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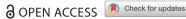
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ABSTRACT

In large-scale group decision-making process, some decision makers hesitate among several linguistic terms and cannot compare some alternatives, so they often express evaluation information with incomplete hesitant fuzzy linguistic preference relations. How to obtain suitable large-scale group decision-making results from incomplete preference information is an important and interesting issue to concern about. After analyzing the existing researches, we find that: i) the premise that complete preference relation is perfectly consistent is too strict, ii) deleting all incomplete linguistic preference relations that cannot be fully completed will lose valid assessment information, iii) semantics given by decision makers are greatly possible to be changed during the consistency improving process. In order to solve these issues, this work proposes a novel method based on Granular computing and optimization model for large-scale group decision-making, considering the original consistency of incomplete hesitant fuzzy linguistic preference relation and improving its consistency without changing semantics during the completion process. An illustrative example and simulation experiments demonstrate the rationality and advantages of the proposed method: i) semantics are not changed during the consistency improving process, ii) completion process does not significantly alter the inherent quality of information, iii) complete preference relations are globally consistent, iv) final large-scale group decision-making result is acquired by fusing complete preference relations with different weights.

1. Introduction

Group decision making is a common decision-making process where a few of decision makers (DMs) (e.g. 3-5 DMs) select the optimal program or several feasible programs among various alternatives (Ju et al., 2020). However, with the rapid development of technological paradigms such as e-democracy (Sundberg, 2019) or

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blockchain (Xiao et al., 2020), dozens to hundreds of DMs are participated in the decision-making process, which constitutes large-scale group decision making (LSGDM) process (Gou et al., 2018; Li et al., 2021; Wu & Xu, 2016).

In real LSGDM process, because DMs usually express uncertain or vague meanings and they would rather qualitative information, linguistic preference relations (LPRs) constructed between any two alternatives are more suitable than quantitative solutions for DMs to provide their evaluations (Li & Wei, 2020; Xu & Wang, 2017). Moreover, since DMs might not have clear opinions about an alternative or they might not be able to compare some alternatives, LPRs are not always complete, that is, DMs may not give all information that they are requested. Thus, it is an important mission to tackle LSGDM problems where some assessment information is missed within LPRs.

From a bird's eye view on literature, when managing incomplete LPRs, some scholars (Ureña et al., 2015) hold the opinions that it takes too much time to complete decision-making information, so they delete incomplete LPRs directly. While most scholars stand for finding missing values according to interactive strategies or static strategies (Alonso et al., 2008; Chuu, 2011; Jiang et al., 2010; Liu et al., 2019, 2020; Song & Li, 2019; Tang et al., 2019; Wang & Xu, 2016; Xia et al., 2014; Xu, 2006; Xu et al., 2016, 2018, 2019; Zhang & Guo, 2014). These two strategies mainly differ in whether fill the missing values by interacting with DMs or not. For interactive strategies, Chuu (2011) developed some interactive methods to revise LPRs with higher consistency or improve consensus degree of DMs, but not for deriving the missing values within incomplete preference relations. Then, Bargiela and Pedrycz (2003) proposed an interactive method based on a feedback mechanism for completing elements. For static strategies, incomplete preference information is estimated through two approaches: 1) based on their own preferences (Alonso et al., 2008; Jiang et al., 2010; Liu et al., 2019, 2020; Song & Li, 2019; Tang et al., 2019; Xia et al., 2014; Xu, 2006; Xu et al., 2018; Zhang & Guo, 2014), and 2) based on other DM's preferences (Xu et al., 2016, 2019). The former approaches include iterative models (Alonso et al., 2008; Liu et al., 2020; Xia et al., 2014; Xu, 2006; Xu et al., 2018) and optimization technique (Jiang et al., 2010; Song & Li, 2019; Tang et al., 2019; Zhang & Guo, 2014), and most of them are related to additive consistency (Jiang et al., 2010; Liu et al., 2020; Tang et al., 2019; Xu, 2006; Zhang & Guo, 2014) or multiplicative consistency (Alonso et al., 2008; Song & Li, 2019; Xia et al., 2014; Xu, 2006; Xu et al., 2018). Recently, an optimization model based on worst consistency index and best consistency index provides a new angle of view towards solving incomplete LPRs problems (Liu et al., 2020). When compensating incomplete preference matrices based on other DMs' preferences, Xu (2006) proposed a trust model to complete missing assessment information. Then, an individual risk appetite (Xu et al., 2018) and a bounded confidence mechanism (Alonso et al., 2008) were introduced to handle incomplete assessment missions. It is worthy to note that it takes extra time to study the society network among DMs, which prolongs the decision-making process. For convenience, we construct a structure of different methods managing incomplete LPRs, shown in Figure 1:
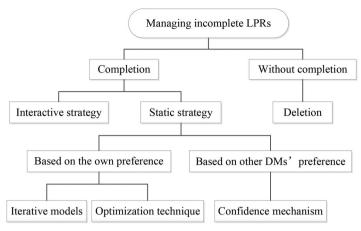
Figure 1. The structure of different methods managing incomplete LPRs.
Source: authors' research.

Although a large number of methods are applied to supple missing linguistic elements, some limitations still remain and motivate us to study:

i.   There is hardly no report about the consistency checking and improving method for incomplete LPRs. Meanwhile, the premise of estimating missing linguistic elements in an incomplete LPR, that a complete preference matrix is perfectly consistent, is too strict in both widely-used iterative models and optimization technique (Alonso et al., 2008; Jiang et al., 2010; Liu et al., 2019, 2020; Song & Li, 2019; Tang et al., 2019; Xia et al., 2014; Xu, 2006; Xu et al., 2018; Zhang & Guo, 2014). We know that the LPR is consistent does not mean it is perfectly consistent, and we should not neglect the importance of original consistency of incomplete LPRs during the LSGDM process, which reflects inherent quality of information provided by DMs.

ii. Deleting all incomplete LPRs that cannot be fully completed will lose lots of valid assessment information. In (Xu, 2006), the incomplete LPR is defined as acceptable if at least one known element exists in each row or column of it, and then delete all unacceptable incomplete LPRs. However, not all unacceptable incomplete LPRs are useless enough to be deleted. For example, a DM cannot give preference information between one object (alternative or attribute) with other objects, but he/she provides other meaningful assessment information among the rest objects, which we can make the most use of.

iii. Semantics given by DMs are possible to be significantly changed by traditional consistency improving methods. Linguistic terms are probably greatly altered during the consistency improving process, and the corresponding semantics are changed at the same time (Gou et al., 2021; Meng et al., 2016; Wu et al., 2019; Zhang et al., 2020; Zhang & Wu, 2014; Zheng et al., 2018; Zhu & Xu, 2014). It is better to find a new way to improve the consistency of preference matrix but do not change the semantics. Moreover, different changes between the initial preference information and the revised consistent preference information should result in various weights of DMs, which is very important in LSGDM process. If

the change of a DM's preference information is small, we should give him/her a high weight, and vice versa.

Based on the above challenges, we find it is interesting to study three research problems:

1. How to change the premise of perfect consistency into original or improved consistency of incomplete LPRs when supplementing missing elements?
2. How to retain more valid assessment information and accurately estimate missing values of incomplete LPR when it can be fully completed or not?
3. How to improve the consistency of preference matrices but not change the semantics given by DMs in the LSGDM process?

Granular Computing, one of constructive and advanced technologies to solve real practical problems with big data (Bargiela & Pedrycz, 2003; Han et al., 2020; Liu et al., 2018), which is similar to LSGDM problems more or less. The core idea of it is to transform the problem into multiple subproblems, solve them separately in each granularity level, and finally combine the solutions of multiple subproblems to form the solution of the original problem. Moreover, Granular computing with linguistic information is a strong weapon to depict the LPRs to be granular rather than numeric through bringing some flexibility to linguistic terms (Pedrycz & Song, 2011). By incorporating Granular computing, we can not only improve the consistency of LPRs without changing the semantics giving by DMs, but also allocate the consistent preference matrices with different weights. The finer the granule is, the greater the weight of the LPR has, and vice versa. Thus, applying the Granular computing in the field of LSGDM will provide a new idea for solving problems, which is rarely reported.

Moreover, both hesitant fuzzy linguistic term set (HFLTS) and hesitant fuzzy linguistic preference relation (HFLPR) are important qualitative expression techniques to enrich the linguistic elicitation based on the fuzzy linguistic approach (Rodriguez et al., 2012; Liao et al., 2015; Ren et al., 2020; Zhang & Wu, 2014; Zheng et al., 2021a, 2021b, 2021c, Chen & Hong, 2014). They can manage the situation where DMs hesitate among several linguistic terms when express evaluation information, which is a common phenomenon in real life. For instance, the phrases, like 'above better than' and 'between a little better than and much better than', etc., can be used in the form of hesitant fuzzy linguistic, and all incomplete LPRs in this paper are in the hesitant fuzzy linguistic context.

Therefore, based on the above analyses, we propose a novel completion method combining Granular computing and optimization model with multi-type incomplete HFLPRs for LSGDM. The primary contributions are concerned with the following:

i. Give a definition of original consistency of incomplete HFLPR, then check and improve the consistency of incomplete HFLPRs based on Granular computing.
ii. In order to retain more valid assessment information given by DMs, we construct an optimization model-based completion method for managing multi-type incomplete HFLPRs that can be fully completed or not.

iii. An illustrative example about assessing the capability of the emergency volunteer services and some simulation comparisons show the application and superiorities of the proposed method: a) semantics given by DMs are not changed when improving the consistency of preference matrices, b) inherent quality of information provided by a certain DM is not significantly changed before and after completion, c) those complete HFLPRs are globally consistent, and d) the final LSGDM result is obtained by fusing complete HFLPRs with different weights.

