ISSN 1330–0008
CODEN FIZAE4

CONVERSION OF ELECTROMAGNETIC WAVE INTO ELECTRON-ACOUSTIC WAVE IN AN INHOMOGENEOUS PLASMA

S. N. PAUL[a,b], G. PAKIRA[c] and B. PAUL[b]

[a]*Inter University Centre for Astronomy and Astrophysics, Post Bag-4, Ganeshkhind, Pune-411007, India*

[b]*Centre for Science Education and Research, B/4, Shyamali Housing Estate, Salt Lake City, Calcutta-700064, India*
E-mail:sailenpaul@rediffmail.com

[c]*Department of Physics, Sripat Sing College, P.O.-Jiaganj, Dist.-Murshidabad, West Bengal, India*

Received 6 October 2003; revised manuscript received 14 July 2005
Accepted 12 September 2005 Online 13 February 2006

The transverse electromagnetic waves and the longitudinal electron-acoustic waves are coupled to each other by the gradients of density or temperature in a plasma. There are also other coupling factors, such as the static magnetic field or nonlinearities existing in the medium. Due to coupling of waves, excitation of one leads to the generation of the other. This results in mutual transfer of power. In the present paper, the expression for the energy flux of acoustic wave due to conversion of electromagnetic wave passing through an inhomogeneous plasma has been obtained using the W. K. B. method. It is suggested that expansion of ionized shell of hot stars and mass loss would be possibly due to the energy conversion process.

PACS numbers: 52.35.-g, 52.40.Db UDC 537.531
Keywords: electromagnetic wave, electron acoustic wave, inhomogeneous plasma

1. Introduction

The phenomenon of coupling between longitudinal and transverse waves in a plasma medium gives some important as well as interesting results in the context of astrophysical phenomena [1]. The parameters like magnetic field, rotation, inhomogeneity, etc., are the sources of creating the coupling phenomena. It is found that the nature of coupling of waves in an inhomogeneous plasma is of some interest. In astrophysical bodies, generation of radio waves may be possible due to coupling between longitudinal and transverse waves in the presence of inhomogeneous density and magnetic field [2]. Considering the effect of the gradients of electron concentration and temperature, Tidman [3] obtained the expressions for scattered transverse waves. Later, Chakraborty [4 − 6] studied the coupling phenomena in a warm nonuniform plasma and derived the expression for the radiating electromagnetic energy due to energy conversion of acoustic waves. He suggested that some astrophysical phenomena, e.g. radio noise in the ionosphere, may be one of the consequences of the energy conversion process. Paul [7] derived the energy flux of the electron acoustic wave due to the conversion of electromagnetic wave in a hot and inhomogeneous plasma under far-field approximation, suggesting mass loss from hot astrophysical bodies when strong electromagnetic wave emitted from the interior of the stellar bodies is converted into electron-acoustic wave. It is to be mentioned that the role of inhomogeneous density in the phenomenon of coupling between acoustic and transverse wave is important if the inhomogeneity is a slowly-varying function of space, and the characteristic length of variation is greater than the wavelength of the transmitted wave. Under these circumstances, it is appropriate to follow the WKB method [8].Using the WKB method, Khan and Paul [9] investigated the conversion of dispersive longitudinal oscillations into reflected and transmitted electromagnetic fields when equilibrium density changes continuously and slowly with space. They numerically estimated the energy of the electromagnetic wave radiated in ionospheric F-region due to the conversion of energy of electron-acoustic wave.

In the present paper, conversion of energy from electromagnetic wave into electron-acoustic wave in a slowly varying plasma has been investigated using the WKB method. The coupling equations of the transverse and longitudinal waves are solved using the WKB method, and the expression for the energy flux carried away by the particles due to conversion of electromagnetic wave are derived. It is seen that the energy fluxes of the particles become significant in a plasma having large density gradient and for the waves of low frequency.

2. *Basic equations*

We consider a stationary, unmagnetized plasma where the velocity of the particles is nonrelativistic and forces due to collision, gravitational effect, etc., are negligibly small in comparison with other forces in the medium. The plasma has inhomogeneous density, i.e. $|\boldsymbol{\nabla} N_0| \neq 0$. With the above assumptions, the linearized plasma equations are

$$\rho_0 \frac{\partial \boldsymbol{u}^{(1)}}{\partial t} = -\boldsymbol{\nabla} P^{(1)} - \frac{e}{m}\rho_0 \boldsymbol{E}^{(1)}\,, \tag{1}$$

$$\frac{\partial \rho^{(1)}}{\partial t} + \boldsymbol{\nabla}\cdot(\rho_0 \boldsymbol{u}^{(1)}) = 0\,, \tag{2}$$

