text{and} \quad q_{2,r}^C = \frac{4(c-1)(1-d)}{7d^2-8d-4}. \tag{1}$$

Furthermore, we obtain the firms' profits

$$\pi_{1,r}^C = \frac{(1-c)^2(2-d)(d-6)}{2(7d^2-8d-4)}, \qquad \pi_{2,r}^C = \frac{16(1-c)^2(1-d)^2}{(7d^2-8d-4)^2},$$

the consumer surplus

$$CS_r^C = \frac{(1-c)^2(9d^4+12d^3-76d^2+80)}{2(7d^2-8d-4)^2}, \tag{2}$$

and the social welfare

$$W_r^C = \frac{(1-c)^2(d^4+38d^3-94d^2+80)}{(7d^2-8d-4)^2}. \tag{3}$$

By comparing the total profit of the innovator firm F_1 obtained by a royalty licensing with the profit obtained when it does not license, using standard computations, we get that[5]

$$\pi_{1,r}^C - \pi_{1,nl}^C > 0, \ \forall\ d \in (0, d_1), \quad \text{and} \quad \pi_{1,r}^C - \tilde{\pi}_{1,nl}^C > 0, \ \forall\ d \in (d_1, 1). \tag{4}$$

By comparing the total profit of the licensee firm F_2 obtained if it accepts the license by paying a royalty to the innovator firm with the profit obtained when it does not accept the license, we obtain that

$$\pi_{2,r}^C - \pi_{2,nl}^C > 0, \ \forall\ d \in (0, d_1), \quad \text{and} \quad \tilde{\pi}_{2,r}^C - \tilde{\pi}_{2,nl}^C > 0, \ \forall\ d \in (d_1, 1). \tag{5}$$

So, we have the following result.

Theorem 3.1. *A royalty licensing strictly dominates no-licensing.*

We observe that, for the licensee, a royalty licensing is also always better than no-licensing.

3.1.1. Comparative static analysis

Now, we evaluate the effects of the degree of the differentiation of the goods over: (i) the optimal innovation size; (ii) the optimal royalty rate; (iii) the difference between the profits that the firms get in the cases of royalty licensing and no-licensing; (iv) the consumer surplus; and (v) the social welfare.

Let d_2, $0<d_2<1$, be such that $7d_2^4-16d_2^3+20d_2^2-80d_2+64=0$.[6] From (1), it is easy to see that

$$\frac{\partial k_r^C}{\partial d}<0,\ \forall\ d\in(0,1),\ \frac{\partial r^C}{\partial d}<0,\ \forall\ d\in(0,d_2),\quad \text{and}\quad \frac{\partial r^C}{\partial d}>0,\ \forall\ d\in(d_2,1).$$

Furthermore, for the innovator firm, based on (4), standard computations yield that

$$\frac{\partial(\pi_{1,r}^C-\pi_{1,nl}^C)}{\partial d}<0,\ \forall\ d\in(0,d_1),\quad \text{and}\quad \frac{\partial(\tilde{\pi}_{1,r}^C-\tilde{\pi}_{1,nl}^C)}{\partial d}<0,\ \forall\ d\in(d_1,1).$$

Now, based on (2) and (3) we obtain that

$$\frac{\partial CS_r^C}{\partial d}<0,\ \forall\ d\in(0,1),\quad \text{and}\quad \frac{\partial W_r^C}{\partial d}<0,\ \forall\ d\in(0,1).$$

Let $g(d) = 231d^{15}-2898d^{14}+10084d^{13}+3516d^{12}-79608d^{11}+87760d^{10}+225632d^{9}-512544d^{8}-37760d^{7}+994304d^{6}-815616d^{5}-382976d^{4}+753664d^{3}-86016d^{2}-245760d+90112$. Now, let d_3, d_4, $0<d_3<d_4<1$, be such that $g(d_3)=0$ and $g(d_4)=0$.[7] For the licensee firm, from (5), we obtain that

$$\frac{\partial(\pi_{2,r}^C-\pi_{2,nl}^C)}{\partial d}<0,\ \forall\ d\in(0,d_3)\cup(d_4,d_1),$$

and

$$\frac{\partial(\pi_{2,r}^C-\pi_{2,nl}^C)}{\partial d}>0,\ \forall\ d\in(d_3,d_4).$$

Hence, based on the above results, we can state the following.

Theorem 3.2. *If there exists a technology transfer based on a royalty licensing, then:*

i. As the goods become more differentiated, the optimal innovation size becomes higher;
ii. For $d\in(0,d_2)$ (respectively, $d\in(d_2,1)$), as the goods become more differentiated (respectively, more homogenous), the optimal royalty rate increases;
iii. In both non-drastic and drastic innovation cases, as the goods become more differentiated, the innovator firm becomes more interested in licensing its technology;
iv. As the goods become more differentiated, the consumer surplus becomes higher;
v. As the goods become more differentiated, the social welfare becomes higher.

We remark that, in the non-drastic innovation case, i.e. $d\in(0,d_1)$, for $d\in(0,d_3)\cup(d_4,d_1)$ (respectively, $d\in(d_3,d_4)$), as the goods become more differentiated (respectively, more homogenous), the licensee firm becomes more interested in accepting the new technology by a per-unit royalty.

