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RESTRICTIONS ON THE UNIFICATION GAUGE GROUPS FROM THE NEUTRAL CURRENT INTERACTIONS DESCRIPTION IN COUPLING CONSTANT SPACE

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The formalism of polar coupling constants, as applied to the neutral current interactions, indicates that the partial true unification of interactions associated with the gauge group $SU(3)_c \times SU(2)_L \times G$, where G is the generalization of $U(1)_Y$, can go via two independent branches; these branches we call the Glashow and Weinberg-Salam branches. The uniqueness of the photon, vector bosons and neutral current interactions, in general, are guaranteed with respect to these two branches only if $G = U(1)_2 \times \cdots \times U(1)_n$, where the indices are associated with gauge coupling constants $g_2, ..., g_n$ (g_1 is associated with $SU(2)_L$); this follows from the fact that only for this G exists a transformation connecting coupling constants and group generators between the two branches. This fact severely limits the possibilities of unification at higher energy scales. At the energy scale just beyond the Standard Model, $G = U(1)_2 \times U(1)_3$; the effects of this G on some leptonic processes are briefly discussed.

1. Introduction

Here we carry out the polar coupling constants formulation of the partial true unification of interactions involving the gauge group $SU(3)_c \times SU(2)_L \times G$, where G, as yet unknown, is the generalization of Abelian $U(1)_Y$. The polar coupling constants provide an approach to unification of interactions in which the gauge group is not specified beforehand but, rather, its form is determined from the consistency conditions within the approach itself. In this formalism, the coupling angles that come from the $SU(2)_L \times G$ gauge coupling constants, cease being coupling parameters when they are identified as mixing angles, and the spherical neutral vector bosons are identified as physical particles. This, in fact, is a partial true unification of gauge interactions since, in addition to the $SU(3)_c$ coupling constant, only the "radial" coupling constant is a bona fide coupling parameter. Only two spherical vector bosons can play the role of the photon. The one case (the Glashow branch) allows partial true unification of many gauge models. The other case (Weinberg-Salam branch) is very restrictive with G allowed to be $U(1)_2 \times \cdots \times U(1)_n$, where indices are associated with gauge coupling constants $g_2, ..., g_n$ (g_1 is associated with $SU(2)_L$). Now the form for G is determined by simply requiring that the physics be the same regardless of the branch. On the formal level, this means that the photon, vector bosons and neutral current interactions, in general, do not change as one transitions from one branch to the other. Clearly, then $G = U(1)_2 \times \cdots \times U(1)_n$, since only for this G exists a transformation connecting coupling constants and group generators when going from the Glashow to the Weinberg–Salam branch, and vice–versa. If, as it is customary, one assumes that under $G = U(1)_2 \times \cdots \times U(1)_n$ fermions are defined to behave in the same way as in the Standard Model, then the only additional fermion interaction is given in terms of the weak fermion hypercharge, providing that there exists a $U(1)_3$ non–elementary fermion particle. Effectively, as far as fermion interactions are concerned, the gauge group is now $G = U(1)_2 \times U(1)_3$ which represents unification at the energy scale that is just beyond the Standard Model.

The new fermion hypercharge neutral current interaction is mediated by new neutral vector boson Z'. The coupling constant of this new neutral current interaction is allowed to be even smaller than the electromagnetic coupling constant. Also, from mass relations, $M_{z'}$, the mass of Z'–boson, need not be very large. As a consequence, the effects of this neutral current interaction on the phenomenological level (with amplitudes evaluated in the Born approximation) on, say, leptonic processes, is essentially of the same order of magnitude as the effect of the usual electroweak radiative corrections.

Here, the effects of this new hypercharge fermion neutral current interaction are analysed on just few leptonic processes. One concludes that this interaction would alter angular and energy distributions and total cross–sections of $\overset{(-)}{\nu}_e e \to \overset{(-)}{\nu}_e e$ and $e^+e^- \to \nu\overline{\nu}'s$ reactions. From the phenomenological analysis of $\overset{(-)}{\nu}_e e \to \overset{(-)}{\nu}_e e$ Born approximation cross–sections, we conclude that the mass of the Z' neutral vector boson should be in the range $M_Z < M_{Z'} < 7M_Z$, where M_Z is the Standard Model neutral vector boson mass.

