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CHIRAL ANOMALY ROUTE TO THE $K_L \to \pi^+\pi^-\gamma$ DECAY

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Dedicated to Professor Mladen Paić on the occasion of his 90$^{\mathbf{th}}$ birthday

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The increasing body of experimental data motivates us to investigate a new contribution to the radiative $K_L \to \pi^+\pi^-\gamma$ decay, which is $\mathcal{O}(p^4)$ within the framework of the chiral perturbation theory. The new contribution refers to the $\Delta S = 1$ WZW action term, established recently for the direct, $\mathcal{O}(p^4)$, $K_L \to \gamma\gamma(-)$ decay amplitude. We present additional evidence in support of such flavour-changing generalization of the WZW action term as a viable parametrisation of the anomalous radiative non-leptonic kaon decays.

1. Introduction

In a search for the origin of CP violation in particle physics, the neutral kaon system still seems to be the most prosperous laboratory. A distinguished feature of this system is the fact that it embodies two distant scales of energy: pseudo-Goldstone kaons have a long-distance (LD) appearance, whereas CP-violating K-decay amplitudes probe the short-distance (SD) scale.Namely, since K contains only quarks of the first and second generation, CP violation requires the appearance of (heavy) quarks of the third generation in loops. Thus, CP-violating K-decays add to an eminent class of physical phenomena which experience many scales of distance.

By exploring nature by this route, one might reveal some of its most profound features. An example is the appearance of quantum mechanical processes on the macroscopic scale (like low-temperature superconductivity), normally thought to proceed at the microscopic scale only. The other known example of a phenomenon spreading over different distance scales (and employed in the present account) is known as the chiral, triangle, or the WZW anomaly. How it is termed depends on the theoretical framework and the corresponding distance scale at which this phenomenon is considered. Its history started with a VVA fermion (proton) triangle loop accounting for the $\pi^0 \to 2\gamma$ decay. Effectively, the anomaly leads to the electromagnetic transition in a situation where one would expect none (π^0 is electrically neutral). In this way, the anomaly represents a signature of a substructure (at the level of charged constituents, the electromagnetic transition appears naturally). Further direct manifestation of the chiral anomaly one can find in the low-energy interactions of the pseudoscalar meson octet. Once quarks have been established as fundamental fermions, the anomalous triangle received its "chiral" counterpart towards ultraviolet (UV), and its "WZW" counterpart towards infra-red (IR). Then, it is almost inevitable to recognize the *anomaly matching principle* in a way in which 't Hooft [1] did it: one requires the anomaly appearing at any scale (with its particular degrees of freedom), including the eventual preonic (constituents of the quark) level.

In this paper we exploit the fact that (in the SM) at the electroweak scale the triangle anomaly originates at the UV/SD scale, the same scale at which the CP-violating K-decay amplitude in SM model seem to originate. In general, the theoretical analysis in the K-system is plagued by LD uncertainties, and the separation of the SD and the LD parts faces almost insurmountable difficulties. However, by focusing on the selected anomaly-governed amplitudes at the mesonic scale, we could possibly infer features of some SD (CP-violating) K-decay amplitudes. In this sense, recognising the anomalous portions of K-decay amplitudes might be essential in decision making in experimentation: among the rare processes which are on the border of feasibility, the anomalous ones have a chance to persist from their SD (short-wavelength) origin to the LD (long-wavelength) scale of experimentation.

Following previous investigations of the potentially large CP-violating $K_S \to \gamma\gamma(-)$ amplitude [2,3], we now focus on the $K_L \to \pi^+\pi^-\gamma$ amplitude. Thereby we elaborate in more detail a recent observation [3] that the decay amplitudes for these processes contain an anomalous piece. In fact, a microscopic treatment of the $K_L \to \gamma\gamma$ decay (first [3] within the chiral quark model, followed by a confirmation [4] in the bound-state calculation) revealed the previously unknown *direct-decay* amplitude. On basis of such microscopic experience Eeg and Picek [3] suggested a generalization of the Wess-Zumino-Witten [5] term to the $\Delta S = 1$ $K_L \to \gamma\gamma$ transition. In this paper we investigate whether such WZW description, *generalized* to a wider class of anomalous procesess, could shed some light on the nature of the direct amplitude measured recently in the $K_L \to \pi^+\pi^-\gamma$ decay [6].

