Printed ISSN 1330–0008
Online ISSN 1333–9125
CD ISSN 1333–8390
CODEN FIZAE4

EFFECT OF IONIC TEMPERATURES ON ION-ACOUSTIC SOLITARY WAVES IN A DRIFT NEGATIVE ION PLASMA WITH SINGLE TEMPERATURE ELECTRON

SANKAR CHATTOPADHYAY

Centre for Theoretical Plasma Research, 346, R. K. Pally (Balaka), P. O. Sonarpur, Kolkata – 150, West Bengal, India E-mail:sankar_djln@yahoo.com

Department of Mathematics, Daria J.L.N Vidyalaya, West Bengal, India

Received 11 July 2007; Revised manuscript received 16 December 2009
Accepted 27 January 2010 Online 19 April 2010

By using the Sagdeev pseudopotential method, ion-acoustic solitary waves are studied in a collisionless, unmagnetised single electron-temperature plasma with adiabatic positive and negative ion drifts having equal ion-temperatures (particular case). The effect of ionic temperature and negative ions on solitary waves is discussed for the plasmas having (He^+, O^-), (He^+, Cl^-) and (H^+, O^-) ions.

PACS numbers: 52.35.Fp, 52.35.Sb UDC 533.951
Keywords: ion-acoustic solitary waves, drift motion

1. Introduction

Linear and non-linear characteristics of electron and ion-acoustic waves through a plasma consisting of single electron temperature, positive as well as negative ions with electron - inertia [1 − 2], warm magnetized plasma [3 − 4] and with relativistic plasma [5 − 6] have been the object of intense research in past few years and many interesting physical phenomena have been studied by many groups of physicists [7 − 24]. The evolution of small but finite-amplitude solitary waves has been studied by K-dV equation for warm ions and hot electrons in a collisionless, unmagnetized plasma. But this method is not suitable for large amplitude solitary waves [25]. Nakamura et al. [26] used the pseudopotential method for large-amplitude solitary waves in a plasma with positive and negative ions. In order to explain the experimental results, many authors have incorporated different important plasma parameters, e.g. temperature of ions, density gradient, temperature gradient, ion beam, non-isothermality, instability, two-temperature electrons, Landau damping, wave-parameter shift, magnetic field etc. Moreover, solitary waves (compressive and rarefactive) have interesting characteristics in presence of negative ions together with positive ions in plasma. The existence of solitary waves depends on finite ion-temperature effect, while both laboratory and space plasmas have the same effect. Tagare and Tappert [27 − 28] used the reductive perturbation method for small-amplitude solitary wave solution in a warm plasma with single electron species. Sakanaka [29] has also found the solitary-wave solution by Sagdeev pseudopotential method for warm ion fluid corresponding to warm ion density. But the warm ion density has a transcendental form while solving the adiabatic fluid equations. Ghosh et al. [30] have been able to solve analytically this transcendental form by a simple algebra and thus get the exact form of the pseudopotential which determines the nature of the solitary wave structures. We solved earlier [31] this type of transcendental form analytically for cold positive and negative ion plasma and thus obtained the exact pseudopotential form of both compressive and rarefactive solitary waves for a single temperature electron plasma without using any approximations. In the present paper, we investigate mainly the effect of ionic temperature, drift velocities and mass ratios on the formation of ion-acoustic solitary waves with warm positive and negative ions. In Section 2, we derive the required formulations of the exact pseudopotential. Section 3 contains the conditions for existence of the solitary wave solutions, while Section 4 gives the fully non-linear amplitudes (first and second order) of the solitary wave solutions. In Section 5, we discuss the entire problem regarding the Sagdeev pseudopotential nature, amplitudes (first and second order) and minimum value of the Mach number (M_1). These are shown graphically with the variation of different parameters. Finally, concluding remarks are given.

2. *Basic equations and pseudopotential approach*

We consider a plasma consisting of warm electrons and two types of ions (positive and negative ions) with drift velocities. The plasma is assumed to be homogeneous, infinite, collisionless, unmagnetised and isothermal. In this case, the temperature of ions is much lower than that of electrons (i.e. $T_i \ll T_e$) so that Landau damping is neglected. Thus the basic equations of such a plasma for the study of ion-acoustic waves in unidirectional propagation in dimensionless form are [32],

$$\frac{\partial n_\alpha}{\partial t} + \frac{\partial}{\partial x}(n_\alpha u_\alpha) = 0\,, \tag{1}$$

$$\frac{\partial u_\alpha}{\partial t} + u_\alpha \frac{\partial u_\alpha}{\partial x} + \frac{\sigma_\alpha}{Q_\alpha n_\alpha}\frac{\partial p_\alpha}{\partial x} = -\frac{Z_\alpha}{Q_\alpha}\,\frac{\partial \phi}{\partial x}\,, \tag{2}$$

$$\frac{\partial p_\alpha}{\partial t} + u_\alpha \frac{\partial p_\alpha}{\partial x} + 3p_\alpha \frac{\partial u_\alpha}{\partial x} = 0\,, \tag{3}$$

$$\frac{\partial^2 \phi}{\partial x^2} = \mathrm{e}^{\phi} - \sum_\alpha Z_\alpha n_\alpha\,, \tag{4}$$

where n_α, u_α, p_α, Q_α, Z_α and σ_α are the number density, velocity, pressure, mass ratio, charge and temperature of the ions, respectively.

Here $Q_\alpha = m_j/m_i$ (m_i = mass of positive ion, m_j = mass of negative ion), $\sigma_\alpha = T_\alpha/T_e$ (T_α = temperature of ions, T_e = temperature of electron); $Z_\alpha = 1$, $Q_\alpha = 1$ for positive ions (i), while $Z_\alpha = -Z$, $Q_\alpha = Q$ for negative ions (j), ϕ is the electrostatic potential and the subscript $\alpha = i$ is for positive ions and $\alpha = j$ for negative ions.

In the above equations, we have normalized the velocities by the characteristic velocity $\sqrt{kT_e/m_\alpha}$ where m_α is the mass of ion and k is the Boltzmann constant, all densities are normalized by the equilibrium value n_0, the distances by the Debye length $\sqrt{kT_e/4\pi e^2 n_0}$, whereas the potential is normalized to kT_e/e, pressure by ion equilibrium pressure $p_0 = n_0 T_\alpha$, time t by reciprocal ion plasma frequency $\omega_{p\alpha}^{-1}$ ($\omega_{p\alpha} = \sqrt{4\pi n_0 e^2/m_\alpha}$), so that the equations appear totally in dimensionless form.