3.2. Fixed-fee licensing

In this subsection, we consider the case when firm F_1 licenses its technology to firm F_2 by means of a fixed-fee only.[8] Let us suppose that firm F_2 accepts the licensing contract by paying a fixed-fee, denoted by f. This entitles it to produce by using the new technology innovation, which generates the same cost reduction as firm F_1. The profit functions for both firms are, respectively, given by:

$$\pi^C_{1,f} = (1-q^C_{1,f}-dq^C_{2,f}-c+k^C_f)q^C_{1,f}-(k^C_f)^2/2+f^C$$

and

$$\pi^C_{2,f} = (1-q^C_{2,f}-dq^C_{1,f}-c+k^C_f)q^C_{2,f}-f^C.$$

In order to determine the maximum fixed-fee that firm F_1 can charge, we need to consider the two cases: (i) non-drastic innovation case; and (ii) drastic innovation case. This fee is such that the firm's F_2 profit equals its no-licensing profit.

i. *Non-drastic innovation case ($d \in (0, d_1)$)*

For $d \in (0, d_1)$, if the firm's F_2 profit equals its no-licensing profit, $\pi^C_{2,f} = \pi^C_{2,nl}$, then the corresponding cost reduction is

$$k^C_f = \frac{2(1-c)}{d^2+4d+2} \tag{6}$$

and the maximum fixed-fee that firm F_1 can charge is

$$f^C = \frac{4(1-c)^2(d^8+2d^7-9d^6-12d^5+16d^4-24d^3-64d^2+32d+48)}{(d^2+4d+2)^2(d^4-8d^2+8)^2}. \tag{7}$$

Hence, we get the same optimal outputs for both firms, given by

$$q^C_{1,f} = q^C_{2,f} = \frac{(1-c)(2+d)}{d^2+4d+2}.$$

Therefore, the firms' profits are, respectively, given by

$$\pi^C_{1,f} = \frac{(1-c)^2 h(d)}{(d^2+4d+2)^2(d^4-8d^2+8)^2}$$

and

$$\pi^C_{2,f} = \frac{(1-c)^2(d^6-4d^5-4d^4+24d^3-32d+16)}{(d^4-8d^2+8)^2},$$

where $h(d) = d^{10} + 4d^9 - 10d^8 - 56d^7 + 12d^6 + 272d^5 + 96d^4 - 608d^3 - 448d^2 + 384d + 320$.

Consumer surplus and social welfare are, respectively, given by

$$CS_f^C = \frac{(1-c)^2(1+d)(2+d)^2}{(d^2+4d+2)^2} \tag{8}$$

and

$$W_f^C = \frac{(1-c)^2(d^3+7d^2+16d+10)}{(d^2+4d+2)^2}. \tag{9}$$

Let d_5, $0<d_5<1$, be such that $3d_5^8 + 4d_5^7 - 40d_5^6 - 32d_5^5 + 152d_5^4 + 16d_5^3 - 240d_5^2 + 96 = 0$.[9] Then, for the innovator firm it is imposed one restrictive condition: it will license its technology if, and only if, its total profit (market profit + fixed-fee) will exceed the profit it makes with no-licensing, i.e. $\pi_{1,f}^C > \pi_{1,nl}^C$. Standard computations yield that this happens for all $d \in (0, d_5)$ and does not happen for any $d \in [d_5, d_1)$. So, in this case it is not always better for the innovator firm to license its technology - it depends on the degree of the differentiation of the goods.

ii. *Drastic innovation case ($d \in (d_1, 1)$)*

For $d \in (d_1, 1)$, if the firm F_2's profit equals its no-licensing profit, i.e. $\tilde{\pi}_{2,f}^C = \tilde{\pi}_{2,nl}^C = 0$, then the corresponding cost reduction is the same as in the non-drastic innovation case, i.e. $\tilde{k}_f^C = k_f^C$. Hence, the maximum fixed-fee that the firm F_1 can charge is

$$\tilde{f}^C = \frac{(1-c)^2(2+d)^2}{(d^2+4d+2)^2}.$$

Then, the firm F_1's profit in the drastic innovation case is

$$\tilde{\pi}_{1,f}^C = \frac{2(1-c)^2(d^2+4d+3)}{(d^2+4d+2)^2}$$

and, obviously, $\tilde{\pi}_{2,f}^C = 0$. Therefore, consumer surplus and social welfare are, respectively, given by

$$\tilde{CS}_f^C = \frac{(1-c)^2(2+d)^2}{2(d^2+4d+2)^2} \quad \text{and} \quad \tilde{W}_f^C = \frac{(1-c)^2(5d^2+20d+16)}{2(d^2+4d+2)^2}.$$

Again, for the innovator firm it is imposed the restrictive condition that it will license its technology if, and only if, its total profit (market profit + fixed-fee) will exceed the profit it makes with no-licensing, i.e. $\tilde{\pi}_{1,f}^C > \tilde{\pi}_{1,nl}^C$. Standard computations yield that this does not happen for any $d \in (d_1, 1)$. So, in the drastic innovation case, the licensor never licenses its technology by a fixed-fee only. Therefore, we can state the following result.

Theorem 3.3.

i. *For* $d \in (0, d_5)$, *fixed-fee licensing strictly dominates no-licensing;*
ii. *For* $d \in (d_5, 1)$, *the licensor firm never licenses its technology by a fixed-fee only.*

3.2.1. Comparative static analysis

We conclude that, for $d \in (d_5, 1)$, the licensor firm never licenses its technology by a fixed-fee only. Therefore, in what follows, we evaluate the effects of the degree of the differentiation of the goods over: (i) the optimal innovation size; (ii) the maximum fixed-fee that can be charged by the innovator firm; (iii) the difference between the profits that the innovator firm gets in the cases of fixed-fee licensing and no-licensing; (iv) the consumer surplus; and (v) the social welfare, only in the non-drastic innovation case and for $d \in (0, d_5)$.

From (6), it is easy to see that $\frac{\partial k_f^C}{\partial d} < 0, \ \forall \ d \in (0, d_5)$. Also, from (7), we get that $\frac{\partial f^C}{\partial d} < 0, \ \forall \ d \in (0, d_5)$.

For the innovator firm, standard computations yield that

$$\frac{\partial(\pi_{1,f}^C - \pi_{1,nl}^C)}{\partial d} < 0, \quad \forall \ d \in (0, d_5).$$

Furthermore, based on (8) and (9), we obtain that $\frac{\partial CS_f^C}{\partial d} < 0, \ \forall \ d \in (0, d_5)$, and $\frac{\partial W_f^C}{\partial d} < 0, \ \forall \ d \in (0, d_5)$.