2. Direct CP violation outside $K \to \pi\pi$

After thirty years, the evidence for CP violation in particle physics has remained restricted to the measurement of the value of the K_L to K_S ratios:

$$|\eta_{+-}| \approx |\eta_{oo}| \sim 2 \times 10^{-3} \ .$$

The ratio for charged pions

$$\eta_{+-} = \frac{A(\mathrm{K}_L \to \pi^+\pi^-)}{A(\mathrm{K}_S \to \pi^+\pi^-)} = \epsilon + \epsilon' \tag{1}$$

and for neutral pions

$$\eta_{oo} = \frac{A(\mathrm{K}_L \to \pi^o\pi^o)}{A(\mathrm{K}_S \to \pi^o\pi^o)} = \epsilon - 2\epsilon' \tag{2}$$

differ in the parameter ϵ' . This parameter measures $\Delta S = 1$ (direct) CP violation, whereas ϵ is a measure of the $\Delta S = 2$ (indirect) CP violation. The present data on the parameter ϵ' quantifying direct CP violation are inconclusive. Whereas the Fermilab [7] measurement is consistent with zero,

$$\mathrm{Re}\ \frac{\epsilon'}{\epsilon} = (7.4 \pm 5.2 \pm 2.9) \times 10^{-4} \quad \text{[E731]},$$

the NA31 experiment at CERN [8] gives a 3σ evidence,

$$\mathrm{Re}\ \frac{\epsilon'}{\epsilon} = (23.0 \pm 3.6 \pm 5.4) \times 10^{-4} \quad \text{[NA31]}.$$

The sensitivity at the 10^{-4} level is expected from the forthcoming facilities at Fermilab (E832), CERN (NA48), and the Frascati Φ-factory (DAΦNE).

Therefore, a study of other (rare) K-decay modes might elucidate the existence of direct CP violation. There are several promising candidate decays advocated in recent reviews [6,9,10]. For the $\mathrm{K_S} \to 3\pi$ decays among them, CPLEAR (CERN) has already provided first results. The η_{+-o} parameter related to the $\mathrm{K_S} \to \pi^+\pi^-\pi^o$ decay mode (having both the CP-conserving and CP-violating pieces) is

$$\mathrm{Re}\ (\eta_{+-o}) = (5 \pm 2 \pm 7) \times 10^{-3} \quad \text{[CPLEAR]}$$

and

$$\mathrm{Im}\ (\eta_{+-o}) = (1.6 \pm 2.4 \pm 1.8) \times 10^{-2} \quad \text{[CPLEAR]}.$$

The $\mathrm{K_S} \to 3\pi^o$ decay mode (having only the CP-violating piece) differs from the previous one in the direct-CP-violation piece:

$$\eta_{ooo} = \epsilon + \epsilon'_{ooo} \ ; \ \ \eta_{+-o} = \epsilon + \epsilon'_{+-o} \ .$$

Still, a foreseeable sensitivity both of CPLEAR and for the DAΦNE will stay at the level of ϵ, so that the direct CP violation issue remains inconclusive.

For similar reasons, one of the authors started to study the radiative $\mathrm{K}_{L,S} \to 2\gamma$ decays [2] in the hope of telling direct CP violation from other signals in these processes.

The parameter for $CP = +1$ photons,

$$\eta_+ \equiv \eta_{\gamma\gamma(+)} = \frac{A(\mathrm{K}_L \to \gamma\gamma(+))}{A(\mathrm{K}_S \to \gamma\gamma(+))} = \epsilon + \epsilon'_{\gamma\gamma(+)} \approx \epsilon \quad , \tag{3}$$

is essentially determined by indirect CP violation. This is due to the fact that, in chiral perturbation theory, $\mathrm{K}_S \to 2\gamma$ first appears through the pion loop. Such a structure,

$$\mathrm{K}_S \to 2\pi \to 2\gamma,$$

indicates that the CP-violating amplitude of this decay has the same origin as in $\mathrm{K}_S \to 2\pi$, implying $\epsilon'_{\gamma\gamma(+)} \approx \epsilon'$ or $\eta_{\gamma\gamma(+)} \approx \eta_{+-}$ in Eq. (1), where the direct CP-violating parameter ϵ' is inhibited by a factor of 22 coming from the $\Delta I = 1/2$ rule. Thus, direct CP violation will be manifested through $\epsilon'_{\gamma\gamma(-)}$, entering the ratio of the physical amplitudes for $\mathrm{K}_{L,S} \to \gamma\gamma(-)$ decays (where $(-)$ refers to the $CP = -1$ state of the photons),

$$\eta_- \equiv \eta_{\gamma\gamma(-)} = \frac{A(\mathrm{K}_S \to \gamma\gamma(-))}{A(\mathrm{K}_L \to \gamma\gamma(-))} = \epsilon + \epsilon'_{\gamma\gamma(-)}. \tag{4}$$

