Author’s earlier investigations on gravitational maser model for the QSO suggest that some quasars are feasibly clusters of black holes of masses $10^{-1} M_\odot$. These black holes are considered to be the compact subunits in the Hoyle-Fowler cluster model for the quasars and the large quasar redshifts are interpreted as significantly intrinsic. Following Bell and Fort the quasars are considered as intermediate distance objects. Utilizing their concept of redshift periodicity, the quasars are conceived as passing through stages of definite redshifts, decreasing in steps of the redshift wavelength. Each stage contains a number of similar black holes which reduce to half their number in the succeeding stages through the appropriate coalescence in pairs. We consider spherical accretion (swallowing of dust) by the constituent black holes and look at the quasar radio luminosity as it should evolve with the black hole’s coalescence process and find it in accord with the Bell and Fort conjecture. Assuming reasonable correlation between the radio and optical emission of

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a quasar, the Bell-Fort absolute magnitude \( M_v = -20.4 + 1.67 z_x \) is reasonably justified. The apparent optical magnitude of a quasar should spread over 5 units in agreement with the observation. The high redshift quasars, if assigned as primitive, should be at comparatively closer distances. This is consistent with the theory of Bell and Fort, and also with the analysis by Hoyle, Burbidge and Sargent. This leads to the assessment that quasars are locally born (Arp and Terrell) with very high (intrinsic) redshift and fly apart as they continue de-reddening. Subsequently, we estimate quasar masses, and interpret the collapsed old quasars as massive holes of galactic nuclei (radio galaxies).

1. Introduction

In earlier investigations [1,2] we considered the possibility that some quasars are gravitational masers, i.e. masers pumped or powered by gravitational radiation [3]. Some quasi-stellar objects (QSO) are likely clusters of black holes. By spherical accretion, black holes of masses \( 10^{-1} M_\odot \) may power the gravitational maser [2] which would be a copious source of primary photons at frequencies \( 10^4 - 10^5 \) Hz. The process may be boosted and diffused into the frequency range \( 10^8 - 10^{10} \) Hz through the Colgate’s mechanism (References 4,5; other Refs. in [1]). Indeed, as we look the relevant photon density curve (Fig. 1 in Ref. 5) shifted 3 to 4 logarithmic units of frequency towards the origin of the figure, we find that the secondary spectrum may duly span the QSO radio emission band of \( 10^8 - 10^{10} \) Hz with a spectral index \((-1)\) what is rather close to that of the steep-spectrum class of the radio quasars.

These black holes are considered as the compact subunits of the Hoyle-Fowler cluster model [6] for the quasar. (Holes are obviously accreting and they may be small masers as well). Zapolsky [7] and others criticize the model on the basis of large quasar redshifts. We interpret the large quasar redshifts as significantly intrinsic or gravitational in origin. This meets a criterion required by the Bell-Fort [8] idea.

The early investigations on redshift periodicity of quasars by Cowan, Burbidge and others [9-13] lead to the concept of “quantized” redshift of Bell and Fort [8]. We follow their idea of quasars as “intermediate distance” objects. In accordance with their concept of “quantized” redshift and magnitude, it is assumed that a quasar will pass through some 30 stages during its passage through the quasistellar phase. The number of stages is obtained by dividing the expected range of value of about 3, i.e. the maximum value of the quasar intrinsic redshift \( z_x \) (see Table 1 in Ref. 8) by the redshift “quantum” 0.1 as assumed by Bell and Fort [8]. In each of these quasar stages a definite redshift should persist, and the redshift should jump in value by 0.1 from one stage to the next. It is further assumed that at any stage the cluster remains formed of similar black holes, and their number decreases to one-half in each stage by coalescence in pairs. The black hole masses would obviously increase.

