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A COMPARATIVE STUDY ON THE BREAK–UP OF DEUTERONS BY ALPHA PARTICLES

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Experimental triple correlation cross–sections in the reaction $\mathrm{d} + \alpha \rightarrow \alpha + \mathrm{p} + \mathrm{n}$, have been analysed following different approaches, in the framework of single level R–matrix theory (RM). Concentrating on the spectra dominated by α–neutron final–state–interaction (αn FSI), a rather large amount of data have been investigated. Fits due to existing Faddeev type calculations have been compared and relative merits discussed.

1. Introduction

In few–body physics, alpha deuteron system plays an important role by presenting itself as a testing ground of studying three–body effects in nuclear reactions. Apart from direct three–body break–up ($\alpha + \mathrm{d} \rightarrow \alpha + \mathrm{p} + \mathrm{n}$), the possible reaction mechanisms are sequential decay through states of ^{5}He and ^{5}Li, i.e. αn and αp final state interaction (FSI), pn FSI and αp and αn quasi–free scattering (QFS). One may select experimental variables in order to concentrate on each mechanism separately or on several mechanisms together. Thus, one can choose a particular quasi–two–body reaction and study its features in the presence of the third particle. The most investigated candidate seems to be the αn FSI in the presence of the spectator proton.

The R–matrix (RM) formula is often used to theoretically reproduce the spectra dominated by αn FSI. Reasonably good to excellent fits have been obtained using RM, at least, qualitatively. The structure of the RM formula used, however, is not unique when applied to different incident energies. There are several approaches [1-4], especially in the choice of the functional form of the relevant angular variable. The situation becomes more intricate when one seeks to obtain qualitative as well as quantitative agreement. It may be pointed out that the conclusions made on the successes of RM (in describing the αn FSI spectra) were based on analyses of rather small amounts of data. So we set out to test the validity and limitations of the said approaches, exploiting a rather large amount of data involving different incident energies. To observe the spectator effect, existing Faddeev–type fits are also compared. Three–body picture of the αd system is assumed to be valid below the α–break–up threshold [5]. We restrict ourselves to the part of existing data where the spectra under investigation satisfy the following conditions:

(a) They are dominated solely by the αn FSI so that the R–matrix theory is expected to be valid.

(b) Higher incident energy (near or above the α–break–up threshold) are excluded so that Faddeev theoretical three–body assumption remains valid.

We avoid lower energy (near deuteron break–up threshold) data because at lower energies, several final state mechanisms overlap almost everywhere in the allowed phase space. We choose energies below 43 MeV (incident α) because in that domain the only reaction channel opened is $\alpha + \mathrm{d} \rightarrow \alpha + \mathrm{p} + \mathrm{n}$.

2. Theoretical background

The amplitude describing the α–induced deuteron break–up process, in the three–body formalism, can be written as [6]

$$U = I_{\alpha\mathrm{p}} + I_{\alpha\mathrm{n}} + M_{\alpha\mathrm{p}} + M_{\alpha\mathrm{n}} + M_{\mathrm{np}}, \tag{1}$$

where $I_{\alpha\mathrm{p}}$ and $I_{\alpha\mathrm{n}}$ represent the impulse terms and M's are the multiple scattering terms in which the pairs indicated by the subscripts interact in the final state of the reaction. The rigorous Faddeev theoretical calculations take into account all terms in Eq. (1). However, when the spectrum under consideration is dominated solely by particular two–body interaction, one simply picks up the relevant two–body term neglecting the others. Thus, in the presence of a strong αn FSI, which is assumed to be in the $P_{3/2}$ state, one applies RM following the procedure of Werntz [7] and writes the cross–section as

$$\frac{\mathrm{d}^3\sigma}{\mathrm{d}\Omega_\alpha \mathrm{d}\Omega_\mathrm{p} \mathrm{d}E_\alpha} = N|\Psi_d(E_\mathrm{p})|^2 \sin^2\delta_1 \frac{F_1^2(ka) + G_1^2(ka)}{(ka)^2} \rho f(\chi), \tag{2}$$

