TORQUE METHOD INVESTIGATION OF WOLFROM GEAR TRAINS

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ARTICLE INFO

***Article history*:**
Received: 21.02.2023.
Received in revised form: 02.05.2023.
Accepted: 02.05.2023

Keywords:
Efficiency
Planetary gear train
Speed ratio
Torque method
Wolfrom gear train
3-K planetary gear train

DOI: https://doi.org/10.30765/er.2099

Abstract:

This article considers the Wolfrom planetary gear train, its most used variants and areas of application. It is emphasized that the train is a compound one, and not a simple one (single-carrier). It is demonstrated how a kinematic and power (efficiency) analysis of the gear train under consideration may be performed using the lever analogy and the torque method based on it. Equations for efficiency determination useful in the optimization procedures of Wolfrom planetary gear train have been developed, and propositions for its application have been given.

1 Introduction

Planetary gear trains (PGTs) may be classified as simple (single-carrier) or compound (multi-carrier) [1], [2]. There are many types of simple PGTs but the train with one external and one internal meshing, and one-rim planets has become established due to its indisputable advantages (Figure1a). This PGT is known as 2K-H according to Kudryavtsev’s classification [3] in many countries. According to the more comprehensive and more clear classification by Tkachenko [4], the designation $\overline{\mathrm{AI}}$ is used in this paper for such PGTs. It stands for one external meshing (A), one internal meshing (I), and one-rim planet (overlining). This PGT has three external shafts (connected to sun gear 1, ring gear 3, and carrier H), the torques of which are in a constant ratio determined by the ratio of the teeth numbers of central gears (z_3/z_1). This ratio is the same as the ratio of forces acting upon a lever (Figure 1c). Two of those torques are unidirectional (T_1 and T_3), while the third acts in opposite direction (T_H) and is equal to the sum of both unidirectional ones. This torque is called the summation torque ($T_H \equiv T_\Sigma$), while the other torques are called difference torques, the smaller torque $T_1 \equiv T_{Dmin}$ and the greater torque $T_3 \equiv T_{Dmax}$. The ratio $t = T_{Dmax}/T_{Dmin}$ is the so-called *torque ratio* of the PGT [1], [5], and this ratio is shown for the $\overline{\mathrm{AI}}$-PGT in Figure 1. Component PGTs of complex multi-carrier PGTs may be conveniently depicted with Wolf-Arnaudov symbols for research purposes. In this case a PGT is presented as circle with three external shafts [6] depicted with different lines according to the values of their torques – the sun gear shaft (with the smallest torque) with a thin line, the ring gear shaft with a thick line, and the carrier shaft (with the greatest torque) with a double line (Figure1b) [7]. Depending on the basic speed ratio i_0 (operation with fixed carrier) PGTs are *positive* ($i_0 > 0$) and *negative* ($i_0 < 0$).

The $\overline{\mathrm{AI}}$-PGT is a positive PGT with maximal speed ratio when operating in fixed ring gear conditions $i_{1H(3)} = i_0 + 1$. The maximum obtainable speed ratio of $\overline{\mathrm{AI}}$ with three planets (most common configuration due to good load sharing conditions between planets) is 13, and a compound PGT is required to obtain a greater speed ratio. A compound (multi-carrier) PGT is obtained by connecting two or more simple PGTs together.