It will be shown that these assumptions justify the Bell-Fort results and also lead to other interesting observations.
2. Luminosity

In a cluster, the black holes may often collide and coalesce in pairs. When two black holes, each of a mass $M/2$, coalesce, the mass of the final black hole (all the black holes are of the Schwarzschild type) is (see Ref. 14, p. 886)

$$M_{\text{final}} \geq \frac{M}{\sqrt{2}}.$$  

(1)

Also, the upper bound of the amount of energy emitted in the collision is

$$E_{\text{radiated}} \leq M - \frac{M}{\sqrt{2}}.$$  

(2)

Some of the vast amount of radiation liberated during the black hole coalescence is assumed in the form of gravitational waves. Main part of the emission is thermal, optical, X-rays, etc. and ejection of particles. In Ref. 1 we proposed (see also Ref. 3) that the radio luminosity of a quasar originates in the gravitational maser powered by the gravitational radiation through the steady-state accretion (swallowing of dusts; see Refs. 14,15) by the black holes in the cluster. The accretion rate for a single hole should be proportional to the surface area of the Schwarzschild sphere of the black hole, i.e. to

$$\sim (\text{mass of the black hole})^2.$$  

(3)

Therefore, for the emission power $P$ from an initial black hole of the mass $M_{\text{initial}} = M/2$ we obtain

$$P(\text{one hole}) \sim \left( \frac{M}{2} \right)^2.$$  

(4)

For a cluster of $N$ such holes, the power of emissions would be

$$P \sim N \left( \frac{M}{2} \right)^2.$$  

(5)

Therefore, the quasar luminosity $L_\alpha$ (the initial luminosity) should obey the relation

$$L_\alpha \sim P.$$  

(6)

i.e. (using Eqs. (5) and (6))

$$L_\alpha \sim N \left( \frac{M}{2} \right)^2.$$  

(7)

According to the assumption in Sec. 1, the number of black holes in the cluster reduces to half of their initial number (i.e. to $N/2$) through pair coalescence when
passing from the initial stage and further so in the succeeding stages of the quasar. When the pair coalescence follows the critical mass value, i.e., when

\[ M_{\text{final}} = (M/\sqrt{2}) = \sqrt{2}(M/2) = \sqrt{2}M_{\text{initial}}, \tag{8} \]

we have in the succeeding stages the cluster of black holes with masses \( \sqrt{2}(M/2) \).

Thus by replacing \( N \) by \( N/2 \) and \( M/2 \) by \( \sqrt{2}(M/2) \) in Eq. (7), we get the quasar luminosity at the next stages as

\[ L \sim \left( \frac{N}{2} \right)^2 \left( \frac{M}{2} \right)^2 = N \left( \frac{M}{2} \right)^2. \tag{9} \]

Assuming the relevant constants in Eqs. (7) and (9) unchanged, one obtains

\[ L = L_0, \tag{10} \]

i.e., no change of the quasar luminosity should occur when the coalescence follows the critical mass value.

When the masses exceed the critical value, the luminosity will change and should be increasing. We note, since the resulting black hole mass can not be less than the critical value, decrease of the intrinsic luminosity is not expected. As the young and the primitive quasar would evolve, the intrinsic luminosity would be increased in accordance with the Bell and Fort conjecture \cite{8}.

With the coalesced mass 7.9\% higher than the critical value the luminosity evolved in one stage (1st stage) would amount to (cf. Eq. (10)):

\[ L = (1.079)^2L_0 = 1.1642L_0, \tag{11} \]

i.e., the luminosity would increase by 16.4\%.

So far only radio luminosity was considered. Should a reasonable correlation (cf. observations by Efanov et al. \cite{16}) between the optical and the radio luminosity of the quasars be assumed, similar relations should follow for the optical luminosity.