Hence, we have the following result.

Theorem 3.4. *If there exists a technology transfer based on a fixed-fee licensing contract (i.e.* $d \in (0, d_5)$*), then:*

i. *As the goods become more differentiated, the optimal innovation size becomes higher;*
ii. *As the goods become more differentiated, the maximum fixed-fee that can be charged by the licensor firm increases;*
iii. *As the goods become more differentiated, the licensor firm becomes more interested in licensing its technology by a fixed-fee licensing;*
iv. *As the goods become more differentiated, the consumer surplus becomes higher;*
v. *As the goods become more differentiated, the social welfare becomes higher.*

3.3. Comparison between the different licensing schemes: fixed-fee and per-unit royalty

In the Cournot model, let us assume that there can exist a technology transfer between firms F_1 and F_2. We will do a comparison of the licensing cases previously studied, in order to state in which case it is indicated for the innovator firm to license its technology. In terms of the non-innovator firm, we will conclude which contract is better to accept in the non-drastic innovation case because, obviously, in the drastic innovation case the profit of the leader firm is null.

A. *Non-drastic innovation (i.e.* $d \in (0, d_1)$*)*

For the innovator firm, standard computations yield that

$$\pi^C_{1,f} - \pi^C_{1,r} < 0, \forall\ d \in (0, d_1),$$
$$\frac{\partial(\pi^C_{1,f} - \pi^C_{1,r})}{\partial d} > 0,\ \forall\ d \in (0, d_6), \quad \text{and} \quad \frac{\partial(\pi^C_{1,f} - \pi^C_{1,r})}{\partial d} < 0,\ \forall\ d \in (d_6, d_1),$$

where d_6, $0 < d_6 < d_1$, is such that $12d_6^{20} + 51d_6^{19} - 318d_6^{18} - 2090d_6^{17} + 1352d_6^{16} + 28676d_6^{15} + 19376d_6^{14} - 182728d_6^{13} - 201024d_6^{12} + 621984d_6^{11} + 726688d_6^{10} - 1255744d_6^{9} - 1141632d_6^{8} + 1709568d_6^{7} + 851712d_6^{6} - 1385472d_6^{5} - 457728d_6^{4} + 393216d_6^{3} + 94208d_6^{2} + 16384d_6 + 16384 = 0.$ [10]

For the non-innovator firm, standard computations yield that

$$\pi^C_{2,f} - \pi^C_{2,r} < 0, \forall\ d \in (0, d_1).$$

Therefore, we have the following result.

Theorem 3.5. *In the Cournot model, if the goods are sufficiently differentiated ($d \in (0, d_1)$), then the innovator firm prefers more to license its technology by a royalty contract than by a fixed-fee one. Furthermore, this incentive increases with the differentiation of the goods, if the goods are close to be homogenous; and decreases, if the goods are close to be independent.*

We observe that, for the non-innovator firm it is always better a royalty contract than a fixed-fee one. Furthermore, the incentive of the non-innovator firm to accept the new technology by a royalty contract instead of a fixed-fee decreases with the differentiation of the goods.

B. *Drastic innovation (i.e. $d \in [d_1, 1)$)*

From the fact that $\tilde{\pi}^C_{1,r} = \pi^C_{1,r},\ \forall\ d \in [d_1, 1)$, we conclude that

$$\tilde{\pi}^C_{1,f} - \tilde{\pi}^C_{1,r} < 0,\ \forall\ d \in [d_1, 1), \quad \text{and} \quad \frac{\partial(\tilde{\pi}^C_{1,f} - \tilde{\pi}^C_{1,r})}{\partial d} < 0,\ \forall\ d \in [d_1, 1).$$

Therefore, we have the following result.

Theorem 3.6. *In the Cournot model, if the innovation is drastic, then the innovator firm prefers more to license its technology by a royalty contract than by a fixed-fee one. Furthermore, this incentive increases with the differentiation of the goods.*

4. Bertrand competition

Previously, we studied the case when there can exist a technology transfer from firm F_1 (the innovator) to firm F_2, based on a per-unit royalty or a fixed-fee licensing contract, in a differentiated Cournot duopoly model. Now, we will study the same issue, but in a differentiated Bertrand duopoly model.[11]

We recall that Li and Ji (2010) studied the cases of pre-licensing and licensing by means of a two-part tariff. From their paper we know that, in the pre-licensing equilibrium, two cases appear: non-drastic innovation (for $d \in (0, d_7)$)[12] and drastic innovation (for $d \in (d_7, 1)$), where $d_7 \in (0, 1)$ is such that $4-4d_7-4d_7^2 + d_7^3 + d_7^4 = 0$. Throughout the paper, all the results for the pre-licensing and licensing by means of a two-part tariff are considered the ones obtained by Li and Ji (2010).

4.1. Per-unit royalty licensing

We recall that, in the case of per-unit royalty licensing, the unitary production costs of firm F_1 and firm F_2 are, respectively, given by $c - k$ and $c-k + r$, where r denotes the per-unit royalty. It is obvious that if $r \geq k$ it is not convenient for firm F_2 to accept the licensing, so the following restriction is imposed[13]: $r < k$. The direct demand functions of both firms are given by

$$q_i^B = \frac{1-d-p_i + dp_j}{1 - d^2},$$

with $i, j = 1, 2,\ i \neq j,\ d \in (0, 1)$. In this case, the profits of the firms F_1 and F_2 are, respectively, given by

$$\pi_{1,r}^B = (p_{1,r}^B - c + k_r^B)q_{1,r}^B - (k_r^B)^2/2 + r^B q_{2,r}^B$$

and

$$\pi_{2,r}^B = (p_{2,r}^B - c + k_r^B - r^B)q_{2,r}^B.$$

Standard computations yield the optimal cost reduction, optimal royalty and optimal prices given, respectively, by

$$k_r^B = \frac{(1-c)(2 + d)(d^2 - d + 6)}{d^3 + d^2 + 12d + 4}, \tag{10}$$

$$\begin{aligned} r^B &= \frac{(1-c)(1 + d)(2 + d)(d^2 - 2d + 4)}{d^3 + d^2 + 12d + 4}, \\ p_{1,r}^B &= \frac{c(3d^3 + d^2 + 6d + 8) - 2(d^3 - 3d + 2)}{d^3 + d^2 + 12d + 4} \end{aligned} \tag{11}$$

and

$$p_{2,r}^B = \frac{c(d^4 + d^3 + 4d^2 + 8d + 4) - d(d^3 + 3d - 4)}{d^3 + d^2 + 12d + 4}.$$

