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INELASTIC CROSS SECTIONS IN TWO-STATE APPROXIMATION OF SUB- AND ABOVE-BARRIER FUSION OF HEAVY IONS

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Coupled channel equations for barrier penetration problems have been separated using the theory of coupled differential equations. The corresponding proper phase shifts have been obtained numerically using JWB approximation. The effect of magnitude of coupling strength was studied. The Gaussian form for the coupling and diagonal potentials in three dimensional space was used to calculate the inelastic cross section for the system ^{58}Ni–^{58}Ni in a two-state approximation.

1. Introduction

The reactions between complex nuclei at low energies are often classified into three categories: quasielastic (for very soft collisions), deeply-inelastic and fusion reactions (for the hard collisions). Theoretical models for description of the reaction types involve extensive use of conservative ion-ion potential in conjunction with the dissipation of energy, i.e. the transformation of the translational energy into intrinsic energies of the nuclei. Often it is difficult to identify whether the reaction of interest is mainly controlled by the ion-ion potential or by the dissipation of energy. However, for the elastic and complete fusion, the ion-ion potential has been granted the central position in most theoretical approaches [1].

The investigation of the fusion of two heavy nuclei below the Coulomb barrier is interesting because it can have important implications concerning the potential energy surface governing the process and it is also important for astrophysics. The big surprise in this field was that the fusion cross sections below barrier are much higher than expected [2]. In particular, for fusion at near barrier energies, the incidence of a particle on a one dimensional potential barrier is the basic aspect of most models [3]. However, a number of recent experiments [4] have clearly shown that the use of a local one dimensional potential is quite inadequate for the understanding of sub-barrier fusion.

A very good review concerning the interpretation of sub-barrier fusion has been given by Dasso et al. [2]. They described the approach based on the well-known channel coupling procedure, formulated in the frame of two coupled differential equations for the s state in one dimensional space. The purpose of the present paper is to extend this approach to three dimensions and to enlarge the investigation to states with angular momentum $l \neq 0$ [5].

2. *Coupled channel formulation*

The coupled channel formalism for direct reaction processes is given in detail in Ref. 6. The total wave function Ψ is expanded in terms of channel states ϕ_α and the radial functions $G_\alpha(r)$ and substituted in the Schrödinger equation. One obtains

$$\frac{\mathrm{d}^2 G_\alpha}{\mathrm{d}r^2} + \frac{2\mu_\alpha}{\hbar^2}\left(E_\alpha - V_\alpha^{ef}(r)\right) G_\alpha = \frac{2\mu_\alpha}{\hbar^2} \sum_{\beta \neq \alpha} V_{\beta\alpha}^{cpl}(r) G_\beta \tag{1}$$

with

$$V_\alpha^{ef}(r) = \frac{\hbar^2}{2\mu_\alpha} \frac{l_\alpha(l_\alpha + 1)}{r^2} + < \varphi_\alpha \mid V \mid \varphi'_\alpha > \tag{2}$$

$$V_{\alpha\beta}^{cpl}(r) = < \varphi_\alpha \mid V \mid \varphi_\beta > \quad \alpha \neq \beta. \tag{3}$$

V is the interaction energy while, for a given channel α, μ_α is the reduced mass, l_α is the angular momentum and E_α is the relative energy:

$$E_\alpha = E + Q_\alpha \tag{4}$$

where Q_α is the reaction Q-value.

The solutions of the coupled equations (1) are usually obtained by requiring the boundary condition $G_\alpha \to 0$ at the origin and matching to the asymptotic form of ingoing wave of unit norm in the entrance channel and of outgoing radial waves in other channels. The coefficients of the outgoing waves then determine the various fusion cross sections.