Since such a study awaits a dedicated experiment at CPLEAR [6], the rest of this paper will be devoted to the $\mathrm{K}_L \to \pi^+\pi^-\gamma$ channel where CP violation has already been measured. It required the use of the γ energy spectrum in order to separate contributions from internal bremsstrahlung (IB) from those of direct emission (DE). The first measurement by E731 [11]

$$|\eta_{+-\gamma}| = (2.15 \pm 0.26 \pm 0.17) \times 10^{-3} \quad \text{[E731]} \quad ,$$

confirmed recently by E773 [16]

$$|\eta_{+-\gamma}| = (2.414 \pm 0.065 \pm 0.062) \times 10^{-3} \quad \text{[E773]} \quad ,$$

refers actualy to the IB contribution to the quantity which in total acquires the form [12]

$$\eta_{+-\gamma} = \frac{A(\mathrm{K}_L \to \pi^+\pi^-\gamma)_{IB+E1}}{A(\mathrm{K}_S \to \pi^+\pi^-\gamma)_{IB+E1}} \approx \eta_{+-} + \epsilon'_{\pi\pi\gamma} \frac{A(\mathrm{K}_S \to \pi^+\pi^-\gamma)_{E1}}{A(\mathrm{K}_S \to \pi^+\pi^-\gamma)_{IB}} \quad . \tag{5}$$

Although the interesting direct CP-violating $\epsilon'_{\pi\pi\gamma}$ is suppresed by a small factor A_{E1}/A_{IB}, it is not suppresed by the factor dictated by the $\Delta I = 1/2$ rule, and might be improved by the time evolution measurement at DAΦNE.

However, these experimental achievements and prospects are confronted with rather poor theorethical predictions [13]. Therefore we are trying to attack this problem form another side, by employing the anomalous nature of the decay under consideration.

3. *Three facets of anomaly in non-leptonic K-decays*

(i) A major source of the theoretical uncertainty in predicting the $\mathrm{K}_L \to \pi^+\pi^-\gamma$ amplitude originates from the cancellation of the potentially leading contributions. Those are termed the *reducible* contributions, where the photonic couplings to mesons originate from the WZW term

$$\mathcal{L}_{WZW} = -\frac{\mathrm{i}N_c}{48\pi^2}\epsilon^{\mu\nu\alpha\beta}\, Tr[W(U,l,r)_{\mu\nu\alpha\beta} - W(I,l,r)_{\mu\nu\alpha\beta}]\ , \tag{6}$$

where N_c is the number of colours. Such term closes the gauging procedure imposed on the original bosonic Lagrangian represented non-linearly by the 3×3 matrix:

$$U \equiv \exp\left(\frac{2\mathrm{i}}{f}\Pi\right)\ . \tag{7}$$

Here $\Pi = \sum_a \pi^a\lambda^a/2$ is given by the eight Goldstone fields $\pi^a(a=1,..,8)$, and f can be identified with the pion decay constant $f = f_\pi = (92.4\pm 0.2)$ MeV ($= f_K$, in the chiral limit).

For the purely electromagnetic (A_μ – field) gauging

$$l_\mu = r_\mu = a_\mu \equiv -eA_\mu Q; \quad Q = \frac{1}{3}\mathrm{diag}(2,-1,-1), \tag{8}$$

the tensor in Eq. (6) acquires the form (symmetric under $U \leftrightarrow U^\dagger, \Sigma^L_\mu = U^\dagger\partial_\mu U \leftrightarrow \Sigma^R_\mu = U\partial_\mu U^\dagger$)

$$\begin{aligned} W(U,a)_{\mu\nu\alpha\beta} \ &=\ \Sigma^L_\mu U^\dagger\partial_\nu a_\alpha U a_\beta + \Sigma^L_\mu a_\nu\partial_\alpha a_\beta \\ &\quad +\Sigma^L_\mu\partial_\nu a_\alpha a_\beta - \mathrm{i}\Sigma^L_\mu\Sigma^L_\nu\Sigma^L_\alpha a_\beta - (L\leftrightarrow R)\ . \end{aligned} \tag{9}$$