$$\boldsymbol{\nabla}\times \boldsymbol{E}^{(1)} = -\frac{1}{c}\frac{\partial \boldsymbol{H}^{(1)}}{\partial t}\,, \tag{3}$$

$$\boldsymbol{\nabla} \times \boldsymbol{H}^{(1)} = -\frac{1}{c}\frac{\partial \boldsymbol{E}^{(1)}}{\partial t} - \frac{4\pi e \rho_0 \boldsymbol{u}^{(1)}}{mc}\,, \tag{4}$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{E}^{(1)} = -4\pi e N^{(1)}\,, \tag{5}$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{H}^{(1)} = 0\,, \tag{6}$$

where

$$P^{(1)} = mV_s N^{(1)}, \quad \rho^0 = mN_0, \quad \rho^{(1)} = mN^{(1)}\,. \tag{7}$$

V_s = thermal speed = $(\chi T/m)^{1/2}$, χ = Boltzmann constant, T is the temperature, m and N are the mass and number density of electrons, the variables with superscript $^{(1)}$ correspond to the first-order of approximation, while ρ_0, P_0 and N_0 are the equilibrium values of mass density, pressure and number density, respectively.

Assuming the time variation of the perturbed quantities as $\exp(-i\omega t)$, ω being the wave frequency, the linear coupling equations of transverse field variables and longitudinal pressure perturbation may be derived as

$$(\nabla^2 + k_t^2)\boldsymbol{H}^{(1)} - \frac{[\boldsymbol{\nabla}\mu \times (\boldsymbol{\nabla} \times \boldsymbol{H}^{(1)})]}{\mu} = \frac{4\pi e \mathrm{i}}{\omega m c \mu}[\boldsymbol{\nabla}\mu \times \boldsymbol{\nabla} P^{(1)}]\,, \tag{8}$$

$$(\nabla^2 + k_l^2)P^{(1)} - \frac{[\boldsymbol{\nabla}\mu \cdot \boldsymbol{\nabla} P^{(1)}]}{\mu} = \mathrm{i}\frac{\omega m c}{4\pi e \mu}[\boldsymbol{\nabla}\mu \cdot (\boldsymbol{\nabla} \times \boldsymbol{H}^{(1)})]\,, \tag{9}$$

where

$$\mu = 1 - \frac{\omega_p^2}{\omega^2}, \quad k_t^2 = \frac{(\omega^2 - \omega_p^2)}{c^2}, \quad k_l^2 = \frac{(\omega^2 - \omega_p^2)}{V_s^2}, \quad \omega_p^2 = \frac{4\pi N_0 e^2}{m}\,. \tag{10}$$

ω_p is the plasma frequency, and k_l and k_t are the wave vectors along the longitudinal and transverse direction, respectively.

3. W.K.B. solution of the coupling equation

Suppose a strong electromagnetic wave is incident on zx-plane and it is the only source creating a pressure perturbation in the medium. If this wave propagates along the z-direction with an x-dependence of the field variables according to $\exp(\mathrm{i}k_0 x)$, k_0 = constant, then the coupling Eqs. (9) and (10) become (the superscript $^{(1)}$ is omitted hereafter)

$$\frac{\mathrm{d}^2\boldsymbol{H}(z)}{\mathrm{d}z^2} + k_t'^2 \boldsymbol{H}(z) = 0\,, \tag{11}$$

and

$$\frac{\mathrm{d}^2 P(z)}{\mathrm{d}z^2} + k_l'^2 P(z) = \frac{\mathrm{i}\,\omega c m}{4\pi e \mu}[\boldsymbol{\nabla}\mu \cdot (\boldsymbol{\nabla} \times \boldsymbol{H})]\,, \tag{12}$$

where, ${k'_t}^2 = {k_t}^2 - {k_0}^2$, ${k'_l}^2 = {k_l}^2 - {k_0}^2$, and

$$\boldsymbol{H} = \boldsymbol{H(z)}\exp(ik_0x), \quad P = P(z)\exp(\mathrm{i}k_0x)\,. \tag{13}$$

Let us assume, the equilibrium number density of electrons in the plasma is a slowly varying function of space. So, k_l (or k'_l) and k_t (or k'_t) will be functions of space. Under this situation, it is appropriate to use the W. K. B method for solving the coupled equations. Therefore, Eq. (11) gives

$$\boldsymbol{H}(z) = \frac{\boldsymbol{A_1}}{\sqrt{k'_t}}\exp\left(\mathrm{i}\int k'_t\mathrm{d}z - \mathrm{i}\omega t\right) + \frac{\boldsymbol{A_2}}{\sqrt{k'_t}}\exp\left(-\mathrm{i}\int k'_t\mathrm{d}z - \mathrm{i}\omega t\right), \tag{14}$$

where $\boldsymbol{A_1}$ and $\boldsymbol{A_2}$ are constants.