3. Two-state approximation

We assume equal coupling to the ground state V^{cpl} for all channels, and we neglect other cross-channel couplings. Then the coupled equations (1) can be replaced by the following system:

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}r^2} + f_1(r)\right) G_1(r) = B_{12}(r) G_2(r) \tag{5}$$

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}r^2} + f_2(r)\right) G_2(r) = B_{12}(r) G_1(r) \tag{6}$$

where

$$f_\alpha(r) = \frac{2\mu_\alpha E_\alpha}{\hbar^2} - \frac{l_\alpha(l_\alpha+1)}{r^2} - \frac{2\mu_\alpha}{\hbar^2} V_{\alpha\alpha}(r) \tag{7}$$

$$B_{\alpha\beta}(r) = \frac{2\mu_\alpha}{\hbar^2} V^{cpl}_{\alpha\beta}(r), \quad \alpha \neq \beta. \tag{8}$$

In this paper, we shall start from the results obtained by a separation of the equations (7) to show that simplifications of this problems is, in principle, possible with a direct determination of various cross sections. Three cases can be considered:

(a) the non-resonance case when $E_1 \neq E_2$, $V_{11} \neq V_{22}$;

(b) the near-resonance case with $E_1 = E_2$, $V_{11} \neq V_{22}$, $l_\alpha = l_\beta$;

(c) the exact resonance case $E_1 = E_2$, $V_{11} = V_{22}$.

The present study is restricted to the last two cases for which the system of coupled equations (5) and (6) can be completely separated. The scattering problem is solved via two separated equations which are obtained using the following transformation [8]:

$$X(a) = \begin{bmatrix} 1-a & 1+a \\ -(1+a) & 1-a \end{bmatrix} \tag{9}$$

where the quantity a is the root of the equation:

$$\left(f_2(r) - f_1(r)\right) a^2 - 4B_{12}(r) a - \left(f_2(r) - f_1(r)\right) = 0. \tag{10}$$

a is also defined by:

$$a = 2\gamma \pm \sqrt{1 + 4\gamma^2} \tag{11}$$

with

$$\gamma = \frac{B_{12}(r)}{f_2(r) - f_1(r)}. \tag{12}$$

The separated equations of the system (5), (6) are

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}r^2} + F^{\pm}(r)\right) Z^{\pm} = 0 \tag{13}$$

where

$$F^{\pm}(r) = \frac{1}{2}(f_1 + f_2) \pm \frac{1}{2}\sqrt{(f_1 - f_2)^2 + 4B_{12}^2}. \tag{14}$$

The functions G_1 and G_2 in Eqs. (5) and (6) may then be recovered by the inverse transformation :

$$\begin{bmatrix} G_1 \\ G_2 \end{bmatrix} = X^{-1} \begin{bmatrix} Z^+ \\ Z^- \end{bmatrix}. \tag{15}$$

We assume that at large distance the potentials $V_{\alpha\alpha}(r)$ and $V_{\alpha\beta}^{cpl}(r)$ are generally expected to become negligible compared to the centrifugal term, so that the asymptotic form of $Z^{\pm}$ will be :

$$Z^{\pm} \sim \sin\left(kr_\alpha - \frac{l_\alpha \pi}{2} + \eta_{l_\alpha}^{\pm}\right) \tag{16}$$

where $\eta_{l\alpha}^{\pm}$ are the proper phase shifts and k_α is the asymptotic wave number

$$k_\alpha^2 = \frac{2\mu_\alpha E_\alpha}{\hbar^2}. \tag{17}$$

The JWB approximation of the uncoupled integral equations determining the proper phase shift gives [11]:

$$\eta_{l_\alpha}^{\pm} = (2l+1)_\alpha \frac{\pi}{4} - k_\alpha r_0^{\pm} + \int\limits_{r_0^{\pm}}^{\infty} \left[F_1^{\pm^{1/2}} - k_\alpha \right] \mathrm{d}r \tag{18}$$

where $F_1^{\pm}$ differs from $F^{\pm}$ only in the replacement of $l_\alpha(l_\alpha + 1)$ by $(l_\alpha + 1/2)^2$, and $r_0^{\pm}$ are the zeros of $F_1^{\pm}$.

In the matrix notation, the asymptotic forms are

$$Z \sim e^{-\mathrm{i}k_\alpha r} I - e^{\mathrm{i}k_\alpha r} S \tag{19}$$

where $S = \exp(-2\mathrm{i}\eta_{l\alpha}^{\pm})$. Using the inverse transformation the asymptotic forms of the functions $G(r)$ are :

$$G \approx e^{-\mathrm{i}k_\alpha r} I - e^{\mathrm{i}k_\alpha r} S', \tag{20}$$

where

$$S' = XSX^{+}. \tag{21}$$

With the transmission matrix T defined by

$$T^{\pm} = 1 - \exp(2\mathrm{i}\eta_l^{\pm}) \tag{22}$$

we obtain :