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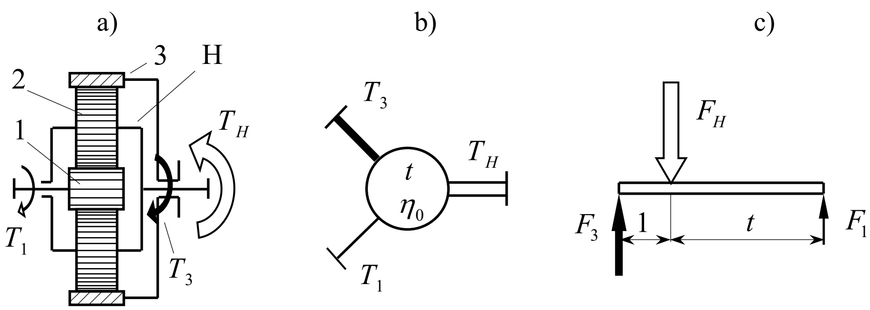
Most transmission ratios in industrial machinery can be covered by two-carrier PGTs [8], while three-carrier PGTs can be used to cover almost all other applications. A special group of so called reduced PGTs in which the two carriers are merged (reduced) into a single carrier can be recognized between the numerous types of two-carrier PGTs. The capability of these PGTs to be built with only one carrier, or even with no carrier in some cases, misguides some authors to classify it as a simple PGT, which is completely wrong. The most commonly used reduced PGTs are the Wolfrom and Ravigneaux gear trains. Due to its large transmission ratio and compactness, the Wolfrom PGT [9] is used in some special cases in which its low efficiency is not a problem – in low-power transmissions in robotics, for example [10], [11], or in mechanisms with non-continuous operation (travel mechanism of handling machines, for example) [1], etc. [12], [13]. It can be concluded that the area of application of this interesting gear train will be broadened due to improvements in contemporary technologies for gear manufacturing [14] and advances in the determination of meshing parameters [15], [16].

This article intends to present a simple and clear way for the investigation of the transmission ratio and efficiency of Wolfrom PGTs by means of the torque method.



$$\eta_0 = \eta_{13(H)} \approx \eta_{31(H)} = 1$$

$$t = \frac{T_3}{T_1} = \frac{F_3}{F_1} > +1$$

$$T_1 < T_3 < |T_H| \qquad F_1 < F_3 < |F_H|$$

$$T_1 : T_3 : T_H = +1 : +t : -(1+t) \qquad F_1 : F_3 : F_H = +1 : +t : -(1+t)$$

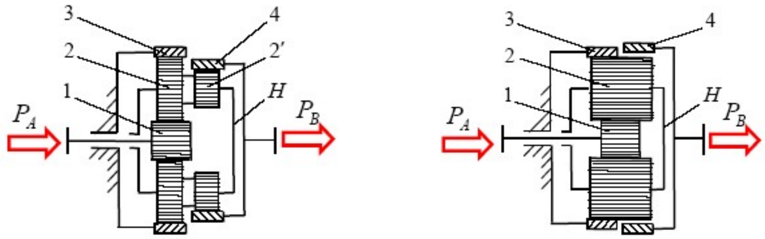
$$\sum T_i = T_1 + T_3 + T_H = 0 \qquad \sum F_i = F_1 + F_3 + F_H = 0$$

Figure 1. Simple $\overline{\text{AI}}$-PGT with its external torques (a), symbol of Wolf-Arnaudov (b) and a lever analogy (c) 1 – sun gear, 2 – planets, 3 – ring gear, H – carrier.

2 Wolfrom PGT arrangement

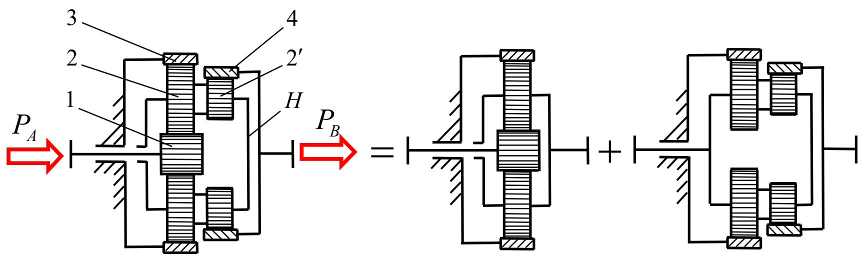
The Wolfrom PGT (Figure 2) is a PGT with three central gears – one with external gearing (1) and two with internal gearing (3 and 4) – which are in mesh with planets 2. The planets may be with two (Figure 2a) or with one (Figure 2b) rim. The planet carrier has no external shaft and its only purpose is to carry the planets. There are even some arrangements without the carrier – central gears hold the planets [1], [16]. One-rim planets are easier for machining and make the PGT more compact. However, this results in the most complex case of meshing in PGTs – a planet having to mesh with three gears [17], [18].

