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MODULATION INSTABILITY OF HIGH-FREQUENCY SURFACE WAVES AT A PLASMA – VACUUM INTERFACE

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Nonlinear propagation of surface waves in a cold electron-plasma half-space is theoretically investigated by using the method of multiple scale. It is shown that high-frequency surface waves are modulationally unstable at a plasma – vacuum interface. The growth of the modulational instability of the surface waves is discussed. It is also shown that the electric field associated with a finite amplitude surface wave can take the form of an envelope soliton which propagates along the plasma – vacuum interface with a velocity independent of the soliton height.

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1. Introduction

The possibility of plasma heating by means of large-amplitude waves has created great interest in the study of nonlinear wave propagation in plasma. Moreover, the surface wave modes promise a wide range of applications in many other fields such as laser plasma interaction, plasma diagnostics, microelectronics, etc. In recent years, with a view to comparing with experimental results, investigations on the nonlinear propagation of surface waves in bounded plasma have increased considerably [1–10]. Yu and Zhelyazkov [5] made electromagnetic treatment for providing dispersion relation even for a cold homogeneous plasma at a nearly electrostatic limit of frequency range. They derived a nonlinear Schrödinger equation by making a Taylor expansion of the linear dispersion relation which cannot account fully for the effect of nonlinearity on the wave attenuation. On the other hand, Nikerson and Johnston [6] analytically examined the existence of solitary surface plasma waves in the electrostatic limit, including ponderomotive-force effects. The resulting solitary wave has a somewhat arbitrary shape but must move with a particular velocity parallel to the surface. From the surface waves in an unmagnetized semi-infinite plasma, Gradov and Stenflo [7] showed the existence of high-frequency surface wave solitons. A high-power circularly-polarized electromagnetic wave interacting with an electron plasma can give rise to solitary wave structures of which the density profile contains a depression at the centre, together with the shoulders of density excess on the sides [8]. It is understood from this investigation that the density shoulders are due to charge separation effects, since thermal dispersion has been neglected. Lindgren et al. [9] considered the problem of nonlinear boundary conditions relevant to the generation and evolution of surface modes in spatially bounded plasma configurations. Propagation of nonlinear high-frequency TM surface waves in a thin unmagnetized plasma layer bounded by vacuum has been investigated by Zakharov and Shabat [10]. They showed that envelope-surface-wave solitons exist from the modulation of finite amplitude electron waves by slow ion acoustic motion. They also discussed the non-existence of solitons with purely electronic modulation. Azarenkov et al. [11] studied the surface wave that produces and sustains the microwave gas discharge propagating along an external magnetic field and has an eigenfrequency in the range in between electron-cyclotron and electron plasma frequency. They obtained analytically and numerically the spatial distributions of the produced plasma density, electromagnetic field, energy flow density, phase velocity and reverse skin-depth of surface waves. Subsequently Sita and Dasgupta [12] have studied the electrostatic surface waves propagating along the interface between a warm magnetized plasma and vacuum. They investigated the general dispersion relation in a closed form and certain special cases particularly when the magnetic field is directed parallel and perpendicular to the boundary surface. A planar plasma wave-guide with a single interface between an isotropic homogeneous plasma, considered as a nonlinear medium, and a linear dielectric (vacuum) has been studied theoretically by Georgieva and Shivarova [13] with respect to the nonlinear effects of self-action of surface waves. Electrostatic surface waves at the interface between a low-temperature nonisothermal dusty plasma and a metallic wall have been investigated by Ostrikov and Yu [14] when a plasma contains massive negatively-charged impurities or dust particles. They showed that impurities can significantly alter the characteristics and damping of surface waves by reducing their phase velocity and causing charge-related damping. Alam et al. [15] have studied the propagation of surface waves at the interface of a semi-infinite dusty plasma considering the effect of dust-charge fluctuation in a dusty plasma. They have shown that there exists a modified low-frequency mode of propagation in the plasma. They have evaluated numerically the value of the attenuation coefficient of the wave and showed the variation of the attenuation coefficient graphically for different values of the dust density and charge variation.

In the present paper, we develop a detailed electro-magnetic treatment of nonlinear surface waves in a cold homogeneous electron plasma half-space. By the method of multiple scales [1], we derive a nonlinear Schrödinger equation describing the nonlinear evolution of surface waves. From this equation, the criteria of instability are obtained. Numerical computations are presented which show that surface waves are modulationally unstable throughout the whole electro-magnetic region of their existence when one disregards the ion motion. The growth rate of this instability and existence of solitary waves have been discussed.

2. *Formulation*

We start from the following equation of continuity equation of motion of electrons and Maxwell's equation which governs the wave propagation in a cold plasma

$$\frac{\partial n}{\partial t} + N_0 \nabla \cdot \boldsymbol{u} = -\nabla \cdot (n\boldsymbol{u}), \tag{1}$$

$$\frac{\partial \boldsymbol{u}}{\partial t} + \frac{e}{m_e}\left[\boldsymbol{E} + \frac{1}{c}(\boldsymbol{u} \times \boldsymbol{B})\right] = -(\boldsymbol{u} \cdot \nabla)\boldsymbol{u}, \tag{2}$$

$$\nabla \times \boldsymbol{E} + \frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t} = 0, \tag{3}$$

$$\nabla \times \boldsymbol{B} = \frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t} - \frac{4\pi e(N_0 + n)\boldsymbol{u}}{c}, \tag{4}$$

$$\nabla \cdot \boldsymbol{E} = -4\pi e n, \tag{5}$$

where the notations are standard (thus n is the perturbation of the uniform density, N_0, of electrons, $\boldsymbol{u}$ is the fluid velocity). Ions will be assumed to form a uniform stationary background having density N_0.

We consider a plasma which occupies the half-space $x > 0$ and is bounded by vacuum. Let us consider a transverse electromagnetic (TEM) wave propagating parallel to the interface (x-y plane) along the z-direction. The wave magnetic field is along the y-direction and the wave electric field lies in the (x-z) plane [16]. For vacuum, $x < 0$, Maxwell's equations corresponding to vacuum are to be used.

To derive the dispersion relation for surface waves, the usual boundary conditions, namely, the continuity of the tangential components of electric and magnetic fields are used.