$$T_{12} = T_{21} = \frac{1-a^2}{1+a^2}\left[\frac{1}{2}(T^+ - T^-)\right] \tag{23}$$

$$T_{11} = \frac{1}{2}(T^+ + T^-) - \frac{a}{1+a^2}(T^+ - T^-) \tag{24}$$

$$T_{22} = \frac{1}{2}(T^+ + T^-) + \frac{a}{1+a^2}(T^+ - T^-). \tag{25}$$

These results lead directly to the elastic and partial inelastic cross sections:

$$Q_{l_\alpha}^{12} = \frac{\pi}{k_\alpha^2}(2l_\alpha + 1)\left[\frac{1-a^2}{1+a^2}\right]^2 \sin^2(\eta_{l_\alpha}^+ - \eta_{l_\alpha}^-) \tag{26}$$

$$Q_{l_\alpha}^{11} = \frac{2\pi}{k_\alpha^2}(2l_\alpha + 1)\left[\frac{(1-a)^2}{1+a^2}\sin^2\eta_{l_\alpha}^+ + \frac{(1+a)^2}{1+a^2}\sin^2\eta_{l_\alpha}^-\right] - Q_{l_\alpha}^{12}. \tag{27}$$

4. *Model calculations*

In this Section we use the above model to estimate the magnitude of effects that are expected in heavy-ion fusion reactions. We assume $l_\alpha = l$, and $V_{\alpha\alpha}$ and $B_{\alpha\beta}$ of a Gaussian form:

$$V_{\alpha\alpha} = V_\alpha \exp(-r^2/2\sigma^2), \quad \alpha = 1,\ 2 \tag{28}$$

$$B_{\alpha\beta} = F\exp(-r^2/2\sigma^2), \quad \alpha \neq \beta. \tag{29}$$

In the following we shall use units of amu, MeV and fm for mass, energy and distance, respectively.

The calculation procedure is to evaluate first the roots $r_0^{\pm}$ of $F_i^{\pm}$ by using the fixed-point iteration method, then to calculate the proper phase shifts, which are defined in Eq. (18) by using the Simpson integration method, and finally to obtain the cross sections from Eqs. (26) and (27).

4.1. *Effects of chanell coupling*

First we chosse a reference set of parameters: $\mu = 1$, $V_1 = V_2 = 0$, $E_1 = 10$ and $Q = 0$ which characterize the exact resonance case. From Eq. (10), we deduce that $a = 0$ and Eqs. (5) and (6) can always be separated. The corresponding result will merely serve for comparision to the results derived in other cases.