For a forward-going wave, take

$$\boldsymbol{H} = \frac{\boldsymbol{A_1}}{\sqrt{k'_t}}\exp\left(\mathrm{i}\,k_0x + \mathrm{i}\int k'_t\mathrm{d}z - \mathrm{i}\omega t\right). \tag{15}$$

Expression (15) is our incident electromagnetic wave which is propagating in the forward Oz-direction. Using (15) in (12), the pressure P can be evaluated from Eq. (12). To obtain the general solution for P, the right-hand side of (12) is taken equal to zero and a complementary function is obtained as

$$P(z) = \frac{B_1}{\sqrt{k'_l}}\exp\left(\mathrm{i}\int k'_l\mathrm{d}z - \mathrm{i}\omega t\right) + \frac{B_2}{\sqrt{k'_l}}\exp\left(-\mathrm{i}\int k'_l\mathrm{d}z - \mathrm{i}\omega t\right), \tag{16}$$

where B_1 and B_2 may be functions of space.

Now, using the method of variation of parameters [10, 11], the pressure P obtained from equation (12) is given by

$$P(z) = -\frac{\mathrm{i}\,k_0cm\omega_p^2A_{1y}}{4\pi e\omega L\sqrt{k'_l}}\left[\int_0^z \frac{\exp(\mathrm{i}\int K_1\mathrm{d}z)}{K_3}\mathrm{d}z\right]\exp(\mathrm{i}\int k'_l\mathrm{d}z - \mathrm{i}\omega t)$$

$$+\frac{\mathrm{i}\,k_0cm\omega_p^2A_{1y}}{4\pi e\omega L\sqrt{k'_l}}\left[\int_0^z \frac{\exp(\mathrm{i}\int K_2\mathrm{d}z)}{K_3}\mathrm{d}z\right]\exp(-\mathrm{i}\int k'_l\mathrm{d}z - \mathrm{i}\omega t)\,, \tag{17}$$

where

$$K_1 = k'_t - k'_l, \quad K_2 = k'_t + k'_l, \quad K_3 = \mu\sqrt{k'_tk'_l)}\,, \quad \boldsymbol{\nabla}\mu = -\left|\frac{\omega_p^2}{\omega^2L}\right|\hat{e_z}\,.$$

$\hat{e}_z$ is the unit vector along OZ-direction, L is the characteristic length of variation of electron density in Z-direction ($L \gg \lambda$), and

$$\left| \frac{1}{k'^3_l} \frac{d^2 k'_l}{dz^2} - \frac{3}{4k'^4_l} (\frac{dk'_l}{dz}) \right| \ll 1 \, .$$

But when density gradient lies perpendicular to the direction of wave propagation, and is given by $\boldsymbol{\nabla}\mu = -|\omega_p^2/(\omega^2 L')|\hat{e}_x$, $\hat{e}_x$ being the unit vector along the OX-direction, where L' is the characteristic length of variation of electron density in X-direction, then the value of $P(z)$ becomes

$$P(z) = \frac{mc\omega_p^2 A_{1y}}{8\pi e\omega L' \sqrt{k'_l}} \left[\int_0^z \left\{ \frac{\mathrm{i}k'_t - \frac{1}{2k'_t}\frac{\mathrm{d}k'_t t}{\mathrm{d}z}}{K_3} \exp\left(\mathrm{i} \int K_1 \mathrm{d}z \right) \right\} \mathrm{d}z \right] \exp(\mathrm{i}k'_l t - \mathrm{i}\omega t)$$

$$- \frac{mc\omega_p^2 A_{1y}}{8\pi e\omega L' \sqrt{k'_l}} \left[\int_0^z \left\{ \frac{\mathrm{i}k'_t - \frac{1}{2k'_t}\frac{\mathrm{d}k'_t t}{\mathrm{d}z}}{K_3} \exp\left(\mathrm{i} \int K_2 \mathrm{d}z \right) \right\} \mathrm{d}z \right] \exp(-\mathrm{i}k'_l t - \mathrm{i}\omega t) \, . \tag{18}$$