3. *Lowest-order solutions for first, second and zero-th harmonic components*

To obtain the equations for modulated surface waves, we make the following Fourier expansion for the field quantities

$$W\big(n, u_x, u_z, E_x, E_z, B_y, E_{vx}, E_{vz}, B_{vy}\big),$$

$$W = \epsilon^2 W_0 + \sum_{s=1}^{\infty} \epsilon^s \big[W_s \exp(\mathrm{i}s\psi) + W_s^* \exp(-\mathrm{i}s\psi)\big] \tag{6}$$

in which $\psi = kz - \omega t$, (ω is the wave frequency and k is the wave number) and W_0, W_s and W_s^* are functions of x, ζ, τ where

$$\zeta = \epsilon(z - C_\mathrm{g} t), \qquad \tau = \epsilon^2 t. \tag{7}$$

Here the subscript v denotes the field quantities in vacuum ($x < 0$). In Eq. (7), $C_\mathrm{g} = \mathrm{d}\omega/\mathrm{d}k$ is the group velocity and ϵ is a small parameter.

Substituting the expansion (6) in Eqs. (1)–(4) and the Maxwell's equations corresponding to free space, and then equating on both sides the coefficients of $\exp(\mathrm{i}\psi)$ and $\exp(2\mathrm{i}\psi)$, we get two sets of equations in component forms which we call I and II, respectively. In the set of equations I and II only terms containing ϵ with power up to 3 and 2, respectively, are to be retained as other terms thus neglected will have no influence on the evolution in the lowest order.

We now make the following perturbation expansion for the field quantities W_0, W_s and W_s^*, which we denote by P_j ($j = 0, 1, 2, \ldots$)

$$P_j = P_j^{(1)} + \epsilon P_j^{(2)} + \epsilon^2 P_j^{(3)} + \ldots . \tag{8}$$

Keeping in mind that for a surface wave field, the quantities decrease exponentially as we move away from the interface due to the self-consistent bunching of the surface charges, we solve the lowest order equations obtained from the set of equations I after substituting the expansion (8). Thus we get for the first harmonic quantities in the lowest order the following solutions

$$n_1^{(1)} = 0,$$

$$u_{x1}^{(1)} = \alpha \frac{e\omega k}{m_\mathrm{e} k_\perp (\omega_\mathrm{e}^2 - \omega^2)} \mathrm{e}^{-\beta x}, \quad u_{z1}^{(1)} = -\mathrm{i}\alpha \frac{e\omega\beta}{m_\mathrm{e} k_\perp (\omega_\mathrm{e}^2 - \omega^2)} \mathrm{e}^{-\beta x},$$

$$E_{x1}^{(1)} = \mathrm{i}\alpha \frac{k\omega^2}{k_\perp (\omega_\mathrm{e}^2 - \omega^2)} \mathrm{e}^{-\beta x}, \qquad E_{z1}^{(1)} = \alpha \frac{\omega^2 \beta}{k_\perp (\omega_\mathrm{e}^2 - \omega^2)} \mathrm{e}^{-\beta x},$$

$$B_{y1}^{(1)} = -\mathrm{i}\alpha \frac{\omega}{k_\perp c} \mathrm{e}^{-\beta x}, \qquad E_{\mathrm{v}x1}^{(1)} = -\mathrm{i}\alpha \frac{k}{k_\perp} \mathrm{e}^{k_\perp x},$$

$$E_{\mathrm{v}z1}^{(1)} = \alpha\, \mathrm{e}^{k_\perp x}, \qquad B_{\mathrm{v}y1}^{(1)} = -\mathrm{i}\alpha \frac{\omega}{k_\perp c} \mathrm{e}^{k_\perp x}, \tag{9}$$

and the linear dispersion relation

$$\beta \quad = \quad -k_\perp \varepsilon(\omega) \tag{10}$$

where

$$\begin{aligned} \varepsilon(\omega) &= 1 - \frac{\omega_e^2}{\omega^2}, \qquad \beta^2 = k^2 - \frac{\omega^2}{c^2}\varepsilon(\omega), \\ k_\perp^2 &= k^2 - \frac{\omega^2}{c^2}, \qquad \omega_e^2 = \frac{4\pi N_0 e^2}{m_e}. \end{aligned} \tag{11}$$

β and $k_\perp$ are the wave attenuation coefficients in plasma and vacuum semi-spaces, respectively. ω_e is the electron plasma frequency and α represents the amplitude of the wave field $E_z^{(1)}$ at the surface. We seek an evolution equation for α. Similarly, solving the lowest-order equations obtained from the set of equations II, after substituting the expansion (8), we get the following solutions for the second harmonic components, where we use the solutions (9),

$$\begin{aligned} n_2^{(1)} &= -\alpha^2 \frac{c^2 a_2(\beta^2 - k^2)}{2\pi e(\omega_e^2 - 4\omega^2)} e^{-2\beta x}, \\ u_{x2}^{(1)} &= i\,\alpha^2 \frac{e}{2m_e\omega}\left[\frac{2k}{\beta_2} C_1 e^{-\beta_2 x} + \beta C_2 e^{-2\beta x}\right], \\ u_{z2}^{(1)} &= \alpha^2 \frac{e}{2m_e\omega}\left[C_1 e^{-\beta_2 x} + k C_2 e^{-2\beta x}\right], \\ E_{x2}^{(1)} &= -\alpha^2 \left[\frac{2k}{\beta_2} C_1 e^{-\beta_2 x} + \frac{a_2 c^2 \beta}{\omega_e^2 - 4\omega^2} e^{-2\beta x}\right], \\ E_{z2}^{(1)} &= i\,\alpha^2 \left[C_1 e^{-\beta_2 x} + \frac{a_2 c^2 k}{\omega_e^2 - 4\omega^2} e^{-2\beta x}\right], \\ B_{y2}^{(1)} &= \alpha^2 \frac{c(\beta_2^2 - 4k^2)}{2\omega\beta_2} C_1 e^{-\beta_2 x}, \end{aligned} \tag{12}$$

where

$$\begin{aligned} a_2 &= \frac{e\omega^2\omega_e^2(\beta^2 - k^2)}{m_e c^2 k_\perp^2(\omega_e^2 - \omega^2)^2}, \qquad C_1 = \frac{2k a_2 \omega^2 c^2 \beta_2}{k_\perp(\omega_e^2 - 4\omega^2)^2 - 2\omega^2\beta_2(\omega_e^2 - 4\omega^2)}, \\ C_2 &= \frac{4a_2\omega^2 c^2}{\omega_e^2(\omega_e^2 - 4\omega^2)}, \qquad \beta_2^2 = 4k^2 + \frac{(\omega_e^2 - 4\omega^2)}{c^2}. \end{aligned} \tag{13}$$