Transforming to stationary wave frame for solitary wave solution, we introduce a single independent variable η defined by

$$\eta = x - Vt\,, \tag{5}$$

where x and t have their usual meanings and V is the velocity of the solitary waves. The boundary conditions are:

$$u_\alpha = u_{\alpha 0}, \quad n_\alpha = n_{\alpha 0}, \quad p_\alpha = p_{\alpha 0}, \quad \text{and} \quad \phi \to 0 \quad \text{as} \quad x \to \pm\infty\,, \tag{6}$$

and the charge neutrality condition of the plasma is

$$\sum_\alpha n_{\alpha 0} Z_\alpha = 1\,. \tag{7}$$

From Eqs. (1) to (4), after using (5),(6) and (7), we get

$$p_\alpha = \frac{p_{\alpha 0}(u_{\alpha 0} - V)^3}{(u_\alpha - V)^3}\,, \tag{8}$$

$$n_\alpha = \sqrt{\frac{Q_\alpha n_{\alpha 0}^3}{6 p_{\alpha 0}\sigma_\alpha}} \sqrt{\left(b_{1\alpha} - \frac{2Z_\alpha}{Q_\alpha}\,\phi\right) - \sqrt{\left(b_{1\alpha} - \frac{2Z_\alpha}{Q_\alpha}\,\phi\right)^2 - b_{2\alpha}^2}}\,, \tag{9}$$

where

$$b_{1\alpha} = (V - u_{\alpha 0})^2 + \frac{3\sigma_\alpha p_{\alpha 0}}{Q_\alpha n_{\alpha 0}}\,, \tag{10}$$

$$b_{2\alpha}^2 = \frac{12\sigma_\alpha p_{\alpha 0}}{Q_\alpha n_{\alpha 0}}(V - u_{\alpha 0})^2, \tag{11}$$

and

$$\frac{\partial^2\phi}{\partial\eta^2} = \mathrm{e}^\phi - \sum_\alpha Z_\alpha \sqrt{\frac{Q_\alpha n_{\alpha 0}^3}{6 p_{\alpha 0}\sigma_\alpha}} \sqrt{\left(b_{1\alpha} - \frac{2Z_\alpha}{Q_\alpha}\,\phi\right) - \sqrt{\left(b_{1\alpha} - \frac{2Z_\alpha}{Q_\alpha}\,\phi\right)^2 - b_{2\alpha}}}\,, \tag{12}$$

Now for n_α to be real the following restrictions on ϕ must be fulfilled

$$-\frac{Q}{2Z}\left[V-u_{j0}-\sqrt{\frac{3\sigma_j p_{j0}}{Qn_{j0}}}\right]^2<\phi<\frac{1}{2}\left[V-u_{i0}-\sqrt{\frac{3\sigma_i p_{i0}}{n_{i0}}}\right]^2. \tag{13}$$

Relation (13) is the more general form and is similar to the condition obtained by Ghosh et al. [33] for warm positive ion only (when negative ion concentration $n_{j0}=0$).

Equation (12) can be written in the form

$$\frac{\partial^2\phi}{\partial\eta^2}=-\frac{\partial\psi}{\partial\phi}\,, \tag{14}$$

where ψ is the Sagdeev pseudopotential function. Integration of Eq. (14) gives "the energy law"

$$\frac{1}{2}\left(\frac{\partial\phi}{\partial\eta}\right)^2+\psi(\phi)=0\,, \tag{15}$$

where

$$\psi(\phi)=(1-\mathrm{e}^{\phi})-\frac{1}{6}\sum_\alpha\sqrt{\frac{n_{\alpha0}^3Q_\alpha^3}{3p_{\alpha0}\sigma_\alpha}}\left[\left(b_{1\alpha}-\frac{2Z_\alpha}{Q_\alpha}\,\phi+b_{2\alpha}\right)^{3/2}\right.$$
$$\left.-\left(b_{1\alpha}-\frac{2Z_\alpha}{Q_\alpha}\,\phi-b_{2\alpha}\right)^{3/2}+(b_{1\alpha}-b_{2\alpha})^{3/2}-(b_{1\alpha}+b_{2\alpha})^{3/2}\right]. \tag{16}$$

In the absence of negative ion ($n_{j0}=0$), the Sagdeev pseudopotential function $\psi(\phi)$ in Eq. (16) exactly reduces to the result in Ref. [42] for single electron temperature plasma and this Sagdeev pseudopotential function $\psi(\phi)$ is always real within the range of ϕ defined in Eq. (13).

In our present case, the compressive ion acoustic solitary waves are found where $\phi>0$ is maintained everywhere for this compressive mode. The boundary condition on Sagdeev potential for compressive soliton is $\psi'(\phi)>0$, where $\phi=(1/2)\big[V-u_{i0}-\sqrt{3\sigma_i p_{i0}/u_{i0}}\big]^2$. By using Eq. (16), the above condition, when simplified, gives

$$\mathrm{e}^{\frac{1}{2}\left(M-\sqrt{\frac{3\sigma_i p_{i0}}{u_{i0}}}\right)^2}+\frac{1}{4}\sqrt{\frac{n_{i0}^3}{3\sigma_i p_{i0}}}\left[\left\{\left(M-\sqrt{\frac{3\sigma_i p_{i0}}{n_{i0}}}\right)^2-\phi\right\}^{\frac{1}{2}}-\left\{\left(M+\sqrt{\frac{3\sigma_i p_{i0}}{n_{i0}}}\right)^2-\phi\right\}^{\frac{1}{2}}\right]$$
$$+\frac{Z}{2}\sqrt{\frac{n_{j0}^3}{3Q\sigma_j p_{j0}}}$$
$$\times\left[\left\{\left(V-u_{j0}+\sqrt{\frac{3\sigma_j p_{j0}}{Qn_{j0}}}\right)^2+\frac{2Z}{Q}\phi\right\}^{\frac{1}{2}}-\left\{\left(V-u_{j0}-\sqrt{\frac{3\sigma_j p_{j0}}{Qn_{j0}}}\right)^2+\frac{2Z}{Q}\phi\right\}^{\frac{1}{2}}\right]<0\,,$$

where $M = V - u_{io}$ is the Mach number in the ion rest frame (see Ref. [30]).