The choice of the meshing parameters of all four gears is a complex procedure, the optimization of which is presented in [19]. This PGT is known as 3K in countries using Russian-based nomenclature because of its three central gear wheels (in Russian wheel – колесо). Even though it is commonly built as single carrier, the Wolfrom PGT is a compound PGT which consists of two simple (single-carrier) PGT as shown in Figure 3 (for an AII train with two-rim planets) and Figure 4 (for an $\overline{\text{AII}}$ train with one-rim planets).



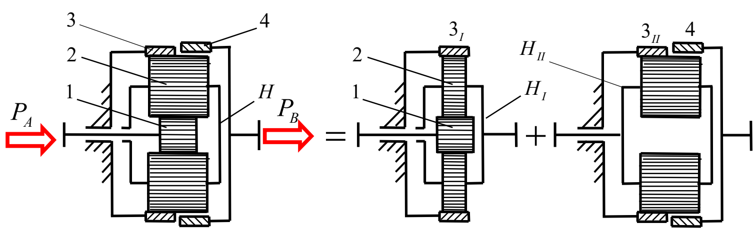
a) b)

Figure 2. Wolfrom PGT with two-rim planets (a) and one-rim planets (b).



$$i_{14(3)} = \left(1+\frac{z_3}{z_1}\right)\frac{1}{1-\dfrac{z_2'}{z_4}\cdot\dfrac{z_3}{z_2}} \qquad i_{1H(3)} = 1+\frac{z_3}{z_1} \qquad i_{H4(3)} = \frac{1}{1-\dfrac{z_2'}{z_4}\cdot\dfrac{z_3}{z_2}}$$

Figure 3. Wolfrom PGT with two-rim planets as a compound PGT.



$$i_{14(3)} = \left(1+\frac{z_3}{z_1}\right)\frac{1}{1-\dfrac{z_3}{z_4}} \qquad i_{1H(3)} = 1+\frac{z_3}{z_1} \qquad i_{H4(3)} = \frac{1}{1-\dfrac{z_3}{z_4}}$$

Figure 4. Wolfrom PGT with one-rim planets as a compound PGT.

The first component PGTs is $\overline{\text{AI}}$-PGT (a negative train with high efficiency but average transmission ratio) while the second one is II-PGT (a positive train with high transmission ratio but low efficiency). The result is a PGT with a high transmission ratio and acceptable efficiency. The corresponding speed ratios of the gear trains are shown in Figure 3 and Figure 4.

The first index shows the input shaft, the second index – the output shaft, and the index in brakes – fixed shaft. For example, $i_{14(3)}$ means input shaft – sun gear 1, output shaft – ring gear 3, and fixed shaft – ring gear 3. z_1, z_3, and z_4 are the teeth numbers of corresponding central gears. z_2 and z'_2 are the teeth numbers of both rims of a two-rim planet (z_2 – of an one-rim planets, respectively). Generally speaking, every three shaft PGT has six operating modes with F = 1 degree of freedom (three as a reducer and three as multiplier) and six operating modes with F = 2 degrees of freedom (as a summation or as a division differential) [1]. Despite this, a Wolfrom PGT is usually operated as shown in Figure 2 (as a reducer) – input through sun gear 2, output through ring gear 4, and fixed ring gear 3.

3 Speed ratio determination

In comparison with the classical methods for PGT kinematic analysis of Willis and Kutzbach, the torque method (based on the lever analogy, see Figure 1) possesses some advantages [1], [5]:

- It has more applications, not only determination of the speed ratio, as in Willis and Kutzbach, but determination of the internal power flows in size and direction and using this to define the efficiency; also, determination of the loads acting on the individual gear train elements.
- The method is simple, particularly clear, and easy to use.
- It combines the precision of the analytical method with the clarity of the graphic method.
- It allows for easy checking of the calculations by the sum of the external torques.
- It breaks the designer's dependence on literary sources, so one can act independently.