In 30 stages the luminosity (optical, noted with a dash) should be

\[ L' = (1.079)^{2\times30}L'_0, \tag{12a} \]

or, taking logarithm and multiplying by -2.5 on both sides

\[ -2.5 \log L' = -4.953 - 2.5 \log L'_0. \tag{13} \]

Eq. (13) shows that with \( L' \) and \( L'_0 \) measured at the source, there should occur a magnitude change of 4.95 in quasar optical intrinsic luminosity during the quasi-stellar phase. According to Bell and Fort \cite{8}, quasars have a large scale evolution e.g. quasar (primitive to old; we call it the quasi-stellar phase \( \rightarrow \) N-galaxy \( \rightarrow \) radio galaxy.) This may be a reasonable measure for the spread in quasar optical magnitude.
3. Magnitudes

According to our assumption (Sec. 1), the primitive quasar would start its life with an intrinsic redshift 3. The shift would decrease in steps of 0.1 per stage and the quasar would evolve through 30 stages into the final state of intrinsic redshift 0.

Quasars with an intrinsic redshift $z_x$ in the notation of Bell and Fort [8], should have passed $(30 - z_x/0.1)$ stages. Consequently, the optical luminosity expected assuming the critical factor of 1.079, would be

$$L' = 1.079^{2(30-z_x/0.1)} L'_o . \quad (14a)$$

$L'_o$ measured at the source represents the quasar optical luminosity in the primitive stage. We further assume the same luminosities of each object, or

$$L' = 10^{(1.981-0.660z_x)} L'_o . \quad (14)$$

Proceeding as in (13) we get

$$-2.5 \log L' = -4.953 + 1.650z_x - 2.5 \log L'_o . \quad (15)$$

The absolute optical magnitude is given by

$$M_V = -2.5 \log \left[ \frac{L'}{(3.0 \cdot 10^{28} W)} \right] \quad (16)$$

(cf. formula (1), p. 786, Ref. 14), thus

$$M_V = -4.953 + 1.650z_x + M_{V,0} . \quad (17)$$

$M_{V,0}$ is the absolute optical magnitude of the primitive quasar (characterised by $z_x = 3$).

Our next step is to determine $M_{V,0}$. We choose 355 Mpc as the mean quasar distance. This distance is suggested by the average value of the “distance” part of the redshift $z_c$ in Table 1 (Ref. 8), namely 0.119, with the Hubble constant $H=100$ km s$^{-1}$ Mpc$^{-1}$ as assumed in the analysis of Bell and Fort [8] (see “Errata” [17]). It is agreeably “intermediate” as stipulated by these authors [17] (compare the “local”, 1-10 Mpc and the characteristic “cosmological”, 1000 Mpc distance in Ref. 18). Using the formula (2), p. 786 from Ref. 14 and the mean distance 355 Mpc, it follows,

$$\langle m_V - M_V \rangle_{\text{mean}} = 37.753, \quad (18)$$

$m_V$ denotes the apparent (visual/optical) magnitude.

With the mean value $\langle m_V \rangle_{\text{mean}}$=18.578 (Table 1. Ref. 8), we get from Eq. (18)

$$\langle M_V \rangle_{\text{mean}} = -19.373 . \quad (19)$$
Again, \((M_V)_{\text{mean}}\), as obtained from Eq. (17) with \((z_x)_{\text{mean}}=0.792\) as suggested in Table 1. Ref. 8, is
\[
(M_V)_{\text{mean}} = -3.646 + M_{V,0} .
\]
Equating Eqs. (19) and (20) we get
\[
M_{V,0} = -15.727 .
\]
Thus the absolute magnitude of the primitive quasar is obtained. Substituting Eq. (21) in Eq. (17) it follows
\[
M_V = -20.680 + 1.650 z_x .
\]
Relation (22) is close to the complete expression for the absolute (optical/visual) magnitude \((M_V = -20.4 + 1.67 z_x)\) as assumed by Bell and Fort [8].

That the magnitude for the final quasar \((z_x = 0)\) is given by (see Eq. (17))
\[
M_{V,\text{final}} = -4.953 + M_{V,0} .
\]
Therefore, the spread in absolute optical magnitudes of the quasi-stellar objects, as implied by Eq. (23) is about 5. With all quasars assumed at the same distance, this would also be the spread of their apparent optical magnitude \(m_V\) (cf. Fig. 10, Ref. 8; Fig. 1, Ref. 19; also Ref. 20).