In order to keep the ion density real, and thus prevent wave breaking, we impose the following extra condition [33], $\psi(\phi) > 0$, where $\phi = \frac{1}{2}\left[V - u_{i0} - \sqrt{3\sigma_i p_{i0}/n_{i0}}\right]^2$. This condition gives

$$1 - \mathrm{e}^{\frac{1}{2}\left(M - \sqrt{\frac{3\sigma_i p_{i0}}{u_{i0}}}\right)^2} - \frac{1}{6}\sum_\alpha \sqrt{\frac{n_{\alpha 0}^3 Q_\alpha}{3p_{\alpha 0}\sigma_\alpha}}\left[\left\{b_{1\alpha} + b_{2\alpha} - \frac{Z_\alpha}{Q_\alpha}\left(M - \sqrt{\frac{3\sigma_i p_{i0}}{n_{i0}}}\right)^2\right\}^{3/2}\right.$$

$$\left. - \left\{b_{1\alpha} - b_{2\alpha} - \frac{Z_\alpha}{Q_\alpha}\left(M - \sqrt{\frac{3\sigma_i p_{i0}}{n_{i0}}}\right)^2\right\}^{3/2} + (b_{1\alpha} - b_{2\alpha})^{3/2} - (b_{1\alpha} + b_{2\alpha})^{3/2}\right] > 0,$$

After simplification, the above inequality gives

$$\mathrm{e}^{\frac{1}{2}\left(M - \sqrt{\frac{3\sigma_i p_{i0}}{u_{i0}}}\right)^2} < 1 + n_{i0}M^2 - \frac{4}{3}\left(3n_{i0}^3\sigma_i p_{i0}\right)^{1/4} M^{3/2} + \sigma_i p_{i0}$$

$$-\frac{1}{6}\sqrt{\frac{Q^3 n_{j0}^3}{3\sigma_j p_{j0}}}\left[\left\{\left(V - u_{j0} + \sqrt{\frac{3\sigma_j p_{j0}}{Q n_{j0}}}\right)^2 + \frac{Z}{Q}\left(M - \sqrt{\frac{3\sigma_i p_{i0}}{n_{i0}}}\right)^2\right\}^{3/2}\right. \tag{17}$$

$$\left. - \left\{\left(V - u_{j0} - \sqrt{\frac{3\sigma_j p_{j0}}{Q n_{j0}}}\right)^2 + \frac{Z}{Q}\left(M - \sqrt{\frac{3\sigma_i p_{i0}}{n_{i0}}}\right)^2\right\}^{3/2} - 6\,(V - u_{j0})\sqrt{\frac{3\sigma_j p_{j0}}{Q n_{j0}}} - 2\sqrt{\frac{3\sigma_j^3 p_{j0}^3}{Q^3 n_{j0}^3}}\right].$$

Inequality (17) gives us the idea about the upper limit for M. With $\sigma_i = \sigma_j = \sigma$, the above condition (17) reduces to

$$\mathrm{e}^{\frac{1}{2}\left(M - \sqrt{\frac{3\sigma p_{i0}}{u_{i0}}}\right)^2} < 1 + n_{i0}M^2 - \frac{4}{3}\left(3n_{i0}^3\sigma p_{i0}\right)^{1/4} M^{3/2} + \sigma p_{i0}$$

$$-\frac{1}{6}\sqrt{\frac{Q^3 n_{j0}^3}{3\sigma p_{j0}}}\left[\left\{\left(V - u_{j0} + \sqrt{\frac{3\sigma p_{j0}}{Q n_{j0}}}\right)^2 + \frac{Z}{Q}\left(M - \sqrt{\frac{3\sigma p_{i0}}{n_{i0}}}\right)^2\right\}^{3/2}\right.$$

$$\left. - \left\{\left(V - u_{j0} - \sqrt{\frac{3\sigma p_{j0}}{Q n_{j0}}}\right)^2 + \frac{Z}{Q}\left(M - \sqrt{\frac{3\sigma p_{i0}}{n_{i0}}}\right)^2\right\}^{3/2} - 6\,(V - u_{j0})\sqrt{\frac{3\sigma p_{j0}}{Q n_{j0}}} - 2\sqrt{\frac{3\sigma^3 p_{i0}^3}{Q^3 n_{i0}^3}}.\right.$$

In absence of negative ion ($n_{jo} = 0$), the above condition reduces to

$$e^{\frac{1}{2}\left(M-\sqrt{\frac{3\sigma p_{i0}}{u_{i0}}}\right)^2} < 1 + n_{i0}M^2 - \frac{4}{3}\left(3n_{i0}^3\sigma p_{i0}\right)^{1/4}M^{3/2} + \sigma p_{i0},$$

which exactly reduces to the result of Ref. [33] for $n_{i0} \to 1$ and $p_{i0} \to 1$, and when $\sigma \to 0$, the above condition reduces to the result of Ref. [31].

3. *Solitary wave solution*

For solitary wave solution the pseudopotential $\psi(\phi)$ in Eq. (16) must satisfy the following conditions [34],

$$\psi(\phi) = \frac{\partial\psi(\phi)}{\partial\phi} = 0 \quad \text{for} \quad \phi = 0\,; \qquad \frac{\partial^2\psi(\phi)}{\partial\phi^2} < 0 \quad \text{for} \quad \phi = 0\,,$$

$$\psi(\phi) = 0 \quad \text{for} \quad \phi = \phi_m\,, \tag{18}$$

$$\psi(\phi) < 0 \quad \text{in the region} \quad 0 < |\phi| < |\phi_m|\,,$$

where $|\phi_m|$ is the amplitude of the solitary wave.