To find the zero-th harmonic (i.e low-frequency) components of field quantities, we neglect the wave magnetic field and introduce the electric potential ϕ through the relation $\boldsymbol{E} = -\nabla\phi$. Thus we obtain from Eqs. (1) – (5) the following equations governing low-frequency surface waves

$$\frac{\partial n}{\partial t} + N_0 \nabla \cdot \boldsymbol{u} = -\nabla \cdot \big(n\boldsymbol{u}\big), \tag{14}$$

$$\frac{\partial \boldsymbol{u}}{\partial t} - \frac{e}{m_{\rm e}}\nabla\phi = -\big(\boldsymbol{u}\cdot\nabla\big)\boldsymbol{u}, \tag{15}$$

$$\nabla^2\phi = 4\pi e n. \tag{16}$$

For vacuum

$$\nabla^2\phi_{\rm v} = 0. \tag{17}$$

Substituting the expansion (6) and a similar expansion for ϕ, $\phi_{\rm v}$ in Eqs. (14) – (17), and then equating the terms independent of ψ on both sides, we get the set of equations III. The lowest-order equations, obtained from the set of equations III after substituting perturbation expansion like (8) for the field quantities, are solved by applying the usual boundary conditions (continuity of ϕ and normal electric displacement), and keeping in mind that for surface waves field quantities decay away from the interface. Thus we obtain the following solution in the lowest order for the zero-th harmonic components

$$\begin{aligned} n_0^{(1)} &= \alpha\alpha^* \frac{2\omega^2 k^2 \beta^2}{\pi m_{\rm e} K_I^2 (\omega_{\rm e}^2 - \omega^2)^2} {\rm e}^{-2\beta x}, \\ u_{x0}^{(1)} &= 0, \\ u_{z0}^{(1)} &= \alpha\alpha^* \frac{e^2\omega^2(\beta^2 - 2k^2)}{m_{\rm e} k_\perp^2 (\omega_{\rm e}^2 - \omega^2)^2 C_{\rm g}} {\rm e}^{-2\beta x}. \end{aligned} \tag{18}$$

Other quantities, which we do not write explicitly, will not be useful in our analysis.

4. Derivation of the modulation equation

Collecting the coefficient of ϵ^3 from both sides of the set of equations I, after substituting the perturbation expansion (8), we get a set of equations for the first harmonic quantities in the third order which can be put in the following matrix form

$$\begin{pmatrix} -\mathrm{i}\omega & N_0\dfrac{\partial}{\partial x} & \mathrm{i}kN_0 & 0 & 0 & 0 \\ 0 & -\mathrm{i}\omega & 0 & \dfrac{e}{m_\mathrm{e}} & 0 & 0 \\ 0 & 0 & -\mathrm{i}\omega & 0 & \dfrac{e}{m_\mathrm{e}} & 0 \\ 0 & 0 & 0 & \mathrm{i}k & \dfrac{\partial}{\partial x} & -\dfrac{\mathrm{i}\omega}{c} \\ 0 & \dfrac{4\pi eN_0}{c} & 0 & \dfrac{i\omega}{c} & 0 & -\mathrm{i}k \\ 0 & 0 & \dfrac{4\pi eN_0}{c} & 0 & \dfrac{\mathrm{i}\omega}{c} & \dfrac{\partial}{\partial x} \end{pmatrix} \begin{pmatrix} n_1^{(3)} \\ u_{x1}^{(3)} \\ u_{z1}^{(3)} \\ E_{x1}^{(3)} \\ E_{z1}^{(3)} \\ B_{y1}^{(3)} \end{pmatrix} = \begin{pmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \\ M_5 \\ M_6 \end{pmatrix}, \tag{19}$$

$$\begin{pmatrix} \mathrm{i}k & -\dfrac{\partial}{\partial x} & -\dfrac{\mathrm{i}\omega}{c} \\ \dfrac{i\omega}{c} & 0 & -\mathrm{i}k \\ 0 & \dfrac{\mathrm{i}\omega}{c} & \dfrac{\partial}{\partial x} \end{pmatrix} \begin{pmatrix} E_{\mathrm{v}x1}^{(3)} \\ E_{\mathrm{v}z1}^{(3)} \\ B_{\mathrm{v}y1}^{(3)} \end{pmatrix} = \begin{pmatrix} M_{\mathrm{v}1} \\ M_{\mathrm{v}2} \\ M_{\mathrm{v}3} \end{pmatrix}, \tag{20}$$

where

$$\begin{aligned} M_1 &= m_1 - \frac{\partial n_1}{\partial \tau} - N_0 \frac{\partial u_{z1}^{(2)}}{\partial \xi} + C_\mathrm{g} \frac{\partial n_1^{(2)}}{\partial \xi}, \\ M_2 &= m_2 - \frac{\partial u_{x1}^{(1)}}{\partial \tau} + C_\mathrm{g} \frac{\partial u_{x1}^{(2)}}{\partial \xi}, \\ M_3 &= m_3 - \frac{\partial u_{z1}^{(1)}}{\partial \tau} + C_\mathrm{g} \frac{\partial u_{z1}^{(2)}}{\partial \xi}, \\ M_4 &= m_4 - \frac{1}{c}\frac{\partial B_{y1}^{(1)}}{\partial \tau} - \frac{\partial E_{x1}^{(2)}}{\partial \xi} + \frac{C_\mathrm{g}}{c}\frac{\partial B_{y1}^{(2)}}{\partial \xi}, \\ M_5 &= m_5 + \frac{1}{c}\frac{\partial E_{x1}^{(1)}}{\partial \tau} + \frac{\partial B_{y1}^{(1)}}{\partial \xi} - \frac{C_\mathrm{g}}{c}\frac{\partial E_{x1}^{(2)}}{\partial \xi}, \\ M_6 &= m_6 + \frac{1}{c}\frac{\partial E_{z1}^{(1)}}{\partial \tau} - \frac{C_\mathrm{g}}{c}\frac{\partial E_{z1}^{(2)}}{\partial \xi}, \\ m_1 &= -\frac{\partial}{\partial x}\big[n_2^{(1)} u_{x1}^{(1)} + n_2^{(1)} u_{x1}^{(x1)*}\big] - \mathrm{i}k\big[n_0^{(1)} u_{z1}^{(1)} + n_2^{(1)} u_{z1}^{(1)*}\big], \\ m_2 &= -\frac{\partial}{\partial x}\big[u_{x2}^{(1)} u_{x1}^{(1)*}\big] - \mathrm{i}k\big[u_{z0}^{(1)} u_{x1}^{(1)} + 2u_{z1}^{(1)*} u_{x2}^{(1)} - u_{z2}^{(1)} u_{x1}^{(1)*}\big], \end{aligned}$$