where N is a factor dependent only on the incident energy, $\Psi_d(E_\mathrm{p})$ is the deuteron wave function, δ_1 is the $P_{3/2}$ resonant phase shift, F_1 and G_1 are neutron wave functions, $\hbar k$ represents the αn relative momentum and ρ is the phase space factor given by [8]

$$\rho = \frac{8E_{\rm p}(E_\alpha)^{1/2}}{(E_{\rm p})^{1/2} + (E_\alpha)^{1/2}\cos\theta_{\alpha{\rm p}} - (E_i)^{1/2}\cos\theta_{\rm p}}, \tag{3}$$

where $E_{\rm p}$, E_α and E_i are the lab kinetic energies of the scattered proton and the scattered and incident α–particles, respectively. The phase shift δ_1 is calculated using the relation

$$\tan\delta_1 = \frac{\Gamma}{2}\left(E_0 + \Delta_1 - E_{\alpha{\rm n}}\right), \tag{4}$$

where

$$\Delta_1 = \frac{\Gamma}{2}(F_1F_1' + G_1G_1') \tag{5}$$

and

$$\frac{\Gamma}{2} = \frac{\gamma^2 ka}{F_1^2 + G_1^2}. \tag{6}$$

E_0, γ^2 and a are the R–matrix parameters, $f(\chi)$ gives the angular dependence, χ being the scattering angle of the neutron in the αn system. This $f(\chi)$ is the factor which led different workers to make several choices of its form to obtain better fits to the data. Different forms used and the respective findings are as follows. Using

$$f(\chi) = \cos^2\chi, \tag{7}$$

which describes the customary angular dependence for p–wave scattering, fits to some of the data at 42 MeV [1] were found very bad. Some of the experimentally observed ^{5}He FSI peaks were found to be completely suppressed when assuming Eq. (7). Analysis of 8.9 MeV (deuteron projectile) data [2] showed that $f(\chi)$ given by Eq. (7) (with Ψ_d = const.) fitted the shapes of the spectra fairly well but badly reproduced the absolute cross-section. The normalization factor ranged from 0.1 to 1.0 over the three correlated angle pairs studied. The authors stressed the presence of dominant multiple processes at lower incident energies and prescribed an empirical relation:

$$f(\chi) = \text{const.} \tag{8}$$

Satisfactory results were obtained in reproducing the absolute cross–sections and the shapes of the spectra. When extended to larger amount of data and at somewhat higher energy (42 MeV α), use of Eq. (8) resulted in better shapes of the spectra than Eq. (7), but no satisfactory quantitative agreement was reached unless the value of the RM parameter (E_0) was altered for different spectra at the same energy [4]. Much more satisfactory fits were reported [3] by choosing

$$f(\chi) = 3\cos^2\chi + 1 \tag{9}$$

which represents the characteristic angular dependence for scattering in the $P_{3/2}$ channel. It may be stressed that the result of Ref. 3 involves investigation of three different angular combinations only. We analyse data distributed over eleven pairs of correlated angles, at two incident energies: $E_d = 18$ MeV and $E_\alpha = 42$ MeV, to better understand the above findings.

3. Results and discussion

Figure 1 displays the results of our calculations for incident deuteron energy of 18 MeV [9] (36 MeV incident α). We denote by R1, R2 and R3 the calculations using Eq. (2) with $f(\chi)$ given by Eqs. (7), (8) and (9), respectively. Parameters used are those recommended by Dodder and Gammel [10]:

$$E_0 = -4.3 \text{ MeV}, \quad \gamma^2 = 6.9 \text{ MeV and } a = 2.9 \text{ fm}.$$