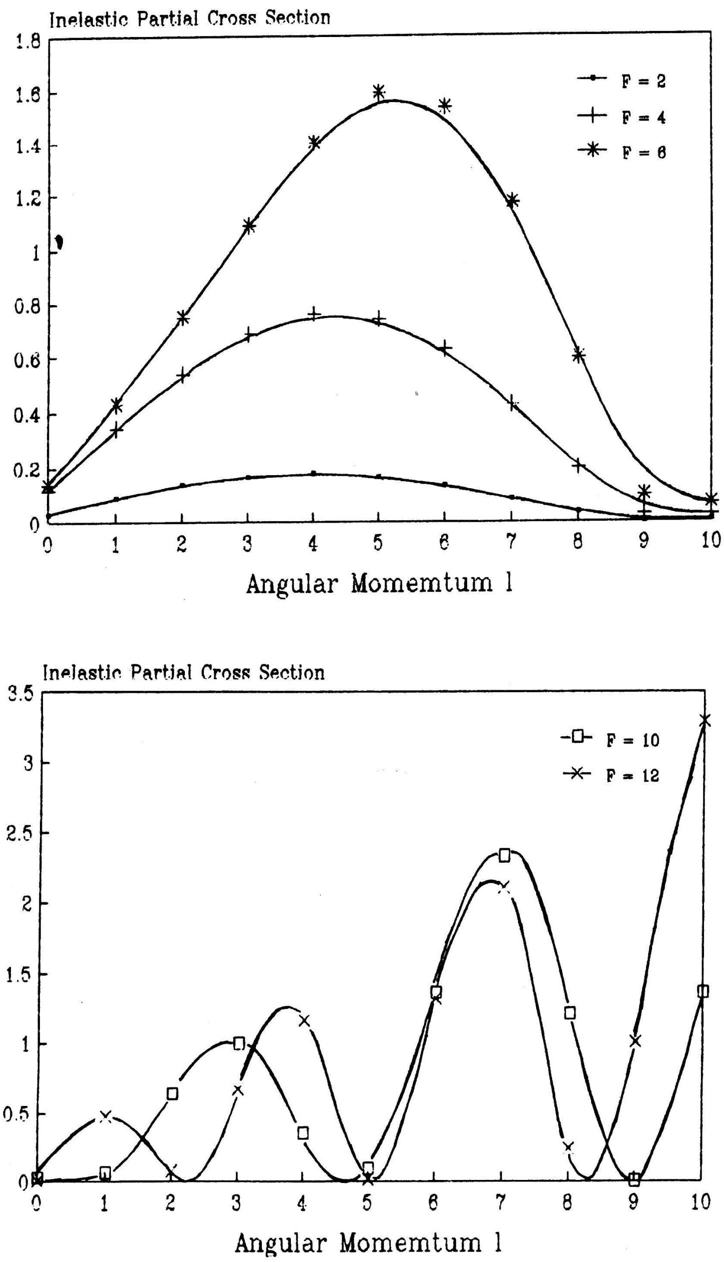

Fig. 1. Partial inelastic cross section Q_l^{12} for different values of the coupling strength F in the exact resonance case. (a) F = 2, 4 and 6; (b) F = 10 and 12. The parameters are: $\mu = 1$, $V_1 = V_2 = 0$, $E_1 = 10$, $\sigma = 6$.

The partial inelastic cross sections of sub- and above-barrier reactions for F=2, 4, 6, 10, 12, are plotted in Fig. 1. It has been shown [1-10] that for exact resonance collisions, as the strenght of coupling is increased from weak coupling, the transfer probability increases rapidly, then reaches a saturation stage, and finally behaves in an oscillatory manner. This result is clearly illustrated in Fig. 2 where the partial inelastic cross section Q_l^{12} is plotted versus F for l=0, 5, 10. The oscillatory behaviour is more pronounced for larger values of l.

Next we fix the entrance channel potential V_1 and the coupling strength F and introduce different potentials V_2 in the excited channel (near-resonance case). In this case the function

$$\gamma = \frac{F}{V_1 - V_2}$$

is independent of r and the quantity a in Eq. (10) is also independent of r. Therefore Eqs. (5) and (6) may be separated. In the case $V_1 = 10$, $V_2 = 8$, 12 the quantity $C = [(1 - a^2)/(1 + a^2)]^2 = 0.8$. If $V_1 = V_2$, we obtain $C = 1$. The partial inelastic cross sections for F= 2, versus the angular momentum l are shown in Fig. 3. One can see the oscillatory behaviour of the partial inelastic cross sections as l increases.