Again, from $\left.\frac{\partial^2\psi(\phi)}{\partial\phi^2} < 0\right|_{\phi=0}$ we get

$$\sum_\alpha \frac{Z_\alpha^2 n_{\alpha 0}^2}{Q_\alpha(V - u_{\alpha 0})^2 n_{\alpha 0} - 3\sigma_\alpha p_{\alpha 0}} < 1\,. \tag{19}$$

This gives the condition for the existence of a solitary wave solution from which we get

$$\frac{Z^2 n_{j0}^2}{Q(V - u_{j0})^2\, n_{j0} - 3\sigma_j p_{j0}} + \frac{n_{i0}^2}{M^2\, n_{i0} - 3\sigma_i p_{i0}} < 1\,, \tag{19a}$$

or

$$M = V - u_{i0} > \left[n_{i0}\left\{1 - \frac{Z^2 n_{j0}}{Q(V - u_{j0})^2 - 3\sigma_j p_{j0}/n_{j0}}\right\}^{-1} + \frac{3\sigma_i p_{i0}}{n_{i0}}\right]^{1/2}. \tag{20}$$

This relation gives us the lower limit of M (say M_1) for warm positive and negative ion plasma. In the absence of negative ion, the inequality (20) reduces to

$$M > \left[n_{i0} + \frac{3\sigma_i p_{i0}}{n_{i0}}\right]^{1/2}.$$

This is more general form than those in Refs. [30, 33, 34]. When $n_{i0} \to 1$, $p_{i0} \to 1$ and $\sigma_i = \sigma$, then $M > \sqrt{1+3\sigma}$, which exactly reduces to the warm-ion condition [30, 33] and Sagdeev's cold ion ($\sigma = 0$) condition [34].

Assuming (particular case) $\sigma_i = \sigma_j = \sigma$, we get from (19a)

$$\frac{Z^2 n_{j0}^2}{Q(V-u_{j0})^2\, n_{j0} - 3\sigma p_{j0}} + \frac{n_{i0}^2}{(V-u_{i0})^2\, n_{i0} - 3\sigma p_{i0}} < 1\,.$$

The critical ion temperature (σ_c) is obtained from the above inequality as

$$\sigma_c = \frac{1}{6p_{i0}p_{j0}} \tag{21}$$

$$\times\left[D_1 \pm \sqrt{D_1^2 + D_2\{-Q(V-u_{i0})^2(V-u_{j0})^2 + Z^2(V-u_{i0})^2 + Qn_{i0}(V-u_{j0})^2\}}\right],$$

where

$$D_1 = Qn_{j0}p_{i0}(V-u_{j0})^2 + n_{i0}p_{j0}(V-u_{i0})^2 - n_{i0}^2 p_{j0} - n_{j0}p_{i0}Z^2$$

and

$$D_2 = 4p_{i0}p_{j0}n_{i0}n_{j0}\,.$$

4. *Amplitude of the solitary wave solution*

For the determination of soliton amplitude, we have from (16) $\psi(\phi_m) = 0$, where ϕ_m is the amplitude of the solitary waves. Thus we get,

$$1 - \mathrm{e}^{\phi_m} - \frac{n_{i0}^{3/2}}{6\sqrt{3\sigma_i}}\left[\left\{\left(V-u_{i0}+\sqrt{\frac{3\sigma_i p_{i0}}{n_{i0}}}\right)^2 - 2\phi_m\right\}^{3/2} - \left\{\left(V-u_{i0}-\sqrt{\frac{3\sigma_i p_{i0}}{n_{i0}}}\right)^2 - 2\phi_m\right\}^{3/2}\right.$$

$$\left. - \left(V-u_{i0}+\sqrt{\frac{3\sigma_i p_{i0}}{n_{i0}}}\right)^3 + \left(V-u_{i0}-\sqrt{\frac{3\sigma_i p_{i0}}{n_{i0}}}\right)^3\right] \tag{22}$$

$$-\frac{Q^{3/2}n_{j0}^{3/2}}{6\sqrt{3\sigma_j}}\left[\left\{\left(V-u_{j0}+\sqrt{\frac{3\sigma_j p_{j0}}{Qn_{j0}}}\right)^2 + \frac{2Z}{Q}\phi_m\right\}^{3/2} - \left\{\left(V-u_{j0}-\sqrt{\frac{3\sigma_j p_{j0}}{Qn_{j0}}}\right)^2 + \frac{2Z}{Q}\phi_m\right\}^{3/2}\right.$$

$$\left. - \left(V-u_{j0}+\sqrt{\frac{3\sigma_j p_{j0}}{Qn_{j0}}}\right)^3 + \left(V-u_{j0}-\sqrt{\frac{3\sigma_j p_{j0}}{Qn_{j0}}}\right)^3\right] = 0\,.$$

This is an exact and nonlinear equation in ϕ_m which gives a fully nonlinear amplitude (ϕ_m) showing nonlinear curves only and from this, the first (Φ_{01}) and the second order (Φ_{02}) K-dV amplitudes are obtained after truncation of the Sagdeev pseudopotential (ψ) function up to third order. For this we expand $\psi(\phi)$ in power series of ϕ and taking terms up to third order (ϕ^3) we have from (12) and (14)

$$\frac{\mathrm{d}^2\phi}{\mathrm{d}\eta^2} = -\frac{\partial\psi}{\partial\phi} = A\phi - B\phi^2 + C\phi^3\,, \tag{23}$$

where

$$A = 1 - \frac{1}{2}\sqrt{\frac{n_{i0}^3}{3\sigma_i p_{i0}}}\left[\left(V - u_{i0} - \sqrt{\frac{3\sigma_i p_{i0}}{n_{i0}}}\right)^{-1} - \left(V - u_{i0} + \sqrt{\frac{3\sigma_i p_{i0}}{n_{i0}}}\right)^{-1}\right]$$

$$+\frac{1}{2}\sqrt{\frac{Z^4 n_{j0}^3}{3Q\sigma_j p_{j0}}}\left[\left(V - u_{j0} - \sqrt{\frac{3\sigma_j p_{j0}}{Qn_{j0}}}\right)^{-1} - \left(V - u_{j0} + \sqrt{\frac{3\sigma_j p_{j0}}{Qn_{j0}}}\right)^{-1}\right], \tag{24}$$

$$B = -\frac{1}{2} + \frac{1}{4}\sqrt{\frac{n_{i0}^3}{3\sigma_i p_{i0}}}\left[\left(V - u_{i0} - \sqrt{\frac{3\sigma_i p_{i0}}{n_{i0}}}\right)^{-3} - \left(V - u_{i0} + \sqrt{\frac{3\sigma_i p_{i0}}{n_{i0}}}\right)^{-3}\right]$$

