Printed ISSN 1330–0008
Online ISSN 1333–9125
CD ISSN 1333–8390
CODEN FIZAE4

MOTION OF TWO DISCS JOINED BY STRINGS IN S-TYPE CONNECTION

RANKO ARTUKOVIĆ

Gymnasium Jurja Barakovića, Park V. Nazora 3/1, Zadar, Croatia

Received 28 May 2008; Accepted 17 December 2008
Online 4 March 2009

Motion of a pair of discs under the action of the centrifugal force and string tension is studied. The discs can slide along a guide-bar and rotate around their axes. Initially, each disc is held symmetrically with respect to the main axis of rotation by a string which is wound around it and tied to a cross-bar. While the assembly rotates around the main axis, the discs are released by cutting the two strings in a way that the configuration resembles the letter S. The discs start moving from the main axis and change rotation around their axes. Their motion is analysed using equations of motion and the law of conservation of angular momentum. The results are verified using the law of conservation of energy. It is found that after a long time, rotation of the guide-bar stops, and the two discs move symmetrically from the main axis with the same speed and rotate with the same angular velocity. An interesting result is that the angle of rotation of the guide-bar does not depend on the initial angular velocity of the assembly.

PACS number: 45.40.-f UDC 531.17, 531.382

Keywords: pair of discs on a guide-bar, motion under centrifugal force and string tension

1. Introduction

The problem of motion of rigid bodies is very old and many textbooks, monographs and articles treat many systems under various conditions. Well known classical textbook by Klein and Sommerfeld [1] gives rather detailed description of rather complicated motion of spinning tops and gyroscopes. Relatively simple problems of motion of bodies of rotational symmetry, rotating without the change of orientation of the axis of rotation, or simple precession of spinning tops, may be found in standard textbooks [2]. Motion of coils, wheels and discs around which a string or tape is wound, under the action of gravity and the string or tape tension, has been treated in many textbooks [3].

This article is dealing with the motion of discs around which the string is wound, under the action of the string tension and the centrifugal force. It is related to the so-called yo-yo, a wheel acted upon by the gravitation and string tension [3]. The description of the systems and equations of motion are presented in Sec. 2. Solution of the equations of motion are given in Sec. 3, and the discussion of the results in Sec. 4. Conclusion is presented in Sec. 5.

2. *The system of a pair of discs – equations of motion*

We consider the system of two discs mounted on a guide-bar (see Figs. 1). The discs can slide along the guide-bar and rotate around their axes. Around each disc, a long string is wound and the ends of the strings are tied to a cross-bar fixed to the guide-bar at right angle. The strings keep the discs symmetrically at a fixed distance from the main axis of rotation while the system rotates (as a rigid body) around the main axis with the initial angular velocity ω_0 (see Figs. 1a and b). At the moment $t = 0$, the strings are cut as indicated in Fig. 1a and in Fig. 1b. The former case we name the progressive pair of discs and the latter the regressive pair of discs.

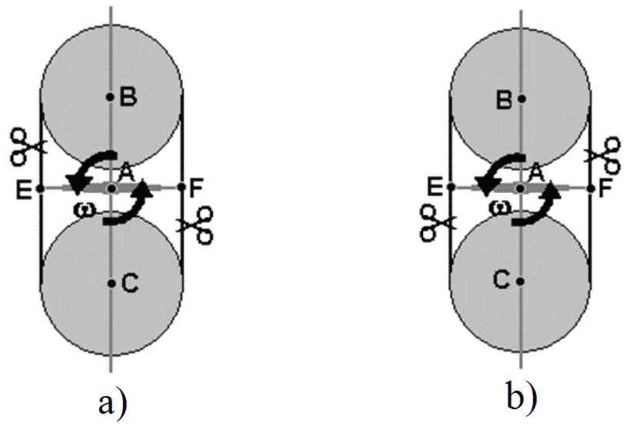

Fig. 1. A pair of discs is mounted on the guide-bar BAC. They can rotate about the points B and C and slide along the guide-bar. Around each disc, a long string is wound, and the ends of the strings are fixed at points E and F to the cross-bar EF which is fixed to the guide-bar. The whole assembly rotates around the point A, the main axis of the system, in the indicated direction. The strings keep the discs at equal distances from the main axis. At the moment $t = 0$, the two strings are cut simultaneously, resulting in radial motion and change of rotation of the two discs. The cutting shown in (a) results in the "progressive" motion and that shown in (b) in the "regressive" motion of the discs.

