ISSN 1846-6168 (Print), ISSN 1848-5588 (Online)
https://doi.org/10.31803/tg-20220604112718

Original scientific paper
Received: 2020-06-04, Accepted: 2022-08-10

Quick Review: Uncertainty of Optimization Techniques in Petroleum Reservoir Management

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Abstract: The notable increase in petroleum demand, together with a decline in discovery rates, has highlighted the desire for efficient production of existing oil wells worldwide. Mainly, the productivity of the existing large oil fields makes us consider the principles of managing reservoirs to make the most of extraction. At the same time, many different uncertainties in the course of the developing oil field, including geological, operational, and economic uncertainties, have a detrimental impact on the reservoir's effective production, which is why dealing with uncertainty is crucial for maximizing output. There is a broad variety of studies on managing oil reservoirs under uncertainty information in the literature. In this study a short review of earlier works has been done on optimization strategies and management of uncertainty in reservoir production.

Keywords: management of reservoir; management of uncertainty; optimization approaches

1 INTRODUCTION

For many years, oil has defined the world economy, this trend will most likely continue in the coming years. Demand for oil has increased with the population growth, and this demand needs to be met with proper optimization. Optimization management between the two factors of oil field development and oil production methods should be done that overall demand does not face problems. Proper planning for optimization requires ground and underground, data these data always have uncertainties that should be considered.

Uncertainty is due to incomplete and imprecise knowledge as a result of limited sampling of the subsurface heterogeneities. Well data and seismic data have incomplete coverage and finite resolution. Sub ground and oil Reservoirs are heterogeneous and difficult to predict away from wells or seismic data. Ignoring uncertainty and locking in important model parameters and choices amounts to an assumption of perfect knowledge and is generally an unacceptable approach. One way to reduce the effects of uncertainty on optimizations is to model them. Understanding the (1) sources of uncertainty, (2) methods to represent uncertainty, (3) the formalisms of uncertainty, and (4) uncertainty modeling methods and workflows were essential for the integration of all reservoir information sources and providing good models for decision making in the presence of uncertainty and improvement of optimization.

Geophysical prospecting consists of making a quantitative inference about subsurface properties from geophysical measurements. Due to many ineluctable difficulties, observed data are almost always insufficient to uniquely specify the rock properties of interest. Hence, inevitable uncertainty remains after the estimation. The sources of the uncertainty arise from many factors: inconsistency in data acquisition conditions, insufficient available data as compared to the subsurface complexities, limited resolution, imperfect dependence between observed data and target rock properties, and our limited physical knowledge. While the uncertainty has been identified for a long time, quantitative framework to discuss the uncertainty has not been well established.

In this study we examine several sources of uncertainty in the development of the oil field make the estimation of the future productivity of a reservoir inaccurate. In general, uncertainty in information about the management of reservoirs falls into four categories.

2 LITERATURE REVIEW

2.1 Conceptual Model

Based on field studies the following conceptual model has been proposed to explain the subject.

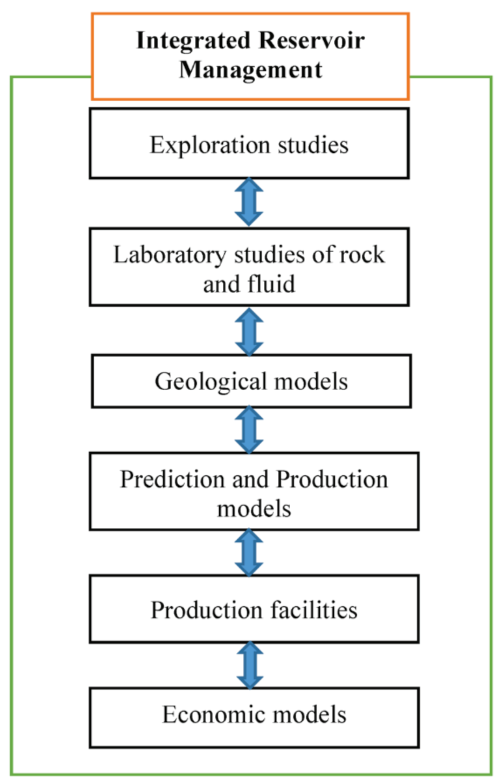

Figure 1 Conceptual model of the present research

2.2 Uncertainty

2.2.1 Engineering Data Uncertainty

Methods such as gas-injection, intermittent water and gas injection (WAG), smart water, and thermal and polymeric methods are commonly used to increase oil recovery. Injecting water into reservoirs is one of the oldest oil recovery methods that has been used for many years. Over time, this method has undergone many alterations and improvements, which has led to its use as the most popular oil recovery method in sandstone and carbonate reservoirs.