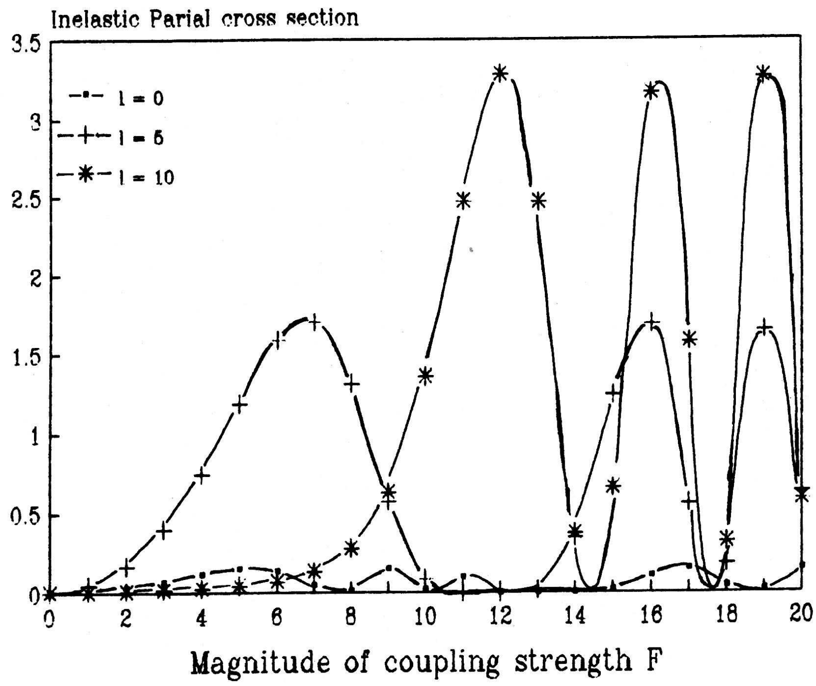

Fig. 2. Partial inelastic cross section Q_l^{12} for $l = 0$, 5, 10 versus the magnitude of the coupling strength F in the exact resonance case. The parameters are: $\mu = 1$, $V_1 = V_2 = 0$, $E_1 = 10$, $\sigma = 6$.

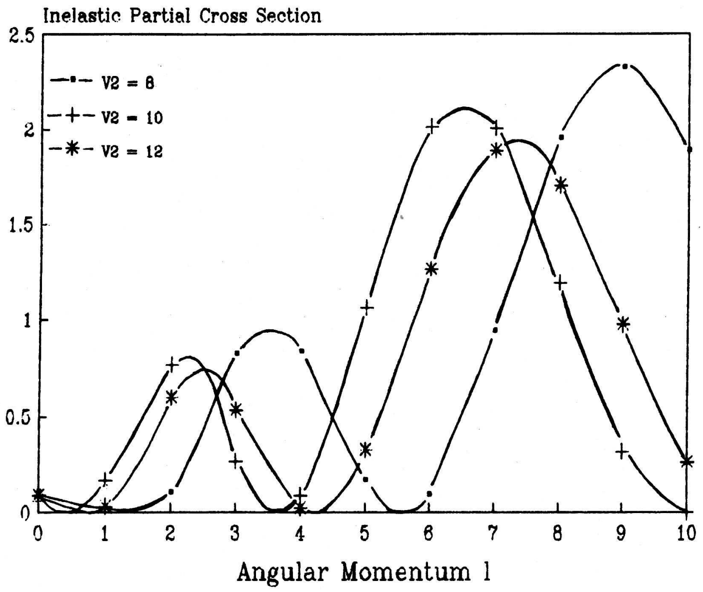

Fig. 3. Inelastic partial cross section Q_l^{12} for different values of the barrier height V_2 versus the angular momentum l. The parameters are: $\mu = 1$, $V_1 = 10$, $E_1 = 10$, $V_2 = 8$, 10, 12, $\sigma = 6$.

4.2. Estimates for sub-barrier fusion reactions

In order to illustrate the present method with our schematic model, we consider a set of parameters designed to model the collision of two Ni isotopes at energies around the barrier region of 100 MeV. Let us further assume that the Q value corresponding to these two channel is almost zero.

Table 1 shows typical values of the parameters r_b, V_b and l_{cut} for these system ^{58}Ni+^{58}Ni. The reaction has been studied experimentally [10].

TABLE 1.
Assumed values of parameters in the study of ^{58}Ni+^{58}Ni collisions.

Barrier radius r_b (fm)	10.8
Barrier height V_b (MeV)	98.0
Number of contributing partial waves l_{cut}	5

In Fig. 4 we display the inelastic cross section versus the angular momentum l in the exact resonance case (Q=0) for F=2, 4, 6. It can be seen how interactions of few

MeV can change the behaviour of the inelastic cross section function. Interactions of such strength inside the barrier are not unrealistic.