In either case, we assume that the motion of the discs is frictionless and without the action of an external force, i.e. the total energy and angular momentum of the system is conserved. We also neglect the masses of the strings and of the bars. Therefore, we consider only the masses and moments of inertia of the two discs.

We use the following notations: R is the radius of the discs, m is the mass of each disc and I its moment of inertia, r is the distance of the disc axes from the main axis. We named the two bodies discs, but other shapes of rotational symmetry are equally possible. In all figures, the two bodies are shown as discs.

Initially, the whole system rotates with the angular velocity $\omega = \omega_0 > 0$, as shown in Figs. 1 when looking at the system from above. That is initially also the angular velocity of the discs about their own axes. The strings keep the discs at the distance $r = r_0$ and the whole system behaves as a rigid body. The total angular momentum of the pair of discs is given by the sum of the angular momenta due to the rotation about their own axes and due to the rotational motion of the discs around the main axis,

$$L_0 = 2(\omega_0 I + \omega_0 m r_0^2) = 2\omega_0 (I + m r_0^2)\,. \tag{1}$$

The total energy of the system is equal to

$$E_0 = \omega_0^2 (I + m r_0^2)\,. \tag{2}$$

We study the motion of the discs after the strings are cut as shown in Figs. 1. Due to the centrifugal forces, the discs begin to slide with a radial velocity $v_r = \mathrm{d}r/\mathrm{d}t$ in opposite directions, radial distances r of disc axes from the main axis increase, the strings unwind and angular velocities of the discs change (see Figs. 2).

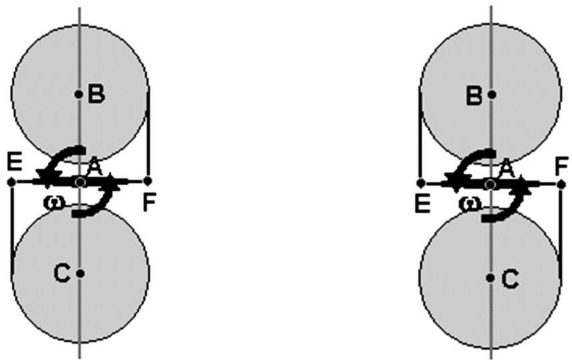

Fig. 2. Initial state of motion after the cutting of the strings of (left) the progressive and (right) the regressive pair of discs.

Looking in the coordinate system bound to the rotating slidebar, the discs do not rotate initially, but after $t = 0$, due to unwinding of the strings, they begin to rotate. In the case of progressive pair, in the opposite direction than is the rotation of the system, and in the case of regressive pair, in the same direction. We denote by ω_{rot} the angular velocity of the discs in the rotating system at time t. So after the cutting of the strings, angular velocities od the discs in the laboratory system are

$$\omega' = \omega \mp \omega_{\mathrm{rot}}\,. \tag{3}$$

where the upper sign referres to the progressive pair (Fig. 2 (left)) and the lower sign to the regressive pair (Fig. 2 (right)). This applies also to all equations in the following whenever two signs appear. It should be noted that with the change of the direction of the main rotation (negative ω), the progressive pair of discs would become the regressive pair.

The total angular momentum of the system at time t is given by

$$L = 2(m\omega r^2 + I\omega') = 2[m\omega r^2 + I(\omega \mp \omega_{\rm rot})] = 2\omega(I + mr^2) \mp I\omega_{\rm rot} \mp I\omega_{\rm rot} \quad (4)$$

$$= L_A \mp L_B \mp L_C \, .$$

where L_A is the angular momentum due to the rotation of the discs around the main axis, and L_B and L_C are the angular momenta due to the rotation of the discs around their axes.

From Eqs. (1), (3) and (4), one obtains the relation between the angular velocities of the system in the laboratory and the rotating frame, $\omega(r)$ and $\omega_{\rm rot}(r)$, and the radial distance of the discs r,

$$\omega(r) = \frac{\omega_0(I + mr_0^2) \pm \omega_{\rm rot}(r)I}{I + mr^2} \, . \quad (5)$$

Radial motion of the discs is determined by two forces. Each disc is acted upon by the centrifugal force F_C and by the string tension F_N (see Figs. 3),

$$m\frac{{\rm d}^2 r}{{\rm d}t^2} = F_C - F_N \, , \quad (6)$$

since the centrifugal force tends to accelerate the discs from the main axis of rotation and the strings pull the discs in the opposite directions.