Furthermore, we obtain the optimal outputs

$$q_{1,r}^{B} = \frac{(1-c)(d^3+d^2+2d+8)}{d^3+d^2+12d+4}, \qquad q_{2,r}^{B} = \frac{2(1-c)(d^2+2)}{d^3+d^2+12d+4},$$

the firms' profits

$$\pi_{1,r}^{B} = \frac{(1-c)^2(d+2)(d^2-d+6)}{2(d^3+d^2+12d+4)}, \qquad \pi_{2,r}^{B} = \frac{4(1-c)^2(2+d^2)(2-d^2-d^4)}{(d^3+d^2+12d+4)^2},$$

the consumer surplus

$$CS_{r}^{B} = \frac{(1-c)^2(5d^6+6d^5+25d^4+60d^3+52d^2+96d+80)}{2(d^3+d^2+12d+4)^2}, \tag{12}$$

and the social welfare

$$W_{r}^{B} = \frac{(1-c)^2(80+128d+58d^2+46d^3+9d^4+4d^5-d^6)}{(d^3+d^2+12d+4)^2}. \tag{13}$$

By comparing the total profit of the innovator firm F_1 obtained by a royalty licensing with the profit obtained when it does not license, using standard computations, we get that[14]

$$\pi_{1,r}^{B}-\pi_{1,nl}^{B}>0,\ \forall\ d\in(0,d_7), \quad \text{and} \quad \pi_{1,r}^{B}-\tilde{\pi}_{1,nl}^{B}>0,\ \forall\ d\in(d_7,1). \tag{14}$$

By comparing the total profit of the licensee firm F_2 obtained if it accepts the license by paying a royalty to the innovator firm with the profit obtained when it does not accept the license, we obtain that

$$\pi_{2,r}^{B}-\pi_{2,nl}^{B}>0,\ \forall\ d\in(0,d_7), \quad \text{and} \quad \pi_{2,r}^{B}-\tilde{\pi}_{2,nl}^{B}>0,\ \forall\ d\in(d_7,1). \tag{15}$$

So, we have the following result.

Theorem 4.1. *A royalty licensing strictly dominates no-licensing.*

We observe that, for the licensee firm a royalty licensing is also always better than no-licensing.

4.1.1. Comparative static analysis

Now, we evaluate the effects of the degree of the differentiation of the goods over: (i) the optimal innovation size; (ii) the optimal royalty rate; (iii) the difference between the profits that the firms get in the cases of royalty licensing and no-licensing; (iv) the consumer surplus; and (v) the social welfare.

From (10) and (11), it is easy to see that

$$\frac{\partial k_r^B}{\partial d}<0 \quad \text{and} \quad \frac{\partial r^B}{\partial d}<0, \ \forall \ d \in (0,1).$$

Furthermore, for the innovator firm, based on (14), standard computations yield that

$$\frac{\partial(\pi_{1,r}^B-\pi_{1,nl}^B)}{\partial d}<0, \ \forall \ d \in (0,d_7), \quad \text{and} \quad \frac{\partial(\tilde{\pi}_{1,r}^B-\tilde{\pi}_{1,nl}^B)}{\partial d}<0, \ \forall \ d \in (d_7,1).$$

Now, based on (12) and (13), we obtain that

$$\frac{\partial CS_r^B}{\partial d}<0 \quad \text{and} \quad \frac{\partial W_r^B}{\partial d}<0, \ \forall \ d \in (0,1).$$

Let $i(d) = 4d^{25} + 85d^{24}-30d^{23}-1581d^{22} + 5d^{21} + 13707d^{20} + 3423d^{19}-62047d^{18}-26937d^{17} + 130108d^{16} + 39039d^{15}-58380d^{14} + 191128d^{13}-207972d^{12} -683496d^{11} + 577216d^{10} + 839872d^9-1356384d^8-601472d^7 + 2115072d^6 + 351744d^5-1726464d^4-167936d^3 + 638976d^2 + 40960d-90112$. Now, let d_8 and d_9, $0<d_8<d_9<1$, be such that $i(d_8) = 0$ and $i(d_9) = 0$.[15] For the licensee firm, from (15), we obtain that

$$\frac{\partial(\pi_{2,r}^B-\pi_{2,nl}^B)}{\partial d}<0, \ \forall \ d \in (0,d_8) \cup (d_9,d_7),$$

and

$$\frac{\partial(\pi_{2,r}^B-\pi_{2,nl}^B)}{\partial d}>0, \ \forall \ d \in (d_8,d_9).$$

Hence, based on the above, we can state the following.

Theorem 4.2. *If there exists a technology transfer based on a royalty licensing, then:*

i. *As the goods become more differentiated, the optimal innovation size becomes higher;*
ii. *As the goods become more differentiated, the optimal royalty rate increases;*
iii. *In both non-drastic and drastic innovation cases, as the goods become more differentiated, the innovator firm becomes more interested in licensing its technology;*
iv. *As the goods become more differentiated, the consumer surplus becomes higher;*
v. *As the goods become more differentiated, the social welfare becomes higher.*

We remark that, in the non-drastic innovation case (i.e. $d \in (0,d_7)$), for $d \in (0,d_8) \cup (d_9,d_7)$ (respectively, $d \in (d_8,d_9)$), as the goods become more differentiated (respectively, more homogenous), the licensee firm becomes more interested in accepting the new technology by a per-unit royalty.