$$
\begin{aligned}
& +\frac{e}{m_{\mathrm{e}}c}\big[u_{z0}^{(1)}B_{y1}^{(1)}+u_{z1}^{(1)*}B_{y2}^{(1)}-u_{z2}^{(1)}B_{y1}^{(1)*}\big],\\
m_3 &= -\left[u_{x1}^{(1)}\frac{\partial u_{z0}^{(1)}}{\partial x}+u_{x2}^{(1)}\frac{\partial u_{z1}^{(1)*}}{\partial x}+u_{x1}^{(1)*}\frac{\partial u_{z2}^{(1)}}{\partial x}\right]-\mathrm{i}k\big[u_{z0}^{(1)}u_{z1}^{(1)}+u_{z1}^{(1)*}u_{z2}^{(1)}\big]\\
& -\frac{e}{m_{\mathrm{e}}c}\big[u_{x1}^{(1)}B_{y2}^{(1)}+u_{x2}^{(1)}B_{y1}^{(1)*}\big],\\
m_4 &= 0,\qquad m_5 = -\frac{4\pi e}{c}\big[n_0^{(1)}u_{x1}^{(1)}+n_2^{(1)}u_{x1}^{(1)*}\big],\\
m_6 &= -\frac{4\pi e}{c}\big[n_0^{(1)}u_{z1}^{(1)}+n_2^{(1)}u_{z1}^{(1)*}\big],\\
M_{\mathrm{v}1} &= -\frac{1}{c}\frac{\partial B_{\mathrm{v}y1}^{(1)}}{\partial\tau}-\frac{\partial E_{\mathrm{v}x1}^{(2)}}{\partial\xi}+\frac{C_{\mathrm{g}}}{c}\frac{\partial B_{\mathrm{v}y1}^{(2)}}{\partial\xi},\\
M_{\mathrm{v}2} &= \frac{1}{c}\frac{\partial E_{\mathrm{v}x1}^{(1)}}{\partial\tau}+\frac{\partial B_{\mathrm{v}y1}^{(2)}}{\partial\xi}-\frac{C_{\mathrm{g}}}{c}\frac{\partial E_{\mathrm{v}x1}^{(2)}}{\partial\xi},\\
M_{\mathrm{v}3} &= \frac{1}{c}\frac{\partial E_{\mathrm{v}z1}^{(1)}}{\partial\tau}-\frac{C_{\mathrm{g}}}{c}\frac{\partial E_{\mathrm{v}z1}^{(2)}}{\partial\xi}.
\end{aligned}
\tag{21}
$$

The boundary conditions are

$$
\begin{aligned}
\big(E_{z1}^{(3)}\big)_{x=0^+} &= \big(E_{\mathrm{v}z1}^{(3)}\big)_{x=0^-},\\
\big(B_{y1}^{(3)}\big)_{x=0^+} &= \big(B_{\mathrm{v}y1}^{(3)})\big)_{x=0^-}.
\end{aligned}
\tag{22}
$$

We multiply Eq. (14) from the left side by the row matrix $[f_1, f_2, f_3, f_4, f_5, f_6]$, where the quantities f_m are assumed to be functions of x and vanish at $x=\infty$, and integrate the resulting equation with respect to x from 0 to ∞. Similarly, we multiply Eq. (20) by the row matrix $[f_{\mathrm{v}1}, f_{\mathrm{v}2}, f_{\mathrm{v}3}]$ and integrate the resulting equation with respect to x from $-\infty$ to 0 where the functions $f_{\mathrm{v}l}$ are assumed to vanish at $x=-\infty$. On addition, the two equations thus obtained give the following equation

$$
\int_0^\infty \sum_{m=1}^{6} f_m M_m \mathrm{d}x + \int_{-\infty}^{0} \sum_{l=1}^{3} f_{\mathrm{v}l} M_{\mathrm{v}l} \mathrm{d}x = 0\,,
\tag{23}
$$

provided we use the boundary conditions (22), and choose the functions f_m, $f_{\mathrm{v}l}$ in such a way that they can satisfy the following equations and boundary conditions

$$
\begin{aligned}
& f_1 = 0, \qquad -\mathrm{i}\omega f_2 + \frac{4\pi e N_0}{c} f_5 = 0, \qquad -\mathrm{i}\omega f_3 + \frac{4\pi e N_0}{c} f_6 = 0,\\
& \frac{e}{m_{\mathrm{e}}} f_2 + \mathrm{i}k f_4 + \mathrm{i}\frac{\omega}{c} f_5 = 0, \qquad \frac{e}{m_{\mathrm{e}}} f_3 + \frac{\partial f_4}{\partial x} + \mathrm{i}\frac{\omega}{c} f_6 = 0,
\end{aligned}
$$

$$-\mathrm{i}\frac{\omega}{c}f_4 - \mathrm{i}kf_5 - \frac{\partial f_6}{\partial x} = 0, \qquad \mathrm{i}kf_{\mathrm{v}1} + \frac{\mathrm{i}\omega}{c}f_{\mathrm{v}2} = 0,$$
$$\frac{\partial f_{\mathrm{v}1}}{\partial x} + \mathrm{i}\frac{\omega}{c}f_{\mathrm{v}3} = 0, \qquad -\mathrm{i}\frac{\omega}{c}f_{\mathrm{v}1} - \mathrm{i}kf_{\mathrm{v}2} - \frac{\partial f_{\mathrm{v}3}}{\partial x} = 0, \tag{24}$$
$$(f_4)_{x=0^+} = (f_{\mathrm{v}1})_{x=0^-} \quad \text{and} \quad (f_6)_{x=0^+} = (f_{\mathrm{v}3})_{x=0^-}. \tag{25}$$

The solutions of Eqs. (24) under the boundary conditions (25) are easily found to be the following

$$f_1 = 0, \qquad f_2 = -A\,\frac{4\pi e N_0 k\omega}{ek_\perp(\omega_{\mathrm{e}}^2-\omega^2)}\mathrm{e}^{-\beta x},$$
$$f_3 = -\mathrm{i}A\,\frac{4\pi e N_0 \omega\beta}{ek_\perp(\omega_{\mathrm{e}}^2-\omega^2)}\mathrm{e}^{-\beta x}, \qquad f_4 = -\mathrm{i}A\,\frac{\omega}{k_\perp c}\,\mathrm{e}^{-\beta x},$$
$$f_5 = -\mathrm{i}A\,\frac{k\omega^2}{k_\perp(\omega_{\mathrm{e}}^2-\omega^2)}\mathrm{e}^{-\beta x}, \qquad f_6 = A\frac{\beta\omega^2}{k_\perp(\omega_{\mathrm{e}}^2-\omega^2)}\mathrm{e}^{-\beta x},$$
$$f_{\mathrm{v}1} = -\mathrm{i}A\frac{\omega}{k_\perp c}\mathrm{e}^{k_\perp x}, \qquad f_{\mathrm{v}2} = \mathrm{i}A\frac{k}{k_\perp}\mathrm{e}^{k_\perp x}, \qquad f_{\mathrm{v}3} = A\mathrm{e}^{k_\perp x}, \tag{26}$$

where A is some arbitrary constant independent of x.