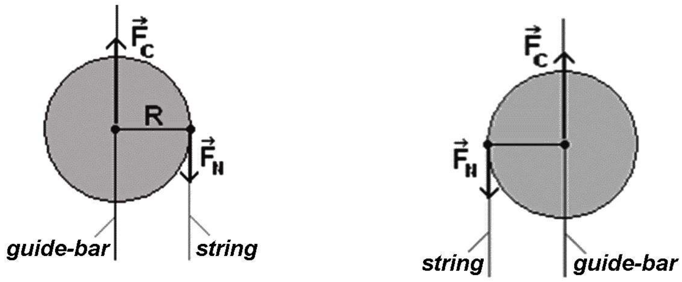

Fig. 3. Forces acting on a disc in the case of (left) the progressive and (right) the regressive pair of discs.

The centrifugal force is given by

$$F_C = m\omega^2 r \, , \quad (7)$$

while the string tension, causing the change of the angular velocity of each disc as viewed in the laboratory frame, is obtained from the basic law relating the torque of the force to the change of the angular momentum of a body. In our case

$$F_N R = \mp I \frac{\mathrm{d}\omega'}{\mathrm{d}t} = I \left(\frac{\mathrm{d}\omega_{\mathrm{rot}}}{\mathrm{d}t} \mp \frac{\mathrm{d}\omega}{\mathrm{d}t} \right) . \tag{8}$$

We introduce the following relations:

$$\frac{\mathrm{d}\omega_{\mathrm{rot}}}{\mathrm{d}t} = \frac{\mathrm{d}\omega_{\mathrm{rot}}}{\mathrm{d}r} \frac{\mathrm{d}r}{\mathrm{d}t} = \omega_{\mathrm{rot}} R \frac{\mathrm{d}\omega_{\mathrm{rot}}}{\mathrm{d}r} , \tag{9}$$

because the radial velocity, due to the unwinding of the strings, is given by

$$\frac{\mathrm{d}r}{\mathrm{d}t} = \omega_{\mathrm{rot}} R . \tag{10}$$

Similarly, we have

$$\frac{\mathrm{d}\omega}{\mathrm{d}t} = \frac{\mathrm{d}\omega}{\mathrm{d}r} \frac{\mathrm{d}r}{\mathrm{d}t} = \omega_{\mathrm{rot}} R \frac{\mathrm{d}\omega}{\mathrm{d}r} . \tag{11}$$

From Eqs. (8) to (11), one obtains for the string tension

$$F_N(r) = I\omega_{\mathrm{rot}} \left(\frac{\mathrm{d}\omega_{\mathrm{rot}}}{\mathrm{d}r} \mp \frac{\mathrm{d}\omega}{\mathrm{d}r} \right) \tag{12}$$

$$= \frac{mIr\omega_{\mathrm{rot}}(r)}{(I+mr^2)^2} \Big(L_0 \pm 2I\omega_{\mathrm{rot}}(r) \Big) \left(\frac{\Big(L_0 \mp 2I\omega_{\mathrm{rot}}(r) \Big) mr^2}{4\omega_{\mathrm{rot}}(r) \Big[(I+mR^2)(I+mr^2) - I^2 \Big]} \pm 1 \right)$$

From Eqs. (9) and (10), we have for the radial acceleration

$$\frac{\mathrm{d}^2 r}{\mathrm{d}t^2} = R^2 \omega_{\mathrm{rot}} \frac{\mathrm{d}\omega_{\mathrm{rot}}}{\mathrm{d}r} . \tag{13}$$

Using Eqs. (5) to (13), after some calculation, one obtains the following differential relation between the radial position of the discs, r, and the angular velocity of the discs in the rotating frame, ω_{rot},

$$\frac{\omega_{\mathrm{rot}} \mathrm{d}\omega_{\mathrm{rot}}}{\omega_0^2 (I+mr_0^2)^2 - \omega_{\mathrm{rot}}^2 I^2} = \frac{r\mathrm{d}r}{(I+mr^2)\big[R^2 I + (I+mR^2)r^2\big]} . \tag{14}$$

This equation is valid for both the progressive and regressive pair of discs.