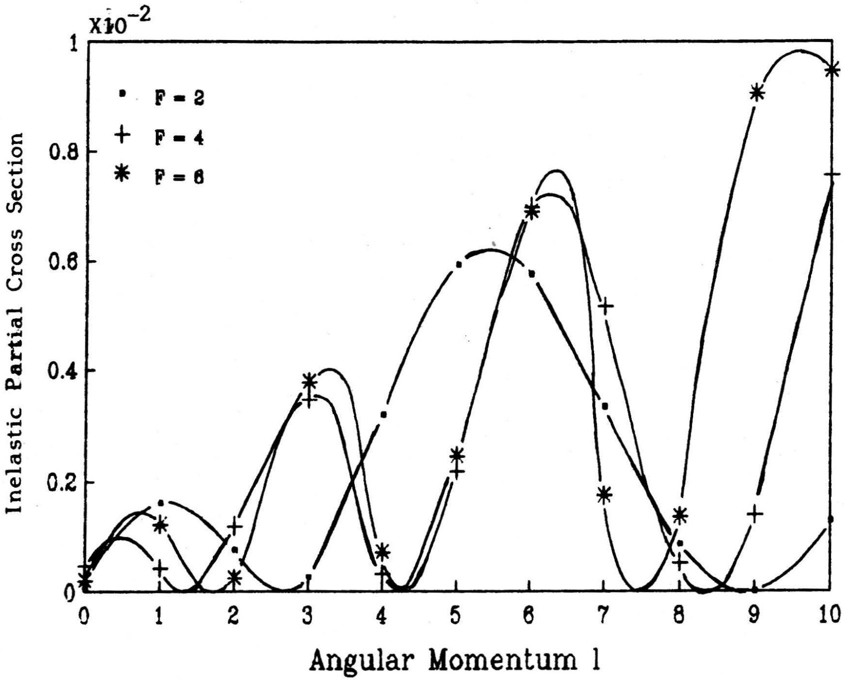

Fig. 4. Effect of the magnitude of coupling strength F on the inelastic cross section in the exact resonance. The parameters are simulated to the s-wave potential for ^{58}Ni -^{58}Ni*:* $\mu = 29$, $V_1 = V_2 = 100$, $F = 2,\ 4,\ 6$, $\sigma = 3$, $E_1 = 100$, $Q = 0$.

The results of calculations shown in Fig. 5 were obtained by fixing the coupling strength F ($F = 2$) and by varying barrier V_2 of the excited channel. The results show the same general features as those shown in Fig. 3. The enhancement and attenuation of the inelastic cross section in two-state model is really a general effect and may affect the magnitude of the conventional resonances or the threshold effect in fusion.

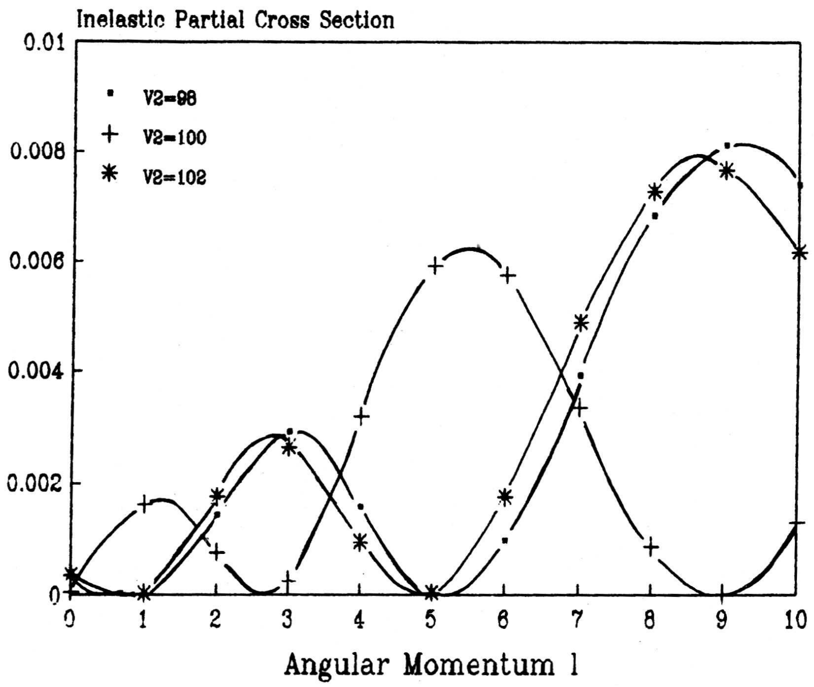

Fig. 5. Effect of the barrier height variation on inelastic partial cross section Q_l^{12} keeping the entrance channel barrier fixed. The parameters are: $\mu = 29$, $V_1 = 100$, $V_2 = 98$, 100, 102, $F = 2$, $\sigma = 3$, $E_1 = 100$.