In Section 2 the partial true unification of interactions is discussed as occurring simultaneously in the two branches of the coupling constant space, the Glashow and Weinberg–Salam branches. Here it is shown that these two branches are the consequence of the fact that the photon can be described by two independent neutral vector boson (coupling constant) polar components. Higher energy scale partial true unification is discussed in Section 3, where the uniqueness of the interactions demands $G = U(1)_2 \times \cdots \times U(1)_n$. Section 4 is devoted to deriving the mass relations among vector bosons. Just beyond the Standard Model, a new fermion hypercharge neutral current interaction is derived in Section 5, where it is assumed that fermion quantum numbers are the same as in the Standard Model. Here also the effects of this fermion hypercharge interaction is evaluated for $\overset{(}{\overline{\nu}}\overset{)}{}_e e \to \overset{(}{\overline{\nu}}\overset{)}{}_e e$ and $e^+e^- \to \nu\overline{\nu}'s$ reactions. In Section 6 the discussion and conclusion are given.

2. Partial true unification with two independent branches

Following Georgi and Weinberg [1], we start the discussion of the partial true unification of the gauge group $SU(3)_c \times SU(2)_L \times G$ by writing the expression for the charge operator (defined in terms of generators of the gauge group $SU(2)_L \times G$) and the neutral part of gauge interaction Lagrangian involving fermions:

$$Q = T_{3L} + \frac{Y}{2}, \quad \frac{Y}{2} = \sum_{i=2}^{n} K_i T_i, \quad \mathcal{L}(\text{neutral}) = \sum_{i=1}^{n} g_i A_i^{\mu} \overline{\Psi} \gamma_{\mu} t_i \Psi, \qquad (2.1, 2.2)$$

where Y is the weak hypercharge operator. $SU(2)_L \times G$ involves n gauge coupling constants: $g_1 \equiv g$ from the $SU(2)_L$ and $g_2, ..., g_n$ from G. Here A_i^{μ}, $i = 1, ..., n$ are neutral gauge fields, t_i are matrices (which may depend on γ_5) that represent T_i^f, the fermion part of T_i; T_i are Hermitean electrically neutral generators of $SU(2)_L$ and G. A_1^{μ} is the third component of the $SU(2)_L$ vector boson triplet: W_i^{μ}; $i = 1, 2, 3$; $A_1^{\mu} = W_3^{\mu}$, $T_1 = T_{3L}$ with T_{iL}, $i = 1, 2, 3$, being the generators of $SU(2)_L$. If G contains Abelian subgroup $U(1)_l$ (where subscript l is associated with gauge coupling constant g_l) with generator $T_l \equiv Y_l$, then, because the structure constants are absent, one may, consistent with expression (2.1) for Q, define a new $\tilde{Y}_l$, and $\tilde{g}_l$, as

$$g_l Y_l = \tilde{g}_l \tilde{Y}_l; \ \ \tilde{g}_l = \frac{g_l}{K_l}, \ \ \tilde{Y}_l = K_l Y_l. \qquad (2.3)$$

Relations (2.3) reflect the "invariance" of the $U(1)_l$ gauge coupling.

The Cartesian, $g_1, ..g_n$, and polar, $\theta_0 \equiv r, \theta_1, ..., \theta_{n-1}$, components of coupling constant vector $\vec{g}$ are related as

$$g_i = \theta_0 C_{i\theta_0}, \ \ i = 1, ..., n; \ \ \theta_0^2 \equiv r = \sum_{i=1}^{n} g_i^2,$$

$$C_{1\theta_0} = \cos\theta_1, \ \ C_{i\theta_0} = \sin\theta_1 \cdots \sin\theta_{i-1} \cos\theta_i, \ \ i = 2, ..., n-1,$$

$$C_{n\theta_0} = \sin\theta_1 \cdots \sin\theta_{n-1}. \qquad (2.4a)$$

The non–zero (diagonal) components of the metric tensor in the (orthogonal) polar coupling constant space are