3. Solution of the equations of motion

Using the identity

$$\frac{1}{(a+bx)(f+gx)} = \frac{1}{bf-ag}\left(\frac{b}{a+bx} - \frac{g}{f+gx}\right), \tag{15}$$

one can write part of the right-hand side of Eq. (14) as follows,

$$\frac{1}{(I+mr^2)\left[R^2I+(I+mR^2)r^2\right]} = \frac{1}{I^2}\left[\frac{I+mR^2}{R^2I+(I+mR^2)r^2} - \frac{m}{I+mr^2}\right]$$

$$= \frac{1}{I^2}\left[\frac{1}{\dfrac{R^2I}{I+mR^2}+r^2} - \frac{1}{\dfrac{I}{m}+r^2}\right].$$

When we use this relation in Eq. (14), we get

$$\frac{\omega_{\rm rot}\mathrm{d}\omega_{\rm rot}}{\dfrac{\omega_0^2(I+mr_0^2)^2}{I^2} - \omega_{\rm rot}^2} = \frac{r\mathrm{d}r}{\dfrac{R^2I}{I+mr^2}+r^2} - \frac{r\mathrm{d}r}{\dfrac{I}{m}+r^2}. \tag{16}$$

One can integrate this equation using the relation

$$\int \frac{x\mathrm{d}x}{a\pm x^2} = \pm\frac{1}{2}\ln(a\pm x^2). \tag{17}$$

Taking integration limits from 0 to $\omega_{\rm rot}$ at the left-hand side of Eq. (16) and from r_0 to r at right, and after taking the antilogarithms, one obtains

$$\frac{\dfrac{\omega_0^2(I+mr_0^2)^2}{I^2}}{\dfrac{\omega_0^2(I+mr_0^2)^2}{I^2} - \omega_{\rm rot}^2} = \frac{\dfrac{I}{m}+r_0^2}{\dfrac{I}{m}+r^2}\,\frac{\dfrac{R^2I}{I+mR^2}+r^2}{\dfrac{R^2I}{I+mR^2}+r_0^2}. \tag{18}$$

Rearranging this relation leads to the following solution of the differential equation (14)

$$\omega_{\rm rot} = \omega_0\sqrt{\frac{(r^2-r_0^2)(I+mr_0^2)}{R^2I+r^2(I+mR^2)}}. \tag{19}$$

This solution is the same for the progressive and regressive pair of discs, i.e. in the two cases the dependence of the angular velocity, $\omega_{\rm rot}$, in the frame rotating with the guide-bar has the same dependence on the radial position of the discs, r. It also satisfies the initial condition $\omega_{\rm rot}(r_0) = 0$.

Using Eqs. (3) and (5), one can easily calculate $\omega'(r)$ and $\omega(r)$ as functions of the radial positions of the discs.

Total energy of the system is a sum of three terms

$$E(r) = mr^2(\omega(r))^2 + I\Big[\omega(r) \mp \omega_{\rm rot}(r)\Big]^2 + mR^2(\omega_{\rm rot}(r))^2 = E_1(r) + E_2(r) + E_3(r)\,. \tag{20}$$

The first term is the kinetic energy due to rotation of the discs around the main axis, the second term is the energy due to rotation of the discs around their centres of mass and the third term is the energy of radial motion of the discs. Equation (20) is the same for both the progressive and regressive rotation, even though Eq. (5) gives different expressions. Using Eqs. (5), (19) and (20), one can calculate the total energy of the system at any moment. We have

$$E(r) = \frac{\omega_0^2(I+mr_0^2)^2 - \omega_{\rm rot}(r)^2 I^2}{I+mr^2} + \omega_{\rm rot}(r)^2 I^2 (I+mR^2)\,. \tag{21}$$

Using $\omega_{\rm rot}(r)$ from Eq. (19), one obtains

$$\begin{aligned} E(r) &= \frac{1}{I+mr^2}\Big[\omega_0^2(I+mr_0^2)^2 - I^2\omega_{\rm rot}(r)^2 + \omega_{\rm rot}(r)^2(I+mR^2)(I+mr^2)\Big] \\ &= \frac{1}{I+mr^2}\Big\{\omega_0^2(I+mr_0^2)^2 + \omega_{\rm rot}(r)^2\Big[(I+mR^2)(I+mr^2) - I^2\Big]\Big\} \\ &= \frac{1}{I+mr^2}\left\{\omega_0^2(I+mr_0^2)^2 + m\omega_0^2\frac{(r^2-r_0^2)(I+mr_0^2)}{R^2I + r^2(I+mR^2)}\Big[R^2I + r^2(I+mR^2)\Big]\right\} \\ &= \frac{\omega_0^2(I+mr_0^2)}{I+mr^2}\Big[(I+mr_0^2) + m(r^2-r_0^2)\Big] = \omega_0^2(I+mr_0^2) = E_0\,. \end{aligned} \tag{22}$$

This result was expected because the system is closed and without losses of energy. It represents a check of the results of integration of the equation of motion.