Then, by expanding the expression in the parenthesis in Eq. (6)

$$[W(U,a)_{\mu\nu\alpha\beta} - W(I,a)_{\mu\nu\alpha\beta}]\ =\ V_{\mu\nu\alpha\beta}\ , \tag{10}$$

to the wanted order, one generates the anomalous meson-photon couplings containing the well-known flavour-diagonal term of order $\mathcal{O}(p^4)$ responsible for $\pi^0\to\gamma\gamma$

$$\begin{aligned} \mathcal{L}^{(4)}_{WZW} \ &=\ -\frac{N_c\alpha}{24\pi f_\pi}\epsilon_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}\left(\pi^0 + \frac{\eta}{\sqrt{3}}\right) \\ &+\frac{\mathrm{i}N_c e}{12\pi^2 f_\pi^3}\epsilon_{\mu\nu\rho\sigma}A^\mu\partial^\nu\pi^+\partial^\rho\pi^-\partial^\sigma\left(\pi^0+\frac{\eta}{\sqrt{3}}\right)\ . \end{aligned} \tag{11}$$

Such couplings, in conjunction with the non-leptonic $\Delta S = 1$ transition Eq. (16), lead to the LD appearance of the chiral anomaly in the radiative non-leptonic K-decays. There is nonvanishing tree level contribution to $\mathrm{K}_L \to \pi^0\pi^0\gamma\gamma$ (Fig. 1a), the real representative of $\mathcal{O}(p^4)$ reducible anomalous process.

However, very similarly to $\mathrm{K}_L \to \gamma\gamma$ (Fig. 1b), the pole diagrams for $\mathrm{K}_L \to \pi^+\pi^-\gamma$ (Fig. 2a), generated by the chiral WZW term (6), vanish in leading order chiral perturbation theory(χPT). There is nonvanishing tree level contribution to $\mathrm{K}_L \to \pi^0\pi^0\gamma\gamma$ (Fig. 1a), the real representative of $\mathcal{O}(p^4)$ reducible anomalous process. The cancellation at this $\mathcal{O}(p^4)$ order of the reducible amplitudes for procesess adresses us to the study of less known direct amplitudes.

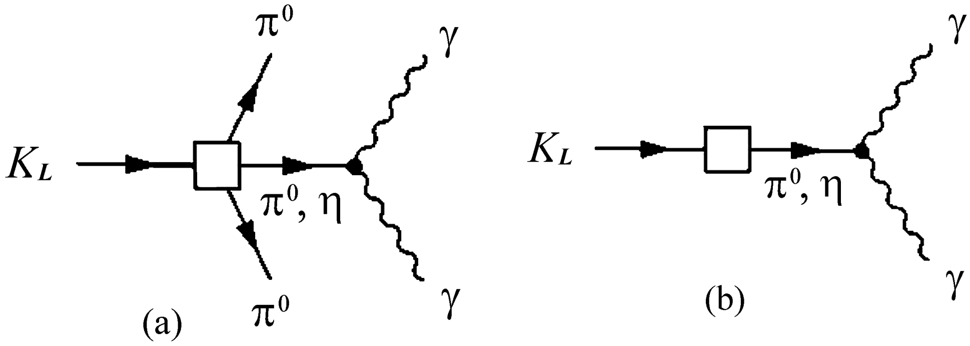

Fig. 1. Long distance contributions to (a) $\mathrm{K}_L \to \pi^0\pi^0\gamma\gamma$ *and (b)* $\mathrm{K}_L \to \gamma\gamma$ *induced by the leading WZW term.*