$$\alpha = 0: \quad \eta_{\theta_0\theta_0} = 1; \alpha = 1: \quad \eta_{\theta_1\theta_1} = \theta_0^2,$$

$$\alpha = 2, ..., n-1: \quad \eta_{\theta_\alpha\theta_\alpha} = \sum_{i=1}^{n} \left(\frac{\partial g_i}{\partial \theta_\alpha}\right)^2 = \theta_0^2 \prod_{\beta=2}^{\alpha} \sin^2\theta_{\beta-1}. \qquad (2.4b)$$

Since $C_{i\theta_0} = \partial g_i/\partial\theta_0$, one introduces $C_{i\theta_\alpha}$ as the transformation coefficients that relate the Cartesian and polar unit vectors, $\hat{g}\cdot\hat{\theta}_\alpha = C_{i\theta_\alpha}$:

$$C_{i\theta_\alpha} = \frac{1}{\sqrt{\eta_{\theta_\alpha\theta_\alpha}}}\frac{\partial g_i}{\partial\theta_\alpha} = \sqrt{\eta_{\theta_\alpha\theta_\alpha}}\frac{\partial\theta_\alpha}{\partial g_i}, \qquad (2.4c, d)$$

where the second equality is due to the invariance of the line element. Explicitly,

$$C_{i\theta_\alpha} = \frac{1}{\sqrt{\eta_{\theta_\alpha\theta_\alpha}}}\left(\delta_{0\alpha}C_{i\theta_0} + \theta_0\frac{\partial C_{i\theta_0}}{\partial\theta_\alpha}\right) = \frac{1}{\sqrt{\eta_{\theta_\alpha\theta_\alpha}}}C_{i\theta_0}\cdot$$

$$\cdot\left\{\delta_{0\alpha} + \theta_0(1-\delta_{0\alpha})\left[(\delta_{\alpha(i-1)} + \delta_{\alpha(i-2)} + ...)\cot\theta_\alpha - (1-\delta_{in})\delta_{i\alpha}\tan\theta_\alpha\right]\right\},$$

$$\alpha = 0, ..., n-1, \;\; i = 1, ..., n, \;\; C_{i\theta_\alpha} = 0, \;\; \alpha > i \neq n; \;\; C_{n\theta_\alpha} = 0, \;\; \alpha \geq n. \qquad (2.4e)$$

From relations (2.4c,d), one deduces straightforwardly the orthonormal relations:

$$\sum_{i=1}^{n} C_{i\theta_\alpha}C_{i\theta_\beta} = \delta_{\alpha\beta}, \quad \sum_{i=1}^{n-1} C_{i\theta_\alpha}C_{j\theta_\alpha} = \delta_{ij}. \qquad (2.4f)$$

Next, Cartesian, A_i^μ, and polar, $Z_{\theta_\alpha}^\mu$, components of the (free) neutral vector boson–vector in the n–dimensional coupling constant space are related as ($\xi_i^2 = 1$)

$$Z_{\theta_\alpha}^\mu = \sum_{i=1}^{n} C_{i\theta_\alpha}\xi_i A_i^\mu, \quad A_i^\mu = \xi_i\sum_{\alpha=0}^{n-1} C_{i\theta_\alpha}Z_{\theta_\alpha}^\mu. \qquad (2.5a, b)$$

Assigning $Z_{\theta_\alpha}^\mu$ as the physical neutral vector boson fields, one has for $\mathcal{L}$(neutral)

$$\mathcal{L}(\text{neutral}) = \sum_{\alpha=0}^{n-1}\mathcal{L}(NC;\theta_\alpha), \quad \mathcal{L}(NC;\theta_\alpha) = \overline{\Psi}\gamma_\mu(NC)_\alpha\Psi Z_{\theta_\alpha}^\mu, \qquad (2.6a, b)$$

where $(NC)_\alpha$ is the generic fermion neutral current projection operator:

$$(NC)_\alpha = \theta_0\sum_{i=1}^{n}\xi_i T_i^f C_{i\theta_0}C_{i\theta_\alpha} \qquad (2.7a)$$

$$= \theta_0 \frac{1}{\sqrt{\eta_{\theta_\alpha \theta_\alpha}}} \sum_{i=1}^{n} \xi_i T_i^f (C_{i\theta_0})^2 \cdot$$

$$\cdot \left\{ \delta_{0\alpha} + \theta_0 (1 - \delta_{0\alpha}) \left[(\delta_{\alpha(i-1)} + \delta_{\alpha(i-2)} + ...) \cot\theta_\alpha - \delta_{\alpha i} \tan\theta_\alpha \right] \right\}, \qquad (2.7b)$$

where $C_{i\theta_\alpha}$ was used in the form (2.4e).

At this point one chooses the photon field to be, say, $Z^\mu_{\theta_\beta}$. One must have that

$$(NC)_\beta = \xi_e e Q_f, \quad e > 0; \quad \mathcal{L}(em) = \mathcal{L}(NC; \theta_\beta) = \xi_e e \overline{\Psi} \gamma_\mu Q_f \Psi A^\mu, \quad A^\mu \equiv Z^\mu_{\theta_\beta}, \qquad (2.8a, b)$$

where Q_f is the fermionic part of Q and $\xi_e^2 = 1$. Relation (2.7a) for $\alpha = \beta$ yields the condition

$$\theta_0(\beta) \xi_i(\beta) C_{i\theta_0}(\beta) C_{i\theta_\beta}(\beta) = e \xi_e(\beta) K_i(\beta), \qquad (2.9a)$$

which insures that $(K_1(\beta) \equiv 1)$,

$$\sum_{i=1}^{n} K_i(\beta) T_i^f = Q_f. \qquad (2.9b)$$

However, since $C_{1\theta_\beta} = 0$, $\beta > 1$ and $C_{1\theta_\beta} \neq 0$, $\beta = 0, 1$, one can only have that,

$$Z^\mu_{\theta_\beta} = A^\mu, \quad \text{for} \quad \beta = 0, 1 \quad \text{only}. \qquad (2.9c)$$

This defines two branches of partial true unification: $Z^\mu_{\theta_0} = A^\mu$ and $Z^\mu_{\theta_1} = A^\mu$, which we call the Glashow ($\beta = 0$) and Weinberg–Salam ($\beta = 1$) branches, respectively. As a consequence, the quantities will be different for $\beta = 0$ and $\beta = 1$ and, as such, formally dependent on β; $\theta_\alpha(\beta)$, $K_i(\beta)$, etc., which was already started in relation (2.9a). However, regardless of the branch, consistent with relations (2.1) and (2.2), the Weinberg angle is defined as

$$s_W = \frac{e}{g_1(\beta)} \to e = \theta_0(\beta) c_1(\beta) s_W, \quad \beta = 0, 1; \quad g_1(0) = g_1(1) \equiv g, \qquad (2.9d)$$

where in general $s_\alpha(\beta) = \sin\theta_\alpha(\beta)$, etc., with $\beta = 0, 1$.

Relations (2.4) and (2.9) determine coefficients K for the branches. Taking that $\xi_i(0) = \xi_e(0)$, from (2.9a), one obtains for the Glashow branch:

$$K_i(0) = \frac{(C_{i\theta_0})^2}{c_1(0) s_W}, \quad K_1(0) = 1 \to c_1(0) = s_W. \qquad (2.10a, b)$$