It will be found subsequently that Eq. (23) leads to the nonlinear Schrödinger equation. We are yet to determine the first harmonic quantities in the second order. To deal with these, we take the help of a similar but linear problem with slightly different wave number $k' = k + \epsilon$. Following Blenerhasset [17], the problem is formulated as follows

$$-\mathrm{i}\omega' F_1 + N_0\frac{\partial F_2}{\partial x} + \mathrm{i}k' N_0 F_3 = 0, \quad -\mathrm{i}\omega' F_2 + \frac{e}{m_e}F_4 = 0,$$
$$-\mathrm{i}\omega' F_3 + \frac{e}{m_e}F_5 = 0, \qquad -\mathrm{i}k' F_4 - \frac{\partial F_5}{\partial x} - \frac{\mathrm{i}\omega'}{c}F_6 = 0,$$
$$\frac{4\pi e N_0}{c}F_2 + \mathrm{i}\frac{\omega'}{c}F_4 - \mathrm{i}k' F_6 = 0, \qquad \frac{4\pi e N_0}{c}F_3 + \mathrm{i}\frac{\omega'}{c}F_5 + \frac{\partial F_6}{\partial x} = 0,$$
$$\mathrm{i}k' F_{\mathrm{v}1} - \frac{\partial F_{\mathrm{v}2}}{\partial x} - \mathrm{i}\frac{\omega'}{c}F_{\mathrm{v}3} = 0, \qquad \mathrm{i}\frac{\omega'}{c}F_{\mathrm{v}1} - \mathrm{i}k' F_{\mathrm{v}3} = 0,$$
$$\mathrm{i}\frac{\omega'}{c}F_{\mathrm{v}2} + \frac{\partial F_{\mathrm{v}3}}{\partial x} = 0, \tag{27}$$

where

$$k' = k + \epsilon, \qquad \text{and} \qquad \omega' = \omega(k+\epsilon).$$

F_m and $F_{\mathrm{v}l}$ are functions of x such that F_m vanishes at $x = \infty$ and $F_{\mathrm{v}l}$ vanishes at $x = -\infty$. Further, they satisfy the following boundary conditions

$$\big(F_5\big)_{x=0^+} = \big(F_{\mathrm{v}2}\big)_{x=0^-},$$

$$\left(F_6\right)_{x=0^+} = \left(F_{\mathrm{v}3}\right)_{x=0^-}. \tag{28}$$

Now make the following expansions

$$\begin{aligned} F_m &= F_m^{(1)} + \epsilon F_m^{(2)} + \epsilon^2 F_m^{(3)} + \ldots, \\ F_{\mathrm{v}l} &= F_{\mathrm{v}l}^{(1)} + \epsilon F_{\mathrm{v}l}^{(2)} + \epsilon^2 F_{\mathrm{v}l}^{(3)} + \ldots, \\ \omega' &= \omega + \epsilon C_{\mathrm{g}} + \epsilon^2 \frac{1}{2}\frac{\mathrm{d}C_{\mathrm{g}}}{\mathrm{d}k}. \end{aligned} \tag{29}$$

Substituting the expansions (29) in Eqs. (27) and (28), and then equating the coefficients of like powers of ϵ from both sides, we get equations and boundary conditions satisfied by different order quantities $F_m^{(j)}$, $F_{\mathrm{v}l}^{(j)}$ ($j = 1, 2, 3, \ldots$). Thus $F_m^{(1)}$, $F_{\mathrm{v}l}^{(1)}$ are found to satisfy a set of equations and boundary conditions identical in form to those satisfied by $\Phi_m^{(1)}$ and $\Phi_{\mathrm{v}l}^{(1)}$ where

$$\begin{aligned} \Phi_m^{(1)} &= \left[n_1^{(1)}, u_{x1}^{(1)}, u_{z1}^{(1)}, E_{x1}^{(1)}, E_{z1}^{(1)}, B_{y1}^{(1)}\right], \\ \Phi_{\mathrm{v}l}^{(1)} &= \left[E_{\mathrm{v}x1}^{(1)}, E_{\mathrm{v}z1}^{(1)}, B_{\mathrm{v}y1}^{(1)}\right]. \end{aligned}$$

So, we can set

$$\begin{aligned} F_m^{(1)} &= \Phi_m^{(1)}, \\ F_{\mathrm{v}l}^{(1)} &= \Phi_{\mathrm{v}l}^{(1)}. \end{aligned} \tag{30}$$

Setting

$$F_m^{(2)} = \alpha \Psi_m^{(2)}, \qquad F_{\mathrm{v}l}^{(2)} = \alpha \Psi_{\mathrm{v}l}^{(2)} \tag{31}$$

in the equations satisfied by $F_m^{(2)}$, $F_{\mathrm{v}l}^{(2)}$ and

$$\Phi_m^{(2)} = -\mathrm{i}\frac{\partial \alpha}{\partial \xi}\chi_m^{(2)}, \qquad \Phi_{\mathrm{v}l}^{(2)} = -\mathrm{i}\frac{\partial \alpha}{\partial \xi}\chi_{\mathrm{v}l}^{(2)}, \tag{32}$$

where

$$\Phi_m^{(2)} = \left[n_1^{(2)},\, u_{x1}^{(2)},\, u_{z1}^{(2)},\, E_{x1}^{(2)},\, E_{z1}^{(2)},\, B_{y1}^{(2)}\right], \quad \Phi_{\mathrm{v}l}^{(2)} = \left[E_{\mathrm{v}x1}^{(2)},\, E_{\mathrm{v}z1}^{(2)},\, B_{\mathrm{v}y1}^{(2)}\right] \tag{33}$$

in equations satisfied by $\Phi_m^{(2)}$, $\Phi_{\mathrm{v}l}^{(2)}$, which are obtained from the set of equations I after substituting the expansion (8) and equating the coefficients of ϵ^2 from both sides, we find that $\psi_m^{(2)}$, $\psi_{\mathrm{v}l}^{(2)}$ and $\chi_m^{(2)}$, $\chi_{\mathrm{v}l}^{(2)}$ satisfy same set of equations and boundary conditions. Hence, we can identify $\psi_m^{(2)}$ with $\chi_m^{(2)}$ and $\Psi_{\mathrm{v}l}^{(2)}$ with $\chi_{\mathrm{v}l}^{(2)}$, i. e.