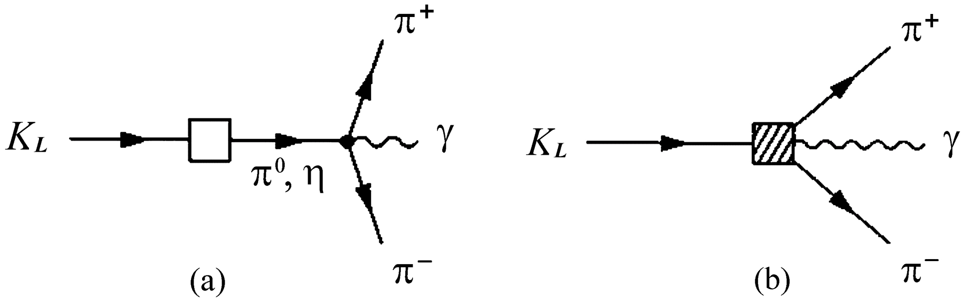

Fig. 2. (a) Reducible contribution to the $\mathrm{K}_L \to \pi^+\pi^-\gamma$ *which vanishes in the* $SU(3)$ *symmetry limit. (b) The same process viewed from the standpoint of an* $\Delta S = 1$ *extended WZW term, encapsulating contributions from all scales.*

(ii) In fact, the process $K_L \to \pi^+\pi^-\gamma$ we are focusing to, is accustomed to be a representative of a class of *direct anomalous* process. The respective odd intrinsic parity weak lagrangian at the order $\mathcal{O}(p^4)$, offered by Bijnens, Ecker and Pich (BEP) [14], is as follows:

$$\begin{aligned}\mathcal{L}_{\rm An}^{\Delta S=1} &= -{\rm i}e\frac{G_8 f_\pi^2}{16\pi^2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}\Big[a_2{\rm Tr}(\lambda_-[U^\dagger QU, L_\mu L_\nu]) \\ &+ 3a_3{\rm Tr}(\lambda_- L_\mu){\rm Tr}((Q+U^\dagger QU)L_\nu) \\ &+ a_4{\rm Tr}(\lambda_- L_\mu){\rm Tr}((Q-U^\dagger QU)L_\nu)\Big] + h.\,c., \end{aligned} \tag{12}$$

where a_2, a_3 and a_4 are coupling constants, $\lambda_\pm = (\lambda_6 \pm {\rm i}\lambda_7)/2$ is the matrix that projects out $\Delta S = \pm 1$ transitions, and $L_\mu = {\rm i}U^\dagger D_\mu U$ is expressed through the covariant derivative $D_\mu U = \partial_\mu U - {\rm i}eA_\mu[Q,\ U]$. The overall coupling constant $|G_8| = 9 \times 10^{-6}{\rm GeV}^{-2}$ is the same as that appearing in the standard lowest-order $\Delta S = 1$ term displayed later in Eq. (16). The origin of the direct anomalous term (12) can be traced back to the bosonization of four-quark operators in the odd-intrinsic parity sector [14,15], by using the functional derivative of the action $S = \int {\rm d}^4x\mathcal{L}$ as an identification of the quark current:

$$\bar{q}\gamma^\mu Lq \sim \frac{\delta S}{\delta l_\mu}\ . \tag{13}$$

To obtain the direct anomalous terms, BEP [14] started from a four-quark effective hamiltonian essentially involving products of two weak currents (in the factorizable limit)

$$\mathcal{L}_{\rm An} \sim G_F\left(\frac{\delta S^{(2)}}{\delta l_\mu}\right)\left(\frac{\delta S_{WZW}}{\delta l^\mu}\right), \tag{14}$$

where $S^{(2)}$ is the normal action $\mathcal{O}(p^2)$

$$\mathcal{L}^{(2)}_{strong/em} = \frac{f_\pi^2}{4}\,{\rm Tr}(D^\mu U D_\mu U^\dagger) \quad . \tag{15}$$

The result, written in the form (12), contributes to $\bar{K}^0 \to \pi^+\pi^-\gamma$ but not to $\bar{K}^0 \to \gamma\gamma$, because the commutator $[\Pi, Q]$ doesn't contain neutral kaons (Q acts as the unit matrix in the s, d sector, and $UQU^\dagger = Q + [\Pi, Q] + ...$). To find a contribution giving the $\bar{K}^0 \to \gamma\gamma$ amplitude in this way, one is addressed to go further to the $\mathcal{O}(p^6)$ terms.