$$-\frac{1}{4}\sqrt{\frac{Z^6 n_{j0}^3}{3Q^3\sigma_j p_{j0}}}\left[\left(V - u_{j0} - \sqrt{\frac{3\sigma_j p_{j0}}{Qn_{j0}}}\right)^{-3} - \left(V - u_{j0} + \sqrt{\frac{3\sigma_j p_{j0}}{Qn_{j0}}}\right)^{-3}\right],$$

$$C = \frac{1}{6} - \frac{1}{4}\sqrt{\frac{n_{i0}^3}{3\sigma_i p_{i0}}}\left[\left(V - u_{i0} - \sqrt{\frac{3\sigma_i p_{i0}}{n_{i0}}}\right)^{-5} - \left(V - u_{i0} + \sqrt{\frac{3\sigma_i p_{i0}}{n_{i0}}}\right)^{-5}\right]$$

$$-\frac{1}{4}\sqrt{\frac{Z^8 n_{j0}^3}{3Q^5\sigma_j p_{j0}}}\left[\left(V - u_{j0} - \sqrt{\frac{3\sigma_j p_{j0}}{Qn_{j0}}}\right)^{-5} - \left(V - u_{j0} + \sqrt{\frac{3\sigma_j p_{j0}}{Qn_{j0}}}\right)^{-5}\right].$$

Now, the first order K-dV amplitude [25] of the solitary wave solution is

$$\Phi_{01} = \frac{3A}{2B}\,, \tag{25}$$

and the second order amplitude of the solitary wave solution is

$$\Phi_{02} = \frac{6A}{2B + \sqrt{4B^2 - 18AC}}\,. \tag{26}$$

5. *Discussion*

We have investigated the occurrence and characteristics of the existence of ion-acoustic solitary waves and amplitudes (the first and the second order) with positive and negative ion drifts for single-temperature electron plasma using Sagdeev's pseudopotential approach. We have also done the numerical calculation for the following three cases:

(i) (He^+, O^-) – plasma corresponding to $Q = 4$.

(ii) (He^+, Cl^-) – plasma corresponding to $Q = 8.875$.

(iii) (H^+, O^-) – plasma corresponding to $Q = 16$.

For different values of the parameters, the results of our calculations are shown in Figs. 1 – 9.

Figure 1 shows the nature of the Sagdeev pseudopotential curves [$\psi(\phi)$ vs. ϕ] for compressive solitary waves ($\phi > 0$) with the variation of negative ion concentration (n_{j0}) for some fixed values of ($V - u_{\alpha 0}$), σ ($\sigma = \sigma_i = \sigma_j$), Q_α, Z_α and $p_{\alpha 0}$. It is seen from the figure that the amplitude (ϕ_m) decreases as the negative ion concentration (n_{j0}) is increased, while the other parameters are kept fixed.

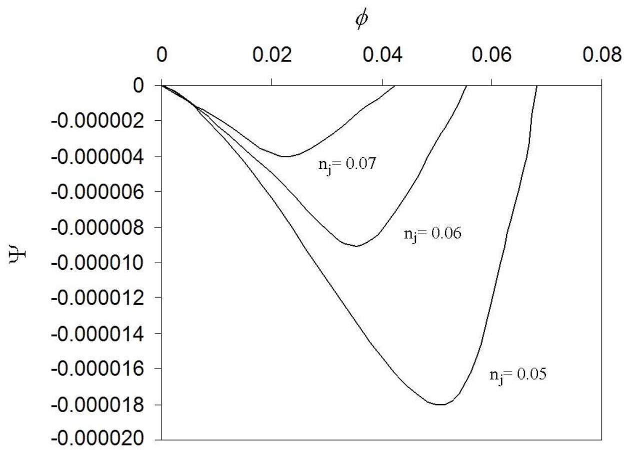

Fig. 1. Solitary wave profiles (ψ) versus electrostatic potential (ϕ) with the variation of negative ion concentration (n_{j0}) for some fixed values of ($V - u_{\alpha 0}$), σ_α and Q_α.

Figure 2 shows the Sagdeev pseudopotential curves [$\psi(\phi)$ vs. ϕ] for compressive solitary waves ($\phi > 0$) with the variation of ($V - u_{\alpha 0}$) for $\sigma = \sigma_i = \sigma_j = 1/30$, $Q = 4$, $n_{j0} = 0.05$, $p_{i0} = p_{j0} = 1$ and $Z = 1$. As the values of ($V - u_{\alpha 0}$) increase, the amplitude increases while keeping the other parameters fixed.

Figure 3 shows the Sagdeev pseudopotential curves [$\psi(\phi)$ vs. ϕ] for compressive solitary waves ($\phi > 0$) with the variation of mass ratio Q_α ($= m_j/m_i$) for some particular values of ($V - u_{\alpha 0}$), σ ($\sigma = \sigma_i = \sigma_j$), n_{j0}, $p_{i0} = p_{j0} = 1$ and $Z = 1$. It is also seen from the figure that the amplitude of the solitary waves increases when Q_α increases.

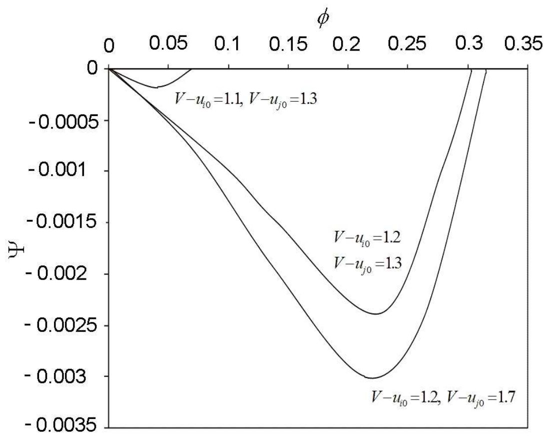

Fig. 2. Solitary wave profiles (ψ) versus electrostatic potential (ϕ) with the variation of ($V - u_{\alpha 0}$) for constant values of n_{j0}, σ_α and Q_α.