$$\psi_m^{(2)} = \chi_m^{(2)} \quad \text{and} \quad \psi_{\mathrm{v}l}^{(2)} = \chi_{\mathrm{v}l}^{(2)}. \tag{34}$$

Setting $F_m^{(3)} = \alpha\psi_m^{(3)}$ and $F_{\mathrm{v}l}^{(3)} = \alpha\psi_{\mathrm{v}l}^{(3)}$ in the equations obtained from Eq.(27) after substituting the expansion (29) and equating the coefficient of ϵ^2 from both sides, we obtain the following equations satisfied by $\psi_m^{(3)}, \psi_{\mathrm{v}l}^{(3)}$, where we use the relations (30) and (31),

$$\begin{pmatrix} -\mathrm{i}\omega & N_0\dfrac{\partial}{\partial x} & \mathrm{i}kN_0 & 0 & 0 & 0 \\ 0 & -\mathrm{i}\omega & 0 & \dfrac{e}{m_\mathrm{e}} & 0 & 0 \\ 0 & 0 & -\mathrm{i}\omega & 0 & \dfrac{e}{m_\mathrm{e}} & 0 \\ 0 & 0 & 0 & \mathrm{i}k & \dfrac{\partial}{\partial x} & -\dfrac{i\omega}{c} \\ 0 & \dfrac{4\pi eN_0}{c} & 0 & \dfrac{\mathrm{i}\omega}{c} & 0 & -\mathrm{i}k \\ 0 & 0 & \dfrac{4\pi eN_0}{c} & 0 & \dfrac{\mathrm{i}\omega}{c} & \dfrac{\partial}{\partial x} \end{pmatrix} \begin{pmatrix} \psi_1^{(3)} \\ \psi_2^{(3)} \\ \psi_3^{(3)} \\ \psi_4^{(3)} \\ \psi_5^{(3)} \\ \psi_6^{(3)} \end{pmatrix} = \begin{pmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \\ N_5 \\ N_6 \end{pmatrix}, \tag{35}$$

$$\begin{pmatrix} \mathrm{i}k & -\dfrac{\partial}{\partial x} & -\dfrac{\mathrm{i}\omega}{c} \\ \dfrac{\mathrm{i}\omega}{c} & 0 & -\mathrm{i}k \\ 0 & \dfrac{\mathrm{i}\omega}{c} & \dfrac{\partial}{\partial x} \end{pmatrix} \begin{pmatrix} \psi_{\mathrm{v}1}^{(3)} \\ \psi_{\mathrm{v}2}^{(3)} \\ \psi_{\mathrm{v}3}^{(3)} \end{pmatrix} = \begin{pmatrix} N_{\mathrm{v}1} \\ N_{\mathrm{v}2} \\ N_{\mathrm{v}3} \end{pmatrix}, \tag{36}$$

where

$$N_1 = \mathrm{i}\big(C_\mathrm{g}\psi_1^{(2)} - N_0\psi_3^{(2)} + P\theta_1^{(1)}\big), \qquad N_2 = \mathrm{i}\big(C_\mathrm{g}\psi_2^{(2)} + P\theta_2^{(1)}\big),$$

$$N_3 = \mathrm{i}\big(C_\mathrm{g}\psi_3^{(2)} + P\theta_3^{(1)}\big), \qquad N_4 = \mathrm{i}\Big(- \psi_4^{(2)} + \frac{C_g}{c}\psi_6^{(2)} + \frac{1}{c}P\theta_6^{(1)}\Big),$$

$$N_5 = \mathrm{i}\Big(- \frac{C_\mathrm{g}}{c}\psi_4^{(2)} + \psi_6^{(2)} - \frac{1}{c}P\theta_4^{(1)}\Big), \quad N_6 = \mathrm{i}\Big(- \frac{C_\mathrm{g}}{c}\psi_5^{(2)} - \frac{1}{c}P\theta_5^{(1)}\Big),$$

$$N_{\mathrm{v}1} = \mathrm{i}\Big(- \psi_{\mathrm{v}1}^{(2)} + \frac{C_\mathrm{g}}{c}\psi_{\mathrm{v}3}^{(2)} + \frac{1}{c}P\theta_{\mathrm{v}3}^{(1)}\Big), \quad N_{\mathrm{v}2} = \mathrm{i}\Big(\psi_{\mathrm{v}3}^{(2)} - \frac{C_\mathrm{g}}{c}\psi_{\mathrm{v}1}^{(2)} - \frac{1}{c}P\theta_{\mathrm{v}1}^{(1)}\Big),$$

$$N_{\mathrm{v}3} = \mathrm{i}\Big(- \frac{C_\mathrm{g}}{c}\psi_{\mathrm{v}2}^{(2)} - \frac{1}{c}P\theta_{\mathrm{v}2}^{(1)}\Big), \tag{37}$$

in which

$$P = \frac{1}{2}\frac{\mathrm{d}C_\mathrm{g}}{\mathrm{d}k}, \qquad \theta_m^{(1)} = \frac{\Phi_m^{(1)}}{\alpha}, \qquad \theta_{\mathrm{v}l}^{(1)} = \frac{\Phi_{\mathrm{v}l}^{(1)}}{\alpha}. \tag{38}$$

The boundary conditions satisfied by $\psi_m^{(3)}$, $\psi_{\mathrm{v}l}^{(3)}$ are

$$\begin{aligned}\psi_5^{(3)}\,|_{x=0^+} &= \psi_{\mathrm{v}2}^{(3)}\,|_{x=0^-}\,,\\ \psi_6^{(3)}\,|_{x=0^+} &= \psi_{\mathrm{v}3}^{(3)}\,|_{x=0^-}\,.\end{aligned} \tag{39}$$

Following the same procedure as adopted in deriving the equation (23) from the set of Eqs. (19) – (22), we obtain from Eqs. (34) – (39) the following equation

$$\int_0^\infty \sum_{m=1}^{6} f_m N_m \mathrm{d}x + \int_{-\infty}^{0} \sum_{l=1}^{3} f_{\mathrm{v}l} N_{vl} \mathrm{d}x = 0. \tag{40}$$