4.2. Fixed-fee licensing

In this subsection, we consider that the innovator firm F_1 offers a licensing contract by means of a fixed-fee. This entitles firm F_2 to produce by using the new technology innovation, which generates the same cost reduction as firm F_1. In this case, the profits of the firms F_1 and F_2 are, respectively, given by

$$\pi_{1,f}^B = (p_{1,f}^B - c + k_f^B)q_{1,f}^B - (k_f^B)^2/2 + f^B$$

and

$$\pi_{2,f}^B = (p_{2,f}^B - c + k_f^B)q_{2,f}^B - f^B.$$

In order to determine the maximum fixed-fee that the firm F_1 can charge, we need to consider the two cases: (i) non-drastic innovation case; and (ii) drastic innovation case. This fee is such that the firm F_2's profit equals its no-licensing profit.

i. *Non-drastic innovation case ($d \in (0, d_7)$)*

For $d \in (0, d_7)$, if the firm F_2's profit equals its no-licensing profit, $\pi_{2,f}^B = \pi_{2,nl}^B$, then the corresponding cost reduction is

$$k_f^B = \frac{2(1-c)(1-d)}{d^3 - 3d^2 + 2d + 2} \tag{16}$$

and the maximum fixed-fee that firm F_1 can charge is

$$f^B = \frac{4(1-c)^2 j(d)}{(d^3 - 3d^2 + 2d + 2)^2 (d^6 - 7d^4 + 16d^2 - 8)^2}, \tag{17}$$

where $j(d) = d^{14} - 3d^{13} - 8d^{12} + 30d^{11} + 25d^{10} - 133d^9 - 21d^8 + 314d^7 - 57d^6 - 424d^5 + 188d^4 + 280d^3 - 176d^2 - 64d + 48$.

Hence, we get the same optimal outputs and the same optimal prices for both firms, given, respectively, by

$$q_{1,f}^B = q_{2,f}^B = \frac{(1-c)(2-d)}{d^3 - 3d^2 + 2d + 2}$$

and

$$p_{1,f}^B = p_{2,f}^B = \frac{d(d^2 - 2d + 1) - c(d^2 - d - 2)}{d^3 - 3d^2 + 2d + 2}.$$

Therefore, the firms' profits are, respectively, given by

$$\pi_{1,f}^B = \frac{(1-c)^2(1-d)k(d)}{(d^3 - 3d^2 + 2d + 2)^2 (d^6 - 7d^4 + 16d^2 - 8)^2}$$

and

$$\pi^B_{2,f} = \frac{(1-c)^2 l(d)}{(d^6-7d^4+16d^2-8)^2},$$

where $k(d) = d^{15}-3d^{14}-16d^{13}+52d^{12}+93d^{11}-351d^{10}-258d^9+1234d^8+324d^7-2404d^6-112d^5+2608d^4-96d^3-1472d^2+64d+320$ and $l(d) = 16-32d-32d^2+72d^3+32d^4-56d^5-23d^6+18d^7+8d^8-2d^9-d^{10}$.

Consumer surplus and social welfare are, respectively, given by

$$CS^B_f = \frac{(1-c)^2(1+d)(2-d)^2}{(d^3-3d^2+2d+2)^2} \tag{18}$$

and

$$W^B_f = \frac{(1-c)^2(10-4d-11d^2+9d^3-2d^4)}{(d^3-3d^2+2d+2)^2}. \tag{19}$$

Let d_{10}, $0<d_{10}<1$, be such that $2d_{10}^{14}-4d_{10}^{13}-23d_{10}^{12}+48d_{10}^{11}+120d_{10}^{10}-272d_{10}^{9}-303d_{10}^{8}+820d_{10}^{7}+292d_{10}^{6}-1304d_{10}^{5}+176d_{10}^{4}+880\ d_{10}^{3}-336d_{10}^{2}-192d_{10}+96=0.$[16] Then, for the innovator firm it is imposed one restrictive condition: it will license its technology if, and only if, its total profit (market profit + fixed-fee) will exceed the profit it makes with no-licensing, i.e. $\pi^B_{1,f}>\pi^B_{1,nl}$. Standard computations yield that this happens for all $d \in (0, d_{10})$ and does not happen for any $d \in (d_{10}, d_7)$. So, in this case it is not always better for the innovator firm to licensee its technology - it depends on the degree of the differentiation of the goods.

i. *Drastic innovation case ($d \in (d_7, 1)$)*

For $d \in (d_7, 1)$, if the firm F_2's profit equals its no-licensing profit, i.e. $\tilde{\pi}^B_{2,f} = \tilde{\pi}^B_{2,nl} = 0$, then the corresponding cost reduction is

$$\tilde{k}^B_f = \frac{4(1-c)(1-d)}{d(d^2-3d+4)}, \tag{20}$$

and the maximum fixed-fee that the firm F_1 can charge is

$$\tilde{f}^B = \frac{(1-c)^2(1+d)(1-d)(2-d)^2}{d^2(d^2-3d+4)^2}. \tag{21}$$

Hence, we get the optimal output and optimal price for firm F_1, given, respectively, by

$$\tilde{q}^B_{1,f} = \frac{(1-c)(2-d)}{d(d^2-3d+4)}$$

and

$$\tilde{p}^B_{1,f} = \frac{c(2+d-d^2)+d^3-2d^2+3d-2}{d(d^2-3d+4)}.$$

Furthermore, we get that the firm F_1 profit in the drastic innovation case is

$$\tilde{\pi}^B_{1,f} = \frac{2(1-d)(1-c)^2}{d(d^2-3d+4)}$$

and, obviously, $\tilde{\pi}^B_{2,f} = 0$. Therefore, consumer surplus and social welfare are, respectively, given by

$$\tilde{CS}^B_f = \frac{(1-c)^2(2-d)^2}{2d^2(d^2-3d+4)^2} \tag{22}$$

and

$$\tilde{W}^B_f = \frac{(1-c)^2(4+12d-27d^2+16d^3-4d^4)}{2d^2(d^2-3d+4)^2}. \tag{23}$$