This, with the help of relation (2.4a), specify $K_i(0)$ as:

$$K_\alpha(0) = (\cot\theta_W s_2(0) s_3(0) \cdots s_{\alpha-1}(0) c_\alpha(0))^2, \quad \alpha = 2, ..., n-1,$$

$$K_n(0) = (\cot\theta_W s_2(0)s_3(0)\cdots s_{n-1}(0))^2, \qquad (2.10c,d)$$

$$\cot^2\theta_W \geq K_i(0) \geq 0, \quad i = 2, ..., n. \qquad (2.10e)$$

Similarly, from relations (2.4a) and (2.9a) for the Weinberg–Salam branch one deduces:

$$K_i(1) = \frac{\xi_i(1)\xi_e(1)C_{i\theta_0}(1)C_{i\theta_1}(1)}{c_1(1)s_1(1)}$$

$$= \frac{\xi_i(1)\xi_e(1)(C_{i\theta_0}(1))^2}{c_1(1)s_1(1)}\left\{\left[\delta_{1(i-1)} + \delta_{1(i-2)} + ...\right]\cot\theta_1(1) - (1-\delta_{in})\delta_{i1}\tan\theta_1(1)\right\}. \qquad (2.11a)$$

From this equation

$$K_1(1) = 1 \to -\xi_1(1) = \xi_e(1). \qquad (2.11b)$$

Next, with $C_{1\theta_0}(1) = -s_1(1)$, combining relations (2.9e) and (2.11b) with (2.9a), one obtains

$$-\xi_1(1)\theta_0(1)c_1(1)s_1(1) = \xi_e(1)\theta_0(1)c_1(1)s_W \to s_1(1) = s_W. \qquad (2.11c)$$

This now allows writing down detailed expressions for $K_i(1)$:

$$K_\alpha(1) = [s_2(1)s_3(1)\cdots c_\alpha(1)]^2, \quad \alpha = 2, ..., n-1, \qquad (2.11d)$$

$$K_n(1) = [s_2(1)s_3(1)\cdots s_{n-1}(1)]^2, \quad 1 \geq K_i(1) \geq 0, \;\; i = 2, ..., n. \qquad (2.11e,f)$$

Here, we generalized $-\xi_1(1) = \xi_e(1)$ into $-\xi_1(1) = \xi_2(1) = ... = \xi_n(1) = \xi_e(1)$ and one should notice that the $K(1)$'s do not depend on $s_1(1) = s_W$. The relation with theory of Glashow, Weinberg, and Salam is, of course, only through the first angle: $c_1(0) = s_1(1) = s_W$ [2].

Using relations (2.7) one writes down the generic fermion neutral current projection operators for the Glashow branch as:

$$(NC(0))_0 = \theta_0(0)\xi_e(0)\sum_{i=1}^{2}(C_{i\theta_0}(0))^2 T_i^f(0) = e\xi_e(0)\sum_{i=1}^{n}K_i(0)T_i^f(0) = e\xi_e(0)Q_f, \qquad (2.12a)$$

$$\alpha \geq 1: \quad (NC(0))_\alpha = (\eta_{\theta_\alpha\theta_\alpha}(0))^{-1/2}\xi_e(0)e\theta_0(0)\cdot$$

$$\cdot\left\{\left[K_{\alpha+1}(0)T_{\alpha+1}^f(0) + K_{\alpha+2}(0)T_{\alpha+2}^f(0) + \cdots\right]\cot\theta_\alpha(0) - K_\alpha(0)T_\alpha^f(0)\tan\theta_\alpha(0)\right\}$$

$$= \frac{(\eta_{\theta_\alpha\theta_\alpha}(0))^{-1/2}\xi_e(0)e\theta_0(0)}{c_\alpha(0)s_\alpha(0)}\cdot$$

$$\cdot\left\{\left[Q_f-\left(T^f_{3L}+K_2(0)T^f_2(0)+\cdots+K_{\alpha-1}(0)T^f_{\alpha-1}(0)\right)\right]c^2_\alpha(0)-K_\alpha(0)T^f_\alpha(0)\right\}, \tag{2.12b}$$

where Q_f denotes the fermion charge operator regardless of the branch. Specifically, for $\alpha = 1$ one obtains,

$$(NC(0)_1\equiv(NC(0))_{NC}=-\frac{\xi_e(0)e}{s_W c_W}[T^f_{3L}-s^2_W Q_f],\quad Z^\mu_{\theta_1}\equiv Z^\mu. \tag{2.12c}$$