Establish a relationship between different types of oil production techniques that consider all factors on oil production for proper optimization is necessary. Uncertainties of Geological, fluid mobility, laboratory, and field heterogeneity are the engineering uncertainties that we will examine in the following.

2.2.2 Uncertainty of Geophysical and Geological Information

Two main types of uncertainty affect our confidence in the results from numerical models: parametric uncertainty and structural uncertainty. Parametric uncertainty arises because of incomplete knowledge of model parameters such as empirical quantities, defined constants, initial conditions, and boundary conditions. Structural uncertainty in models arises because of inaccurate treatment of dynamical, physical, and chemical processes, inexact numerical schemes, and inadequate resolutions. Uncertainty from geology is usually related to seismic data, which is classified as Structural uncertainty.

Seismic data used in the construction of a reservoir system are unclear. These uncertainties relate to data collection, analysis, and statistical explanation. Typically, uncertainties occur as a result of errors in data collection, conflicting explanations, error in converting depth data, error in preliminary explanation, and the wavelength map error that has to do with the crest of the reservoir.

Probably uncertainties are mostly geological. In geological information, uncertainties arise due to sedimentation, rock nature (lithology), rock extension region, and rock physical properties, which leads to the following uncertainties:

- of the reservoir's gross volume
- of the size and direction of sedimentation
- of difference in extension of rock type
- of porosity data
- of the net/gross ratio
- of contact between fluids.

These uncertainties have an impact on the assessment of on-site hydrocarbons and the movement of fluid across the reservoir.

2.2.3 Uncertainty of Dynamic Information

Petroleum reservoirs are very heterogeneous and it is difficult to predict the movement of fluid in them and always cause uncertainties that prevent proper and practical optimization. To reduce these uncertainties, various models and simulation methods should be used to perform the correct optimization to compensate for global oil demand. For example one of this method is grey bootstrap is proposed to resolve some problems about evaluation of the uncertainty in the process of dynamic. This method can evaluate the uncertainty without any prior information about probability distribution of random variables, separating trends with known and unknown law.

At this point, uncertainties of all variables that have an impact on the flow of a fluid through the reservoir are addressed. These variables include absolute, vertical-horizontal, and relative permeability (the measurement of a rock's ability, to transmit fluids), fault transmissibility, rate of injection, productivity index, pores, and skin (around a wellbore), capillary pressure curve, aquifers (water-bearing portions). Such uncertainties influence both the calculation of the reserve and the change of flow rates with time.

2.2.4 Uncertainty of PVT Information

PVT data are known to be the least uncertain data. Lack of certainty of PVT data affects the capacity of process units, hydrocarbon transportation, and marketing. Among the uncertainties in this category are as follows:

- Uncertainty of fluid tests
- Uncertainty of fluid structure
- Uncertainty of the calculation of PVT properties
- Uncertainty of interfacial tension.

2.2.5 Uncertainty of Field Performance Information

The well drilled in Pennsylvania was the first example of a well that showed the earth's layers, but with the drilling of other wells, a variety of different layers of the earth emerged, and engineers concluded that the earth's layer was different in each basin. Therefore, information of different layers and surface data should be used to minimize uncertainties

Furthermore, data on field performance might as well be swayed by the following uncertainties:

- The cost of oil production is usually calculated systematically and accurately; nevertheless, calculations of the water-oil ratio (WOR) and the gas-oil ratio (GOR) are occasionally performed;
- The rate of production fluctuation is normally evened out as it can occur at short durations; the rate of gas is not calculated correctly, especially if parts of it is burned;
- Injection information is less accurate than production information as a result of errors in the calculation stage, loss of fluid at a different time because of leakage in the skin or flow behind the piping system; and
- Pressures gauged at a certain phase of the flow analysis are generally less reliable than those acquired during shut-in.