4. Results and discussion

As an illustration, a realistic example has been calculated and the results are presented in figures and table. We assumed the following values of the parameters

$$m = 10\ \text{kg}, \quad \omega_0 = 20\pi\ \text{s}^{-1}, \quad r_0 = 0.2\ \text{m}, \quad R = 0.1\ \text{m},\ \text{and} \quad I = 2\ \text{kgm}^2. \tag{23}$$

With these parameters, the results of numerical calculations for the dependence on r of the angular velocities ω and ω' (laboratory frame angular velocities) for the progressive and regressive pairs of discs are shown in Figs. 4. In either case, the rotation of the guide-bar ceases eventually, so the discs just rotate about their axes and move radially. This state is reached much more quickly in the case of the regressive pair of discs (Fig. 4 (right)).

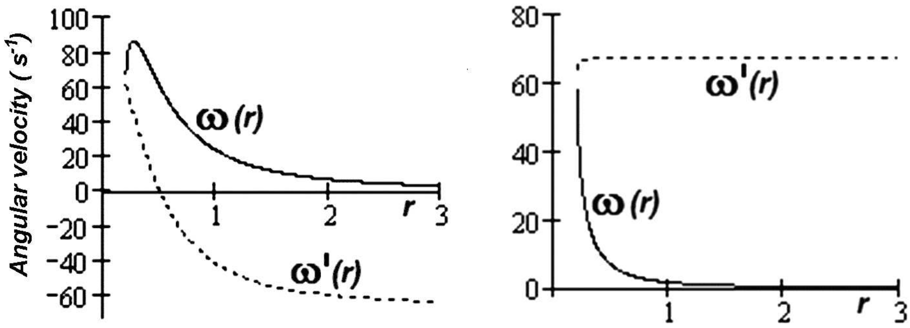

Fig. 4. Variation of the angular velocities $\omega(r)$ and $\omega'(r)$ of rotation of the guide-bar and the discs, respectively, with the distance of the discs from the main axis, r, in the case of (left) the progressive and (right) the regressive pair of discs. The shown values have been calculated using Eqs. (3), (5) and (19) and parameters given in (23).

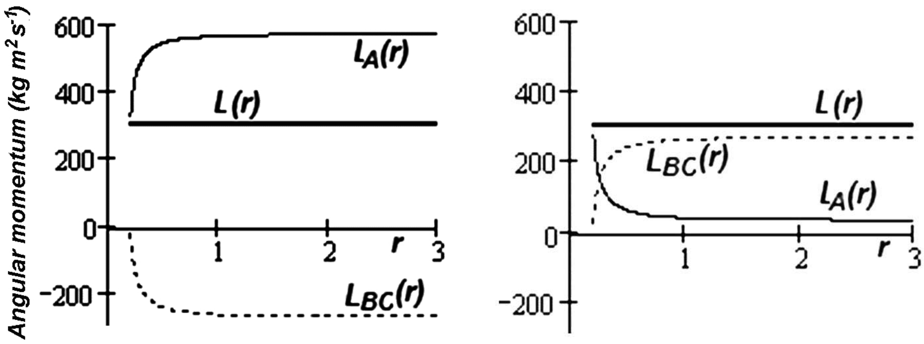

Fig. 5. Variation of the angular momenta $L_A(r)$ and $L_{AB}(r) = L_B(r) + L_C(r)$ with the distance of the discs from the main axis, r, in the case of the progressive (left) and the regressive pair of discs (right). In either case, $L(r) = \text{const}$ is the total angular momentum. The shown values have been calculated using Eqs. (4), (5) and (19) and parameters given in (23).

Figures 5 show similarly the dependence of angular momenta L_A due to the motion of the discs around point A and $L_\mathrm{BC} = L_B + L_C$ due to the motion of the discs around their centres of masses B and C. Also their sum is shown (which is constant).

Results calculated using the above set of parameters for the energy terms E_1, E_2 and E_3 (see Eq. (20)) are shown for the progressive and regressive pairs of discs in Figs. 6. As in the previous considerations, the results in the two cases are very different. The latter case reaches a steady state much quicker.