Figure 6. shows the results of calculation where the number of contributing partial waves was fixed ($l_{cut} = 5$). The value of $\sigma = 3$ was determined by fitting a typical Ni-Ni s-wave potential barrier which has a height close to 100 MeV [10]. We noted that between the two values of energies $E_1 = V_1 \pm F$ the inelastic cross section Q_{tot}^{12} $(V_1 > V_2)$, while the opposite is true outside this energy range. This agrees with the intuitive idea that the system prefers the tunelling to the channel which presents the lowest barrier. This result is similar to that obtained by Dasso et al. [10] for the total probability for transmission. Fig. 6 was plotted in a logarithmic scale to illustrate the enhancement of inelastic cross section below the barrier as opposed to the reduction above.

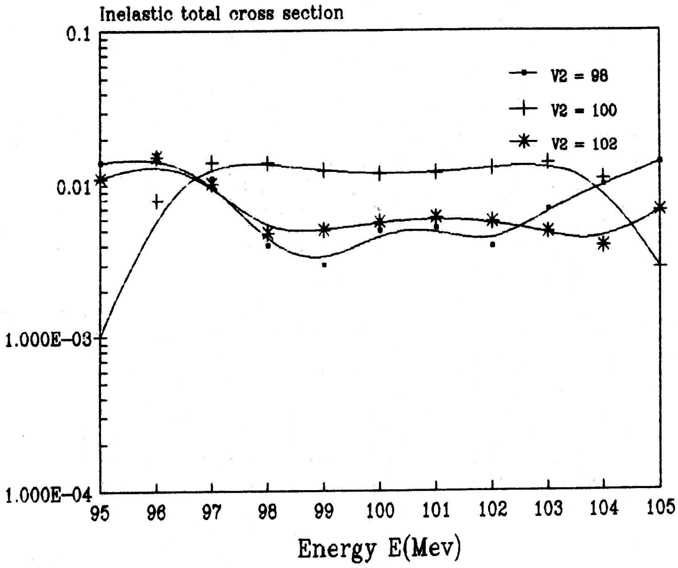

Fig. 6. Effect of the barrier height variation on the total inelastic cross section $\sum_l Q_l^{12}$ ($l_{cut} = 5$) keeping the entrance channel barrier fixed in the exact resonance case. The parameters are: $\mu = 29$, $V_1 = 100$, $V_2 = 98$, 100, 102, $F = 2$, $\sigma = 3$.

5. *Conclusions*

In this work we consider a coupled-channel framework for computing inelastic cross section for heavy ions.

For the Gaussian forms of the diagonal and coupling potentials, we have shown the effect of the magnitude of coupling strength for heavy ions. The treatment was extended to the three dimensional case that allows the investigation to states with $l = 0$.

The above description of the "channel coupling procedure" is by necessity very schematic. Only two levels were considered and Q was assumed to be zero. However, it still presents the essential features of the sub-barrier fusion. For more details we have calculated the sub-barrier inelastic cross section for the system ^{58}Ni+^{58}Ni by using the schematic model calculations.

The inelastic cross section turns out to be the average of two uncoupled equations, Eqs. (5) and (6). It follows that the existence of two channels, coupled to each other with a strength F, can be expressed in terms of the initial configurations of the separated potential barrier. The first one corresponds to an increase and the second one to a decrease of F.

More generally speaking, the results obtained in this work are useful in the sense that they can set up a larger base to approach the problem of coupled differential equations, and to provide a method of avoiding the burden of computational work.

The method is described in the frame of exact and non-exact resonance cases. However, extension to a non resonance case is possible at the cost of more complications, and is in progress. It may be noted that the two-state approximation is flexible enough to be adapted to many cases [12].

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NEELASTIČNI UDARNI PRESJECI U APROKSIMACIJI DVAJU STANJA PRI FUZIJI TEŠKIH IONA ISPOD I IZNAD BARIJERE

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Jednadžbe vezanih kanala za penetraciju barijere razdvojene su upotrebom teorije vezanih diferencijalnih jednadžbi, a fazni pomaci brojčano izvrijednjeni u JWBK aproksimaciji. Studiran je efekt jačine vezanja. Dijagonalni potencijali i potencijal vezanja u tri dimenzije reprezentirani su Gaussovom formom u računu neelastičnog udarnog presjeka za ^{58}Ni–^{58}Ni sistem u aproksimaciji dva stanja.