2.2.6 Uncertainty of Economic Information

In all different parts of human life, the economy is the most effective factor. In the oil industry, drilling a well or using an oil production method or using new technology is only used when it has an economic benefit. The main purpose of optimization is to reduce the risks that may impede the economic benefits of an operation. By accurately recognizing the uncertainties, the most optimal model for an economic operation can be used. For example, the use of nanoparticles to wettability Alteration and reduce sand production is a technique that has received much attention from researchers in the past few years, but since its economic uncertainty is very high, it has not been widely used in industry.

Production management is challenged by some uncertain risks of going up or down in oil prices. Conventional production enhancement approaches concentrate on net present value over time. The lack of reliability of long-term predictions is the key problem of many strategies. Given the time-dependent nature and the instability of oil prices, more often than not, it makes oil risky production [1, 2].

2.2.7 Uncertainty of Political Information

The uncertainty of the political system is a key feature affecting the local investment climate, which firms and entrepreneurs must consider when deciding to start, expand, or contract their businesses. Given the impact of oil on the economy and relations between countries and its key role in determining world powers, it is governed by very complex policies. Investors and entrepreneurs engaged in oil trade in different countries must act in accordance with that country's policy and also consider the impact of factors such as sanctions.

In order to predict the future of their business, these investors must pay close attention to the behavior of countries and world powers in order to avoid destructive global policies. Many countries have a major source of oil revenue, which, given the lifespan of their reservoirs, which are in the semi-finals, is forced to use techniques that reduce uncertainties and are able to meet demand. Therefore, their policy should be formulated in such a way as to give the leading companies investment security so that they do not face any problems.

2.2.8 Uncertainty of Environmental Information

Coping with the above uncertainty would be a serious factor in the production of the oil field. Being less sensitive to uncertainty along with the implementation of calculations are two methods that have been found to be exceedingly contradictory in some research. In this study, however, we present a small report of certain optimization techniques in the development of the petroleum field, based on the data on uncertainty to be able to both reduce uncertainty and sensitivity to its data.

In this study, we present an up-to-date analysis of optimization methods used to solve these problems by first analyzing the cause of uncertainty and then reviewing some practical optimization techniques against technical problems.

3 RESEARCH METHOD

The principles of robust optimization have been introduced primarily in the research papers of engineering design.

The first approach to deal with uncertainties is probably stochastic (linear) programming in the form of a risk factor to manage robustness, whereas robust optimization is known to be more useful in engineering fields, according to the study by Mulvey and Bai [3, 4].

The person who discussed the great implications regarding Robust Optimization in Engineering [5] was Taguchi, who is well recognized for developing a leading design strategy and has gained a lot of interest in the last few years.

Risk management should be addressed, too, when we deal with oil project investments. The recovery of oil is severely affected by geological, financial, and technical risks of exploration & production operations. The major elements of risk reduction [6] are the collection of relevant data and flexibility.

As a result of risk quality dynamics and the amount of risk exposure (RE) that threaten risk management's efficiency, it has been recommended that dynamic risk assessment in oil production and robust optimization programs be examined [7].

Van Essen et al. (2009) introduced the Robust Optimization (RO) approach to minimize the risk of geological uncertainties inherent in the development stage of the oil field, by implementing a series of discoveries that explain a range of possible geological systems to address geological uncertainty data [8].

NPV included a single objective with fixed oil pricing, and it was the related objective function. They also used a standard gradient-based optimization strategy wherein they access the gradients through an adjoin formulation.

Alhuthali et al. (2010) evened up the time of arrival of waterfronts across all producers with the aid of several geological realizations and implemented two optimization parameters of expected value and standard deviation, in linear form with a risk aversion coefficient; actually, they have employed the gradient and the analytical form of Hessian calculation of the objective function [9].

Almeida et al. (2010) attempted, under technological and geological uncertainties, to generate a pro-active approach and specify a project using a genetic algorithm to optimize the single objective NPV [10].

Chen and Hoo (2012) present a link between the Markov chain Monte Carlo (MCMC) and the Kalman filter ensemble (EnKF) in trying to collect changes of certain variables to monitor the amount of water pumped to a reservoir entitled the water-flooding program. They accomplished this by employing an efficient model-based system involving the uncertain parameter changes and a specific low-order model developed from a first principle model [11].