(iii) We shall instead explore another route [3] which is close to the one used by Cronin [17] to generalize the $\mathcal{O}(p^2)$ strong/electromagnetic term (16) to the corresponding non-leptonic $\Delta S = 1$ term [17]

$$\mathcal{L}^{(2)}_{\Delta S=1} = G_8\,{\rm Tr}(\lambda_+ D^\mu U D_\mu U^\dagger)\ . \tag{16}$$

The new flavour–changing WZW-like term [3], predicting uniquely $\bar{K}^0 \to 2\gamma$ at the $\mathcal{O}(p^4)$, was obtained from the ordinary WZW term by the simple substitution $1 \to \mathcal{G}_{\rm WZW}\, \lambda_+$ in front of $V_{\mu\nu\alpha\beta}$ in Eq. (10):

$$\mathcal{L}_{\rm WZW}^{\Delta S=1} \;=\; -\frac{{\rm i}N_c}{48\pi^2}\mathcal{G}_{\rm WZW}\,\epsilon^{\mu\nu\alpha\beta}\,Tr(\lambda_+ V_{\mu\nu\alpha\beta})\;. \tag{17}$$

The effective coupling $\mathcal{G}_{\rm WZW}$ accounts for both the CP-conserving (*even*) and CP-violating (*odd*) transitions, by the respective terms in

$$\mathcal{G}_{\rm WZW} \;=\; \mathcal{G}_{\rm WZW}^{even} \;+\; {\rm i}\,\mathcal{G}_{\rm WZW}^{odd}.$$

In the present paper we go a step further in exploiting the anomalous $\mathcal{L}_{\rm WZW}^{\Delta S=1}$ terms in order to predict other radiative non-leptonic processes. We argue that also for a class of flavour-changing anomalous processes, there should be no new coupling strengths when we go from one anomalous process to another. This justifies relation (17), providing a predictive link among processes involving an odd number of Goldstone bosons. The expression (17), taken at its face value, accounts for the transitions $\bar{K}^0 \to \gamma\gamma$, $\pi^+\pi^-\gamma$, $\pi^0\pi^0\gamma$, $\pi^+\pi^-\gamma\gamma$ and $\pi^0\pi^0\gamma\gamma$:

$$\begin{aligned}\mathcal{L}_{\rm WZW}^{\Delta S=1}(4) \;=\; &\mathcal{G}_{\rm WZW}\frac{N_c\alpha}{24\pi f_\pi}\frac{2\sqrt{2}}{3}\epsilon_{\mu\nu\rho\sigma}A^\nu F^{\rho\sigma}\partial^\mu \bar{K}^0 \\ &+\mathcal{G}_{\rm WZW}\frac{{\rm i}N_c e}{12\pi^2 f_\pi^3}\frac{\sqrt{2}}{3}\epsilon_{\mu\nu\rho\sigma}A^\sigma\partial^\mu \bar{K}^0[\partial^\nu\pi^-\partial^\rho\pi^+] \\ +\mathcal{G}_{\rm WZW}\frac{N_c\alpha}{128\sqrt{2}\pi f_\pi^3}&\epsilon_{\mu\nu\rho\sigma}F^{\nu\rho}A^\sigma[\bar{K}^0\pi^0\partial_\mu\pi^0 - \partial_\mu\bar{K}^0\pi^0\pi^0] + h.c.\end{aligned} \tag{18}$$

Provided that (18) explains the total $\mathrm{K}_L \to \gamma\gamma$ rate, an extraction of the coupling $\mathcal{G}_{\rm WZW}$ from the first term in Eq. (18) enables one to determine the $\mathrm{K}_L \to \pi^+\pi^-\gamma$ decay (the second term in (18) shown on Fig. 2b) .

In determining the coupling $\mathcal{G}_{\rm WZW}$ from the K^0 decay into two photons, we have to rewrite our flavour–non–diagonal WZW interaction (17) in the physical (decay-state) basis. We obtain it by combining the $A(\bar{K}^0 \to \gamma\gamma)$ and $A(K^0 \to \gamma\gamma)$ amplitudes:

$$\mathcal{L}_{\gamma\gamma(-)}^{\Delta S=1} \;=\; \frac{N_c\alpha}{24\pi f_\pi}\frac{2}{3}[\;\mathcal{G}_{\rm WZW}^{even}\;\Phi_{\mathrm{K}_2} - {\rm i}\;\mathcal{G}_{\rm WZW}^{odd}\;\Phi_{\mathrm{K}_1}]\epsilon_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}\;. \tag{19}$$

The two terms in (19) correspond to the CP-conserving $\mathrm{K}_L(\approx \mathrm{K}_2) \to \gamma\gamma(-)$ and the CP-violating $\mathrm{K}_S(\approx \mathrm{K}_1) \to \gamma\gamma(-)$ amplitudes, respectively.