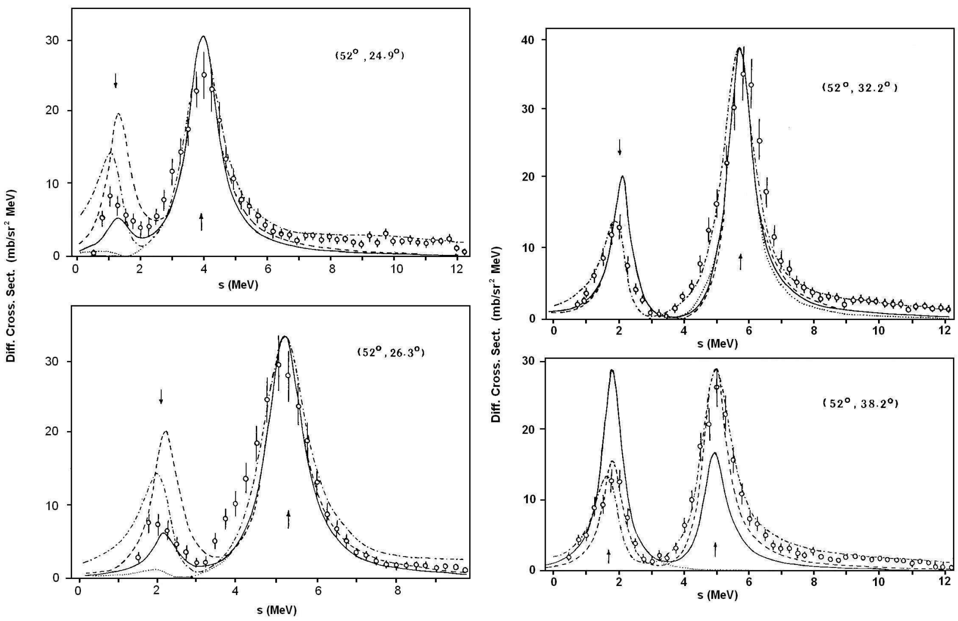

Fig. 1. Triple correlation cross-sections $d^3\sigma/d\Omega_\alpha d\Omega_p ds$ *as functions of arc length S for incident deuteron energy of 18* MeV. *Correlated angles* $(\theta_p, \theta_\alpha)$ *are indicated in the figure.* $(\phi_\alpha, \phi_p) = (180°, 0°)$. *Experimental data points are from Oswald et al. [9]. The present calculations R1, R2 and R3 (as explained in the text), based on R–matrix theory, are shown by dotted, dashed and solid curves, respectively. The dashed–dotted curves are the prediction from Faddeev theoretical calculations (FT) of Ref. 9. The arrow marks denote the kinematically predicted positions of* ^{5}He *FSI peaks.*

R1 (dotted curves) did not reproduce the spectra except one (Fig. 1c.). Almost complete suppression occurs for the smaller FSI peaks at $(\theta_{\rm p}, \theta_\alpha)$ = (52°, 24.9°) and (52°, 26.3°), and the larger peak at (52°, 38.2°). Such was the case for some of the spectra at 42 MeV [1], too. Thus, it seems very unlikely that an overall satisfactory fit using R1 can be obtained.

Results due to R2 (dashed curves) are quite good: the shapes as well as absolute cross–sections of the spectra are reproduced reasonably well (Table 1). The only discrepancies found are for the smaller peaks of the first two spectra (Fig. 1a, b), where calculated cross–sections are almost twice the experimental ones. When analysis was made [4] of the 42 MeV data (comprising about 10 spectra) such an overall good fit was obtained but at the cost of altering the input parameters.

TABLE 1.

The ratios $(\sigma_{exp}/\sigma_{cal})$ of the experimental values of the n–α *FSI peak cross–sections to the calculated ones, in the framework of single–level R–matrix theory, following two different approaches denoted by R2 and R3 (as explained in the text). Parameters:* $a = 2.9$ fm, $\gamma^2 = 6.9$ MeV *and* $E_0 = -4.3$ MeV.