Oil production increased (by 9.0 percent and 8.2 percent with EnKF and MCMC adjustments, respectively), and water production decreased in the final total net present value in the parameter update model. The findings also indicated that maximizing the reservoir's oil production had an effect on the amount of water added to drive the contained oil out, so it is of utmost importance to change the uncertain geological variables (porosity and permeability) to optimize the reservoir's oil production.

In water flooding modeling, Capolei, et al. (2013) also included an open-loop modeling scenario with no input and a closed-loop modeling scenario with geological uncertainty. To bolster the RO technique, they developed an updated robust proposed methodology (modified RO) with larger profits and less risk. The gains were calculated according to the predicted NPV, while the risk was calculated according to the normal NPV deviation [12]. Yasari, et al. (2013) put forth an interesting theory to minimize uncertainty sensitivity while no measurement data were expected to be available [13]. As such, by using a derivative-free Evolutionary Multi-objective Optimization (EMO) technique in the context of an updated Non-dominated Sorting Genetic Algorithm (NSGA), known as NSGA-Il, they established the robust optimization technique to generate several Pareto-optimal alternatives without theoretical deduction of the dynamic reservoir systems. And in 2015, in an effort to obtain the optimal - yet robust - waterflood strategies, they offered multi-objective optimization formulations. Two multi-objective, Pareto-based robust optimization models have been tested to overcome the permeability uncertainties.

The test studies showed that the proposed approach delivered better performance in providing a robust optimal alternative(s based on Pareto (injection policies) against permeability uncertainties that were accurate for the original group of realizations [14].

In contrast with the alternative equivalence of certainty and robust optimization techniques, the mean-variance parameter's potential to help minimize the significant inherent geological uncertainties has been suggested for production optimization.

Through their study, it became clear that maximizing certainty equivalence and robust optimization remain to be risky solutions. Still, the efficiency of mean-variance optimization in risk management and reducing the degree of uncertainty in optimizing efficiency is quite remarkable.

Siraj, et al. (2015) suggested a multi-objective optimization question that takes into account financial and model uncertainties to subsidize the adverse effects, that is, the risk of these uncertainties on production output [15].

Without significantly sacrificing the main priority of economic life-cycle efficiency, they established improved robustness. In order to describe the financial and geological uncertainty domain, they also provided a set of different oil price possibilities and geological system realizations. An average NPV among these groups is the main priority.

Their second goal was to optimize the pace of oil production to minimize risk since the risk of uncertainty grows with time. The multi-objective model was applied separately in a dynamic or lexicographic fashion for both types of uncertainties.

Geological uncertainty greatly affects the optimum well placement strategy and has to be considered in the question of optimization of well placement. A geological realization control mechanism for well placement against geological uncertainty was established by Rahim and Li (2015) [16].

Hanssen and Foss (2015) framed the question of optimization as a two-level stochastic programming question, and the outcome was a technique to run the wells, rather than a single set point acquired by the deterministic problem. The principles of risk theory are super beneficial as a result of high degrees of uncertainty in model-based financial modeling of the water-flooding mechanism in oil reserves. They proposed an inverted risk management system in another study to optimize the lower tail (worst instances) of the distribution of the economic objective function but without seriously sacrificing the upper tail (best instances). Within geological uncertainty, they found the worst robust optimization scenario and Conditional Value-at-Risk (CVaR) measure to optimize the worst problem(s). Also, a deviation method of semi-variance was included in geological and economical uncertainty defined by a set of geological system realizations and a set of different oil price scenarios to optimize the worst cases [17, 19].

Foroud, et al., and Siraj, et al. (2016, 2017) stressed the geological system as a primary cause of uncertainty in the simulation of petroleum reservoirs that can lower the reliability of optimization process outcomes of simulation. The clustering algorithms such as the Kernel K-means Method (KKM) were suggested to pick a generic subset of geological systems and reduce the overall calculation cost during the process of simulation.

The strategy of some researchers is to control uncertainty in the field design. They proposed new methodologies to quantify and minimize risks based on an efficient production plan and decision-making procedure. It is called risk management. In the construction of elaborate petroleum fields, Santos, et al. (2017), for instance, considered the robust risk assessment strategy by introducing resilience to the production mechanism and developing a robust production plan.