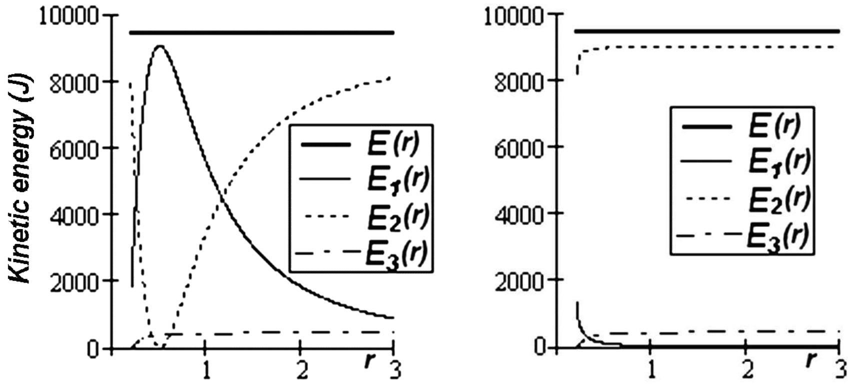

Fig. 6. Variation of the kinetic energies $E_1(r)$, $E_2(r)$ and $E_3(r)$ with the distance of the discs from the main axis, r, in the case of the progressive (left) and the regressive pair of discs (right). In either case, $E(r) =$ const is the total kinetic energy. The shown values have been calculated using Eqs. (5), (19) and (20) and parameters given in (23).

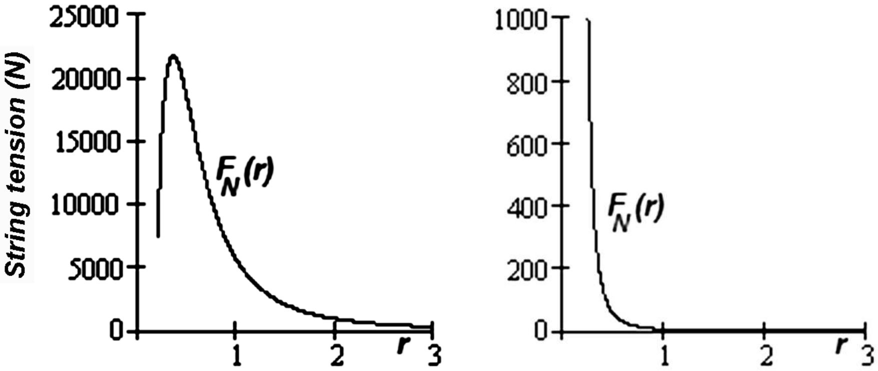

Fig. 7. Variation of the string tension $F_N(r)$ with the distance of the discs from the main axis, r, in the case of the progressive (left) and the regressive pair of discs (right). The shown values have been calculated using Eqs. (12) and (19) and parameters given in (23).

Finally, the string tensions for the progressive and regressive pair of discs are presented in Figs. 7. It should be noted that by choosing different values of the parameters, one finds that the results for the string tension in the case of the regressive pair of discs can be negative, i.e., the string becomes loose. Then the equations for the string tension and also for the other considered quantities are no longer valid.

4.1. Final state of motion

The angular velocity of the guide-bar, $\omega(r)$, ceases in the final state, because even the slightest rotation of the guide-bar would cause string tension (due to the centrifugal forces) and exchange of angular momenta of the discs due to the rotation around the main axis (the point A in Figs. 1) and rotation of the discs around their centres of masses (the points B and C in Figs. 1). Thus the total angular momentum in the final state is shared by the two discs. But not the total energy, because the discs, besides the rotational energy have also the translational kinetic energy as they move away from the central point A. They are given by the terms $E_2(r)$ and $E_3(r)$ in Eq. (20) for $r \to \infty$. Their final ratio is equal to I/mR^2.

The final angular velocity of the discs, as observed in the laboratory frame, is equal to the angular velocity in the frame rotating with the guide-bar, because the rotation of the guide-bar ceases. It is given by

$$(\omega_{\rm rot})_{\rm final}(r \to \infty) = \omega_0 \sqrt{\frac{I + mr_0^2}{I + mR^2}} \tag{24}$$

The final translational velocity of the discs is, therefore, given by $(v_r)_{\rm final} = (\omega_{\rm rot})_{\rm final} R$.