Incident particle	Incident energy (MeV)	Correlated pair of angles $(\theta_\alpha, \theta_{\rm p})$	$(\sigma_{exp}/\sigma_{cal})$	
			R_2	R_3
		(24.9°, 52°)	1.0	1.0
		(26.3°, 52°)	1.08	1.15
d	18	(32.2°, 52°)	1.17	2.34
		(38.2°, 52°)	0.78	3.40
		(9.8°, 40°)		1.0
		(14.8°, 30°)		0.41
		(14.8°, 50°)		1.46
		(14.8°, 60°)		1.32
		(19.8°, 60°)		1.22
α	42	(19.8°, 50°)		1.02
		(19.8°, 30°)		0.33

Considering the shapes only, the fits due to R3 (solid curves) are remarkably better than those due to R1 and R3 for the first three spectra (Fig. 1a, b, c). For the fourth spectrum (Fig. 1d), the relative heights of the FSI peaks are not well reproduced; rather they are reversed in nature. The values of the theoretical cross–sections deviate considerably from the experimental ones, the normalization factors being from 1.0 to 3.4 (Table 1) for the four spectra studied. When we apply R3 to the 42 MeV data (Fig. 2), we find fairly good fits to the shapes of the spectra, but absolute cross–sections are, again, not well reproduced. Normalization factors range from 0.33 to 1.46 (Table 1) over the seven angle pairs studied.

Comparing with the existing Faddeev theory (FT) fits [9,11] (shown by the dashed–dotted curves), one can note that R3 reproduces shapes of the spectra (Figs. 1 and 2) almost as well as FT. The deviations of FT for smaller peaks in

Fig. 1a, b are reduced by R3 while R2 overestimates the same. Fits due to R3 seem to be better than those due to FT for $(\theta_\alpha, \theta_\mathrm{p}) = (14.8°, 60°)$ and $(14.8°, 50°)$ at 42 MeV (Fig. 2a, b). As shown in Fig. 2f, the slope predicted by FT in the region between the two peak positions is opposite to that of the experimental distribution.