For torque method purposes, the Wolfrom PGT must be considered a compound two-carrier PGT (Figure 3 and Figure 4). A breakdown into simple PGTs with Wolf-Arnaudov symbols will be given in Figure 5. The $\overline{\mathrm{AI}}$ -PGT is considered to be the first component PGT, while the second is either the II-PGT (Figure 3) or $\overline{\mathrm{II}}$ -PGT (Figure 4). For each of them a torque ratio is determined:

$$t = \frac{T_{D\max}}{T_{D\min}} > 1, \tag{1}$$

Where:

$T_{D\min}$ is the smaller of difference (unidirectional) external ideal (without losses considering) torques (for $\overline{\mathrm{AI}}$ -PGT it is T_1);

$T_{D\max}$ is the bigger of difference external ideal torques (for $\overline{\mathrm{AI}}$-PGT it is T_3).

For the first component train, the torque ratio according to Eq. (1) is

$$t_I = \frac{T_3}{T_1} = \frac{z_3}{z_1}. \tag{2}$$

To determine $T_{D\min}$ and $T_{D\max}$ in the second component gear some investigation is needed.

From the planet equilibrium, after considering the tangential forces in mesh and gears diameters, the torque ratio is determined as follows:

For II-PGT:

$$t_{II} = \frac{T_3}{T_H} = \frac{1}{i_0 - 1} = \frac{1}{\frac{z_2}{z_3} \cdot \frac{z_4}{z_2'} - 1}; \tag{3}$$

For $\overline{II}$ -PGT:

$$t_{II} = \frac{T_1}{T_H} = \frac{1}{i_0 - 1} = \frac{1}{\frac{z_4}{z_3} - 1}. \tag{4}$$

The external ideal torques in both cases are in the following ratios:

For II-PGT:

$$T_H \equiv T_{D\min} < T_3 \equiv T_{D\max} < |T_1| \equiv |T_\Sigma| = 1 < t < |-(t+1)|; \quad (5)$$

For $\overline{II}$ -PGT:

$$T_H \equiv T_{D\min} < T_1 \equiv T_{D\max} < |T_3| \equiv |T_\Sigma| = 1 < t < |-(t+1)|. \quad (6)$$

An advantage of the torque method is demonstrated by the equality of the right parts of both equations – after determining the torque ratio t of different PGTs the same formula (procedure, software) may be reused for their analysis.

The structural scheme of a Wolfrom PGT as compound gear train is shown in Figure 5. The component PGTs are depicted by Wolf-Arnaudov symbols with coupled carriers on internal shafts. The fixed gear 3 behaves like a compound external shaft with reactive torque T_C. The aim of the torque method is to determine the ratio of external torques in relation to torque ratios t_I and t_{II} of the component PGTs. This can be simplified by choosing a value of +1 for one of the smaller torques $T_{D\min}$ and calculating all other torques in the gear train step by step. This is performed in Figure 6 beginning from the sun gear 1 being the input shaft. The sequence of calculations is given by the numbers in circles.

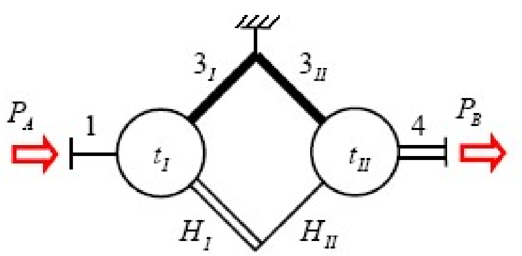


Figure 5. Structural scheme of Wolfrom PGT as compound one.