The integrals $\int_0^z \frac{\exp(\mathrm{i}\int K_1 \mathrm{d}z)}{K_3} \mathrm{d}z$ and $\int_0^z \frac{\exp(\mathrm{i}\int K_2 \mathrm{d}z)}{K_3} \mathrm{d}z$ of equation (18) and the integrals

$$\int_0^z \frac{\mathrm{i}k'_t - \frac{1}{2k'_t}\frac{\mathrm{d}k'_t t}{\mathrm{d}z}}{K_3} \exp\left(\mathrm{i} \int K_1 \mathrm{d}z \right) \mathrm{d}z$$

and

$$\int_0^z \frac{\mathrm{i}k'_t - \frac{1}{2k'_t}\frac{\mathrm{d}k'_t t}{\mathrm{d}z}}{K_3} \exp\left(\mathrm{i} \int K_2 \mathrm{d}z \right) \mathrm{d}z$$

and Eq. (23) can be evaluated by expanding $\exp(\mathrm{i} \int_0^z K_n \mathrm{d}z)$ as [3]

$$\exp\left(\mathrm{i} \int_0^z K_n \mathrm{d}z \right) = \exp\left[\mathrm{i} \left\{ K_n(0) + K'_n(0)\frac{z^2}{2} + \cdots \right\} \right] , \tag{19}$$

where $n = 1, 2$; $z \ll L$, and K'_n is the derivative of K_n with respect to z.

In the right-hand side of the expression (19), second, third and other terms are very small in comparison to the first term. Therefore, we obtain

$$\int_0^z \frac{\exp\left(\mathrm{i}\int K_1 \mathrm{d}z\right)}{K_3} \mathrm{d}z$$

$$= \frac{1}{\mathrm{i}K_1K_3}\left(1-\frac{\mathrm{i}}{\mathrm{K_1K_3}}\frac{\mathrm{d}K_3}{\mathrm{d}z}\right)\{\exp(\mathrm{i}K_1z)-1\}\int\limits_0^z \frac{\exp\left(\mathrm{i}\int K_2\mathrm{d}z\right)}{K_3}\mathrm{d}z$$

$$= \frac{1}{\mathrm{i}K_2K_3}\left(1-\frac{\mathrm{i}}{\mathrm{K_2K_3}}\frac{\mathrm{d}K_3}{\mathrm{d}z}\right)\{\exp(iK_2z)-1\}\int\limits_0^z \frac{1}{K_3}\left(\mathrm{i}k'_t-\frac{1}{2k'_t}\frac{\mathrm{d}k't}{\mathrm{d}z}\right)\exp\left(\mathrm{i}\int K_1\mathrm{d}z\right)\mathrm{d}z$$

$$= \left[\frac{k'_t}{K_1K_3}+\frac{\mathrm{i}}{\mathrm{K_1K_3}}\left\{\left(\frac{1}{2k'_t}-\frac{1}{K_1}\right)\frac{\mathrm{d}k'_t}{\mathrm{d}z}-\frac{k'_t}{K_1K_3}\frac{\mathrm{d}K_3}{\mathrm{d}z}\right\}\right]\{\exp(\mathrm{i}K_1)-1\}$$

$$\times\int\limits_0^z\left\{\frac{1}{K_3}\left(\mathrm{i}k'_t-\frac{1}{2k'_t}\frac{\mathrm{d}k't}{\mathrm{d}z}\right)\ \exp\left(\mathrm{i}\int K_2\mathrm{d}z\right)\right\}\mathrm{d}z$$

$$= \left[\frac{k'_t}{K_2K_3}+\frac{\mathrm{i}}{\mathrm{K_2K_3}}\left\{\left(\frac{1}{2k'_t}-\frac{1}{K_2}\right)\frac{\mathrm{d}k'_t}{\mathrm{d}z}-\frac{k'_t}{K_2K_3}\frac{\mathrm{d}K_3}{\mathrm{d}z}\right\}\right]\{\exp(\mathrm{i}K_2z)-1\},$$

where the terms containing $\mathrm{d}^2K_3/\mathrm{d}z^2$, $(\mathrm{d}K_3/\mathrm{d}z)^2$ etc. have been neglected.