The paper is organized as follows: Section 2 introduces materials and methods, including preliminaries and a novel Granular computing and optimization model-driving completion method for solving LSGDM problems with multi-type incomplete HFLPRs. Then, we provide an illustrative example about assessing the capability of emergency volunteer services and some comparative analyses including simulation experiments in Section 3. Section 4 makes some discussions about the proposed method and findings. Finally, Section 5 covers some concluding remarks.

2. Materials and methods

1.1. Preliminaries

1.1.1. HFLTS, HFLPR and incomplete HFLPR

Computing with words is a more useful tool than numeric values to depict the assessment information (Zadeh, 1975). Subscript-asymmetric linguistic term set (LTS) is shown as $S = \{s_\delta | s_1, s_2, \ldots, s_{2\tau-1}\}$, where s_δ represents the value of a linguistic variable. Especially, s_1 and $s_{2\tau-1}$ indicate the lower bound and the upper bound of s_δ, respectively. For instance, a set of uniformly distributed LTS can be expressed as:

$$S = \begin{cases} s_1 : Extremely \text{ bad}, \ s_2 : Very \text{ bad}, \ s_3 : \text{Bad}, \ s_4 : Slightly \text{ bad}, \\ s_5 : Medium, \ s_6 : Slightly \text{ good}, \ s_7 : Good, \ s_8 : Very \text{ good}, \ s_9 : Extremely \text{ good} \end{cases}$$

In order to depict the natural feature of hesitancy when DMs provide their assessment information, hesitant fuzzy linguistic term set (HFLTS) (Rodriguez et al., 2012) is proposed to enrich the linguistic-expression model, and the mathematic definition (Liao et al., 2015) is shown as:

$$H_S = \big\{ \langle x_i, h_s(x_i) \rangle | x_i \in X \big\}, \tag{1}$$

where $h_s(x_i) = \big\{ s_s^l(x_i) \in S, l \in \{1, 2, \ldots, L\} \big\}$ is made up of some consecutive linguistic terms in S, L is the number of linguistic terms in $h_s(x_i)$. τ is a non-negative integer, denoting the median subscript of linguistic term set. $h_s(x_i)$ is called the hesitant fuzzy linguistic element (HFLE).

Based on the concepts of HFLTS and HFLE, hesitant fuzzy linguistic preference relation (HFLPR) (Zhu & Xu, 2014) is developed to assist DMs to provide preference information between objects (attributes or alternatives). For a set of objects $X = \big\{ x_i | i \in \{1, 2, \ldots, n\} \big\}$, a HFLPR is defined as $B = (b_{ij})_{n \times n} \subset X \times X$, where b_{ij} represents that the object x_i is preferred to x_j with some hesitancy degrees. $b_{ij} =$

$\left\{ b_{ij(l)} | l \in \{1, 2, \ldots, L\} \right\}$ is a HFLE where $b_{ij(l)}$ indicates a certain term s_δ, and L is the number of linguistic terms in b_{ij}. For all $i, j \in \{1, 2, \ldots, n\}$, $b_{ij}(i<j)$ should satisfy the following requirements:

$$b_{ij(l)} \oplus b_{ji(l)} = s_{2\tau}, b_{ii} = \{s_\tau\}, \#b_{ij} = \#b_{ji}, b_{ij(l)} < b_{ij(l+1)}, b_{ji(l+1)} < b_{ji(l)}, \tag{2}$$

where $b_{ij(l)}$ and $b_{ji(l)}$ are the l−th terms in b_{ij} and b_{ij}, respectively. Meanwhile, $\#b_{ij}$ and $\#b_{ij}$ are the number of terms in b_{ij} and b_{ij}, respectively.

It is of inconvenience in computation process if the number of linguistic terms in an element is not the same, so that we should normalize HFLEs in HFLPRs. One of methods is adding some values $b_{\delta'}$ with $\delta' = \mu\delta^+ + (1-\mu)\delta^-$ in such elements with less linguistic terms, where δ^+ and δ^- are the maximum and minimum subscript values of linguistic terms in a HFLE, respectively. For convenience, we often set μ as 0.5.

Especially, for a HFLPR $B = (b_{ij})_{n \times n} \subset X \times X$, if some of elements cannot be provided by DMs, then the B is called an incomplete HFLPR. For convenience, we use 'x' to represent the missing element. And all known elements satisfy that (Ren et al., 2020):

$$b_{ij(l)} \oplus b_{ji(l)} = s_{2\tau}, b_{ii} = \{s_\tau\}, \#b_{ij} = \#b_{ji}, b_{ij(l)} < b_{ij(l+1)}, b_{ji(l+1)} < b_{ji(l)}, \\ \forall b_{ij} \in \text{Known Elements}. \tag{3}$$

2.1.2. Multiplicative consistency measure

Multiplicative consistency is one of the important types to measure transitivity among three objects (Zhang & Wu, 2014), Zheng et al. (2021c) proposed a novel multiplicative consistency measure index for HFLPRs, which is expressed as:

$$CI(B) = CI\big((b_{ij})_{n \times n}\big) = \begin{cases} 0 & \text{if } n<3 \\ CI(B_{3\times3}) & \text{if } n = 3 \\ \dfrac{1}{\Phi} \sum\limits_{i=1}^{\Phi} CI(\Gamma_i) & \text{if } n>3 \end{cases}, \tag{4}$$

for $i, j \in \{1, 2, \ldots, n\}$, where:

a. $CI(B_{3\times3}) = \dfrac{1}{L} \sum\limits_{l=1}^{L} \left(\dfrac{\lambda - \left(\frac{I(b_{13(l)})I(b_{32(l)})I(b_{21(l)})}{I(b_{31(l)})I(b_{23(l)})I(b_{12(l)})} + \frac{I(b_{31(l)})I(b_{23(l)})I(b_{12(l)})}{I(b_{13(l)})I(b_{32(l)})I(b_{21(l)})} \right)}{\lambda - 2} \right)$, and $\lambda = 9^3 + 9^{-3}$;

b. Γ_i is the i−th pair of different 3-by-3 transitivity digraphs in $B_{n \times n}$;

c. Φ is the number of pairs of different 3-by-3 transitivity digraphs in $B_{n \times n}$, and $\Phi = \frac{n!}{3!(n-3)!}$;

d. $I : S \to [1, 2\tau - 1]$ is a function changing S to $[1, 2\tau - 1]$, such that $I(s_\alpha) = \alpha$ for any $s_\alpha \in S$. Meanwhile, $I^{-1} : [1, 2\tau - 1] \to S$ is a function changing $[1, 2\tau - 1]$ to S, such that $I^{-1}(\alpha) = s_\alpha$ for any $\alpha \in [1, 2\tau - 1]$. Obviously, $I(b_{ij(l)})$ is the l−th subscript value of the linguistic term in $b_{ij(l)}$.
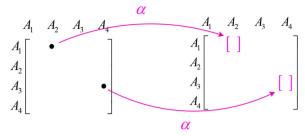
Figure 2. Information granularity in judgment matrix (Pedrycz & Song, 2011).

2.1.3. Granular computing

Since the concept of information granule came into being (Bargiela & Pedrycz, 2003), a great number of scholars were dedicated to research Granular computing from the perspective of theory or application (Liu et al., 2018; Han et al., 2020). Especially, Granular computing with linguistic information and granular matrix of pairwise comparisons (Pedrycz & Song, 2011) creatively extend the preference information into granular matrix with the level of granularity. The main idea of information granularity in judgment matrix is shown in Figure 2.

From Figure 2, we can see that the single linguistic terms in the left preference matrix can be extended to intervals in the right preference matrix by information granularity, where α denotes granularity level. In sum, information granularity can bring flexibility and exploit previous linguistic terms to the fullest possible extent. It reveals the fuzzification of semantics and improves the consistency of preference matrix at the same time.

2.2 A novel completion method based on Granular computing and optimization model

2.2.1. Decision-making problem description and methodology framework

M DMs ($E_m(m \in \{1, 2, \ldots, M\})$) need to select the best one or several feasible programs from n alternatives ($A_i(i \in \{1, 2, \ldots, n\})$). Since the limitation of knowledge or time and the nature of hesitancy often exist during the decision-making process, they provide incomplete HFLPRs ($B^m \{ b^m_{ij(l)} | i, j \in \{1, 2, \ldots, n\}, l \in \{1, 2, \ldots, L\}, m \in \{1, 2, \ldots, M\} \}$) for preference information.

This work proposes a novel completion method based on Granular computing and optimization model for solving LSGDM problems with multi-type incomplete HFLPRs. Facing an incomplete HFLPR B, at first, we need to distinguish its type through five steps of judgements: i. Whether it can be fully completed or not; ii. Whether it is an acceptable incomplete HFLPR or not; iii. Whether it has an original consistency or not; iv. Whether the consistency is acceptable or not; v. Whether its consistency can be improved or not. Specially, if the original consistency of B is unacceptable, we utilize Granular computing to improve its consistency without changing the semantics giving by DMs. Then, we estimate the missing linguistic terms of multi-type incomplete HFLPRs by different optimization models: i. if B does not have an original consistency, we assume that it is perfectly consistent and
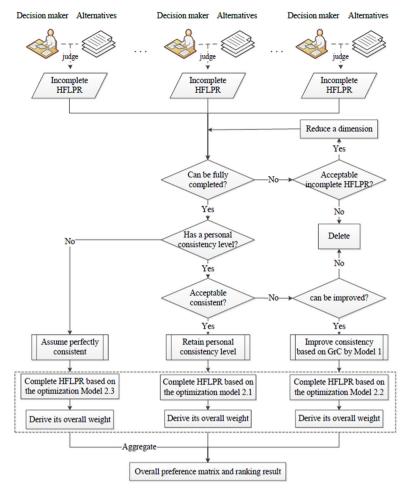
Figure 3. Framework of the proposed granular computing and optimization model-based completion method with multi-type incomplete HFLPRs for LSGDM.
Source: authors' research.

complete it by utilizing optimization model as same as traditional optimization method; ii. if the original consistency of B is acceptable, we consider individual original consistency of the DM rather than perfect consistency during the completion process; iii. if the consistency of B can be improved, we put the improved consistency into the optimization model. After completion, we obtain the final LSGDM results through fusing information with different overall weights. In brief, the whole methodology framework is shown in Figure 3 as follows:

2.2.2. Original consistency of incomplete HFLPRs and its thresholds

As we can see in Figure 3, the first step is to judge whether the incomplete HFLPR can be fully completed or not. We can make judgement through the following definition:

Definition 1. For an incomplete HFLPR B, if every unknown element can be derived through other known elements, then B can be fully completed. Otherwise, B cannot be fully completed.