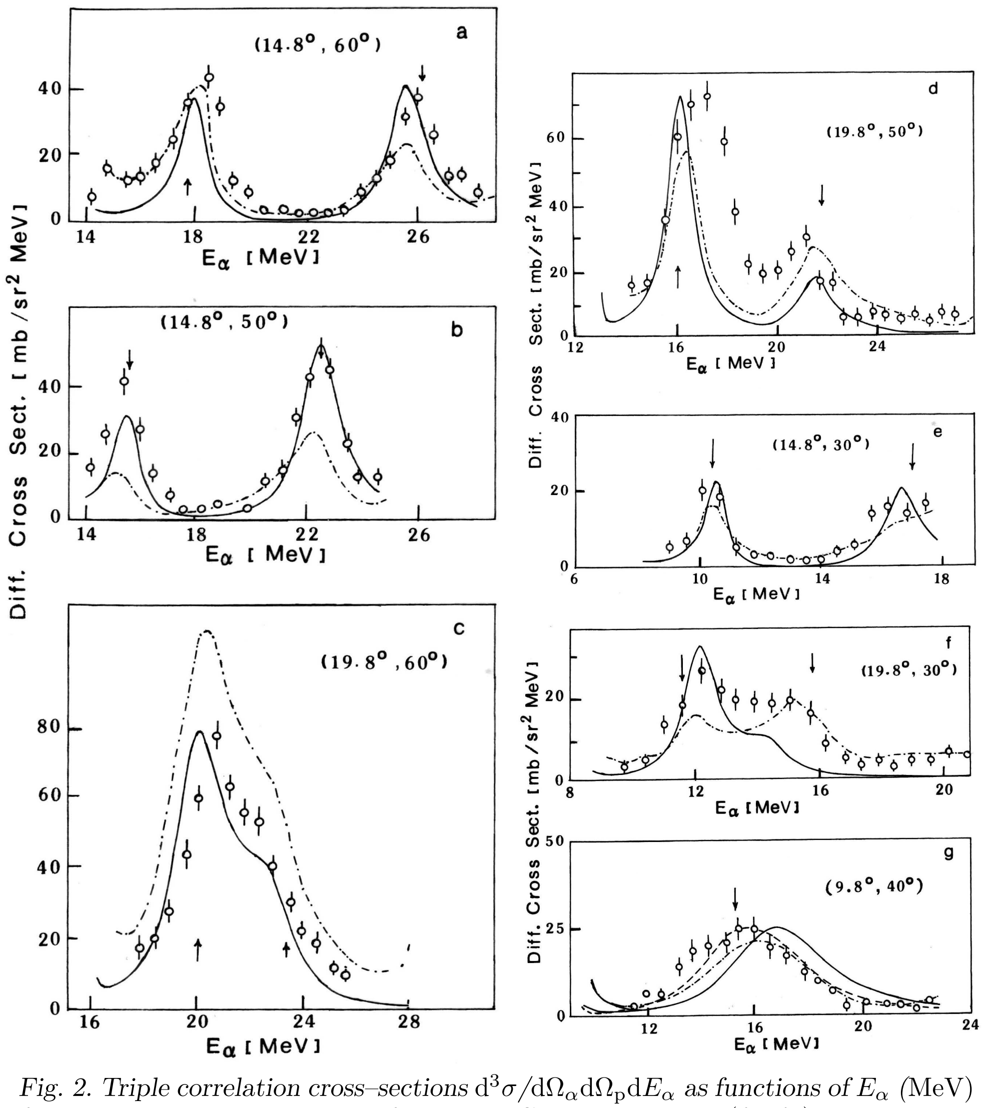

Fig. 2. Triple correlation cross–sections $\mathrm{d}^3\sigma/\mathrm{d}\Omega_\alpha \mathrm{d}\Omega_\mathrm{p} \mathrm{d}E_\alpha$ *as functions of* E_α (MeV) *for incident* α *particle energy of* 42 MeV. *Correlated angles* $(\theta_\alpha, \theta_\mathrm{p})$ *are indicated the figure.* $(\phi_\alpha, \phi_\mathrm{p}) = (180°, 0°)$. *R3 (solid curves) representing present calculations and dashed–doted curves representing FT [11] are as in Fig. 1.*

R3, however, results in too low absolute cross–section around the second peak.

In Fig. 2g, the calculated peak position due to R3 is considerably displaced; FT removes this discrepancy, but the best fit seems to be achieved by R2 (dashed curve). The most difficult case is shown in Fig. 2d. The width of the larger peak is not reproduced by any of the theoretical approaches considered. The distribution of αp and pn relative energies shows that in this region each of αp and pn FSIs plays insignificant role. Similar is the case for αp and αn QFS, as revealed by the distribution of scattered neutron and proton energies, respectively (Fig. 3). Again, the phase space factor (shown by the dashed line), is almost constant in the region of interest. That rules out any enhancement due to kinematic effect. Thus, no apparent reason to such a large discrepancy between theoretical and experimental findings is known.

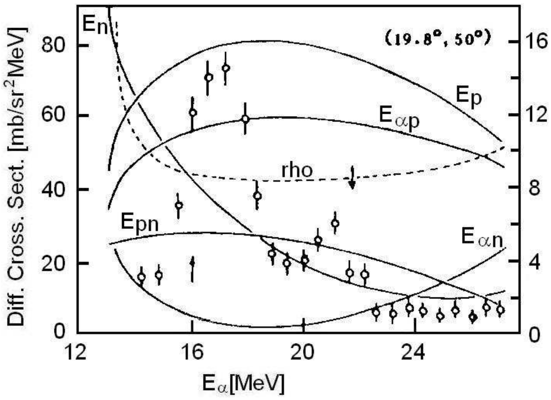

Fig. 3. Differential cross–section as in Fig. 2d. The right scale corresponds to the relative energies of αn, αp *and* pn *pairs and laboratory kinetic energies of scattered neutron and protons. Dashed curve represents phase space factor in arbitrary units.*