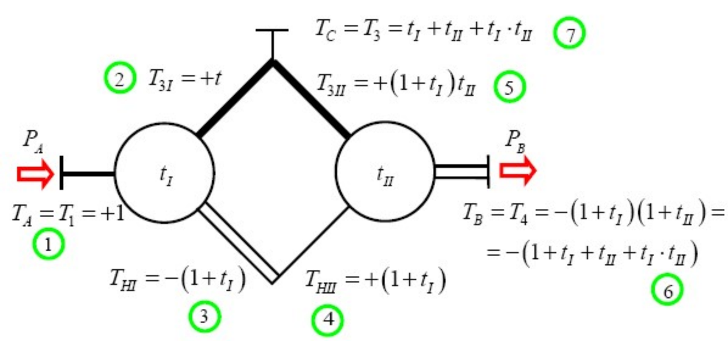


Figure 6. Ideal torques determination in Wolfrom PGT by torque method.

The calculations may be checked as follows:

$$\sum T_i = T_A + T_B + T_C = 1 - 1 - t_I - t_{II} - t_I \cdot t_{II} + t_I + t_{II} + t_I \cdot t_{II} = 0. \quad (7)$$

As it is known [1], [5], [7], [8], the transmission ratio i_k can be determined by ideal external torques (T_A - an input torque and T_B - an output torque) as follows:

$$i_k = -\frac{T_B}{T_A}. \tag{8}$$

For the PGT in question (Figure 6) this means

$$i_k = -\frac{T_B}{T_A} = -\frac{-\left(1+t_I+t_{II}+t_I\cdot t_{II}\right)}{+1} = \left(1+t_I+t_{II}+t_I\cdot t_{II}\right). \tag{8a}$$

Obviously, this is a high value, especially when the second compound PGT is $\overline{\text{II}}$ -PGT with t_{II} by Eq. (4).

4 Efficiency determination

The torque method allows determination of the efficiency η by means of real external torques (T'_A - an input torque and T'_B - an output torque, determined considering the losses) [1], [5], [7], [8] through *torque transformation* i_T

$$i_T = \frac{T'_B}{T'_A}, \tag{9}$$

as follows:

$$\eta = -\frac{i_T}{i_k} = \frac{\frac{T'_B}{T'_A}}{\frac{T_B}{T_A}}. \tag{10}$$

The real torques on all shafts of a compound PGT may be determined in the same sequence as the ideal ones (Figure 6). Their values depend on the basic efficiencies η_{0I} and η_{0II} as well as on the direction of relative (rolling) powers $P_{rel\,I}$ and $P_{rel\,II}$ of the component PGTs.

The basic efficiency is the train efficiency when operating in locked carrier mode (as pseudo-planetary gear train). It depends on losses occurring during the relative movement of PGT elements with respect to the carrier [1]. The main part of these losses are losses in mesh ψ_z which can be calculated more [20], [21] or less precisely [1], [22], depending on the investigation purpose. For design calculations, structural or optimization analysis, etc., a value of η_0= 0,97 is appropriate for $\overline{\text{AI}}$-PGT [1]. For more precise optimization calculations it is recommended to use the following equations [22], [23]:

$$\psi_z = 0.15\left(\frac{1}{z_1}+\frac{1}{z_2}\right)+0.2\left(\frac{1}{z_2}-\frac{1}{z_3}\right); \tag{11}$$

$$\psi_0 = 1.15\psi_z \quad \text{to} \quad 1.30\psi_z; \tag{12}$$

$$\eta_0 = 1-\psi_0, \tag{13}$$

where the basic losses factor ψ_0 is 15 to 30 % bigger than mesh losses factor ψ_z because of different type of losses (in bearings, seals, oil churning, etc.). In $\overline{\mathrm{AI}}$-PGT the losses factor is very low (a few percent) and the change of teeth numbers in Eq. (11) does not affect very much the value of basic efficiency η_0. While the opposite is true for the $\overline{\mathrm{II}}$ -PGT in which the losses are bigger. Like Eq. (11) the following equation may be used [1], [24]:

$$\psi_z = +0.2\left(\frac{2}{z_2} - \frac{1}{z_3} - \frac{1}{z_4}\right). \tag{14}$$