Let d_{11}, $0<d_{11}<1$, be such that $2d_{11}^3-3d_{11}^2+7d_{11}-4=0$.[17] Again, for the innovator firm it is imposed the restrictive condition that it will license its technology if, and only if, its total profit (market profit + fixed-fee) will exceed the profit it makes with no-licensing, i.e. $\tilde{\pi}^B_{1,f}>\tilde{\pi}^B_{1,nl}$. Standard computations yield that this happens for all $d \in (d_7, d_{11})$ and does not happen for any $d \in (d_{11}, 1)$. So, in this case it is not always better for the innovator firm to licensee its technology - it depends on the degree of the differentiation of the goods.

Therefore, we can state the following result.

Theorem 4.3.

i. *For $d \in (0, d_{10}) \cup (d_7, d_{11})$, a fixed-fee licensing strictly dominates no-licensing;*
ii. *For $d \in (d_{10}, d_7) \cup (d_{11}, 1)$, the licensor firm never licenses its technology by a fixed-fee only.*

4.2.1. Comparative static analysis

Now, we evaluate the effects of the degree of the differentiation of the goods over: (i) the optimal innovation size; (ii) the maximum fixed-fee that can be charged by the innovator firm; (iii) the difference between the profits that the innovator firm gets in the cases of fixed-fee licensing and no-licensing; (iv) the consumer surplus; and (v) the social welfare.

In the non-drastic innovation case ($d \in (0, d_7)$), we conclude that, for $d \in (d_{10}, d_7)$, the licensor firm never licenses its technology by a fixed-fee only. Therefore, we evaluate the effects of the degree of the differentiation of the goods only for $d \in (0, d_{10})$.

From (16), it is easy to see that $\frac{\partial k_f^B}{\partial d} < 0,\ \forall\ d \in (0, d_{10})$. Also, from (17), we get that $\frac{\partial f^B}{\partial d} < 0,\ \forall\ d \in (0, d_{10})$.

For the innovator firm, standard computations yield that

$$\frac{\partial(\pi_{1,f}^B - \pi_{1,nl}^B)}{\partial d} < 0,\ \forall\ d \in (0, d_{10}).$$

Furthermore, based on (18), we can note that $\frac{\partial CS_f^B}{\partial d} < 0,\ \forall\ d \in (0, d_{10})$. From (19), we conclude that $\frac{\partial W_f^B}{\partial d} < 0,\ \forall\ d \in (0, d_{10})$.

In the drastic innovation case ($d \in (d_7, 1)$), we saw that the licensor firm will license its technology only for $d \in (d_7, d_{11})$. So, we will make the analysis only for $d \in (d_7, d_{11})$. From (20), we easily get that $\frac{\partial \tilde{k}_f^B}{\partial d} < 0,\ \forall\ d \in (d_7, d_{11})$.

Based on (21), we obtain that $\frac{\partial \tilde{f}^B}{\partial d} < 0,\ \forall\ d \in (d_7, d_{11})$.

Furthermore, for the innovator firm, standard computations yield that

$$\frac{\partial(\tilde{\pi}_{1,f}^B - \tilde{\pi}_{1,nl}^B)}{\partial d} < 0,\ \forall\ d \in (d_7, d_{11}).$$

From (22) and (23), we get that

$$\frac{\partial \tilde{CS}_f^{\ B}}{\partial d} < 0 \quad \text{and} \quad \frac{\partial \tilde{W}_f^{\ B}}{\partial d} < 0,\ \forall\ d \in (d_7, d_{11}).$$

Hence, we have the following result.

Theorem 4.4. *If there exists a technology transfer based on a fixed-fee licensing contract (i.e. $d \in (0, d_{10}) \cup (d_7, d_{11})$), then:*

i. *As the goods become more differentiated, the optimal innovation size becomes higher;*
ii. *As the goods become more differentiated, the maximum fixed-fee that can be charged by the licensor firm increases;*
iii. *As the goods become more differentiated, the licensor firm becomes more interested in licensing its technology by a fixed-fee licensing;*
iv. *As the goods become more differentiated, the consumer surplus becomes higher;*
v. *As the goods become more differentiated, the social welfare becomes higher.*

4.3. Comparison between the different licensing schemes: fixed-fee and per-unit royalty

In the Bertrand model, let us assume that there can exist a technology transfer between firms F_1 and F_2. We will do a comparison of the licensing cases previously studied, in order to state in which case it is indicated for the innovator firm to license its technology. In terms of the non-innovator firm, we will conclude which contract is better to accept in the non-drastic innovation case because, obviously, in the drastic innovation case the profit of the leader firm is null.

A. *Non-drastic innovation (i.e. $d \in (0, d_6)$)*

For the innovator firm, standard computations yield that

$$\pi^B_{1,f} - \pi^B_{1,r} < 0, \forall \ d \in (0, d_6), \quad \text{and} \quad \frac{\partial(\pi^B_{1,f} - \pi^B_{1,r})}{\partial d} > 0, \ \forall \ d \in (0, d_6).$$

For the non-innovator firm, standard computations yield that

$$\pi^B_{2,f} - \pi^B_{2,r} > 0, \forall \ d \in (0, d_6).$$

Therefore, we have the following result.