Under a plausible assumption that the CP-conserving amplitude from Eq. (19) dominates the measured $\mathrm{K}_L \to \gamma\gamma$ decay rate, one can determine $\mathcal{G}_{\rm WZW}^{even}$ [3] from the measured $\mathrm{K}_2\gamma\gamma$ coupling C_{K_2}. The rate for $\mathrm{K}_2 \to \gamma\gamma$ gives (in the limit $f_K \;=\; f_\pi$)

$$|C_{\mathrm{K}_2}| \;=\; \tfrac{2}{3}\,|\mathcal{G}_{\rm WZW}^{even}|\,C_{\pi^0} \;=\; 5.9\times 10^{-11} MeV^{-1}\,,$$

and thus the dimensionless coupling is

$$\mathcal{G}_{WZW} \approx |\mathcal{G}^{even}_{\rm WZW}| \;\approx\; 2\times 10^{-7}\;. \tag{20}$$

Armed with the knowledge of the coupling $\mathcal{G}_{WZW}$, let us analyse the $\Delta S = 1$ direct anomalous term in the $\mathrm{K}_L \to \pi^+\pi^-\gamma$ decay .

4. *Anomalous* $\mathrm{K}_L \to \pi^+\pi^-\gamma$ *decay*

Experimentally, the $\mathrm{K}_L \to \pi^+\pi^-\gamma$ amplitude appears to be suitable to study the DE amplitude. The first step in this direction is represented by subtracting the IB contribution from the total rate. The kinematical cuts in photon energy allow one to eliminate the uninteresting IB predicted fully by gauge invariance.

On the theoretical side, the cancellation of the reducible amplitudes related to the WZW functional leaves some space for the study of less known direct amplitudes. Once we have $\mathcal{G}_{WZW}$ (20), we can predict the WZW-related process $\mathrm{K}_L \to \pi^+\pi^-\gamma$ explicated in the second term of Eq. (18). Transforming this term into the physical K-meson basis, we obtain the following expression for the CP-conserving amplitude which presumably dominates the rate:

$$\mathcal{L}^{\Delta S=1}_{WZW(1\gamma)} \;=\; \mathcal{G}^{even}_{\rm WZW}\frac{N_c e}{12\pi^2 f_\pi^3}\frac{2}{3}\epsilon_{\mu\nu\rho\sigma}A^\mu\partial_\nu\Phi_{\mathrm{K}_2}[\partial_\rho\pi^+\partial_\sigma\pi^-]\;. \tag{21}$$

Let us parametrize the amplitude for the one-gamma emission $\mathrm{K}_L(k) \to \pi^+(p_+)\pi^-(p_-)\gamma(q)$ as follows

$$A(\mathrm{K}_L \to \pi^+\pi^-\gamma) \;=\; -\mathrm{i}eG_{1\gamma}\epsilon_{\mu\nu\rho\sigma}\epsilon^\mu k^\nu p_+^\rho p_-^\sigma\;. \tag{22}$$

This amplitude reproduces the results of the E731 experiment at Fermilab [11]

$$Br(\mathrm{K}_L \to \pi^+\pi^-\gamma) \;=\; (3.19\pm 0.16)\times 10^{-5}\;,$$

with the value of the effective coupling $G_{1\gamma}$

$$|G^{expt}_{1\gamma}| \;\approx\; 4.9\;\times 10^{-6}\;GeV^{-3}. \tag{23}$$

On the other hand, by employing the value (20) for $|\mathcal{G}^{even}_{\rm WZW}|$ in Eq. (21), we predict

$$\begin{aligned}|G^{theo}_{1\gamma}| \;&=\; |\mathcal{G}^{even}_{\rm WZW}|\frac{N_c}{12\pi^2 f_\pi^3}\frac{2}{3}\\ &\approx\; 4.28\times 10^{-6}\;GeV^{-3}\quad ,\end{aligned} \tag{24}$$

which agrees well with the upper experimental value. This indicates that flavour-extended WZW term successfully relates non-leptonic radiative kaon decays.