Property 1. If at least one known element exists in every row or column (except diagonal elements), then B can be fully completed.

Property 2. If all elements in a row or column are unknown (except diagonal elements), then B cannot be fully completed.

Proofs for Properties 1 and 2 are easy so we omit them.

In this paper, one of important ideas is: not all incomplete HFLPRs which cannot be fully completed are unacceptable. We give a new definition of an acceptable incomplete HFLPR:

Definition 2. If only one row or one column of B is completely empty, then B is acceptable. However, if more than two rows or two columns are completely empty, then B is unacceptable.

Remark 1. That is to say if a DM cannot provide preference information between the corresponding two objects, but he/she can provide enough evaluation information among other objects, and the unknown elements within other rows or columns can be estimated through known elements, then the preference information that only one row or column is empty is valid enough. But an incomplete HFLPR with too many unknown elements is not meaningful for decision making, so as to be unacceptable. The idea is much different from the concept of unacceptable incomplete preference relations in (Xu, 2006).

Then, we provide a definition to judge whether an incomplete HFLPR has an original consistency or not.

Definition 3. For an incomplete HFLPR $B = (b_{ij})_{n \times n} \subset X \times X$, if $\exists i, j, k \in \{1, 2, \ldots, n\} \& i \neq j \neq k$, b_{ik}, b_{kj} and b_{ji} are all known elements, then B has an original consistency. Otherwise, B does not have an original consistency.

Remark 2. If the incomplete HFLPR B has an original consistency, then we compute the missing values based on its original consistency, which makes the calculation process of missing values be full of personality and accuracy. Moreover, if the incomplete HFLPR B does not have an original consistency, then it is treated as perfectly consistent. And we calculate its missing values in the way as same as the calculation procedures in traditional iterative or optimization methods.

For an incomplete HFLPR B with an original consistency, we give a definition of original multiplicative consistency index, which is inspired by (Pelaez & Lamata, 2003), shown as follows:

Definition 4. If an incomplete HFLPR B has an original consistency, the original multiplicative consistency index $CI(B)$ is defined based on the known elements within B, shown as:

$$CI(B) = CI\big((b_{ij})_{n \times n}\big) = \begin{cases} 0 & \text{if } n < 3 \\ CI\big((b_{ij})_{3 \times 3}\big) & \text{if } n = 3 \\ \dfrac{1}{\Phi} \sum_{g=1}^{\Phi} CI(\Gamma_g) & \text{if } n > 3 \end{cases}, \tag{5}$$

where:

a. $i, j \in \{1, 2, \ldots, n\}$ and each b_{ij} is a known element;

b. $CI(B_{3\times3}) = \frac{1}{L}\sum_{l=1}^{L}\left(\frac{\lambda - \left(\frac{I(b_{13(l)})I(b_{32(l)})I(b_{21(l)})}{I(b_{31(l)})I(b_{23(l)})I(b_{12(l)})} + \frac{I(b_{31(l)})I(b_{23(l)})I(b_{12(l)})}{I(b_{13(l)})I(b_{32(l)})I(b_{21(l)})}\right)}{\lambda - 2}\right)$, and $\lambda = 9^3 + 9^{-3}$;

c. Γ_i is the i−th pair of different 3-by-3 transitivity digraphs in B and all elements in Γ_i are known;

d. Φ is the number of Γ_i in B;

e. $I : S \to [1, 2\tau - 1]$ is a function changing S to $[1, 2\tau - 1]$, such that $I(s_\alpha) = \alpha$ for any $s_\alpha \in S$. Meanwhile, $I^{-1} : [1, 2\tau - 1] \to S$ is a function changing $[1, 2\tau - 1]$ to S, such that $I^{-1}(\alpha) = s_\alpha$ for any $\alpha \in [1, 2\tau - 1]$. Obviously, $I(b_{ij(l)})$ is the l−th subscript value of the linguistic term in $b_{ij(l)}$.

Property 3. Values of $CI(B)$ fall in the interval $[0, 1]$.

Property 4. $CI(B) = 1$ means that the incomplete HFLPR B is completely multiplicatively consistent.

Proofs for Properties 3 and 4 are shown in Supplemental file.

Remark 3. Definition 4 is similar to Definition 4 in (Zheng et al., 2021c), but there are some differences between them: the value of Φ is not fixed for each n, while it is fixed in (Zheng et al., 2021c) (e. g., $\Phi = C_4^3 = \frac{4!}{3!(4-3)!} = 4$ if $n = 4$, and $\Phi = C_5^3 = \frac{5!}{3!(5-3)!} = 10$ if $n = 5$). For convenience, consistency index threshold of complete HFLPR is denoted as ξ^n, and consistency index threshold of incomplete HFLPR is denoted as ξ_Φ^n.

The determination of threshold ξ^n for consistency measure is quite significant. We establish a Monte Carlo simulation 1000×1000 times to calculate thresholds ξ^n of $CI(B) = CI((b_{ij})_{n\times n})$ based on the percentile that accepting multiplicatively consistent HFLPR at the point with the smoothest change.

Assume that $n = 4$, $l = 3$ and $n = 5$, $l = 3$. After executing the above algorithm (Python 3.7), the thresholds $\xi^4 = 0.9985$ of $CI((b_{ij})_{4\times4})$ and $\xi^5 = 0.9948$ of $CI((b_{ij})_{5\times5})$ are displayed in Figure 4.

Remark 4. The reasons for choosing the thresholds ξ^4 and ξ^5 at that point where the value of $\Delta y/\Delta x$ is the lowest are shown as follows:

a. The percentile as the value for accepting matrices is relatively low. In general, only a small number of multiplicatively consistent HFLPRs exist in randomly generated HFLPRs, and fortunately, for $n = 4$, $l = 3$, just 9.6% of HFLPRs are considered consistent at the chosen point; for $n = 5$, $l = 3$, just 17.1% of HFLPRs are considered consistent at the chosen point.

b. The rate of change at that point is small. We know that the smaller the value of $\Delta y/\Delta x$ is, the more slightly the value of $CI(B)$ changes. A suitable threshold should be chosen at the point with the gentlest change rate of $CI(B)$ value.

For complete HFLPRs, if $n = 4$, $l = 3$ or $n = 5$, $l = 3$, the values of Φ are always 4 or 10, which means that there are four or ten pairs of different 3-by-3 transitivity digraphs in complete HFLPRs. But for incomplete HFLPRs, if $n = 4$, $l = 3$, the
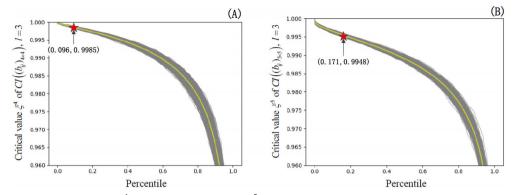
Figure 4. Thresholds ξ^4 of $CI((b_{ij})_{4\times4})$ (A) and ξ^5 of $CI((b_{ij})_{5\times5})$(B).
Source: authors' research.

values of Φ change from 1 to 3; if $n = 5$, $l = 3$, the values of Φ change from 1 to 9. Inspired by the thought that the percentiles of accepting consistent preference matrices for each dimension are similar (Ignacio et al., 2018), we compute different thresholds ξ_Φ^4 and ξ_Φ^5 for various Φ, shown as Tables 1 and 2, respectively:

If the value of original consistency index $CI(B)$ of an incomplete HFLPR is lower than the corresponding threshold ξ_Φ^n, then B is called inconsistent. Thus, we should try best to improve the original consistency of B.

2.2.3. Granular computing-driving consistency improving method

It is worthwhile to note that improving the consistency of preference matrix by the existing consistency-improving techniques (Lin et al., 2014; Meng et al., 2016; Wu et al., 2019; Zhang et al., 2020; Zhang & Wu, 2014; Zheng et al., 2018; Zhu & Xu, 2014), is somewhat a result of a postmortem process where the DM is rather passive to modify his/her initial judgement. Even the revised preference matrix is far from its initial matrix. In this work, we utilize Granular computing to improve the consistency of HFLPRs but without changing the semantics. At first, we introduce the granularity level of HFLE and HFLPR with granularity level.

Definition 5 (Zheng et al., 2021c). For a HFLE b_{ij} of a HFLPR B, the granularity level of HFLE α_{ij} is calculated as:

$$\alpha_{ij} = \max\left\{\left|b''_{ij(l)} - b_{ij(l)}\right| \times 2\right\}, \tag{6}$$

where $l \in \{1, 2, \ldots, L\}$ and $i, j \in \{1, 2, \ldots, n\}$. $b''_{ij(l)}$ is the $l-$th revised linguistic term and $b_{ij(l)}$ is the $l-$th initial linguistic term. In order not to change semantics through introducing flexible linguistic terms, we stipulate that if $b_{ij(l)} = s_\delta(s_\delta \in \{s_2, s_3, \ldots, s_{2\tau-2}\})$, then $b''_{ij(l)} \in [s_{\delta-0.5}, s_{\delta+0.5}]$; if $b_{ij(l)} = s_1$, then $b''_{ij(l)} \in [s_1, s_{1.5}]$; if $b_{ij(l)} = s_{2\tau-1}$, then $b''_{ij(l)} \in [s_{2\tau-0.5}, s_{2\tau-1}]$. τ is the median subscript of linguistic term set.