The proposed method is based on the performance evaluation of all possibilities of an algorithmically optimized production plan, which aims to further evolve the optimization procedure and minimize risk. Multi-Attribute Utility Theory (MAUT) through multiple objectives (technological and financial indicators) is the essence of this system [22]. They recommended systematic, objective methods in subsequent work to quantify the expected value of flexibility (EVF). In the production process, this approach applies to complex reservoirs with several uncertainties shaping the selection of the production strategy [23].

Silva, et al. proposed (2017) a five-stage approach to estimate the importance of flexibility under exogenous and endogenous uncertainties in oil production operations, and each stage is split into certain secondary stages to identify the specifics of problem analysis and strategy development [24].

Measuring the uncertainties of reservoir aquifer response by conducting a complete simulation of fluids flow on a wide range of models also assumes prohibitive intractable calculation of costs and time. Some methods suggest an estimated solution (flow proxies) to address this challenge [25] or organize the realizations inside a multidimensional sphere depending on the flow results received through an estimated (computationally cheaper) design [26].

In order to measure the uncertainties for a broader class of variables, Bardy 2019 employed both methods and combined the complex performance of the entire group of models [27].

Olalotiti-Lawal provided (2018) a novel technique for calibration of the subsurface system and quantification of uncertainty through Markov chain Monte Carlo (MCMC), wherein proper mixing is improved by contact among parallel Markov chains. This approach substantially increases the convergence of the sampling performance without loss [2].

In 2018, Zanbouri and Salahshoor proposed a novel robust modeling approach to specify a group of robust surrogate systems with unorganized uncertainty for economical performance estimation of an uncertain petroleum reservoir during the water flooding phase, based on geological uncertainty as a serious hurdle in the development of petroleum fields. In this process, the MIMO surrogate system combined with the desired nonlinear NPV objective function was recognized to produce a new updated, robust surrogate system in a configuration form of multi-input single-output (MISO) and allow direct economic quality evaluation calculation [5].

The robust optimization method was developed by Mudhafar et al. in 2018 to evaluate the optimal intervals of gas injection, soaking, and oil processing in diverse reservoirs within geological uncertainties [9].

In his analysis, the robust optimization method within geological uncertainties showed higher recovery of oil and NPV than nominal realization optimization, providing the decision-maker with a degree of freedom to substantially reduce the plan's risk.

The redevelopment of Brownfield is a highly valued answer to manage the drop in production and to optimally position the infill well to optimize recovery and reduce operating costs given the unstable climate of oil price [11].

A new procedure for robust and efficient well placement optimization within geological uncertainty was suggested by Hutahaean 2019. Multi-objective aided background matching, Bayesian posterior estimation, and well positioning optimization were incorporated into the multi-objective environment through various geological systems in their conceptual workflow. The proposed workflow provides robust and reliable optimal decisions in placing the infill well over multiple history match models [12].

In oilfield production and reservoir operations, well placement efficiency is a serious hurdle because reservoir asymmetries generate deeply non-smooth, discontinuity, non-convex cost functions comprising several local optimums. It is also important to run a massive number of simulations on the reservoir.

Several optimizing strategies can be categorized into two classes of approaches to assess the location of the well: gradient-based ex, and derivative-free ex approaches [23, 24]. Optimization techniques have recently been implemented to design and reduce the computational problem of well-placement optimization within uncertainty [27, 3].

In order to find an infill drilling scheme for vertical or horizontal well positioning optimization, Jesmani 2020 used the Simultaneous Perturbation Stochastic Approximation (SPSA) method, a local optimization technique, which would reduce computation significantly [13].

3.1 Propose an Optimal Model of Integrated Reservoir Management

Basically, there are five main factors in integrated reservoir management: well design and management, reservoir properties, reservoir modeling, surface facility design and economy. The first three cases are surveyed below. The integration of these three makes it possible to propose and design efficient and economical ways to enhance production from petroleum reservoirs (as schematically shown in Fig. 2).