Let us compare our prediction to the direct (factorizable) contribution [14] derived from Eq. (12) of (ii) part of the preceeding section.

$$\mathcal{L}_{BEP}^{\Delta S=1} = \frac{G_8 e}{4\sqrt{2}\pi^2 f_\pi}(a_2 + 2a_4)\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}\partial^\mu \bar{K}^0 \partial^\nu \pi^- \pi^+ \quad . \tag{25}$$

A direct comparison of this and our contribution to the $K_L \to \pi^+\pi^-\gamma$ (18) implies

$$a_2 + 2a_4 \approx 1 \quad ,$$

which is consistent with a claim [14]

$$a_2 \text{ and } a_4 \lesssim 1 \quad ,$$

drawn from the analysis of the $K^+ \to \pi^+\pi^0\gamma$ decay. This represents another successful prediction of the flavour-changing extension of the WZW, described in part (iii) of the previous section.

5. *Conclusions*

In a situation that the $K_L \to \pi^+\pi^-\gamma$ amplitude is claimed to be unpredictable within the customary analysis within the χPT, we take an alternative look at the direct part of this amplitude governed by the chiral Wess-Zumino term. In fact, we check a viability of the prescription to generate the anomalous non-leptonic interaction terms. We argue that the anomaly mechanism takes care of the uniqueness of such a prescription, i. e. for the univesal coupling $\mathcal{G}_{WZW}$ in Eq. (17). Thus we arrive at the predictive relations linking different radiative non-leptonic K-decays. The viability of the new parametrization is demonstrated here by analysing the available body of the experimental and theoretical data on the $K_L \to \pi^+\pi^-\gamma$ process. An additional support commes from another proces, $K_L \to \pi^0\pi^0\gamma\gamma$, contained in our relation (18). Assuming dominant CP contribution, it is given by

$$\mathcal{L}_{WZW(2\gamma)}^{\Delta S=1} = \mathcal{G}_{\rm WZW}\frac{\alpha N_c}{54\pi f_\pi^2}\epsilon^{\mu\nu\alpha\beta}F_{\nu\alpha}A_\beta[K_2\pi^0\partial_\mu\pi^0 - (\partial_\mu K_2)\pi^0\pi^0] \quad .$$

The explicit comparison to the existing theoretical analysis of the (reducible) contribution [20] shows that we have a comparable prediction for the rate of this process too.

To conclude, the "anomaly road", advocated here, which starts form true-contact (non-separable) SD mechanism, seems to incorporate a whole body of anomalous transitions: reducible, direct (factorizable) and the contact (direct-nonfactorizable). On the top of the previous parametrisations, including links among processes, the presented "anomaly road" incorporates, at the end of the chain, the K to vacuum transition. Relying on the anomaly as a mechanism which has an imprint on a different distance scale, the flavour-changing generalization of the WZW term seems to bear the closest contact with the bosonization of $\Delta S = 1$ non-leptonic radiative K decays. Simultaneously, WZW flavour-changing term seems to provide a unified treatment of such processes in the sense that it incorporates contributions irrespectivelly of the scale from which they originate. In this sense such parametrization is compatible to the χPT framework.

Finally, such an "anomaly road", being able to give a new insight in the SD/contact contributions for selected rare kaon decays, might improve the power of theoretical predictivity. Isolating the direct anomalous/contact terms of radiative non-leptonic K-decays seems to be a necessary ingredient in separating direct from indirect CP violation. In this sense, the processes like $K_L \to \gamma\gamma$ and $K_L \to \pi^+\pi^-\gamma$, where LD (reducible) anomalous amplitudes cancel out to the leading order, seem to have enough sensitivity to the SD part of decay amplitude in order to make progress in further study of CP violation.

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KIRALNA SIMETRIJA I RASPAD $\mathrm{K}_L \to \pi^+\pi^-\gamma$

Novi eksperimentalni rezultati potiču izučavanje doprinosa radijacijskom raspadu $\mathrm{K}_L \to \pi^+\pi^-\gamma$, koji je u kiralnoj perturbacijskoj teoriji reda $\mathcal{O}(p^4)$. Takav potencijalni doprinos nalazimo u članu $\Delta S = 1$ WZW djelovanja, koji je nedavno identificiran kroz proučavanje amplitude $\mathrm{K}_L \to \gamma\gamma(-)$ raspada reda $\mathcal{O}(p^4)$. Tako poopćeno WZW djelovanje na procese s promjenom kvarkovskih vrsta uspješno obuhvaća anomalne radijacijske neleptonske raspade kaona.