4. Primitive quasar

Should we assume an apparent magnitude of 18 for the primitive (the high redshift) quasars \(^2\) and recall the absolute magnitude value \(-15.727\) (see Eq. (21)), we get the distance of 56 Mpc for these quasars. The distance is comparatively close. This should be compared with the distance of 50 Mpc suggested by Burbidge and Hoyle [23] for similar quasars. Highest redshifts are, therefore, assigned to primitive quasars, and these quasars are considered to be at comparatively closer distances. Consistently with the theory and concepts of Bell and Fort [8] and with the present analysis, we find that quasars with high intrinsic redshift \((z_x)\) favour smaller mean distances \((z_x)\) (see Fig. 14 in Ref. 8 and Fig. 1 of the present paper). The values of \(z_x\) and \(z_c\) were determined using relations (1), (2) and (3), Ref. 8, and the source list of 240 quasars with known redshifts. The basic steps in the analysis are: (i) select a value for \(M_0\); (ii) use \(M_0\) to calculate \(z_x\) and \(z_c\) for each quasar from the relations (3) and (1), respectively; (iii) examine the \(z_x\) distribution with the power–spectrum analysis. In step (i) \(M_0\) was chosen in regular intervals in the range +10 to −24.

We note that for one particular value of \(M_0\) (absolute magnitude, \(z_x\) extrapolated to zero) the \(z_x\) distribution is quantized, and the quantization is improved if the

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\(^2\)The apparent magnitude of 17.5–18.5 was assigned for the exceptionally high redshift quasars OH 471, OQ 172 [21,22].
absolute magnitude is given by the relation $M_V = M_0 + S z_x = -20.4 + 1.67 z_x$. In steps (ii) and (iii) an IBM 360 computer was used for the calculation. The slope $S$ was determined by an iteration procedure. Further details can be found in Ref. 8. We account $z_x$ as gravitational.

Fig. 1. The cosmological redshift component $z_c$, plotted against $z_x$ for 240 quasars. The dashed line represents the approximate plate cutoff. Figure reads that quasars with highest intrinsic redshifts ($z_x$) favour the lower mean distances ($z_c$). Diagram follows that of Bell and Fort, Ref. 8, Fig. 14.

Assuming the primitive (high redshift) quasars at the closest distance, it appears that the ideas of Arp [24,25] and Terrell [26,27] are reasonably accommodating to the present scheme. They indicate the direction of quasar evolution, i.e. from high to low, and finally to zero intrinsic redshift. Also, it seems reasonable to accept the large scale evolution of quasars through the related objects to the radio galaxies, as conjectured by Bell and Fort [8]. We conclude the considerations as follows: A quasar originates at some local distance, e.g. from a “peculiar galaxy” of Arp, with the very high intrinsic redshift that has the gravitation as the origin (Hoyle-Fowler model), and gradually moves away reddening less and less, and finally blooming into a radio galaxy at reasonable distances with the appropriate ($z_x = 0$; $z$ not zero) redshift.
5. QSO mass and dying quasars

We consider coalescence of black holes in a quasar continuing stage to stage following the usual second law of black hole mechanics (see Ref. 14, p. 889, 891). The cluster of black holes should ultimately reduce to a single massive hole in the last stage. If $N$ is the initial number of black holes in the primitive quasar, and if we pursue the assumption (Sec. 1) that the black holes attain half their number at each stage, assuming 30 stages of coalescence, one obtains

$$N \approx 2^{30} \approx 10^9.$$  

This gives for the number of black holes in the cluster of the initial quasar as $10^9$, and with the assumed black hole masses of $10^{-1} M_\odot$ the initial mass ($M_Q$) of a quasar would be

$$M_Q = 10^{10} \text{ to } 10^9 M_\odot. \quad (24)$$

This appears to be an acceptable value (see p. 371, Ref. 1). If the equality sign in the coalescence rule (1) is valid in each stage of coalescence, the total mass of the cluster is reduced by the factor 0.707 in each stage. Hence, after 30 stages when a single massive black hole is formed, the mass becomes

$$0.707^{30} M_Q = 3.04 \cdot 10^{-5} M_Q = 3.04 \cdot (10^5 - 10^4) M_\odot. \quad (25)$$