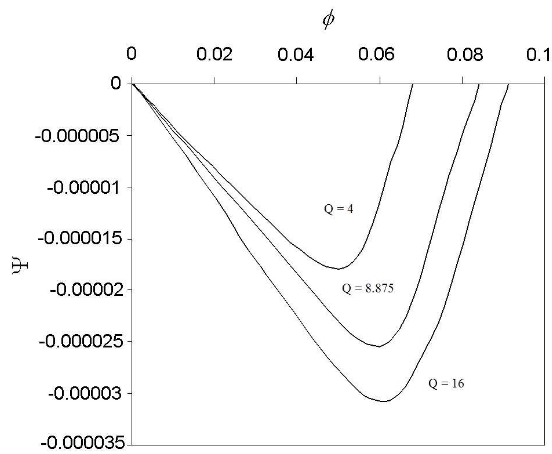

Fig. 3. Solitary wave profiles (ψ) versus electrostatic potential (ϕ) with the variation of mass ratio (Q_α) for some particular values of ($V - u_{\alpha 0}$), σ_α and n_{j0}.

Figure 4 shows the typical Sagdeev pseudopotential curves [$\psi(\phi)$ vs. ϕ] for compressive solitary waves ($\phi > 0$) for $(V - u_{i0}) = 1.1$, $(V - u_{j0}) = 1.3$, $Q = 4$, $n_{j0} = 0.05$, $p_{i0} = p_{j0} = 1$ and $Z = 1$ with the variation of the ion temperature σ ($\sigma = \sigma_i = \sigma_j$). It is observed from this figure that as the value of σ is lowered (remembering the forbidden region because the value of σ may be taken in such a way that no solitary wave solution is found for such a particular value of σ; the region where no solitary wave solution is found is known as forbidden region), the amplitudes of the solitary waves increase, but that amplitude is maximum for cold ion ($\sigma = \sigma_i = \sigma_j = 0$) plasma only. This is consistent with the results of Refs. [31, 33, 34] for positive ions only (cold and warm).

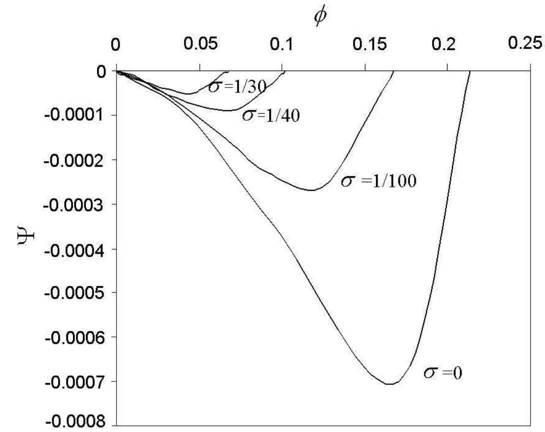

Fig. 4. Solitary wave profiles (ψ) versus electrostatic potential (ϕ) with the variation of ionic temperatures (σ_α) for some fixed values of ($V - u_{\alpha 0}$), Q_α and n_{j0}.

From Eqs. (25) and (26) we have that the first order (Φ_{01}) and the second order (Φ_{02}) amplitudes decrease when the negative ion concentration (n_{jo}) is increased for some fixed values of ($V - u_{\alpha 0}$), Q_α and σ. In Fig. 5, both Φ_{01} and Φ_{02} are decreasing for increasing negative ion concentration (n_{jo}) for different values of Q_α ($Q_\alpha = 4, 8.875, 16$) for some fixed values of ($V - u_{\alpha 0}$), σ, $p_{\alpha 0}$ and Z_α. It is also found that as Q increases, Φ_{01} and Φ_{02} are increasing, which is consistent with the results of Ref. [31]. The first (Φ_{01}) as well as second-order (Φ_{02}) amplitudes

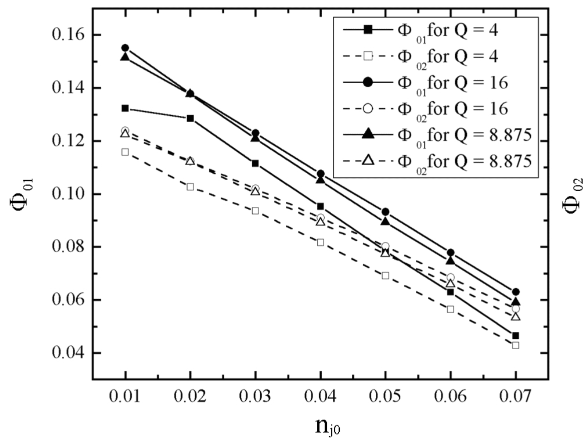

Fig. 5. Variation of amplitudes (first and second order) of solitary waves versus negative ion concentration (n_{j0}) with different values of negative to positive ion mass ratios (Q_α) for some fixed values of ($V - u_{\alpha 0}$) and σ_α.

increase when the ionic temperature σ ($\sigma = \sigma_i = \sigma_j$) decreases, which is consistent with the results of Refs. [22, 27]. This is shown in Fig. 6.

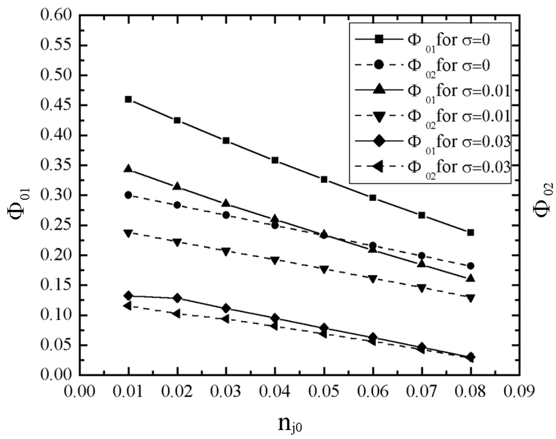

Fig. 6. Variation of amplitudes (first and second order) of solitary waves versus negative ion concentration (n_{j0}) with three different values of ionic temperature (σ) for some fixed values of ($V - u_{\alpha 0}$) and Q.

From Eq. (20), we obtained the minimum value of the Mach number (M_1) and the variations of the Mach number (M_1) against the negative ion concentration (n_{jo}) for different values of negative to positive ion mass ratios (Q), velocities of negative ions including drifts ($V - u_{jo}$) and temperature of ions σ ($\sigma = \sigma_i = \sigma_j$), as shown in Figs. 7−9.