Definition 6 (Zheng et al., 2021c). For a HFLPR B, its granularity level α is the biggest one among all granularity levels α_{ij} of its HFLEs with $\alpha_{ij} \in [0, 1]$ and $\alpha \in [0, 1]$.

Table 1. Thresholds $\xi^4_{\geq\Phi}$ for various $\Phi(\Phi \in \{1,2,3\})$ when $n = 4$ and $l = 3$.

Φ	1	2	3
$\xi^4_{\geq\Phi}$	0.9998	0.9993	0.9988

Source: authors' research.

Table 2. Thresholds $\xi^5_{\geq\Phi}$ for various $\Phi(\Phi \in \{1,2,\ldots,9\})$ when $n = 5$ and $l = 3$.

Φ	1	2	3	4	5	6	7	8	9
$\xi^5_{\geq\Phi}$	0.9987	0.998	0.9973	0.9967	0.9963	0.996	0.9955	0.9952	0.995

Source: authors' research.

For convenience, we show the thought of constructing the HFLPR with granularity level in Figure 5.

Based on the concepts of the granularity level of HFLE and HFLPR with granularity level, we can improve the original consistency of incomplete HFLPR B and calculate the optimal granularity level of B from the following optimization model at the same time.

Firstly, we should define the goal function, which is to find the minimum granularity level of HFLPR, shown as:

$$\min \ \alpha. \tag{7}$$

Because the granularity level α of B is the biggest one among all granularity levels α_{ij} of its HFLEs, Eq. (7) is rewritten as:

$$\min\{\max\alpha_{ij}\}. \tag{8}$$

Then, we give two constraints of this optimization model, including consistency index value of the revised incomplete HFLPR and the revised incomplete HFLEs.

- Consistency index value of the revised incomplete HFLPR

It is definite that the revised incomplete HFLPR should be acceptably consistent, which means that the consistency index value of the revised incomplete HFLPR $CI(B'')$ should be equal or higher than the threshold $\xi^n_{\geq\Phi}$ of the giving n and Φ, that is:

$$CI(B'') \geq \xi^n_{\geq\Phi}, \tag{9}$$

where $CI(B'') = CI((b''_{ij})_{n \times n}) = \begin{cases} 0 & \text{if } n < 3 \\ CI((b''_{ij})_{3\times3}) & \text{if } n = 3 \\ \frac{1}{\Phi}\sum_{g=1}^{\Phi} CI(\Gamma''_g) & \text{if } n > 3 \end{cases}$ and

$$CI(B''_{3\times3}) = \frac{1}{L}\sum_{l=1}^{L}\left(\frac{\lambda - \left(\frac{I(b''_{13(l)})I(b''_{32(l)})I(b''_{21(l)})}{I(b''_{31(l)})I(b''_{23(l)})I(b''_{12(l)})} + \frac{I(b''_{31(l)})I(b''_{23(l)})I(b''_{12(l)})}{I(b''_{13(l)})I(b''_{32(l)})I(b''_{21(l)})}\right)}{\lambda - 2}\right). \quad \lambda = 9^3 +$$

9^{-3} and $i,j \in \{1,2,\ldots,n\}$. Γ''_i is the $i-$th pair of different 3-by-3 transitivity digraphs in B'' and all elements in Γ''_i are known, and Φ is the number of Γ''_i in B''.
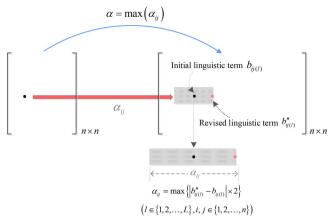
Figure 5. HFLPR with granularity level α (Zheng et al., 2021c).

- Revised incomplete HFLEs

In this model, one of important thoughts is that the HFLEs $b''_{ij(l)}$ in the revised incomplete HFLPR cannot change original semantics provided by a DM. so that the constraint is:

$$b''_{ij(l)} \in \begin{cases} [s_{\delta-0.5}, s_{\delta+0.5}] & \text{if} \quad b_{ij(l)} = s_\delta(s_\delta \in \{s_2, s_3, ..., s_{2\tau-2}\}) \\ [s_1, s_{1.5}] & \text{if} \quad b_{ij(l)} = s_1 \\ [s_{2\tau-0.5}, s_{2\tau-1}] & \text{if} \quad b_{ij(l)} = s_{2\tau-1} \end{cases} \tag{10}$$

where $i, j \in \{1, 2, ..., n\}$, $l \in \{1, 2, ..., L\}$ and τ is the median subscript of linguistic term set.

Hence, the optimization model is constructed to improve the multiplicative consistency of incomplete HFLPR B and calculate the optimal granularity level of B, shown as:

Model 1:

$$\min\{\max\alpha_{ij}\}$$

$$s.t. \begin{cases} \alpha_{ij} = \max\left\{\left|b''_{ij(l)} - b_{ij(l)}\right| \times 2\right\} \\ \alpha_{ij} \in [0, 1] \\ \xi^n_\Phi \leq CI(B'') \leq 1 \\ b''_{ij(l)} \in \begin{cases} [s_{\delta-0.5}, s_{\delta+0.5}] & \text{if} \quad b_{ij(l)} = s_\delta(s_\delta \in \{s_2, s_3, ..., s_{2\tau-2}\}) \\ [s_1, s_{1.5}] & \text{if} \quad b_{ij(l)} = s_1 \\ [s_{2\tau-0.5}, s_{2\tau-1}] & \text{if} \quad b_{ij(l)} = s_{2\tau-1} \end{cases} \\ i, j \in \{1, 2, ..., n\} \\ l \in \{1, 2, ..., L\} \end{cases}$$

Remark 6. We utilize Particle Swarm Optimization (PSO) with constraint support to solve this model. PSO algorithm is one of valid and widely-used evolutionary computation technologies to find the optimal solution through the cooperation and

information sharing among individuals in a group (Pedrycz & Song, 2011). Different from the traditional PSO algorithm used in granular computing (Pedrycz & Song, 2011), we add constraint limitations to the PSO algorithm to ensure that B'' is acceptably consistent. The parameters for the cognitive acceleration coefficient and social acceleration coefficient of the particle are both set as 2, the number of particles in the swarm is set as 400 and the algorithm is run for 100 generations. These values are commonly encountered in the existing researches (Mohammadi-Ivatloo et al., 2012).

Not all preference information has an optimal granularity level, namely that not every HFLPR's consistency can be improved without change of semantics. The following definition can affirm whether the original consistency of incomplete HFLPR can be improved or not.

Definition 7. For an incomplete HFLPR B, if it has a minimum granularity level, then we call the consistency of B can be improved, and vice versa.

Remark 7. If an incomplete HFLPR B has an acceptable improved or original consistency, then it can be completed; however, if an incomplete HFLPR B has an original consistency but the consistency cannot be improved based on the premise of without changing the semantics, then it should be deleted. Moreover, if $CI(B) \geq \xi_\Phi^n$, we call the incomplete HFLPR B is acceptably consistent and the granularity level of it is set as 0.

2.2.4. Optimization model-based method to complete multi-type incomplete HFLPRs

After measuring the consistency condition of incomplete HFLPR B, another optimization model is designed to estimate the missing values. If an incomplete HFLPR B has an acceptable original consistency, then it can be completed by Model 2.1.

At first, we define the goal function of Model 2.1, which is to minimize the distance between consistency index value $CI(\hat{B})$ of complete preference matrix and the transformed acceptable original consistency index value $CI'(B)$, shown as:

$$\min |CI(\hat{B}) - CI'(B)|, \tag{11}$$

where the transformed acceptable original consistency index value $CI'(B)$ is related to the acceptable original consistency index value $CI(B)$:

$$\frac{1 - CI(B)}{1 - \xi_\Phi^n} = \frac{1 - CI'(B)}{1 - \xi^n}, \tag{12}$$

where ξ^n is the consistency index threshold of complete HFLPRs corresponding to different values of n, and ξ_Φ^n is the consistency index threshold of incomplete HFLPRs corresponding to different values of n and Φ. So that we can transfer $CI(B)$ into $CI'(B)$ with the same proportion of ξ_Φ^n to ξ^n. Then, the distance between the consistency index value $CI(\hat{B})$ of complete preference matrix and the relative consistency index value $CI'(B)$ of the revised preference matrix can be calculated directly.
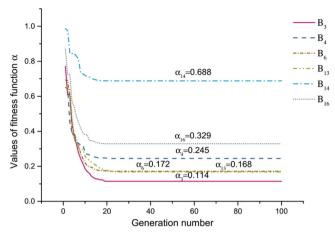
Figure 6. Values of fitness function (also granularity levels) $\alpha_3, \alpha_4, \alpha_6, \alpha_{13}, \alpha_{14}, \alpha_{16}$ calculated by the PSO algorithm.
Source: authors' research.

Constraints are designed from two aspects: consistency index value of the complete HFLPR and the missing values to be estimated.