As for the reproduction of both the absolute cross–sections and shapes of the spectra by R1, R2 and R3, only R2 is successful to some extent. This is revealed from fits of 18 MeV data of Fig. 1, 8.9 MeV data of Ref. 2, 12.87 and 9.847 MeV data as stated by Bruno et al. [3] and our earlier 42 MeV data [4], but allowing the alteration of input parameters. These involve data sets of four, three, three and ten pairs of correlated angles, respectively. General behaviour of the experimental data is best described by FT. This is also reflected by a better reproduction of the peak width. However, there are several notable discrepancies in FT. We refer the fits at $(\theta_\alpha, \theta_\mathrm{p}) = (14.8°, 60°)$, $(14.8°, 50°)$ and $(19.8°, 50°)$ at 42 MeV (Fig. 2a, b, d), $(\theta_\mathrm{p}, \theta_\alpha) = (52°, 24.9°)$ and $(52°, 26.3°)$ at the lower peak position, at 18 MeV (Fig. 1a, b), the data sets of Ref. 3, where normalization factor runs from 0.45 to 1.6, the 11.3 MeV data of Ref. 12 and a significant amount of data of Ref. 13. We cite from Ref. 13 an example, the data for $E = 11.300$ MeV, $(\theta_\alpha, \theta_\mathrm{p}) = (15.0°, 16.4°)$. The full FT calculation does not reproduce the larger peak at all. Again, when the calculations are done without np FSI, the larger peak is rightly reproduced (allowing a shift along the arc length axis) but a large discrepancy in absolute cross–section is observed for the smaller ^{5}He peak (at 2.7 MeV arc length). Thus, it seems that the effect of the third particle ("spectator") remains to be described properly, especially when the three–body exit channel is dominated by a particular two–body interaction (αn, for our case).

Finally, we get a feeling that for the spectra each of which contains two FSI peaks, there is usually a trend of one of the peaks to be rather badly reproduced than the other. This is true whether the fits are due to FT or R1, R2 or R3. Such findings were encountered neither in Ref. 2 nor in Ref. 3, the inferences of which are being tested in the present paper. Actually, there is no question of such relative judgement. Because, in Ref. 2, the existence of more than one αn FSI peak is ruled out kinematically for each of the spectra studied. And in Ref. 3, FSI peaks corresponding to the upper and lower loci were treated separately.

4. Conclusions

We conclude that none of the different approaches (R1, R2, R3) can describe uniquely the three–body reaction dominated by the αn FSI alone. Still, it is apparent that the success of R2 is comparatively better when the results on absolute cross–sections as well as the line shapes of the spectra are considered. Though the overall structure of the three–particle reaction is best described by FT, discrepancies observed in the αn FSI regions are quite significant for several cases. In some cases, they are comparable to those found from R2 and R3. Better understanding of the two–body interaction, proper treatment of αp Coulomb interaction [5] and inclusion of three–body forces [14,15] may improve the situation.

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USPOREDBENO PROUČAVANJE RAZBIJANJA DEUTERONA ALFA ČESTICAMA

Eksperimentalni se tro–korelacijski udarni presjeci reakcije $\mathrm{d}+\alpha \rightarrow \alpha+\mathrm{p}+\mathrm{n}$ analiziraju raznim metodama na osnovi jednorazinske R–matrične teorije. Istražuju se mnogi spektri u kojima prevladava međudjelovanje α–neutron u konačnom stanju. Dobiveni se rezultati uspoređuju s ranijim računima zasnovanim na Faddeevovoj teoriji.