Therefore, the pressure P is given by

$$P=-\frac{k_0cm\omega_p^2A_{1y}}{4\pi e\omega LK_1K_3\sqrt{k'_l}}\left(1-\frac{\mathrm{i}}{\mathrm{K_1K_3}}\frac{\mathrm{d}K_3}{\mathrm{d}z}\right)\{\exp(\mathrm{i}K_1z)-1\}\exp\left(\mathrm{i}k_0x+\mathrm{i}\int k'_l\mathrm{d}z-i\omega t\right)$$
$$+\frac{k_0cm\omega_p^2A_{1y}}{4\pi e\omega LK_2K_3\sqrt{k'_l}}\left(1-\frac{\mathrm{i}}{\mathrm{K_2K_3}}\frac{\mathrm{d}K_3}{\mathrm{d}z}\right)\{\exp(\mathrm{i}K_2z)-1\}\exp\left(\mathrm{i}k_0x-i\int k'_l\mathrm{d}z-\mathrm{i}\omega t\right) \tag{20}$$

where $\boldsymbol{\nabla}\mu$ is assumed to be present in the direction of OZ, and

$$P=\frac{cm\omega_p^2A_{1y}}{8\pi e\omega L'K_1K_3\sqrt{k'_l}}\left[k'_t+\mathrm{i}\left\{\left(\frac{1}{2k'_t}-\frac{1}{K_1}\right)\frac{\mathrm{d}k'_t}{\mathrm{d}z}-\frac{k'_t}{K_1K_3}\frac{\mathrm{d}K_3}{\mathrm{d}z}\right\}\right]$$
$$\times\{\exp(\mathrm{i}K_1z)-1\}\exp\left(\mathrm{i}k_0x+\mathrm{i}\int k'_l\mathrm{d}z-\mathrm{i}\omega t\right)$$
$$-\frac{cm\omega_p^2A_{1y}}{4\pi e\omega L'K_2K_3\sqrt{k'_l}}\left[k'_t+\mathrm{i}\left\{\left(\frac{1}{2k'_t}-\frac{1}{K_1}\right)\frac{\mathrm{d}k'_t}{\mathrm{d}z}-\frac{k'_t}{K_2K_3}\frac{\mathrm{d}K_3}{\mathrm{d}z}\right\}\right]$$
$$\times\{\exp(\mathrm{i}K_2z)-1\}\exp\left(\mathrm{i}k_0x-i\int k'_l\mathrm{d}z-\mathrm{i}\omega t\right), \tag{21}$$

where $\boldsymbol{\nabla}\mu$ lies along the Ox-axis.

So, considering the variation of equilibrium electron number density parallel to the Z-axis, real values of transmitted and reflected part of the perturbed pressures read

$$\mathrm{Re}(P)^t = -\frac{k_0 c m \omega_p^2 A_{1y}}{4\pi e \omega L\ K_1 K_3 \sqrt{k'_l}} \left[\cos\phi_2 - \cos\phi_1 - \frac{1}{K_1 K_3} \frac{\mathrm{d}K_3}{\mathrm{d}z} (\sin\phi_2 - \sin\phi_1) \right], \quad (22)$$

and

$$\mathrm{Re}(P)^r = \frac{k_0 c m \omega_p^2 A_{1y}}{4\pi e \omega L K_2 K_3 \sqrt{k'_l}} \left[\cos\phi_4 - \cos\phi_3 - \frac{1}{K_2 K_3} \frac{\mathrm{d}K_3}{\mathrm{d}z} (\sin\phi_4 - \sin\phi_3) \right]. \quad (23)$$

Similarly, when the variation of electron density is along OX-axis, real values of transmitted an reflected part of the perturbed pressure are

$$\mathrm{Re}(P)^t = \frac{c m \omega_p^2 A_{1y}}{8\pi e \omega L' K_1 K_3 \sqrt{k'_l}} \left[k'_t (\cos\phi_2 - \cos\phi_1) \right.$$
$$\left. - \left\{ \left(\frac{1}{2k'_t} - \frac{1}{K_1} \right) \frac{\mathrm{d}k'_t}{\mathrm{d}z} - \frac{k'_t}{K_1 K_3} \frac{\mathrm{d}K_3}{\mathrm{d}z} \right\} (\sin\phi_2 - \sin\phi_1) \right] \quad (24)$$

and

$$\mathrm{Re}(P)^r = -\frac{c m \omega_p^2 A_{1y}}{8\pi e \omega L' K_2 K_3 \sqrt{k'_l}} \left[k'_t (\cos\phi_4 - \cos\phi_3) \right.$$
$$\left. - \left\{ \left(\frac{1}{2k'_t} - \frac{1}{K_2} \right) \frac{\mathrm{d}k'_t}{\mathrm{d}z} - \frac{k'_t}{K_2 K_3} \frac{\mathrm{d}K_3}{\mathrm{d}z} \right\} (\sin\phi_4 - \sin\phi_3) \right] \quad (25),$$

where

$$\phi_1 = k_0 + \int k'_l \mathrm{d}z - \omega t,$$

$$\phi_2 = k_0 + K_1 + \int k'_l \mathrm{d}z - \omega t,$$

$$\phi_3 = k_0 - \int k'_l \mathrm{d}z - \omega t,$$

$$\phi_4 = k_0 + K_2 - \int k'_l \mathrm{d}z - \omega t,$$

4. *Energy flux carried away by the particles*

The energy carried away by the particles in the direction of wave propagation is given by

$$S'_z = (\mathrm{Re}\, u_z)\,(\mathrm{Re}\, P)\,. \tag{26}$$

Now, from the basic equations (1) - (7), one obtains,

$$\boldsymbol{u} = -\frac{\mathrm{i}\,\omega}{mN_0(\omega^2-\omega_p^2)}\,\boldsymbol{\nabla}\, P\,. \tag{27}$$