It is known that in simple PGTs the relative (rolling) power has only two possible directions – from the sun gear to the ring gear or vice versa. In both cases, if the real torque on the input shaft (of relative power) T'_{inp} is known, then the real torque on the output shaft T'_{outp} is equal to the ideal output torque decreased by losses:

$$T'_{outp} = \eta_0 \cdot T_{outp}. \tag{15}$$

If the output (of relative power) torque T'_{outp} is known, then the real input torque T'_{inp} is equal to ideal input toque increased by losses:

$$T'_{inp} = \frac{1}{\eta_0} T_{inp}. \tag{16}$$

Considering the above, the real torques in the PGT in question are determined in Fig. 7. The directions of relative (rolling) power in both simple PGTs are shown by dotted lines. The relative (rolling) power directions are obvious for both component simple PGTs as they work with F = 1 degree of freedom. According to Eq. (9), the torque transformation is equal to:

$$i_T = \frac{T'_B}{T'_A} = \frac{-\left(1+\eta_{0I}\cdot t_I + \eta_{0II}\cdot t_{II} + \eta_{0I}\cdot\eta_{0II}\cdot t_I\cdot t_{II}\right)}{+1} = -\left(1+\eta_{0I}\cdot t_I + \eta_{0II}\cdot t_{II} + \eta_{0I}\cdot\eta_{0II}\cdot t_I\cdot t_{II}\right). \tag{17}$$

According to Eq. (10) Wolfrom PGT efficiency is equal to

$$\eta = -\frac{i_T}{i_k} = \frac{\left(1+\eta_{0I}\cdot t_I + \eta_{0II}\cdot t_{II} + \eta_{0I}\cdot\eta_{0II}\cdot t_I\cdot t_{II}\right)}{\left(1+t_I+t_{II}+t_I\cdot t_{II}\right)}. \tag{18}$$

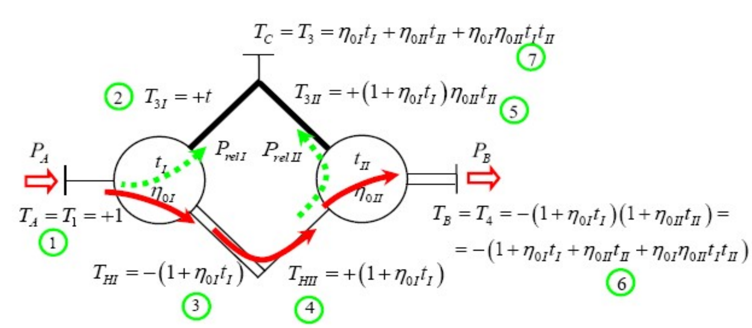


Figure 7. Determination of real torques in Wolfrom PGT by torque method.

5 Discussion

The equations for the speed ratio and efficiency of Wolfrom PGT derived through the torque method are simple, clear, and understandable, and very useful for optimization procedures. By changing the torque ratios t_I and t_{II} (respectively the teeth numbers of gears) of component PGTs it is possible to search for their optimal values by one or more parameters (lower overall dimensions and higher efficiency, for example). The basics of such procedures for other types of compound PGTs are given in [22], [25], and have been used to develop more detailed optimization procedures in [26] - [30].

The existing software for kinematic optimization of Wolfrom PGTs [19] may be improved by efficiency checking the calculations based on of Eq. (18). This is very important because the relatively low efficiency of the Wolfrom gear train is one of the main limitations of its application.

This optimization procedure requires the following:

Input data: Number of planets k, teeth number of sun gear z_1, desired transmission ratio i_k, gear module m, range of change of the modification coefficient of the planets from $x_{2\min}$ to $x_{2\max}$, coefficient of deviation of the operating center distance from the reference center distance $K_{\alpha w}$.