To calculate the total angle of rotation of the guide-bar from the moment of cutting of the strings to infinite time, we introduce the function

$$g(r) = \frac{\omega(r)}{\omega_{\rm rot}(r)} = \frac{{\rm d}\phi/{\rm d}t}{{\rm d}\varphi/{\rm d}t} = \frac{{\rm d}\phi}{{\rm d}\varphi} \tag{25}$$

Since the radial displacement of the discs on the guide-bar is given by

$${\rm d}r = R{\rm d}\varphi = R\omega_{\rm rot}{\rm d}t\,. \tag{26}$$

one obtains

$${\rm d}\phi = \frac{1}{R} g(r){\rm d}r\,. \tag{27}$$

The total angle of rotation of the guide-bar is obtained by the integration of this expression from r_0 to infinity,

$$\phi_{\rm tot} = \frac{1}{R} \int_{r_0}^{\infty} g(r){\rm d}r\,. \tag{28}$$

Numerical integration yields a surprising result. The total angle of rotation of the guide-bar does not depend on the initial angular velocity of the system (i.e. the guide-bar) ω_0. The results using the set of parameters (23) are presented in Fig. 8. They are different in the cases of the progressive and regressive pairs of discs because $\omega(r)$ is different in the two cases (see Eq. (5)). The results of numerical integration, assuming the set of parameters (23), are: $\phi_{\rm tot} = 720°$ in the case of the progressive pair and 131° in the case of the regressive pair of discs.

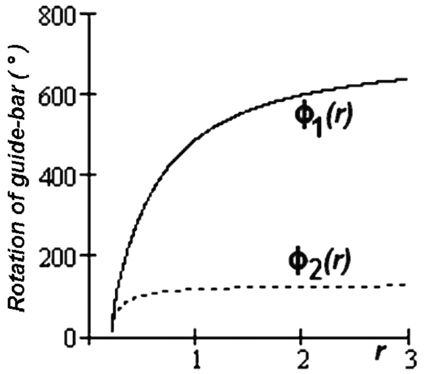

Fig. 8. Variation of the angles of rotation of the guide-bar after the cutting of the strings, $\phi_1(r)$ and $\phi_2(r)$ in the case of the progressive and the regressive pair of discs, respectively, with the distance of the discs from the main axis, r. The shown values have been calculated using Eqs. (5), (19), (25) and (28), and parameters given in (23). The integration was performed numerically.

Using Eq. (26) one can express the relation between the time and the radial distance of the discs from the point A as

$$t(r) = \frac{1}{R} \int_{r_0}^{r} \frac{\mathrm{d}r}{\omega_{\mathrm{rot}}(r)} \, . \tag{29}$$

Since $\omega_{\mathrm{rot}}(r)$ is the same in the case of progressive and regressive pair of discs, the same results are obtained in the two cases. The results for the mirror function $r(t)$, using the set of parameters (23), are shown in Fig. 9.

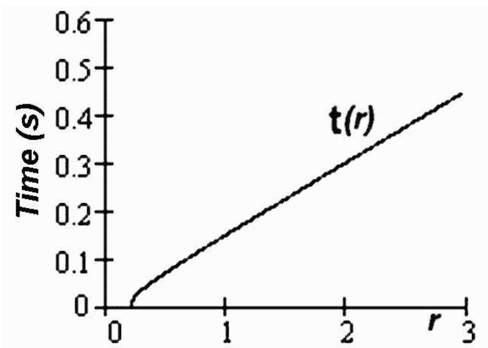

Fig. 9. The relation of the time elapsed after the cutting of the strings, $t(r)$, and the distance of the discs from the main axis, r. The shown values have been calculated using Eqs. (19) and (29), and parameters given in (23). It is the same in the case of the progressive and the regressive pair of discs. The integration was performed numerically.

The results of the numerical calculation are also presented in Table 1.

TABLE 1. Physical quantities in the motion of a pair if discs under the action of centrifugal force and string tension, calculated assuming the parameters (23).

	Progr.	Regr.	Progr.	Regr.	Progr.	Regr.	Progr.	Regr.	Progr.	Regr.
r [m]	0.20		0.40		1.00		2.00		1000	
ω_{rot} [s^{-1}]	0		56.51		65.50		66.75		67.17	
ω [s^{-1}]	62.83	62.83	73.28	10.49	23.48	1.65	6.77	0.41	0	0
ω'[s^{-1}]	62.83	62.83	16.77	67.00	-42.02	67.15	-59.98	67.17	-67.17	67.17
L_A [kgm^2s^{-1}]	301.59	301.59	527.65	75.54	563.6	39.59	568.61	34.58	570.27	32.91
L_{AB} [kgm^2s^{-1}]	0	0	-226.05	226.05	-262.01	262.01	-267.02	267.02	-268.68	268.68
L [kgm^2s^{-1}]	301.59		301.59		301.59		301.59		301.59	
E_1 [J]	1579	1579	8592	176.1	5514	27.20	1832	6.78	0.01	0
E_2 [J]	7895	7895	562	8979	3531	9018	7196	9022	9023	9023
E_3 [J]	0		319.38		429.05		445.61		451.18	
E [J]	9474.82		9474.82		9474.82		9474.82		9474.82	
F_N [N]	7895	7895	21171	129.3	5492	5.28	913.65	0.61	0	0
t [s]	0		0.05		0.15		0.30		148.88	
ϕ [deg.]	0	0	252.15	93.70	490.85	116.88	600.75	124.02	720.28	131.04