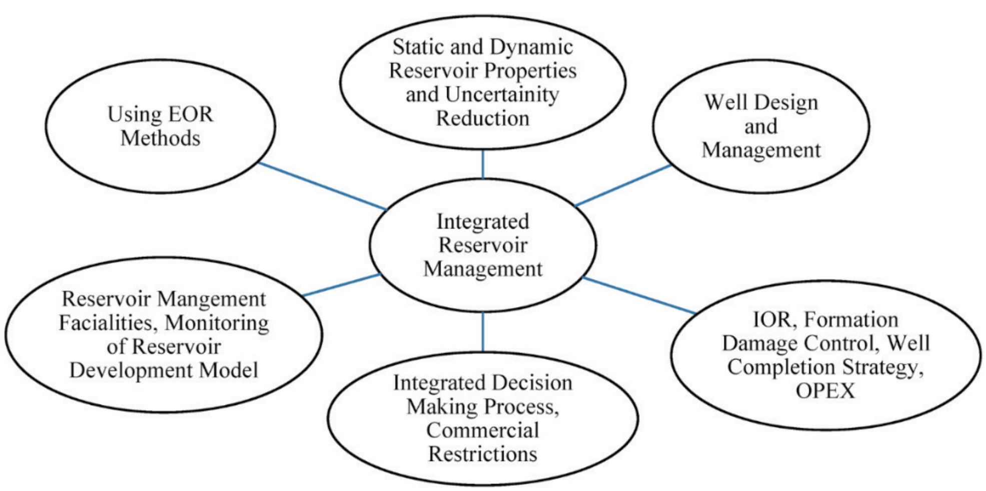

Figure 2 Integrated Reservoir Management

Reservoir properties use two main methods: structural modeling and stratigraphy modeling. In relation to structural modeling, interactive modeling software now helps to examine the compatibility between horizons and seismic faults and observations made in wells. In stratigraphic modeling, core information is used in an integrated manner to extract rock types based on geological and petrophysical criteria. In practice, a multivariate statistical experiment and analysis of well logs is performed, and the resulting crossplots are analyzed jointly with petrophysical core data to identify rock types.

The advantages of this approach are twofold: 1. Using the information available in all wells; 2. Calibration of geological facies in terms of information flow characteristics in several wells, Therefore, the identified rock types remain significant for both the sedimentologist and the reservoir engineer.

The purpose of describing the reservoir is to improve the geological modeling of the reservoir, thereby reducing subsequent uncertainties in the reservoir model and assigning dynamic properties to network blocks on a good scale to reduce uncertainties in production forecasting. These studies include performing the required laboratory tests (mainly measuring relative permeability and capillary pressure) in real reservoir conditions to observe fluid properties, wettability conditions, and saturation endpoints. Although much more complex and time consuming than conventional laboratory studies, SCAL results are much more reliable for calibrating the reservoir model. Special methods and equipment have been developed for this purpose. The role of flow units defined in the scale of the reservoir model can be easily related to the distribution of rock types in the exact geological model [27].

In order to adapt to different development plans, the repository simulator must be implemented with a number of options. Thus, the reservoir simulator must consider the mechanical effects of the rock. Optimization process to simulate hybrid effects such as gas injection and management Due to the complex well adaptation and modeling of heterogeneities, the simulator should be run with unstructured networking facilities. Finally, in order to be able to perform heavy calculations and use it to make quick decisions, the simulator must be able to run on parallel machines.

In addition, the integration of dynamic data greatly contributes to the reliability of the geological model for subsequent reservoir applications. This integration can be done in the early stages of field development using well test resources. Later, new dynamic information from the wells will allow the geological model to be updated. In practice, advanced mathematical methods are now available to model the geological model with good experimental results [19]. They include: Inversion techniques of simulated well experiments, such as the gradient method, to adjust the petrophysical properties. And a gradual deformation technique to adjust the geological model itself, the distribution of facies, reservoir boundaries and fault position. In the field of reservoir properties, there are complete software lines. This is the case for IFP with Reservoir Modeling Line (LMR).

4 CONCLUSION

Uncertainty is due to incomplete and imprecise knowledge as a result of limited sampling of the subsurface heterogeneities. In this study, optimization strategies in petroleum reservoir planning are introduced. It has been shown the output of each method independently. It can be seen that for reservoir management priority, many optimization studies have so far focused on production optimization. Mainly, the efficiency of the existing large petroleum fields causes us to consider the principles of reservoir management in order to increase EOR. There are several sources of uncertainty in the oil field development that misrepresent the future reservoir productivity. In general, uncertainty in information about reservoir management is divided into four categories. At the same time, many different ambiguities in the course of the developing oil field, including geological, operational and economic uncertainties, have a devastating effect on the effective production of this reservoir.

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