So, using the expression (20) in (27), real values of the transmitted and reflected part of the longitudinal velocity of plasma particles are

$$\mathrm{Re}\,(u_z)^t = \frac{\omega_p^2\, k_0\, c\, A_{1y}}{4\pi e N_0\, L\, K_1\, K_3(\omega^2-\omega_p^2)\,\sqrt{k'_l}}\,\Bigg[k'_t\,\cos\phi_2 - k'_l\,\cos\phi_1$$

$$+\left(\frac{1}{2k'_l}\frac{\mathrm{d}k'_l}{\mathrm{d}z} - \frac{k'_l}{K_1K_3}\frac{\mathrm{d}K_3}{\mathrm{d}z}\right)\sin\phi_2 + \left(\frac{1}{2k'_l}\frac{\mathrm{d}k'_l}{\mathrm{d}z} + \frac{1}{K_3}\left(1-\frac{k'_l}{K_1}\right)\frac{\mathrm{d}K_3}{\mathrm{d}z}\right)\sin\phi_1\Bigg] \tag{28}$$

and

$$\mathrm{Re}\,(u_z)^r = -\frac{\omega_p^2\, k_0\, c\, A_{1y}}{4\pi e N_0\, L\, K_2\, K_3(\omega^2-\omega_p^2)\,\sqrt{k'_l}}\,\Bigg[k'_t\,\cos\phi_4 - k'_l\,\cos\phi_3$$

$$+\left(\frac{1}{2k'_l}\frac{\mathrm{d}k'_l}{\mathrm{d}z} - \frac{k'_l}{K_2K_3}\right)\frac{\mathrm{d}k'_l}{\mathrm{d}z}\sin\phi_4 + \left(\frac{1}{2k'_l}\frac{\mathrm{d}k'_l}{\mathrm{d}z} + \frac{1}{K_3}\left(1-\frac{k'_l}{K_2}\right)\frac{\mathrm{d}K_3}{\mathrm{d}z}\right)\sin\phi_3\Bigg]\,, \tag{29}$$

where $\boldsymbol{\nabla}\mu$ has been assumed to be present along the OZ-axis.

Asuming $\boldsymbol{\nabla}\mu$ to be along the OX-axis, we get from the expression (30) and (40) the real values of $(u_z)^t$ and $(u_z)^r$ which are given by

$$\mathrm{Re}(u_z)^t = -\frac{\omega_p^2\, c\, A_{1y}}{8\pi e N_0\, L'\, K_1\, K_3(\omega^2-\omega_p^2)\,\sqrt{k'_l}}\,\Bigg[k'_t\,(k'_t\,\cos\phi_2 - k'_l\,\cos\phi_1)$$

$$-\left(\left(\frac{1}{2k'_l}-\frac{1}{K_1}\right)\frac{\mathrm{d}k'_t}{\mathrm{d}z} - \frac{k'_t}{K_1K_3}\frac{\mathrm{d}K_3}{\mathrm{d}z}\right)(k'_t\sin\phi_2 - k'_l\sin\phi_1)$$

$$-\left\{k'_t\frac{\mathrm{d}}{\mathrm{d}z}\left(\frac{1}{K_1K_3\sqrt{k'_l}}\right) + \frac{1}{K_1K_3\sqrt{k'_l}}\frac{\mathrm{d}k'_t}{\mathrm{d}z}\right\}(\sin\phi_2 - \sin\phi_1)\Bigg] \tag{30}$$

and

$$\mathrm{Re}(u_z)_r = -\frac{\omega_p^2\, c\, A_{1y}}{8\pi e N_0\, L'\, K_2\, K_3(\omega^2-\omega_p^2)\,\sqrt{k'_l}}\,\Bigg[k'_t\,(k'_t\,\cos\phi_2 - k'_l\,\cos\phi_1)$$

$$-\left(\left(\frac{1}{2k'_l}-\frac{1}{K_2}\right)\frac{\mathrm{d}k'_t}{\mathrm{d}z}-\frac{k'_t}{K_2K_3}\frac{\mathrm{d}K_3}{\mathrm{d}z}\right)(k'_t\sin\phi_4-k'_l\sin\phi_3)$$

$$-\left\{k'_t\frac{\mathrm{d}}{\mathrm{d}z}\left(\frac{1}{K_2K_3\sqrt{k'_l}}\right)+\frac{1}{K_2K_3\sqrt{k'_l}}\frac{\mathrm{d}k'_l}{\mathrm{d}z}\right\}(\sin\phi_4-\sin\phi_3)\Bigg]. \tag{31}$$