Using Eqs. (31)–(33) and (38), Eqs. (21) can be rewritten as follows

$$\begin{aligned}
M_1 &= \alpha^2\alpha^*\bar{m}_1 - \frac{\partial\alpha}{\partial\tau}\theta_1^{(1)} - \mathrm{i}\frac{\partial^2\alpha}{\partial\xi^2}\big(C_\mathrm{g}\psi_1^{(2)} - N_0\psi_3^{(2)}\big),\\
M_2 &= \alpha^2\alpha^*\bar{m}_2 - \frac{\partial\alpha}{\partial\tau}\theta_2^{(1)} - \mathrm{i}\frac{\partial^2\alpha}{\partial\xi^2}C_\mathrm{g}\psi_2^{(2)},\\
M_3 &= \alpha^2\alpha^*\bar{m}_3 - \frac{\partial\alpha}{\partial\tau}\theta_3^{(1)} - \mathrm{i}\frac{\partial^2\alpha}{\partial\xi^2}C_\mathrm{g}\psi_3^{(2)},\\
M_4 &= \alpha^2\alpha^*\bar{m}_4 - \frac{1}{c}\frac{\partial\alpha}{\partial\tau}\theta_6^{(1)} - \mathrm{i}\frac{\partial^2\alpha}{\partial\xi^2}\bigg(- \psi_4^{(2)} + \frac{C_\mathrm{g}}{c}\psi_6^{(2)}\bigg),\\
M_5 &= \alpha^2\alpha^*\bar{m}_5 + \frac{1}{c}\frac{\partial\alpha}{\partial\tau}\theta_4^{(1)} - \mathrm{i}\frac{\partial^2\alpha}{\partial\xi^2}\bigg(\psi_6^{(2)} - \frac{C_\mathrm{g}}{c}\psi_4^{(2)}\bigg),\\
M_6 &= \alpha^2\alpha^*\bar{m}_6 + \frac{1}{c}\frac{\partial\alpha}{\partial\tau}\theta_5^{(1)} + \mathrm{i}\frac{\partial^2\alpha}{\partial\xi^2}\frac{C_\mathrm{g}}{c}\psi_5^{(2)},\\
M_{\mathrm{v}1} &= -\frac{1}{c}\frac{\partial\alpha}{\partial\tau}\theta_{\mathrm{v}3}^{(1)} - \mathrm{i}\frac{\partial^2\alpha}{\partial\xi^2}\bigg(- \psi_{\mathrm{v}l}^{(2)} + \frac{C_\mathrm{g}}{c}\psi_{\mathrm{v}3}^{(2)}\bigg),\\
M_{\mathrm{v}2} &= \frac{1}{c}\frac{\partial\alpha}{\partial\tau}\theta_{\mathrm{v}1}^{(1)} - \mathrm{i}\frac{\partial^2\alpha}{\partial\xi^2}\bigg(\psi_{\mathrm{v}3}^{(2)} - \frac{C_\mathrm{g}}{c}\psi_{\mathrm{v}1}^{(2)}\bigg),\\
M_{\mathrm{v}3} &= \frac{1}{c}\frac{\partial\alpha}{\partial\tau}\theta_{\mathrm{v}2}^{(1)} + \mathrm{i}\frac{\partial^2\alpha}{\partial\xi^2}\bigg(\frac{C_\mathrm{g}}{c}\psi_{\mathrm{v}2}^{(2)}\bigg),\\
\bar{m}_1 &= -\frac{\partial}{\partial x}\big(\theta_1^{(0)}\,\theta_2^{(1)} + \theta_1^{(2)}\,\theta_2^{(1)\star}\big) - \mathrm{i}k\big(\theta_1^{(0)}\,\theta_3^{(1)} + \theta_1^{(2)}\,\theta_3^{(1)\star}\big),\\
\bar{m}_2 &= -\frac{\partial}{\partial x}\big(\theta_2^{(0)}\,\theta_2^{(1)*}\big) - \mathrm{i}k\big(\theta_3^{(0)}\,\theta_2^{(1)} + 2\theta_3^{(0)}\,\theta_2^{(2)*} - \theta_3^{(2)}\,\theta_2^{(1)*}\big)\\
&\quad + \frac{e}{m_\mathrm{e}c}\big(\theta_3^{(0)}\,\theta_6^{(1)} + \theta_3^{(1)*}\,\theta_6^{(2)} + \theta_3^{(2)}\,\theta_6^{(1)*}\big),
\end{aligned}$$

$$\begin{aligned}
\bar{m}_3 &= -\big(\theta_2^{(1)}\frac{\partial}{\partial x}\theta_3^{(0)} + \theta_2^{(2)}\frac{\partial}{\partial x}\theta_3^{(1)*} + \theta_2^{(1)*}\frac{\partial}{\partial x}\theta_3^{(2)}\big) - \mathrm{i}k\big(\theta_3^{(0)}\,\theta_3^{(1)} + \theta_3^{(1)*}\,\theta_3^{(2)}\big) \\
&\quad -\frac{e}{m_\mathrm{e}c}\big(\theta_2^{(1)*}\,\theta_6^{(2)} + \theta_2^{(2)}\,\theta_6^{(1)*}\big), \\
\bar{m}_4 &= 0, \qquad\qquad \bar{m}_5 = -\frac{4\pi e}{c}\big(\theta_1^{(0)}\,\theta_2^{(1)} + \theta_1^{(2)}\,\theta_2^{(1)*}\big), \\
\bar{m}_6 &= -\frac{4\pi e}{c}\big(\theta_1^{(0)}\,\theta_3^{(1)} + \theta_1^{(2)}\,\theta_3^{(1)*}\big), \qquad (41)
\end{aligned}$$

where

$$\theta_m^{(2)} = \frac{\Phi_m^{(2)}}{\alpha^2}, \qquad \theta_1^{(0)} = \frac{n_0^{(1)}}{\alpha\alpha^*}, \qquad \theta_3^{(0)} = \frac{u_{z0}^{(1)}}{\alpha\alpha^*}. \qquad (42)$$

Note that the quantities $\theta_m^{(1)}$, $\theta_{\mathrm{v}l}^{(1)}$ are given by Eq. (9), $\theta_m^{(2)}$ by Eq. (12) and $\theta_1^{(0)}$, $\theta_3^{(0)}$ by Eq. (18), all set with $\alpha = 1$.