Theorem 4.5. *In the Bertrand model, if the goods are sufficiently differentiated ($d \in (0, d_6)$), then the innovator firm prefers more to license its technology by a royalty contract than by a fixed-fee one. Furthermore, this incentive decreases with the differentiation of the goods.*

We observe that, for the non-innovator firm it is always better a fixed-fee contract than a royalty one. Furthermore, the incentive of the non-innovator firm to accept the new technology by a royalty contract instead of a fixed-fee decreases with the differentiation of the goods.

A. *Drastic innovation (i.e. $d \in [d_6, 1)$)*

From the fact that $\tilde{\pi}^B_{1,r} = \pi^B_{1,r}, \ \forall \ d \in [d_6, 1),$ we conclude that

$$\tilde{\pi}^B_{1,f} - \tilde{\pi}^B_{1,r} < 0, \ \forall \ d \in [d_6, 1), \quad \text{and} \quad \frac{\partial(\tilde{\pi}^B_{1,f} - \tilde{\pi}^B_{1,r})}{\partial d} < 0, \ \forall \ d \in [d_6, 1).$$

Therefore, we have the following result.

Theorem 4.6. *In the Bertrand model, if the innovation is drastic, then the innovator firm prefers more to license its technology by a royalty contract than by a fixed-fee one. Furthermore, this incentive increases with the differentiation of the goods.*

5. Cournot model vs. Bertrand model

In this section we do a direct comparison between the Cournot duopoly model and the Bertrand duopoly model, based, on one hand, on our results obtained in the cases of licensing by means of a per-unit royalty and licensing by means of a fixed-fee, and, on the other hand, on the results obtained by Li and Ji (2010) in the cases of no-licensing and licensing by means of a two-part tariff. We recall that in the Cournot model, the innovation is non-drastic (respectively, drastic) for $d \in (0, d_1)$ (respectively, $d \in [d_1, 1)$), where $d_1 \simeq 0.806$. In the Bertrand model, the innovation is non-drastic (respectively, drastic) for $d \in (0, d_7)$ (respectively, $d \in [d_7, 1)$), where $d_7 \simeq 0.651$.

We begin by comparing the cost-reduction for those two models.

Let d_{12}, d_{13} and d_{14}, $0<d_{12},d_{13},d_{14}<1$, be such that $d_{12}^5-d_{12}^4-8d_{12}^3+4d_{12}^2+16d_{12}-8=0$, $d_{13}^4-4d_{13}^2-8d_{13}+8=0$ and $3d_{14}^3+3d_{14}^2-4=0$.[18] Direct comparison yields the following result.

Theorem 5.1.

i. *If there exists no technology licensing and the goods are sufficiently differentiated ($d \in (0,d_{12})$) or sufficiently homogenous ($d \in (d_{13},1)$) (respectively, in an intermediate level of differentiation ($d \in (d_{12},d_{13})$)), then the innovating firm invests more (respectively, less) in R&D under Cournot competition than under Bertrand competition;*

ii. *If there exists a technology transfer based on a royalty licensing contract, then the innovating firm invests more in R&D under Bertrand competition than under Cournot competition;*

iii. *If there exists a technology transfer based on a fixed-fee licensing contract and the goods are sufficiently differentiated ($d \in (0,d_7)$) or sufficiently homogenous ($d \in (d_{14},1)$) (respectively, in an intermediate level of differentiation ($d \in (d_7,d_{14})$)), then the innovating firm invests more (respectively, less) in R&D under Cournot competition than under Bertrand competition.*

We recall that, from (Li & Ji, 2010), if there exists a technology transfer based on a two-part licensing contract, then the innovating firm invests more in *R&D* under Bertrand competition than under Cournot competition.

We continue by investigating the profits of the innovator firm F_1. Letting d_{15}, $0<d_{15}<1$, be such that $2d_{15}^5-3d_{15}^4-8d_{15}^3+16d_{15}-8=0$,[19] we get the following result.

Theorem 5.2.

i. *If there exists no technology licensing and the goods are sufficiently differentiated ($d \in (0,d_{15})$) (respectively, sufficiently homogenous ($d \in (d_{15},1)$)), then the profit of the innovator firm is higher (respectively, lower) under Cournot competition than under Bertrand competition;*

ii. *If there exists a technology transfer based on a royalty licensing contract, then the profit of the innovator firm is higher under Bertrand competition than under Cournot competition;*

iii. *If there exists a technology transfer based on a fixed-fee licensing contract, then the profit of the innovator firm is higher under Cournot competition than under Bertrand competition.*

We recall that, based on (Li & Ji, 2010), if there exists a technology transfer based on a two-part licensing contract, then the profit of the innovator firm is higher under Cournot competition than under Bertrand competition.

Furthermore, we make a direct comparison of the consumer surplus for those two models. We obtain the following result.

Theorem 5.3.

i. *If there exists no technology licensing and the goods are sufficiently differentiated ($d \in (0,d_1)$) (respectively, sufficiently homogenous ($d \in (d_1,1)$)), then the* *consumer surplus is higher under Bertrand than under Cournot competition (respectively, the same in both models);*

ii. *If there exists a technology transfer based on a royalty licensing contract, then the consumer surplus is higher under Cournot than under Bertrand competition;*

iii. *If there exists a technology transfer based on a fixed-fee licensing contract and the goods are sufficiently differentiated ($d \in (0, d_7)$) or sufficiently homogenous ($d \in (d_1, 1)$) (respectively, in an intermediate level of differentiation ($d \in (d_7, d_1)$)), then the consumer surplus is higher (respectively, lower) under Bertrand than under Cournot competition.*

We recall that, from (Li & Ji, 2010), if there exists a technology transfer based on a two-part licensing contract, then the consumer surplus is higher under Cournot competition than under Bertrand competition.

Comparing now the social welfare for those two models, and letting d_{16}, d_{17}, $0<d_{17}<d_{16}<1$, be such that $25d_{16}^9-134d_{16}^8+11d_{16}^7-210d_{16}^6+548d_{16}^5+1120d_{16}^4-880d_{16}^3-1184d_{16}^2+448d_{16}+256=0$ and $d_{17}^7+3d_{17}^6-6d_{17}^5-24d_{17}^4+28d_{17}^3+60d_{17}^2-8d_{17}-16=0$[20], we get the following result.