Calculation procedure: Consists of three stages – from teeth numbers of planets and ring gears and calculation to interference check and determination of contact ratios ($\varepsilon_{\alpha 12}$, $\varepsilon_{\alpha 23}$, and $\varepsilon_{\alpha 24}$).

This allows for more precise determination of mesh losses – for example, by following the equation for external and internal meshings [3], [31]:

$$\psi_{z12(H)} = \mu_z \frac{\pi}{z_1} \cdot \frac{u_{12}+1}{u_{12}} \left(1-\varepsilon_{\alpha 12}+\varepsilon_{\alpha 1}^2+\varepsilon_{\alpha 2(1)}^2\right) = f\left(\mu_z\right), \tag{19}$$

$$\psi_{z23(H)} = \mu_z \frac{\pi}{z_2} \cdot \frac{u_{23}-1}{u_{23}} \left(1-\varepsilon_{\alpha 23}+\varepsilon_{\alpha 2(3)}^2+\varepsilon_{\alpha 3}^2\right) = f\left(\mu_z\right), \tag{20}$$

$$\psi_{z24(H)} = \mu_z \frac{\pi}{z_2} \cdot \frac{u_{24}-1}{u_{24}} \left(1-\varepsilon_{\alpha 24}+\varepsilon_{\alpha 2(4)}^2+\varepsilon_{\alpha 4}^2\right) = f\left(\mu_z\right), \tag{21}$$

where

μ_z - meshing friction coefficient;

z_1 and z_2 - teeth numbers of gears;

$\varepsilon_{\alpha 12} = \varepsilon_{\alpha 1} + \varepsilon_{\alpha 2(1)}$, $\varepsilon_{\alpha 23} = \varepsilon_{\alpha 2(3)} + \varepsilon_{\alpha 3}$, and $\varepsilon_{\alpha 23} = \varepsilon_{\alpha 2(3)} + \varepsilon_{\alpha 3}$ - transverse contact ratios of external and both internal meshings;

$\varepsilon_{\alpha 2(1)}$ and $\varepsilon_{\alpha 1}$ - transverse contact ratios of in-front-of-pitch-point and beyond-of-pitch-point meshing of external meshing.

$\varepsilon_{\alpha 3}$ and $\varepsilon_{\alpha 2(3)}$ - transverse contact ratios of in-front-of-pitch-point and beyond-of-pitch-point meshing of internal meshing;

$u_{12} = z_2/z_1 > 1$, $u_{23} = z_3/z_2 > 1$, and $u_{24} = z_4/z_2 > 1$ - teeth ratios of external and both internal meshings.

The simplified determination of losses may be the first step in an optimal choice of parameters for Wolfrom PGT design, before more detailed optimization of the chosen arrangement by known methods [16], [20], [21]. The main point of the previously mentioned optimizations must be the determination of the basic efficiency η_{0II}.

6 Numerical example

A numerical example with dates of a Wolfrom PGT developed for travel mechanism of an electrical hoist is presented. The gear train is with one-rim planet (Figure 2b).

Input date:
Module m = 1.25 mm;
Center distance a_w = 30.30 mm;
Gears teeth numbers: z_1 = 18, z_2 = 22, z_3 = 60, and z_4 = 63;

Kinematic calculations:
Torque ratios of component gears by Eq. (2) and (4): t_I = 3.333 and t_{II} = 20.
From Figure 6 values of the ideal external are

$$T_A = +1$$

$$T_B = -(1 + t_I + t_{II} + t_I \cdot t_{II}) = -(1 + 3.333 + 20 + 3.333 \cdot 20) = -90.933$$

$$T_C = +t_I + t_{II} + t_I \cdot t_{II} = 3.333 + 20 + 3.333 \cdot 20 = 89.933$$

Checking the correctness of calculations by Eq. (7)

$$\sum T_i = T_A + T_B + T_C = 1 - 90.933 + 89.933 = 0$$

Kinematic transmission ratio by Eq. (8):

$$i_k = -\frac{T_B}{T_A} = -\frac{-90.933}{+1} = +90.933.$$

Kinematic transmission ratio by known equation in Figure 4 is the same:

$$i_k \equiv i_{14(3)} = \left(1 + \frac{z_3}{z_1}\right)\frac{1}{1 - \frac{z_3}{z_4}} = \left(1 + \frac{60}{18}\right)\frac{1}{1 - \frac{60}{63}} = 4.333 \cdot 21 = 90.933$$

Proposed torque method procedure is appropriate for Wolfrom PGT kinematic investigation.