Now, using Eqs. (40) and (41), we get from Eq. (23) the following nonlinear Schrödinger equation which describes the nonlinear evolution of finite-amplitude surface waves in a homogeneous cold-electron-plasma half-space,

$$\mathrm{i}\frac{\partial\alpha}{\partial\tau} + P\frac{\partial^2\alpha}{\partial\xi^2} = Q\alpha^2\alpha^*, \qquad (43)$$

where

$$\begin{aligned}
P &= \frac{1}{2}\frac{\mathrm{d}C_\mathrm{g}}{\mathrm{d}k}, \\
Q &= \mathrm{i}\int_0^\infty \mathrm{d}x \sum_{j=1}^{6} \bar{f}_j\bar{m}_j \times \Bigg[\int_0^\infty \mathrm{d}x\Bigg(\sum_{l=1}^{3}\bar{f}_l\theta_l^{(4)} + \frac{1}{c}\bar{f}_4\theta_6^{(1)} - \frac{1}{c}\bar{f}_5\theta_4^{(1)} - \frac{1}{c}\bar{f}_6\theta_5^{(1)}\Bigg) \\
&\quad + \frac{1}{c}\int_{-\infty}^{0}\mathrm{d}x\Bigg(\bar{f}_{\mathrm{v}1}\theta_{\mathrm{v}3}^{(1)} - \bar{f}_{\mathrm{v}2}\theta_{\mathrm{v}1}^{(1)} - \bar{f}_{\mathrm{v}3}^{(1)}\theta_{\mathrm{v}2}^{(1)}\Bigg)\Bigg]^{-1},
\end{aligned}$$

in which

$$\bar{f}_i = \frac{f_j}{A}, \qquad \bar{f}_{\mathrm{v}l} = \frac{f_{\mathrm{v}l}}{A}$$

and are given by Eq. (26) with $A = 1$.

Using the previous solutions, we obtain

$$C_\mathrm{g} = \frac{(2\omega^2 - \omega_\mathrm{e}^2)c^2k}{(2\omega^2 - \omega_\mathrm{e}^2 - k^2c^2)\omega}, \qquad (44)$$

$$P = \frac{(2\omega^2 - \omega_e^2)c^2 + 8\omega k C_g c^2 - (6\omega^2 - \omega_e^2 - 2k^2c^2)C_g^2}{2\omega(2\omega^2 - \omega_e^2 - k^2c^2)}, \quad (45)$$

$$Q = \frac{e^2\omega^3\omega_e^2}{2m_e^2 k_\perp^2(\omega_e^2 - \omega^2)^4(\omega_e^2 - 4\omega^2)}\left[8\omega^2k^4\beta + \frac{\omega^2(\beta^2 - k^2)^3}{\beta} - \frac{16\omega^4\beta k^2(\beta^2 + k^2)}{\omega_e^2} - \frac{2\omega^2\omega_e^4k^2(\beta^2 - k^2)(2k^2 + \beta\beta_2)}{\omega_e^2k_\perp - 2\omega^2(2k_\perp + \beta_2)}\right] \times\left[\frac{k^2}{k_\perp} + \frac{\omega^2}{2\beta c^2} + \frac{\omega^2(\omega_e^2 + \omega^2)(k^2 + \beta^2)}{2\beta(\omega_e^2 - \omega^2)^2}\right]^{-1}. \quad (46)$$

5. *Results and discussion*

Solutions of the nonlinear Schrödinger equation like (43) have been extensively studied in connection with nonlinear propagation of waves of various kinds. It is a well known result that a uniform plasma wave is modulationally stable or unstable depending on whether

$$PQ > 0 \quad \text{or} \quad PQ < 0. \quad (47)$$

So, with the values of P and Q given, respectively, by Eqs. (45) and (46), for $PQ < 0$, one obtains the condition for the modulational instability of a surface wave propagating parallel to the interface of a cold homogeneous electron-plasma half-space and vacuum. Numerical computation shows that the surface wave at a cold homogeneous electron-plasma half-space is modulationally unstable throughout the whole electro-magnetic region where they exist. The maximum growth rate $g_{\max}$ of this instability is given by [18]

$$g_{\max} = \alpha_0^2|Q| \quad (48)$$

where α_0 is a real constant.

By using the inverse scattering method, Zakharav and Shabat [10] solved the nonlinear Schrödinger equation like (43) for an initial value problem and showed that in the modulationally unstable case, an initial distribution tends to evolve into a series of solitary wave packets called envelope solitons. The formation of solitons in the nonlinear stage of instability can be considered as a dynamical balance between the dispersion effects and nonlinear effects. For $PQ < 0$, Eq. (43) has a solution of the form of an envelope soliton. Representing the complex amplitude α by $\rho(\xi,\tau)\exp[i\sigma(\xi,\tau)]$, it can be shown that the envelope soliton solution is given as

$$\rho(\xi - V_g\tau) = \sqrt{2}\rho_0\cosh^{-1}\left[\sqrt{|Q|/|P|}\rho_0(\xi - V_g\tau)\right], \quad (49)$$

where the soliton speed V_g is independent of the soliton amplitude ρ_0. Thus the electric field associated with a finite-amplitude surface wave can take the form of an envelope soliton which propagates along the plasma – vacuum interface with a velocity independent of the soliton height.

6. Summary and concluding remarks

We make detailed electromagnetic treatment of nonlinear surface waves in a cold, homogeneous electron-plasma half-space. It has been shown that these waves are modulationally unstable throughout the whole electromagnetic region where they exist. Thus the detailed inclusion of the effects of nonlinearities does not change the stability character of surface waves [2]. However, there is a definite quantitative change in the growth rate of the instability.

Finally, we point out that our analysis of purely electronic modulation under the rigid boundary assumption is relevant to plasmas bounded by solid-dielectric and solid state plasmas. The above analysis may be conveniently extended to more realistic studies including the temperature effects and ion motion.

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MODULACIJSKA NESTABILNOST VISOKOFREKVENTNIH POVRŠINSKIH VALOVA NA GRANICI PLAZMA – VAKUUM

Teorijski istražujemo nelinearno širenje površinskih valova u poluprostoru s hladnom elektronskom plazmom metodom višestruke ljestvice. Pokazuje se da su visokofrekventni površinski valovi modulacijski nestabilni na granici plazma – vakuum. Raspravlja se rast modulacijske nestabilnosti površinskih valova. Također pokazujemo da električno polje, koje je povezano s površinskim valovima konačne amplitude, može primiti oblik anvelopnog solitona koji se širi duž granice plazma – vakuum brzinom koja ne ovisi o visini solitona.