Therefore, using the values of $\mathrm{Re}(P)$ and $\mathrm{Re}(u_z)$, and then averaging over a time period $2\pi/\omega$, the expression for energy carried away by the particles along the OZ- and OX-directions are obtained as

Case-I: The variation of electron density is along the direction of wave propagation(OZ–direction)

Using the above expressions, the transmitted part of the Z-component of particle energy flux is obtained as

$$|\langle(S'_z)^t_I\rangle|=\frac{\omega^2\omega_p^2c^2k_0^2(k'_t+k'_l)A_{1y}^2}{4\pi L^2k'_tk'^2_l(k'_t-k'_l)^2(\omega^2-\omega_p^2)^2}\{1-\cos(k'_t-k'_l)z\}\,. \tag{32}$$

The reflected part of energy flux carried away by the particles is given by

$$|\langle(S'_z)^r_I\rangle|=\frac{\omega^2\omega_p^2c^2k_0^2(k'_t-k'_l)A_{1y}^2}{4\pi L^2k'_tk'^2_l(k'_t+k'_l)^2(\omega^2-\omega_p^2)^2}\{1-\cos(k'_t+k'_l)z\}\,. \tag{33}$$

Case-II: The variation of density lies perpendicular to the direction of wave propagation

The transmitted and reflected part of the particle energy flux are given by

$$|\langle(S'_z)^t_{II}\rangle|=\frac{\omega^2\omega_p^2c^2k_t(k'_t+k'_l)A_{1y}^2}{32\pi L'^2k'^2_l(k'_t-k'_l)^2(\omega^2-\omega_p^2)^2}\{1-\cos(k'_t-k'_l)z\} \tag{34}$$

and

$$|\langle(S'_z)^r_{II}\rangle|=\frac{\omega^2\omega_p^2c^2k_t(k'_t-k'_l)A_{1y}^2}{32\pi L'^2k'^2_l(k'_t+k'_l)^2(\omega^2-\omega_p^2)^2}\{1-\cos(k'_t+k'_l)z\}\,. \tag{35}$$

Expressions (32) – (35) show that the energy flux of the particles ($\langle S'\rangle$) depends on $\boldsymbol{\nabla}N_0$, ω, z, etc. It may be observed here that $\langle S'\rangle$ becomes significant in a plasma having large density gradient, but it is insignificant for a very high frequency wave. For a wave having frequency very close to the plasma frequency, i.e., $\omega\approx\omega_p$, the particle energy flux would be significantly larger. The role of the distance (z) covered by the wave on $\langle S'\rangle$ gives some interesting results for the energy fluxes. For $z=0$ to $z=2\pi/K_1$ (or $2\pi/K_2$), the average value of the transmitted and reflected energy flux of the particles will be varying in nature. It is seen that when $z=0,2\pi/K_1,4\pi/K_1,\cdots$ etc., the transmitted particle energy fluxes $\langle(S')^t_I\rangle$ and $\langle(S')^t_{II}\rangle$ are minimum. But, when $z=\pi/K_1,3\pi/K_1,5\pi/K_1,\cdots$ etc., both

$\langle (S')^t_I \rangle$ $\langle (S')^t_{II} \rangle$ become maximum.The reflected parts of the energy flux $\langle (S')^r_I \rangle$ and $\langle (S')^r_{II} \rangle$ would be minimum and maximum when $z = 0, 2\pi/K_2, 4\pi/K_2, \cdots$ etc. and $z = \pi/K_2, 3\pi/K_2, 5\pi/K_2, \cdots$ etc. respectively.

5. *Summary and concluding remarks*

In this paper, coupling of electromagnetic wave and electron-acoustic wave in an inhomogeneous plasma has been studied using W. K. B. method for which it is assumed that equilibrium density changes slowly but continuously in such a way that the characteristic length of variation is much greater than the wave length of the transmitted wave. It has been shown that the incident electromagnetic wave would be converted in to electron-acoustic wave due to the coupling of waves in inhomogeneous plasma. The particle energy flux depends on the density gradient, frequency of incident wave and characteristic length of density variation. This energy flux will be significant in a plasma having large density gradient and for low-frequency waves and when the wave frequency approaches to plasma frequency. It is very important to mention that for the present study, both electromagnetic and electron acoustic wave are assumed to be stable during propagation through the inhomogeneous plasma, i.e., instabilities of the waves in inhomogeneous plasma have been neglected.