• Consistency index value of the complete HFLPR

We know that if a complete HFLPR is acceptably consistent, its consistency index value $CI(\hat{B})$ should be equal or higher than the threshold ξ^n of the given n, that is:

$$CI(\hat{B}) \geq \xi^n, \tag{13}$$

where $CI(\hat{B}) = CI\left(\left(\hat{b}_{ij}\right)_{n\times n}\right) = \begin{cases} 0 & \text{if } n<3 \\ CI\left(\left(\hat{b}_{ij}\right)_{3\times3}\right) & \text{if } n=3 \\ \frac{1}{\Phi}\sum_{g=1}^{\Phi} CI\left(\hat{\Gamma}_g\right) & \text{if } n>3 \end{cases}$ and

$CI\left(\left(\hat{b}_{ij}\right)_{3\times3}\right) = \frac{1}{L}\sum_{l=1}^{L}\left(\frac{\lambda - \left(\frac{I\left(\hat{b}_{13(l)}\right)I\left(\hat{b}_{32(l)}\right)I\left(\hat{b}_{21(l)}\right)}{I\left(\hat{b}_{31(l)}\right)I\left(\hat{b}_{23(l)}\right)I\left(\hat{b}_{12(l)}\right)} + \frac{I\left(\hat{b}_{31(l)}\right)I\left(\hat{b}_{23(l)}\right)I\left(\hat{b}_{12(l)}\right)}{I\left(\hat{b}_{13(l)}\right)I\left(\hat{b}_{32(l)}\right)I\left(\hat{b}_{21(l)}\right)}\right)}{\lambda-2}\right).$ $\lambda = 9^3 + 9^{-3}, i,j \in \{1,2,...,n\}$. $\hat{b}_{ij}$ is every completed element in $\hat{B}$. $\hat{\Gamma}_i$ is the i-th pair of different 3-by-3 transitivity digraphs in $\hat{B}$ and all elements in $\hat{\Gamma}_i$ are known, and Φ is the number of $\hat{\Gamma}_i$ in $\hat{B}$.

• Missing values to be estimated

The missing values to be estimated should meet the requirements of both HFLE in HFLPR and the HFLPR with granularity level. The specific constraints are shown as follows:

$$I\left(x_{ij(l)}\right) \in [1, 2\tau - 1], \tag{14}$$

$$I(x_{ij(l)}) + I(x_{ji(l)}) = 2\tau - 1, \tag{15}$$

$$I(x_{ii(l)}) = \tau, \tag{16}$$

$$I(x_{ij(l+1)}) - I(x_{ij(l)}) \in \begin{cases} (0,2) & \text{if } i<j \\ (-2,0) & \text{if } i>j \end{cases}, \tag{17}$$

$$I(x_{ij(l+2)}) - I(x_{ij(l)}) \in \begin{cases} (1,3) & \text{if } i<j \\ (-3,-1) & \text{if } i>j \end{cases}, \tag{18}$$

where $i,j \in \{1,2,\ldots,n\}$ and $l,\ l+1,\ l+2 \in \{1,2,\ldots,L\}$.

Therefore, the whole Model 2.1 to complete the incomplete HFLPR is established as:

Model 2.1

$$\min |CI(\hat{B}) - CI'(B)|$$
$$s.t. \begin{cases} CI(\hat{B}) \geq \xi^n \\ I(x_{ij(l)}) \in [1, 2\tau - 1] \\ I(x_{ij(l)}) + I(x_{ji(l)}) = 2\tau \\ (x_{ii(l)}) = \tau \\ I(x_{ij(l+1)}) - I(x_{ij(l)}) \in \begin{cases} (0,2) & \text{if } i<j \\ (-2,0) & \text{if } i>j \end{cases} \\ I(x_{ij(l+2)}) - I(x_{ij(l)}) \in \begin{cases} (1,3) & \text{if } i<j \\ (-3,-1) & \text{if } i>j \end{cases} \\ i,j \in \{1,2,\ldots,n\} \\ l,\ l+1,\ l+2 \in \{1,2,\ldots,L\} \end{cases}$$

If an incomplete HFLPR B has an original consistency, and can be revised to B'' with acceptable consistency by Model 1, then it can be completed by Model 2.2.

The goal function of Model 2.2 is similar to that of Model 2.1, which is to minimize the distance between consistency index value $CI(\hat{B})$ of the complete preference matrix and the transformed improved consistency index value $CI'(B'')$, shown as:

$$\min |CI(\hat{B}) - CI'(B'')|, \tag{19}$$

where the transformed improved consistency index value $CI'(B'')$ is related to the improved consistency index value $CI(B'')$:

$$\frac{1 - CI(B'')}{1 - \xi_\Phi^n} = \frac{1 - CI'(B'')}{1 - \xi^n}, \tag{20}$$

ξ^n is the consistency index threshold of complete HFLPRs corresponding to different values of n, and ξ_Φ^n is the consistency index threshold of incomplete HFLPRs corresponding to different values of n and Φ.

Constraints of Model 2.2 are as same as those in Model 2.1, so the whole Model 2.2 is constructed as:

Model 2.2:

$$\min \left| CI(\hat{B}) - CI'(B'') \right|$$

$$s.t. \begin{cases} CI(\hat{B}) \geq \xi^n \\ I(x_{ij(l)}) \in [1, 2\tau - 1] \\ I(x_{ij(l)}) + I(x_{ji(l)}) = 2\tau \\ (x_{ii(l)}) = \tau \\ I(x_{ij(l+1)}) - I(x_{ij(l)}) \in \begin{cases} (0,2) & if \quad i<j \\ (-2,0) & if \quad i>j \end{cases} \\ I(x_{ij(l+2)}) - I(x_{ij(l)}) \in \begin{cases} (1,3) & if \quad i<j \\ (-3,-1) & if \quad i>j \end{cases} \\ i,j \in \{1,2,...,n\} \\ l,\ l+1,\ l+2 \in \{1,2,...,L\} \end{cases}$$

Based on Definition 3, we know that if an incomplete HFLPR B does not have an original consistency, then it is treated as perfectly consistent, and it can be completed by Model 2.3.

Model 2.3:

$$\min \left| CI(\hat{B}) - 1 \right|$$

$$s.t. \begin{cases} CI(\hat{B}) \geq \xi^n \\ I(x_{ij(l)}) \in [1, 2\tau - 1] \\ I(x_{ij(l)}) + I(x_{ji(l)}) = 2\tau \\ (x_{ii(l)}) = \tau \\ I(x_{ij(l+1)}) - I(x_{ij(l)}) \in \begin{cases} (0,2) & if \quad i<j \\ (-2,0) & if \quad i>j \end{cases} \\ I(x_{ij(l+2)}) - I(x_{ij(l)}) \in \begin{cases} (1,3) & if \quad i<j \\ (-3,-1) & if \quad i>j \end{cases} \\ i,j \in \{1,2,...,n\} \\ l,\ l+1,\ l+2 \in \{1,2,...,L\} \end{cases}$$

2.2.5. Obtain the final LSGDM results

After revising and completing the incomplete HFLPRs, we should aggregate them to acquire the final LSGDM results. At first, we design an overall weight index for each DM, integrating the influence of granularity level, the number of known elements, and the final multiplicative consistency of each DM. The overall weight index for each DM is shown as:

$$w_m = \frac{\varpi_m \times \omega_m \times \psi_m}{\sum_{m=1}^{M} \varpi_m \times \omega_m \times \psi_m} \tag{21}$$

where:

a. $\varpi_m = \frac{1-\alpha_m}{\sum_{m=1}^{M}(1-\alpha_m)}$ is the weight index for granularity level of each DM;

b. $\omega_m = \frac{N_m}{\sum_{m=1}^{M} N_m}$ is the weight index for known elements of each DM;

c. $\psi_m = \frac{CI(\hat{B}_m)}{\sum_{m=1}^{M} CI(\hat{B}_m)}$ is the weight index for final multiplicative consistency of each DM;

d. $m \in \{1, 2, \ldots, M\}$ and M is the number of DMs in the LSGDM process.

Because the overall weight of a DM is computed from three different aspects: the influence of granularity level, the number of known elements, and the final multiplicative consistency, the weight of DM is more reasonable and comprehensive than that in traditional LSGDM methods (Xu et al., 2016, 2019). The overall weights calculated here play an important theoretical foundation for gathering opinions of DMs in LSGDM. Thus, the final preference matrix can be acquired based on completed HFLPRs and their overall weights, which is displayed as follows:

$$B_{all} = \begin{array}{c} \\ A_1 \\ A_2 \\ \ldots \\ A_i \end{array} \begin{array}{cccc} A_1 & A_2 & \ldots & A_j \\ \left[\begin{array}{cccc} \dfrac{\sum_{m=1}^{M_{11}} \hat{b}_{11}^m \times w_m}{\sum_{m=1}^{M_{11}} w_m} & \dfrac{\sum_{m=1}^{M_{12}} \hat{b}_{12}^m \times w_m}{\sum_{m=1}^{M_{12}} w_m} & \ldots & \dfrac{\sum_{m=1}^{M_{1j}} \hat{b}_{1j}^m \times w_m}{\sum_{m=1}^{M_{1j}} w_m} \\ \dfrac{\sum_{m=1}^{M_{21}} \hat{b}_{21}^m \times w_m}{\sum_{m=1}^{M_{21}} w_m} & \dfrac{\sum_{m=1}^{M_{22}} \hat{b}_{22}^m \times w_m}{\sum_{m=1}^{M_{22}} w_m} & \ldots & \dfrac{\sum_{m=1}^{M_{2j}} \hat{b}_{2j}^m \times w_m}{\sum_{m=1}^{M_{2j}} w_m} \\ \dfrac{\sum_{m=1}^{M_{i1}} \hat{b}_{i1}^m \times w_m}{\sum_{m=1}^{M_{i1}} w_m} & \dfrac{\sum_{m=1}^{M_{i2}} \hat{b}_{i2}^m \times w_m}{\sum_{m=1}^{M_{i2}} w_m} & \ldots & \dfrac{\sum_{m=1}^{M_{ij}} \hat{b}_{ij}^m \times w_m}{\sum_{m=1}^{M_{ij}} w_m} \end{array} \right]_{n \times n} \end{array} \tag{22}$$

It is worthy to note that the final LSGDM result is obtained by aggregating different preference relations provided by DMs, which is much different from clustering methods in other LSGDM models such as (Xu et al., 2016, 2019). Because different final preference matrix can be acquired by various clustering methods and related principle of group forming, the final LSGDM result is obtained by directly aggregating complete HFLPRs with their comprehensive weights in this paper to facilitate the efficient LSGDM process. The whole procedures of obtaining the final LSGDM result based on granular computing and optimization model can be shown as follows:

Algorithm:

Input: DMs provide assessment information in the form of incomplete HFLPRs $B_m (m \in \{1, 2, \ldots, M\})$.