This would be the case if each pair followed the critical mass value. Otherwise, the final massive hole estimate is somewhat larger, in fact about one order higher. With the critical factor of 1.079, as assumed in Sec. 2 and Sec. 3, the mass is obtained by the formula

$$(1.079 \cdot 0.707)^{30} M_Q = 2.97 \cdot (10^6 - 10^5) M_\odot. \quad (26)$$

This represents an estimate of the mass of a black hole at the core of a dying or dead quasar. Considering the large scale evolution of a quasar i.e. quasar (quasi-stellar phase) $\rightarrow$ N-galaxy $\rightarrow$ radio galaxy as conjectured by Bell and Fort [8], we assume collapsed old quasars to be galactic nuclei (radio galaxies) of nuclear mass $2.97(10^6 - 10^5) M_\odot$ (cf. Lynden-Bell [28]).

6. Conclusion

We assume that the clusters of black holes in a quasar evolve in such a manner that at each discontinuous stage the holes coalesce in pairs. This is a hypothesis that lends support to the interesting observations of Bell and Fort [8] and of several other researchers. It resembles a phase-like transition in the cluster. The accretion and the corresponding luminosities are qualitatively different for holes of different masses. That should reveal as the cluster evolves into a massive hole. If the luminosity happens to fall, a compensating source of radio luminosity would be required such as the canonical synchrotron emission [2]. With masses of holes changing, surrounding gas would also change and nebulosity would develop.
Since the work by Bell and Fort [8], many high redshift objects have been discovered and added to the QSO list (see p. 11, Ref 29). Some exceed the value \( z \approx 4 \). Inclusion to the Bell and Fort sample set requires fresh computation. Value of the Hubble constant, as taken in Bell and Fort, might also have to be changed. Bell and Fort favour an increase of the constant. People may argue that the Hubble constant of \( H = 100 \text{ km s}^{-1} \text{ Mpc}^{-1} \) assumed by Bell and Fort is too high. However, it is within the Sandage limit \( 30 < H < 110 \text{ km s}^{-1} \text{ Mpc}^{-1} \) (see p. 788, Ref. [14]).

It is interesting to note that if half of the distance part of the redshift \( z_c \) is attributed to other causes, e.g. to the neutrino effect [30], the effective Hubble constant doubles the accepted value (related to the Hubble expansion). That would take the value of 50 up to the value of 100 km s\(^{-1}\) Mpc\(^{-1}\).

The discoveries of higher and higher redshifts are significant in a sense that they invoke a third part (due to other reasons) in the total redshift value, besides the “intrinsic” part and the Hubble expansion (“cosmological”) part.

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Razmatra se model kvantiziranog crvenog pomaka kvazara pretpostavljajući da su kvazari grozdi crnih jama mase $1 - 10 M_\odot$. Te se crne jame uzimaju da su kompaktné jedinke Hoyle-Fowlerovog modela kvazara te da se veliki crveni pomaci tumac te su dijelom intrinski (gravitacijski). Slijedeći Bella i Forta, kvazari se smatraju objektima na daljinama koje nisu ekstremno velike. Pretpostavljeno je da pri prijelazu u kvazistelarnu fazu kvazari prolaze kroz 30 kvantiziranih međustanja. Pretpostavljajući da su promjene ekvidistantne, dobiveno je zadovoljavajuće slaganje s Bell-Fortovim rezultatima, a izvedeno je također i niz novih rezultata.