Our present analysis may be useful to understand some phenomena in the ionosphere, solar corona and hot stars. It is known that earth's ionosphere is a non-uniform plasma medium and the density is a slowly varying function of distance from earth's surface. Electromagnetic waves propagating through the ionosphere would be converted into acoustic waves due to the coupling of waves and this may be one of the sources of heating of the ionosphere.

Expansion of the ionized shell of hot stars may also be due to the conversion of a transverse wave in to an acoustic wave. When electromagnetic waves, generated in the interior of hot stars, come to the surface and enter its ionized shell, strong acoustic waves would be generated due to the energy conversion process. As a result, the ionized shells of hot stars may be expanded. If the velocity of expansion of the plasma shell is larger than the escape velocity of the plasma particles, some amount of mass would be ejected from the plasma shell to out of the stars. Considering the values of the plasma parameters of the hot stars, the energy carried away by the particles in the outward direction can be numerically estimated.

It is to be noted that the effects of magnetic field, temperature gradient, etc., have not been considered in the present study, though these are very important for the study of the coupling of waves. During astrophysical disturbances (e.g., during solar bursts, magnetic storms, etc.) strength of the magnetic field becomes very high and the plasma particles move with very high velocity of relativistic order. To have the actual picture of the coupling of waves in the inhomogeneous plasma of the astrophysical objects it is necessary to consider the static magnetic field and the relativistic effects for the analysis of the coupling of waves. In this regard, it is important to note that if electrons have streaming motion in the presence of the density gradient in the plasma, equilibrium density becomes time dependent, which may be observed from the continuity equation in unperturbed state. So, we require to perform W. K. B. analysis in time, i.e., the time variation of the perturbed quantities should be assumed as proportional to $\exp[-\mathrm{i}\int \omega \mathrm{d}t]$ instead of $\exp[-\mathrm{i}\omega t]$, ω being the frequency of the wave.

Acknowledgements

This work was supported by the University Grants Commission, New Delhi, India. S. N. Paul is thankful to the Inter University Centre for Astronomy and Astrophysics, Pune, India for providing hospitality for the work. The authors are also thankful to the referee for some useful suggestions which helped to make the manuscript in present form.

References

[1] V. V. Zeleznyakov, *Radio Emission of Sun and Planets*, (Translated from Russian by H. Massey), Pergamon Press (1964).

[2] G. B. Field, Ap. J. **124** (1956) 555.

[3] D. A. Tidman, Phys. Rev. **117** (1960) 362.

[4] B. Chakraborty, J. Math. Phys. **11** (1970) 2570.

[5] B. Chakraborty , J. Math. Phys. **12** (1971) 529.

[6] B.Chakraborty, Appl. Sci. Res. **28** (1973) 466.

[7] S. N. Paul, Czec. J. Phys. B **27** (1977) 873.

[8] K. G. Budden, *Radio Waves in Ionosphere*, Cambridge University Press, New York (1961) Chap.-9.

[9] M. R. Khan and S. N. Paul, Appl. Sci. Res. **33** (1977) 209.

[10] A. R. Forsyth, *A Treatise on Differential Equations*, McMillan and Co., London (1914) Chapters II and IV.

[11] F. Mathew and R. L. Walker, *Mathematical Methods of Physics*, W. A. Benjamin, Inc., New York (1964) Sec. 1.1.

PRETVORBA ELEKTROMAGNETSKOG VALA U ELEKTRONSKO-ZVUČNI VAL U NEHOMOGENOJ PLAZMI

Poprečni elektromagnetski valovi i uzdužni elektronsko-zvučni valovi vezani su međusobno gradijentima gustoće i temperature u plazmi. Djeluju i drugi vezni čimbenici, kao statičko magnetsko polje ili nelinearnost plazme. Zbog povezivanja valova, uzbuda jednih stvara druge. Stoga imamo uzajamni prijenos energije. U ovom se radu, primjenom W. K. B. metode, izvodi izraz za tok energije zvučnog vala zbog pretvorbe elektromagnetskog vala koji prolazi nehomogenom plazmom. Izražava se mogućnost da su širenje ionizirane ljuske vrućih zvijezda i gubitak mase posljedica procesa pretvorbe energije.