Output: The final preference matrix.

Step 1: Judge whether each incomplete HFLPR B_m can be fully completed or not by Definition 1. If it can, then go to Step 3; if it cannot, then go to Step 2.

Step 2: Estimate whether the incomplete HFLPR B_m is acceptable through Definition 2. If it is, then we delete the row and column that is empty so as to reduce a dimension, and then go back to Step 1; otherwise, we delete the incomplete HFLPR B_m and start with another one from Step 1.

Step 3: Judge whether the incomplete HFLPR B_m has an original consistency or not through Definition 3. If it does, then go to Step 4; if not, then go to Step 5.

Step 4: After normalization, we calculate the original consistency of B_m by Definition 4. If it is higher than the threshold, then go to Step 6; if it is lower than the threshold, then go to Step 5.

Step 5: Estimate whether the original consistency of each B_m can be improved through Model 1. If it does, then we calculate the improved consistency of the incomplete HFLPR B_m and go to Step 6; otherwise, we delete the incomplete HFLPR B_m and start with another one from Step 1.

Step 6: Compute the missing values of the incomplete HFLPR B_m. If B_m has an original consistency and it is acceptably consistent, then we estimate the missing values by Model 2.1 and go to Step 7; if B_m has an improved consistency, then we calculate the missing linguistic terms through Model 2.2 and go to Step 7; if B_m does not have an original consistency, then we execute Model 2.3 to estimate the missing values and go to Step 7.

Step 7: Obtain the completed HFLPR $\hat{B}_p (p \in \{1, 2, \ldots, P\})$, $P \leq M$.

Step 8: Calculate the overall weight index for each $\hat{B}_p$ through Eq (21).

Step 9: Acquire the final preference matrix based on Eq. (22).

End.

3. Case study

3.1. Illustrative example of evaluating the capability of the emergency volunteer services

2020 is probably one of the most special years for everyone in the world, due to the explosive eruption of Corona Virus Disease 2019 (COVID-19) around the world. On January 30th, 2020, the World Health Organization declared that the outbreak of COVID-19 is to be a Public Health Emergency of International Concern, posing a high risk to countries with vulnerable health systems (Sohrabi et al., 2020). During the phase of epidemic prevention and control, not only the accurate and efficient medical treatment, but also the social assistance, such as voluntary services, have played an important role to greatly reduce the losses incurred. Voluntary service is an effective form of organization for the public to participate in COVID-19 prevention and control. In China, there are four types of services for volunteers to participate: epidemic prevention and control in community or villages, publicizing the measures in epidemic prevention and control, distribution of daily supplies and post services guarantee. Some related researches have declared that the stronger the capability of emergency volunteer service is, the better and more professional the service can be offered to the influenced persons (Waldman et al., 2018; Wang & Huang, 2020). Thus, it is necessary to comprehensively evaluate the capability of the emergency volunteer services. According to the research of the organization's ability to compete in society that contains three aspects: individual competence, organizational competence and environmental competence (Charles et al., 1995; Matachi, 2006), the service capability of emergency volunteering organization should be evaluated from internal capability and external capability as well. To be specific, on the one hand, internal capability of emergency volunteering service is mainly measured by organization and

Table 3. The detailed explanation of the five indicators that influence the capability of the emergency volunteer services.

Indicators	The detailed explanation
Specialization	The ability that volunteer team offers professional service or assistance
Standardization	The ability that external institution and government provide enough daily supplies and policy supports as the voluntary organizations are voluntary
Coordination	The ability that volunteer teams communicate, cooperate and coordinate with the government and other organizations in service actions due to the dependence of external resources brought by non-profit organizations
Sedimentation	The ability that volunteer teams can gain recognition and participation in social groups because of the free and public welfare nature of voluntary organizations
Organization	The ability that volunteer teams can organize and gather the volunteers together

Source: authors' research.

specialization, because only professional volunteering service can guarantee the succeed of emergency rescue and only efficient group management can keep everything running smoothly in organization. On the other hand, external capability of emergency volunteering service is constructed through standardization, coordination and sedimentation, which mainly reflects in the support and guarantee of the external environment and adjustment and adaptation of the organization itself to the external environment. Therefore, to assess the capability of the emergency volunteer services, an assessment indicator system is necessary, which is constructed from five aspects (Wang & Huang, 2020): 1) *Specialization*, 2) *Standardization*, 3) *Coordination*, 4) *Sedimentation*, and 5) *Organization*. The detailed explanation of these five indicators is exhibited in Table 3.

COVID-19 has broken out in City C so that emergency volunteering service is necessary for medical assistance and society stability. To assess the capacity of emergency volunteering services, and because of the inconvenience to execute several consensus reaching process at special stage of COVID-19, 20 DMs $E_m(m \in \{1, 2, \ldots, 20\})$ are invited to participate in an online group decision making and evaluate the different significances of these five indicators $A_i(i \in \{1, 2, \ldots, 5\})$. Because of the limited time and human nature of hesitation in the decision-making process, DMs might not be able to evaluate the indicators that influence the capability of the emergency volunteer service comprehensively, so they give pairwise comparison matrices in the form of the incomplete HFLPRs ($B_m(m \in \{1, 2, \ldots, 20\})$). Supposing that $S = \{s_1 = $ *extremely inferior*, $s_2 = $ *very inferior*, $s_3 = $ *inferior*, $s_4 = $ *slightly inferior*, $s_5 = $ *medium*, $s_6 = $ *slightly superior*, $s_7 = $ *superior*, $s_8 = $ *very superior*, $s_9 = $ *extremely superior*$\}$. All original assessment information is exhibited in supplemental files. Here, we take the incomplete HFLPR B_3 for example:

$$B_3 = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{array} \begin{array}{ccccc} A_1 & A_2 & A_3 & A_4 & A_5 \\ \left[\begin{array}{ccccc} \{s_5\} & \{s_2, s_3\} & \{s_7\} & \{s_9\} & \{s_6, s_7, s_8\} \\ \{s_8, s_7\} & \{s_5\} & \{s_4, s_5, s_6\} & \{s_5, s_6, s_7\} & \{s_5, s_6\} \\ \{s_3\} & \{s_6, s_5, s_4\} & \{s_5\} & \{s_5, s_6\} & \{s_6\} \\ \{s_1\} & \{s_5, s_4, s_3\} & \{s_5, s_4\} & \{s_5\} & x_{45} \\ \{s_4, s_3, s_2\} & \{s_5, s_4\} & \{s_4\} & x_{54} & \{s_5\} \end{array} \right] \end{array}$$

Step 1: According to Definition 1, we find that only B_{17} cannot be fully completed, so send it to Step 2 and send others to Step 3.

Step 2: After judgement by Definition 2, we know that B_{17} is an unacceptable incomplete HFLPR. So we delete B_{17} and continue the following steps.

Step 3: Through Definition 3, it is easy to find that preference matrices $B_3, B_4, B_6, B_{13}, B_{14}, B_{16}, B_{18}, B_{19}, B_{20}$ have original consistency, so put them into Step 4. While others $B_1, B_2, B_5, B_7, B_8, B_9, B_{10}, B_{11}, B_{12}, B_{15}$ do not have, so send them to Step 6.

Step 4: After normalization, we calculate the original consistency of $B_3, B_4, B_6, B_{13}, B_{14}, B_{16}, B_{18}, B_{19}, B_{20}$ by Definition 4, respectively. If it is higher than the threshold, then go to Step 6; if it is lower than the threshold, go to Step 5. For example, $CI_p(B_3) = 0.9938$, which is lower than the its corresponding threshold $\xi_9^5 = 0.995$, so continue Step 5.

Step 5: By Model 1, we revise the preference matrices $B_3, B_4, B_6, B_{13}, B_{14}, B_{16}$ into $B''_3, B''_4, B''_6, B''_{13}, B''_{14}, B''_{16}$, while delete B_{18}, B_{19}, B_{20} since they cannot be revised. For example, B_3 turns into B''_3, shown as:

$$
B''_3 = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{array}
\begin{array}{c} A_1 \\ \left[\begin{array}{c} \{s_5, s_5, s_5\} \\ \{s_{7.944}, s_{7.496}, s_{6.949}\} \\ \{s_{3.054}, s_{3.054}, s_{3.054}\} \\ \{s_{1.057}, s_{1.057}, s_{1.057}\} \\ \{s_{3.976}, s_{2.976}, s_{2.057}\} \end{array} \right. \end{array}
\begin{array}{c} A_2 \\ \{s_{2.060}, s_{2.504}, s_{3.051}\} \\ \{s_5, s_5, s_5\} \\ \{s_{5.947}, s_{5.057}, s_{4.015}\} \\ \{s_{5.056}, s_{4.057}, s_{2.953}\} \\ \{s_{5.053}, s_{4.529}, s_{4.057}\} \end{array}
\begin{array}{c} A_3 \\ \{s_{6.946}, s_{6.946}, s_{6.946}\} \\ \{s_{4.053}, s_{4.943}, s_{5.985}\} \\ \{s_5, s_5, s_5\} \\ \{s_{4.950}, s_{4.555}, s_{3.982}\} \\ \{s_{3.945}, s_{3.945}, s_{3.945}\} \end{array}
\begin{array}{c} A_4 \\ \{s_{8.943}, s_{8.943}, s_{8.943}\} \\ \{s_{4.944}, s_{5.943}, s_{7.047}\} \\ \{s_{5.050}, s_{5.445}, s_{6.018}\} \\ \{s_5, s_5, s_5\} \\ x_{54} \end{array}
\begin{array}{c} A_5 \\ \{s_{6.024}, s_{7.024}, s_{7.943}\} \\ \{s_{4.947}, s_{5.471}, s_{5.943}\} \\ \{s_{6.055}, s_{6.055}, s_{6.055}\} \\ x_{45} \\ \left. \{s_5, s_5, s_5\} \end{array} \right]
$$

And we calculate the granularity levels $\alpha_3, \alpha_4, \alpha_6, \alpha_{13}, \alpha_{14}, \alpha_{16}$ of $B''_3, B''_4, B''_6, B''_{13}, B''_{14}, B''_{16}$ by the PSO algorithm, respectively, which are shown in Figure 6 as follows:

Step 6: Because the incomplete HFLPRs $B''_3, B''_4, B''_6, B''_{13}, B''_{14}, B''_{16}$ have improved consistency, we estimate their missing values through Model 2.2. Meanwhile, the incomplete HFLPRs $B_1, B_2, B_5, B_7, B_8, B_9, B_{10}, B_{11}, B_{12}, B_{15}$ have acceptable original consistency, so we compute their missing linguistic terms by Model 2.1.