Theorem 5.4.

i. *If there exists no technology licensing, then the social welfare is higher under Bertrand competition than under Cournot competition;*

ii. *If there exists a technology transfer based on a royalty licensing contract and the goods are sufficiently differentiated ($d \in (0, d_{16})$) (respectively, sufficiently homogenous ($d \in (d_{16}, 1)$)), then the social welfare is higher (respectively, lower) under Cournot competition than under Bertrand competition;*

iii. *If there exists a technology transfer based on a fixed-fee licensing contract and the goods are sufficiently differentiated ($d \in (0, d_{17})$) (respectively, sufficiently homogenous ($d \in (d_{17}, 1)$)), then the social welfare is higher (respectively, lower) under Bertrand competition than under Cournot competition.*

We recall that, from (Li & Ji, 2010), if there exists a technology transfer based on a two-part licensing contract, then the social welfare is higher under Cournot competition than under Bertrand competition.

6. Conclusions

The present paper studied the cases of licensing by means of a per-unit royalty and licensing by means of a fixed-fee in both differentiated-good Cournot duopoly model and differentiated-good Bertrand duopoly model, when one of the firms engages in an R&D process that gives an endogenous cost-reducing innovation. We note that the innovation can be either non-drastic or drastic, depending on the degree of the differentiation of the goods (see Li & Ji, 2010). We computed explicitly the main variables of these duopoly models: the optimal innovation size; the optimal outputs; the optimal prices; the profits; the consumer surplus; and the social welfare, in both non-drastic and drastic innovation cases. Furthermore, we did a comparative static analysis. We conclude that the degree of the differentiation of the goods represents a great importance in the results. We also discussed the optimal licensing, meaning that which licensing method is preferred, in each of the duopoly models considered.

Finally, we compared the results obtained in this paper and the results obtained by Li and Ji (2010), for these two cases of duopoly. We note that we get different results, depending if there exists no technology licensing; or if there exists technology licensing by means of a per-unit royalty, by means of a fixed-fee or by means of a two-part tariff.

Concerning the innovation size, on one hand, we saw that in the case of licensing by means of a per-unit royalty, Bertrand competition induces higher *R&D* effort than Cournot competition does, the same as in (Li & Ji, 2010) in the case of licensing by a two-part tariff case. On the other hand, in contrast, we saw that if there exists no technology licensing or if there exists a licensing by means of a fixed-fee, the innovating firm invests more in *R&D* either under Cournot or under Bertrand competition, depending on the degree of the differentiation of the goods.

About the profit of the innovator firm, we conclude that in case of licensing by means of a fixed-fee, the profit is higher under Cournot competition, the same as in (Li & Ji, 2010), in the two-part tariff case. In contrast, in the case of licensing by means of a per-unit royalty, the profit is higher under Bertrand competition. In case there exists no technology licensing, then the profit can be higher either under Cournot or under Bertrand competition, depending on the degree of the differentiation of the goods.

Regarding the consumer surplus, we note that in the case of licensing by a per-unit royalty, this is higher under Cournot competition, the same as in case of licensing by a two-part tariff (Li & Ji, 2010). But, in case there exists no technology licensing or there exists licensing by means of a fixed-fee, the consumer surplus can be higher either under Cournot competition or under Bertrand competition, or can be the same, depending on the degree of the differentiation of the goods.

Finally, concerning the social welfare, we conclude that, in contrast to the result got in the case of licensing by a two-part tariff, in case there exists no technology licensing, this is higher under Bertrand competition than under Cournot competition. In the case of licensing by means of a per-unit royalty or by a fixed-fee we saw that the social welfare can be higher either under Cournot or under Bertrand competition, depending on the degree of the differentiation of the goods.

Furthermore, in each of the duopoly models considered, both Cournot and Bertrand models, we analysed the optimal licensing, meaning that which licensing method is preferred.

Notes

1. Throughout the paper, we use the notation superscript C to refer to the Cournot competition and subscript nl to refer to the pre-licensing case.
2. We note that $d_1 \simeq 0.806$.
3. Throughout the paper, we use the $\sim$ notation for the drastic case.
4. Throughout the paper, we use the notation subscript r to refer to the royalty licensing case.
5. We note that in the royalty licensing case the innovation is non-drastic for all $d \in (0, 1)$.
6. We note that $d_2 \simeq 0.918$.

7. We note that $d_3 \simeq 0.652$ and $d_4 \simeq 0.739$.
8. Throughout the paper, we use the notation subscript f to refer to the fixed-fee licensing case.
9. We note that $d_5 \simeq 0.761$.
10. We note that $d_6 \simeq 0.756$.
11. Throughout the paper, we use the notation superscript B to refer to the Bertrand competition.
12. We note that $d_7 \simeq 0.651$.
13. In the literature, the commonly adopted constraint on licensing is the fixed fee to be positive. If firms compete in quantities, then this constraint will be equivalent to $r < k$. However, if firms compete in prices, then there may be a variance in these two constraints. Although this study adopts the constraint $r < k$, it should be noted that the rationale supporting the findings of this study also stands for the alternative constraint.
14. We note that in the royalty licensing case the innovation is non-drastic for all $d \in (0, 1)$.
15. We note that $d_8 \simeq 0.557$ and $d_9 \simeq 0.609$.
16. We note that $d_{10} \simeq 0.641$.
17. We note that $d_{11} \simeq 0.680$.
18. We note that $d_{12} \simeq 0.502, d_{13} \simeq 0.755$ and $d_{14} \simeq 0.849$.
19. We note that $d_{15} \simeq 0.675$.
20. We note that $d_{16} \simeq 0.733$ and $d_{17} \simeq 0.549$.

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