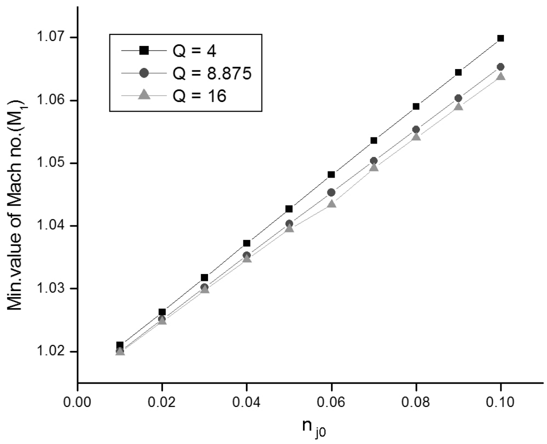

Fig. 7. Variation of minimum value of the Mach number (M_1) versus negative ion concentration (n_{j0}) for three different values of negative to positive ion mass ratios (Q) for $V - u_{j0} = 1.3$ and $\sigma = 0.01$.

In Fig. 7, the minimum value of the Mach number (M_1) against the negative ion concentration (n_{j0}) with the variation of negative to positive ion mass ratios (Q) is shown for some particular value of the velocity including negative ion drift

$(V - u_{j0})$ and temperature of ions σ ($\sigma = \sigma_i = \sigma_j$). When n_{j0} increases, the minimum value of the Mach number (M_1) also increases for all values of Q, and when Q increases ($Q = 4, 8.875, 16$), then the minimum value of the Mach number (M_1) gradually decreases for all values of n_{j0}. The minimum value of the Mach number (M_1) against negative ion concentration (n_{j0}) with the variation of the velocity including negative ion drift $(V - u_{j0})$ for $Q = 4$ and $\sigma = 1/100$ is shown in Fig. 8. When the values of $(V - u_{j0})$ increase $(V - u_{j0}) = 1.3, 1.7$), the minimum value of the Mach number (M_1) decreases for $Q = 4$ and $\sigma = 0.01$.

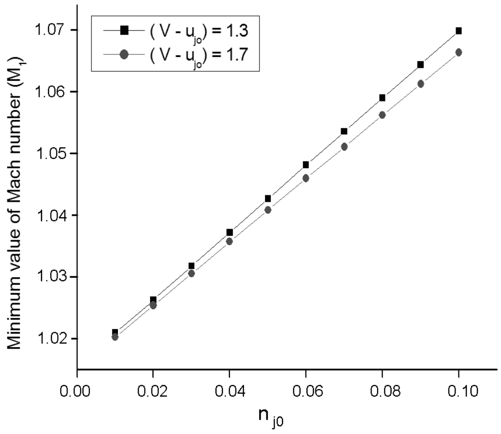

Fig. 8. Variation of minimum value of the Mach number (M_1) versus negative ion concentration (n_{j0}) with the velocity including negative ion drift velocity $(V - u_{j0})$ for $Q = 4$ and $\sigma = 0.01$.

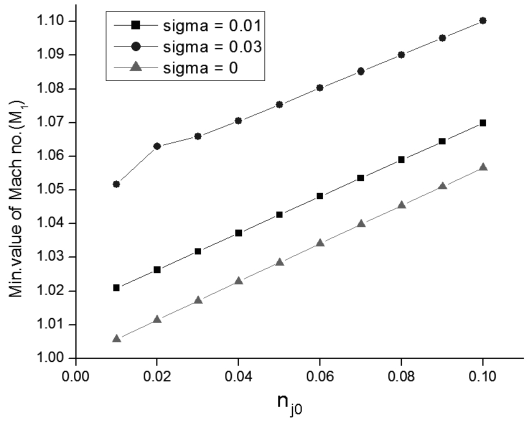

Fig. 9. Variation of minimum value of the Mach number (M_1) versus negative ion concentration (n_{j0}) with different values of the temperature of ions (σ) for $(V - u_{j0}) = 1.3$ and $Q = 4$.

Figure 9 shows the minimum value of the Mach number (M_1) against the negative ion concentration n_{j0} for three different values of the temperature of ions σ ($\sigma = \sigma_i = \sigma_j$) for some values of Q and $(V - u_{j0})$. As n_{j0} increases, the minimum value of the Mach number (M_1) also increases. Moreover, the value of the Mach number (M_1) is minimum for cold ion ($\sigma = \sigma_i = \sigma_j = 0$), and when σ increases ($\sigma = 0, 0.01, 0.03$), then the value of the Mach number (M_1) also increases for some particular value of Q and $(V - u_{j0})$.

6. *Concluding remarks*

We investigated the effect of warm positive and negative ions with warm electrons on the existence of solitary waves, amplitudes and minimum value of the Mach number (say M_1) using Sagdeev's pseudopotential technique. We have derived here an exact form of the pseudopotential function (ψ) for a single temperature electron with warm positive and negative ion plasma. From the exact form of the Sagdeev pseudopotential function, we have derived the analytical condition for the lower limit of M (say M_1) by the inequality (19a) for a compressive solitary wave containing single temperature electron with warm positive as well as negative ions. This gives us a better result than that of Ref. [33] for a single electron-temperature and warm positive ions only. Also, this result exactly reduces to the result of Ref. [31] for cold positive and negative ions with single-temperature electron and also Sagdeev's condition [34] for a single temperature electron with cold ion plasma only. The value of the lower limit of M (say M_1) increases with the increasing value of an appreciably large finite ion-temperature σ ($\sigma = \sigma_i = \sigma_j$). From the inequality (17) with $\sigma \neq 0$ we obtained similarly the upper limit of M for a compressive solitary wave containing single-electron temperature and warm positive as well as negative ions. The basic set of Eqs. (1) to (4) is in normalized form. In all cases of plasma dynamics, it is our advantage to use these non-dimensional or normalized plasma parameters which are used in Eqs. (1) to (4) rather than dimensional parameters. From Eqs. (1) to (4), the unnormalized or dimensional plasma parameters are easily obtained by using the following substitutions:

$$\phi = \frac{kT_e}{e}\bar{\phi}, \quad u_\alpha = \sqrt{\frac{kT_e}{m_\alpha}}\bar{u}_\alpha, \quad E = -\frac{kT_e}{e}\,\mathrm{grad}\,\bar{\phi}, \quad x = \sqrt{\frac{kT_e}{4\pi e^2 n_0}}\bar{x},$$

$$t = \sqrt{\frac{m_\alpha}{4\pi e^2 n_0}}\bar{t}, \quad p_\alpha = p_0\bar{p}_\alpha = n_0 T_\alpha \bar{p}_\alpha, \quad n_\alpha = n_0\bar{n}_\alpha\,.$$

All above parameters under the bar denote have normalized form, while the non-bar parameters have unnormalized form. We have omitted this bar for simplicity reasons in Eqs. (1) to (4) and in the following equations.