Step 7: we get all complete HFLPRs $\hat{B}_p(p \in \{1, 2, \dots, 16\})$. It is worthy to note that there are some virtual linguistic terms in B''_p and $\hat{B}_p$, where they are not the linguistic opinions these DMs directly offer, but the calculation results in revision and completion process, as same as the computation principle in (Wu et al., 2019). For example, B''_3 becomes $\hat{B}_3$, shown as:

$$
\hat{B}_3 = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{array}
\begin{array}{c} A_1 \\ \left[\begin{array}{c} \{s_5, s_5, s_5\} \\ \{s_{7.944}, s_{7.496}, s_{6.949}\} \\ \{s_{3.054}, s_{3.054}, s_{3.054}\} \\ \{s_{1.057}, s_{1.057}, s_{1.057}\} \\ \{s_{3.976}, s_{2.976}, s_{2.057}\} \end{array} \right. \end{array}
\begin{array}{c} A_2 \\ \{s_{2.060}, s_{2.504}, s_{3.051}\} \\ \{s_5, s_5, s_5\} \\ \{s_{5.947}, s_{5.057}, s_{4.015}\} \\ \{s_{5.056}, s_{4.057}, s_{2.953}\} \\ \{s_{5.053}, s_{4.529}, s_{4.057}\} \end{array}
\begin{array}{c} A_3 \\ \{s_{6.946}, s_{6.946}, s_{6.946}\} \\ \{s_{4.053}, s_{4.943}, s_{5.985}\} \\ \{s_5, s_5, s_5\} \\ \{s_{4.950}, s_{4.555}, s_{3.982}\} \\ \{s_{3.945}, s_{3.945}, s_{3.945}\} \end{array}
\begin{array}{c} A_4 \\ \{s_{8.943}, s_{8.943}, s_{8.943}\} \\ \{s_{4.944}, s_{5.943}, s_{7.047}\} \\ \{s_{5.050}, s_{5.445}, s_{6.018}\} \\ \{s_5, s_5, s_5\} \\ \{s_7, s_6, s_5\} \end{array}
\begin{array}{c} A_5 \\ \{s_{6.024}, s_{7.024}, s_{7.943}\} \\ \{s_{4.947}, s_{5.471}, s_{5.943}\} \\ \{s_{6.055}, s_{6.055}, s_{6.055}\} \\ \{s_3, s_4, s_5\} \\ \left. \{s_5, s_5, s_5\} \end{array} \right]
$$

Step 8: We calculate the overall weight index for each complete HFLPR $\hat{B}_p$ through Eq (21). For instance, the overall weight index of $\hat{B}_3$ is $w_3 = 0.071$.

Step 9: The final preference matrix is derived based on Eq. (22), shown as follows:

$$
B_{all} = \begin{array}{c}
\\
A_1 \\
A_2 \\
A_3 \\
A_4 \\
A_5
\end{array}
\begin{bmatrix}
\overset{A_1}{\{s_5, s_5, s_5\}} & \overset{A_2}{\{s_{4.188}, s_{4.735}, s_{5.466}\}} & \overset{A_3}{\{s_{4.418}, s_{5.137}, s_{5.65}\}} & \overset{A_4}{\{s_{4.705}, s_{5.175}, s_{5.636}\}} & \overset{A_5}{\{s_{4.466}, s_{5.084}, s_{5.769}\}} \\
\{s_{5.812}, s_{5.265}, s_{4.534}\} & \{s_5, s_5, s_5\} & \{s_{4.326}, s_{4.8}, s_{5.396}\} & \{s_{4.916}, s_{5.275}, s_{6.057}\} & \{s_{4.517}, s_{5.111}, s_{5.644}\} \\
\{s_{5.582}, s_{4.863}, s_{4.35}\} & \{s_{5.674}, s_{5.2}, s_{4.604}\} & \{s_5, s_5, s_5\} & \{s_{3.958}, s_{4.496}, s_{5.123}\} & \{s_{4.05}, s_{4.6}, s_{5.435}\} \\
\{s_{5.295}, s_{4.825}, s_{4.364}\} & \{s_{5.084}, s_{4.725}, s_{3.943}\} & \{s_{6.042}, s_{5.504}, s_{4.877}\} & \{s_5, s_5, s_5\} & \{s_{4.234}, s_{5.087}, s_{5.677}\} \\
\{s_{5.534}, s_{4.916}, s_{4.231}\} & \{s_{5.483}, s_{4.889}, s_{4.356}\} & \{s_{5.95}, s_{5.4}, s_{4.565}\} & \{s_{5.766}, s_{4.913}, s_{4.323}\} & \{s_5, s_5, s_5\}
\end{bmatrix}
$$

From the final preference matrix, we can easily obtain the ranking result of these five indicators that influence the capability of the emergency volunteer services:

$$A_2 > A_1 > A_5 > A_4 > A_3$$

Therefore, according to the significance of the influence on capability of the emergency volunteer services, the indicators are arranged from the largest to the smallest in order as follows: *Standardization*, *Specialization*, *Organization*, *sedimentation*, and *Coordination*. The result is as same as that in (Wang & Huang, 2020), which shows the reasonability of the proposed method. Specially, the proposed method, not only includes large number of DMs participated in the assessment process, but also reflects the uncertain or hesitant nature of DMs, which is more practical in reality.

3.2. Comparative analyses

Since the consistency improving process and the completion process are both important missions in LSGDM with incomplete preference information, we make comparisons based on the illustrative example and further design three simulation experiments to demonstrate general superiorities of the proposed method.

3.2.1. Superiority in consistency improving process

The majority of existing consistency improving methods including (Lin et al., 2014; Meng et al., 2016; Wu et al., 2019; Zhang et al., 2020; Zhang & Wu, 2014; Zheng et al., 2018; Zhu & Xu, 2014) are based on additive consistency or multiplicative consistency. Although the consistency of matrices can be improved, DMs are rather passive to modify their initial judgements and even the revised preference matrices are far from their initial matrices. Especially, the consistency improving method in (Zhang & Wu, 2014) is based on multiplicative consistency measurement for HFLPRs, which is similar to that in this work, so we make a comparison with it to show the advantages of the consistency improving process in this paper. Different results computed by these two methods based on the illustrative example are shown follows:

Figure 7 shows different amounts of changes in linguistic terms within each revised preference matrix before and after consistency improving by using different methods. We can see that the result calculated by the proposed consistency improving method is much smaller than that by the traditional consistency improving method (Zhang & Wu, 2014). And according to the concept of HFLPR with granularity level, the consistency of incomplete HFLPRs are improved but the semantics are not changed during the proposed consistency improving process.
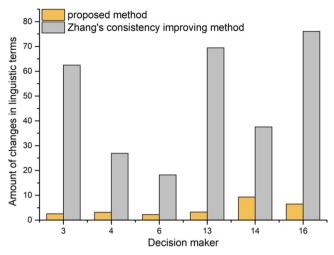
Figure 7. Amount of changes in linguistic terms between the proposed consistency improving method and the Zhang's consistency improving method.
Source: authors' research.

Because the comparison result may be affected by specific data, we execute a Monte Carla Simulation to state the general virtue of the proposed consistency improving process, where 100 groups of data are randomly generated, and each group contains 20 original incomplete HFLPRs. At first, we design an indicator to help to judge the performance:

Indicator 1:

$$I_1 = \sum_{p=1}^{P} \sum_{l=1}^{L} \sum_{j=1}^{n} \sum_{i=1}^{n} \left| b''^{p}_{ij(l)} - b^{p}_{ij(l)} \right|, \tag{23}$$

where $b^{p}_{ij(l)}$ are linguistic terms in the $p-$th original incomplete HFLPR and $b''^{p}_{ij(l)}$ are linguistic terms in the $p-$th revised incomplete HFLPR. $i, j \in \{1, 2, \ldots, n\}$, $l \in \{1, 2, \ldots, L\}$, $p \in \{1, 2, \ldots, P\}$, and P is the number of incomplete HFLPR which can be revised. Indicator 1 measures the total amount of changes in linguistic terms within a group of incomplete HFLPRs before and after consistency improving. The smaller the value of I_1 is, the better the consistency improving method is.

Different results calculated by the proposed consistency improving method and the consistency improving method (Zhang & Wu, 2014) are shown as follows:

From Figure 8 we know that, the total amount of changes of linguistic terms within each group calculated by the proposed consistency improving process is much smaller than that in the consistency improving process (Zhang & Wu, 2014). So the simulation experiment demonstrates the former method is superior to the latter one in the case of improving preference matrix without changing semantics.