5. Conclusion

The motion of a pair of discs mounted on a guide-bar, under the action of the centrifugal force and string tension, is studied. To simplify the calculations, the system is assumed to be frictionless and isolated, i.e., the total energy and the total angular momentum of the system are assumed to be constant. The equation of motion of the discs is derived and its solution is given. Numerical results are calculated for a set of parameters of the system and the results are illustrated in figures and in a table. An unexpected results is obtained. The angle of rotation of the system in the time from the moment of cutting of the strings to infinity does not depend on the initial angular velocity of the system.

Another system that may be named "real" pair of discs in S-connection is shown in Figs. 10. It may be visualized even without the guide-bar, but in weightless space. Again, the progressive and regressive cases are possible. One can formulate the equations of motion, but they are considerably more complicated and finding the analytical solution of this problem would be very dificult.

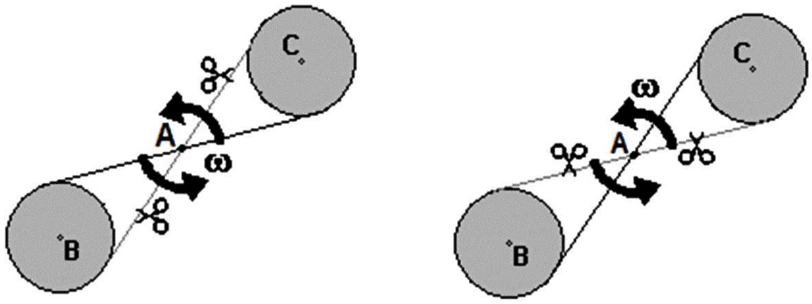

Fig. 10. A system of S-connected discs without the cross-bar. It is mechanically simpler, but the calculation is much more difficult.

Acknowledgements

Author is thankfull to prof. A. Rubčić for the suggestions and support in this work.

References

[1] F. Klein and A. Sommerfeld, Theorie des Kreisels, Teubner, Leipzig (1897 – 1910).

[2] Ch. Kittel, W. D. Knight and M. A. Ruderman, *Berkeley Physics Course, Vol. 1, Mechanics*, 2nd ed., McGraw-Hill, New York (1973);
R. P. Feynman, R. B. Leighton and M. Sands, *The Feynman Lectures in Physics*, Addison-Wesley, Reading (1963);

[3] D. Halliday, R. Resnik and J. Walker, *Fundamentals of Physics*, Wiley (2003) (see p. 250 for yo-yo);
H. D. Young and R. A. Freedman, *University Physics*, 11th ed., Addison-Wesly, San Francisco (2004) (see p. 372 and 374 for yo-yo);
R. W. Pohl, *Mechanik, Akustik und Wärmelehre*, Springer, Berlin (1964) der Maxwell-rad, Fig. 40.

GIBANJE DVAJU POVEZANIH DISKOVA U SPOJU S-TIPA

Proučavamo gibanje para diskova pod djelovanjem centrifugalne sile i napetosti niti. Diskovi su postavljeni na vodilicu duž koje mogu kliziti i mogu se vrtjeti oko svojih osi. Oko svakog diska namotana je nit koja je vezana na prečku učvršćenu na vodilicu i te niti početno drže diskove na jednakoj udaljenosti od glavne osi. Dok se sustav vrti oko glavne osi, diskovi se oslobode istovremenim rezanjem jedne i druge niti (sustav sliči slovu S). Diskovi se počnu udaljavati od glavne osi i mijenjaju svoju vrtnju. Gibanje diskova analizira se primjenom jednadžbi gibanja i zakona sačuvanja momenta impulsa. Provjeru rezultata predstavlja zakon sačuvanja energije. Nakon dugog vremena prestane vrtnja vodilice, dva se diska simetrično udaljuju od glavne osi jednakom brzinom i vrte jednakom kutnom brzinom. Neočekivan rezultat jest neovisnost kuta zakreta vodilice tijekom proučavanog gibanja o početnoj kutnoj brzini sustava.