We studied the effect of ionic temperature of the plasma having positive ion, negative ion and warm electron. This type of model plasma has significant role for some application in laboratory or space plasma. The theoretical investigation gives some new ideas on solitary waves to develop further the infra-structure for the study of solitons in a warm plasma with negative ions. That would be of interest for the laboratory and space plasmas. In laser-induced laboratory plasma, our present investigation of soliton would be relevant from which we get new information about the propagation of waves in such types of plasmas. Our future plan is to develop the pseudopotential curve (ψ) of relativistic plasma for two-temperature electrons with drifting positive and negative ions and electron drifts.

Acknowledgements

The author is grateful to Dhrubajyoti Misra (B. E.) of Salt Lake Engineering College, West Bengal and Dr. Dipankar Chatterjee (M. E.), Department of Mechanical Engineering, Indian Institute of Technology, Khargapur, West Bengal for their numerical computation and to the referee for very valuable comments and suggestions so that the paper takes this present improved form. Also the author would like to thank Dr. D. Ray and Dr. S. N. Paul for their valuable suggestions and constant encouragement.

References

[1] S. N. Paul, S. Chattopadhyay, S. K. Bhattacharya and B. Bera, Pramana **60**, 6 (2003) 1217.

[2] K. Singh, V. Kumar and H. K. Malik, Phys. Plasmas **12** (2005) 072302.

[3] K. K. Ghosh, S. N. Paul and S. Chattopadhyay, Fizika A (Zagreb) **13**, 1 (2004) 33.

[4] D. K. Singh and H. K. Malik, Phys. Plasmas **13** (2006) 082104.

[5] S. K. Bhattacharaya, S. N. Paul and K. K. Mondal, Phys. Plasmas **9** (2002) 4439.

[6] K. Singh, V. Kumar and H. K. Malik, Phys. Plasmas **12** (2005) 052103.

[7] H. Washimi and T. Taniuti, Phys. Rev. Lett. **17** (1966) 996.

[8] A. P. Kakad, S. V. Singh, R. V. Reddy, G. S. Lakhina, S. G. Tagare and F. Verheest, Phys. Plasmas **14** (2007) 052305.

[9] S. Watanabe, J. Phys. Soc. Japan **53**, 3 (1984) 950.

[10] Y. Hase, S. Watanabe and H. Tanaca, J. Phys. Soc. Japan **54** (1985) 4115.

[11] Y. Nakamura and I. Tsukabayashi, J. Plasma Phys. **34** (1985) 3.

[12] F. Verheest, J. Plasma Phys. **39** (1988) 71.

[13] M. Matsukawa and S. Watanabe, J. Phys. Soc. Japan **57** (1988) 2725.

[14] Kh. Ibohanbi Singh and G. C. Das, IEEE Trans. Plasma Sci. **17** (1989) 869.

[15] G. S. Cho, Phys. Fluids B **2** (1990) 2272.

[16] S. S. Chauhan, H. K. Malik and R. P. Dahiya, Phys. Plasmas **3** (1996) 3932.

[17] H. K. Malik and R. P. Dahiya, Phys. Plasmas **1** (1994) 2872.

[18] H. Ikezi, Phys. Fluids **16** (1973) 1668.

[19] K. E. Lonngren, Plasma Phys. **25** (1983) 943.

[20] H. K. Malik and K. Singh, IEEE Trans. Plasma Sci. **33** (2005) 1995.

[21] K. Nishihara and M. Tajiri, J. Phys. Soc. Jpn. **50** (1981) 4047.

[22] G. S. Lakhina, A. P. Kakad, S. V. Singh and F. Verheest, Phys. Plasmas **15** (2008) 062903.

[23] G. S. Lakhina, S. V. Singh, A. P. Kakad, F. Verheest and R. Bharuthram, Nonlin. Processes Geophysics **15** (2008) 903.

[24] H. K. Malik, S. Singh and R. P. Dahiya, Phys. Plasma **1** (1994) 1137.

[25] S. R. Sharma, Yashvir and T. N. Bhatnagar, Phys. Fluids **29** (1986) 442.

[26] Y. Nakamura, I. Tsukabayashi, G. O. Ludwig and J. L. Ferreira, Phys. Lett. A **113** (1985) 155.

[27] S. G. Tagare, J. Plasma Phys. **36** (1986) 301.

[28] F. D. Tappert, Phys. Fluids **15** (1972) 2446.

[29] P. H. Sakanaka, Phys. Fluids **15** (1972) 304.

[30] S. S. Ghosh, K. K. Ghosh and A. N. S. Iyengar, Phys. Plasmas **3** (1996) 3939.

[31] S. Chattopadhyay, Fizika A (Zagreb) **16** (2007) 25.

[32] S. G. Tagare and R. V. Reddy, J. Plasma Phys. **35** (1986) 219.

[33] S. S. Ghosh and A. N. S. Iyengar, Phys. Plasmas **4**, 6 (1997) 2139.

[34] R. Z. Sagdeev, in *Reviews of Plasma Physics*, edited by M. A. Leontovich, Consultants Bureau, New York (1966) Vol. 4, p. 23.

UČINAK IONSKE TEMPERATURE NA IONSKO-ZVUČNE SOLITONSKE VALOVE U POSMIČNOJ PLAZMI S NEGATIVNIM IONIMA I JEDNOM ELEKTRONSKOM TEMPERATUROM

Proučavaju se ionsko-zvučni solitonski valovi u bezsudarnoj nemagnetiziranoj plazmi s jednom elektronskom temperaturom i s adijabatskim pomacima pozitivnih i negativnih iona koji imaju jednu temperaturu (poseban slučaj) primjenom Sagdeeve pseudopotencijalne metode. Razmatra se učinak ionske temperature i negativnih iona na solitonske valove u plazmi s (He^+, O^-), (He^+, Cl^-) i (H^+, O^-) ionima.