Efficiency calculations:

The main point of efficiency calculation through torque method is the basic efficiency determination of both component PGTs (η_{0I} and η_{0II}).

For $\overline{\text{AI}}$-PGT the basic efficiency by Eq. (11), (12), and (13) is calculated

$$\psi_{zI} = 0.15\left(\frac{1}{z_1} + \frac{1}{z_2}\right) + 0.2\left(\frac{1}{z_2} - \frac{1}{z_3}\right) = 0.15\left(\frac{1}{18} + \frac{1}{22}\right) + 0.2\left(\frac{1}{22} - \frac{1}{60}\right) = 0.02091;$$

$$\psi_{0I} = 1.20\psi_z = 0.0251;$$

$$\eta_{0I} = 1 - \psi_{0I} = 1 - 0.0251 = 0.9749 \approx 0.97.$$

For $\overline{\text{I}}\overline{\text{I}}$-PGT the basic efficiency by simplified Eq. (14) and Eq. (12 and (13) is calculated

$$\psi_{zII} = +0.2\left(\frac{2}{z_2} - \frac{1}{z_3} - \frac{1}{z_4}\right) = +0.2\left(\frac{2}{22} - \frac{1}{60} - \frac{1}{63}\right) = 0.0117;$$

$\psi_{0II} = 1.30\psi_{zII} = 0.0152$;

$\eta^*_{0II} \equiv \eta_{34(H)} = 1 - \psi_{0II} = 1 - 0.0152 = 0.9848 \approx 0.98$.

To determine the real torques in Figure 7 the basic efficiency with fixed gear 4 [1] have to be used

$$\eta_{0II} \equiv \eta_{H4(3)} = \frac{\frac{z_4}{z_3} - 1}{\frac{z_4}{z_3} - \eta^*_{0II}} = \frac{\frac{63}{60} - 1}{\frac{63}{60} - 0.98} = 0.7143 \approx 0.71 .$$

Torque transformation by Eq. (17):

$$i_T = \frac{T'_B}{T'_A} = -\left(1 + 0.97 \cdot 3.333 + 0.71 \cdot 20 + 0.97 \cdot 0.71 \cdot 3.333 \cdot 20\right) = -64.3417 .$$

Efficiency by Eq. (10):

$$\eta = -\frac{i_T}{i_k} = -\frac{-64.3417}{90.933} = 0.7076 \approx 0.70 .$$

Precise calculation of the same gear train by classical method, considering Eq. (19), (20), and (21) gives a similar result:

$\eta = 0.709$

Similar is the efficiency value determined by a commonly used diagram in [1].

All the above shous that the proposer calculation procedure through torque method gives adequate results for gear train ratio and efficiency.

7 Conclusion

The torque method is a *simple and practical one*, as it combines the *accuracy* of the method of Willis with the *clarity* of the method of Kutzbach, which can be found separately in each of them. Furthermore, in addition to the transmission ratio, the method allows for the definition of efficiency, so that it allows for more possibilities than the methods of Willis and Kutzbach. The method also has other advantages which are particularly useful in the study of complex compound multi-carrier planetary gear trains, especially where other methods are either difficult, or totally impractical to apply. And finally, unlike other methods, the torque method allows for an easy and quick check of the solution by summation of the ideal external torques (7) – an advantage that it is not negligible. The developed equation for the determination of the efficiency will be important in extending the area of applications of Wolfrom PGTs.

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