3.2.2. Superiority in completion process

We all know that classical techniques based on consistency properties to estimate the missing values in preference matrix can be divided in two different approaches: the iterative approach (Xu et al., 2018) and the optimization approach (Song & Li, 2019).
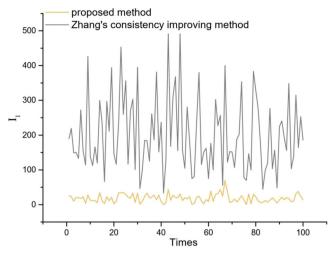
Figure 8. Different results I_1 of the proposed consistency improving method and the Zhang's consistency improving method.
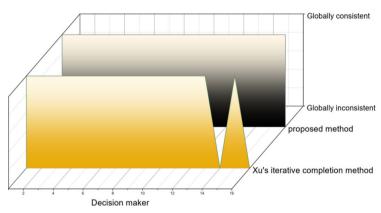Source: authors' research.

Figure 9. Whether the complete HFLPRs are globally consistent by using the proposed completion method or the Xu's iterative completion method.
Source: authors' research.

The former method leads to a locally perfect consistency in a preference matrix, and the latter one is to find the global optimization solution for forming a perfectly consistent preference matrix. They are different so we make comparisons with these two methods and the proposed completion process, respectively.

1) Comparison with the iterative approach

Firstly, we make a comparison based on the data from illustrative example, and the result is shown in Figure 9.

Figure 9 reflects different status whether each complete preference matrix is globally consistent or not. It is easy to see that by using the proposed completion method, all complete preference matrices are globally consistent, whereas two complete preference matrices calculated by iterative completion approach (Xu et al., 2018) are not.
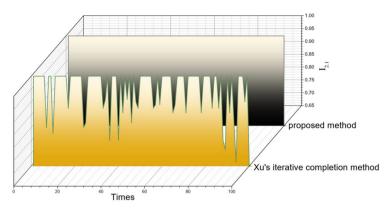
Figure 10. Different results $I_{2.1}$ of the proposed completion method and the Xu's iterative method.
Source: authors' research.

Then, we conduct a simulation experiment based on the generated 100 groups of data. Indicator 2.1 is designed to compare their different results, shown as:

Indicator 2.1:

$$I_{2.1} = N/M \qquad (24)$$

where N is the number of complete preference matrices in a group whose consistency index value is equal or more than the threshold value, and M is the number of incomplete preference matrices in a group. If the value of $I_{2.1}$ reaches 1, then all complete preference matrices are globally consistent, and vice versa. The higher the value of $I_{2.1}$ is, the better the completion process is.

Different results computed by the proposed completion method and the iterative method (Xu et al., 2018) are shown as:

According to Figure 10, we find that almost all complete preference matrices obtained by the proposed completion method are globally consistent, while just a few of complete preference matrices obtained by the iterative method (Xu et al., 2018) are globally consistent. It reflects that the former method is superior to the latter one in terms of ensuring global consistency of preference matrix.

2) Comparison with the optimization approach

Comparison results based on the data from illustrative example are shown as follows:

Figure 11 demonstrates the difference of consistency similarity between original preference matrix and complete preference matrix by utilizing two methods. We find that the result of the proposed completion method is much smaller than that of the optimization completion approach (Song & Li, 2019), which means that the process of estimating the missing elements in the proposed completion method, is based on the inherent quality of information provided by a certain DM instead of perfect consistency.

In addition, we design Indicator 2.2 for further simulation, which is used to compare the result of the proposed completion process with that of the optimization approach (Song & Li, 2019), shown as:
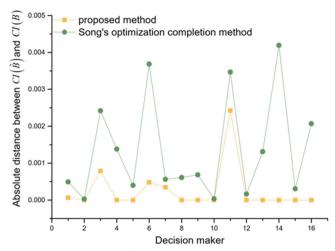
Figure 11. Absolute distance between $CI(\hat{B})$ and $CI(B)$ by using the proposed completion method and the Song's optimization completion method.
Source: authors' research.

Indicator 2.2:

$$I_{2.2} = \sum_{p=1}^{P} \left| CI^p(\hat{B}) - CI^p(B) \right|, \tag{25}$$

where $CI^p(B)$ is the original consistency index value of the $p-$th original preference matrix, and $CI^p(\hat{B})$ is the consistency index value of the $p-$th complete preference matrix. P is the number of complete HFLPRs in a group. The value of $I_{2.2}$ reflects the total amount of changes between the consistency of original preference matrices and that of complete preference matrices in each group. The smaller the value of $I_{2.2}$ is, the better the completion method is.

Based on the indicator $I_{2.2}$, different results are given below:

From Figure 12, it is easy to find that the value of $I_{2.2}$ calculated by the proposed completion method is always lower than that of the optimization method (Song & Li, 2019). Thus, no matter the comparison result based on illustrative example or simulation experiment, it shows the results obtained by the former method are closer to reality than the latter one, considering the inherent evaluation quality and personality of DMs during the decision-making process.

4. Discussions

A method based on Granular computing and optimization model provides a novel perspective for dealing with incomplete HFLPRs and solving LSGDM problems such as emergency volunteer service capability assessment. In the following, we make further discussions in terms of theoretical aspect and application prospect.
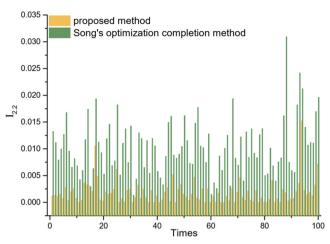
Figure 12. Different results $I_{2.2}$ of the proposed completion method and the Song's optimization completion method.
Source: authors' research.

4.1. Theoretical aspect

The proposed method handles LSGDM problems under the perspective of Granular computing. The characteristics of the method are reflected in two aspects. One is that the multiplicative consistency of incomplete HFLPR is improved without changing semantics, which is because the preference matrix is regarded as granular rather than numeric through bringing some flexibility to linguistic terms (Pedrycz & Song, 2011). By incorporating Granular computing, we can not only improve the consistency of HFLPRs without changing the semantics giving by DMs, but also allocate the consistent preference matrices with different weights. The other is that the original consistency of incomplete HFLPR is considered and multiple types of incomplete HFLPRs are completed in different ways through optimization models during the completion process.

Compared with other typical LSGDM methods (Xu et al., 2016, 2019), we compute overall weight of a DM from three different aspects: the influence of granularity level, the number of known elements, and the final multiplicative consistency. So the weight of DM is more reasonable and comprehensive than that in traditional LSGDM methods (Xu et al., 2016, 2019), which lays an important theoretical foundation for gathering opinions of DMs in LSGDM. To sum up, the proposed method has the following advantages: a) improving preference matrix without changing semantics, b) ensuring global consistency of complete preference matrix, c) considering the inherent information quality given by DMs, and d) taking reasonable weights of DMs into consideration during the process of acquiring LSGDM result.

4.2. Application prospect

The illustrative example of evaluating the capability of the emergency volunteer service, not only takes concerns of hot topic in society, but also clarifies the practicability and superiority of the proposed method. In the same application field, Wang and

Huang (2020) adopted analytical hierarchy process to assess the capability of the emergency volunteer service. However, three differences in the evaluation process show the advantages of the proposed method. Firstly, DMs are allowed to be hesitant or uncertain when provide linguistic preference information under complex and urgent circumstance, so that incomplete HFLPRs are allowed for further decision-making, which is to be more realistic; Second, twenty DMs instead of five DMs are invited to participate in online decision-making process, making the final ranking results of assessment indicators in emergency volunteer service to be more authoritative and comprehensive; At last, after the linguistic preference information has been accurately processed, we can obtain LSGDM results quickly rather than be involved in multiple consensus reaching process.

Our application can be extended in several ways. First of all, five indicators can be subdivided into several secondary indicators, which is convenient for detailed and accurate evaluation. For example, the specialization of organization can be examined from the aspects of the promotion and training of professional knowledge, the actual combat skills, the equipment for professional tools. Moreover, the improvement of the volunteering service organization after assessment, such as management, training, extension, etc., also need further study. Last but not least, the proposed method can be transferred to solve problems in other relevant application fields, e.g., assessing psychological situation of COVID-19 infected persons, choosing the optimal way to protect the personal information while defending against COVID-19. We can then find some interesting results from the LSGDM process.

5. Conclusions

This paper mainly proposes a novel completion method based on Granular computing and optimization model for solving LSGDM problems with multi-type incomplete HFLPRs, considering the original consistency of incomplete HFLPRs and improving the consistency of preference matrices without changing semantics during the completion process. Then, apply the proposed method to assess the capability of the emergency volunteer services. The main contributions are highlighted as follows:

   i. Introduce the concept of original consistency of incomplete HFLPR, and then propose a consistency improving method based on Granular computing to check and improve the consistency of incomplete HFLPRs without changing the semantics given by DMs. Meanwhile, the granularity level of incomplete HFLPR can be obtained for further computation.
  ii. Construct a novel completion method based on various optimization models to estimate missing values within multi-type incomplete HFLPRs. The original consistency of an incomplete HFLPR is taken into consideration and more valid assessment information given by DMs can be retained.
 iii. The illustrative example about evaluating the capability of the emergency volunteer services and simulation experiments demonstrate the rationality and superiorities of the proposed method: 1) semantics of DMs are not changed during consistency improving process, 2) completion process does not significantly alter

inherent information quality levels of DMs, 3) complete HFLPRs are globally consistent, and 4) the final LSGDM result is obtained by fusing complete HFLPRs with different weights.

However, because none of models that can be regarded as the best, the proposed method also has its limitations. As far as we know, In the research area of LSGDM, consensus reaching process is another hot and important issue attracting a large number of researchers. Moreover, the linguistic preference matrix can be allowed to be granular rather than numeric by bringing flexibility to linguistic terms, how about decision-making matrix with the same qualitative assessment problems? In the future, we will focus on solving large-scale group consensus and multi-attribute decision-making problems by using Granular computing or some machine learning technologies.

Disclosure statement

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