Similarly, by taking into account relation (2.11a), one obtains the fermion neutral current projection operators for the Weinberg–Salam branch:

$$\alpha=0:\quad (NC(1))_0=\theta_0(1)\xi_e(1)\left\{-T_{3L}(C_{1\theta_0}(1))^2+s^2_W\sum_{i=2}^n K_i(1)T^f_i(1)\right\}$$

$$=-\frac{\xi_e(1)e}{s_W c_W}[T^f_{3L}-s^2_W Q_f],\quad Z^\mu_{\theta_0}\equiv Z^\mu. \tag{2.13a}$$

$$\alpha\geq 1:\quad (NC(1))_\alpha=(\eta_{\theta_\alpha\theta_\alpha}(1))^{-1/2}\xi_e(1)(\theta_0(1))^2\cdot$$

$$\cdot\left\{\left[\delta_{\alpha 1}T^f_{3L}(C_{1\theta_0}(1))^2-(1-\delta_{\alpha 1})s^2_W K_{\alpha 1}(1)T^f_\alpha(1)\right]\tan\theta_\alpha(1)\right.$$

$$\left.+s^2_W\left[K_{\alpha+1}(1)T^f_{\alpha+1}(1)+K_{\alpha+2}(1)T^f_{\alpha+2}(1)+\cdots\right]\cot\theta_\alpha(1)\right\}, \tag{2.13b}$$

where, specifically for $\alpha = 1$,

$$(NC(1))_1=\theta_0(1)\xi_e(1)c_W s_W\left\{T^f_{3L}+\sum_{i=2}^n K_i(1)T^f_i(1)\right\}=\xi_e(1)eQ_f. \tag{2.13c}$$

2.1. $n = 2$: *The Standard Model*

Here, $n = 2$ and the electromagnetic and Standard Model neutral current interactions are reversed from each other when going from one to another branch. Namely, taking that $\xi_e(0) = \xi_e(1) = \xi_e$, $\xi_e^2 = 1$, one obtains from relations (2.12) and (2.13) that

$$(NC(0))_0=(NC(1))_1\equiv(NC)_{em}=\xi_e eQ,\quad A^\mu=Z^\mu_{\theta_0(0)},Z^\mu_{\theta_1(1)}, \tag{2.14a}$$

$$(NC(0))_1=(NC(1))_0\equiv(NC)_{NC}=-\frac{\xi_e e}{s_W c_W}[T^f_{3L}-s^2_W Q_f],\quad Z^\mu=Z^\mu_{\theta_1(0)},Z^\mu_{\theta_0(1)}. \tag{2.14b}$$

Clearly, these two interactions are of the same importance.

3. Determining the form of the gauge group

Clearly, as the number of coupling constants, n, increases (at higher the energy scale), the two branches will have a potential to significantly differ from each other. This follows from the fact that the K–coefficients are so different in the two branches. To see that, for the sake of simplicity, one assigns common angles for $i \geq 2$ in both branches:

$$g_1(0) = g_1(1) \equiv g; \ \ \theta_i(0) = \theta_i(1) \equiv \theta_i, \ \ i = 2, ..., n, \tag{3.1a}$$

while still having $c_1(0) = s_1(1) = s_W$. Directly from relations (2.4a), (2.10) and (2.11), one obtains,

$$\frac{g_i^2(0)}{g_i^2(1)} = \frac{\theta_0^2(0)s_1^2(0)}{\theta_0^2(1)s_1^2(1)} = \frac{K_i(0)}{K_i(1)}\cot^2\theta_W = \cot^4\theta_W, \quad i = 2, ..., n. \tag{3.1b}$$

Furthermore, taking into account expressions for K's, relations (2.10) and (2.11), we have

$$0 < K_i(\beta) < \delta_{\beta 0}\cot^2\theta_W + \delta_{\beta 1}, \ \ i = 2, ..., n, \ \ n > 2, \ \ \beta = 0, 1, \tag{3.2a}$$

$$\frac{Y}{2} = \sum_{i=2}^{n} K_i(\beta)T_i(\beta), \quad \sum_{i=2}^{n} K_i(\beta) = \delta_{\beta 0}\cot^2\theta_W + \delta_{\beta 1}, \tag{3.2b,c}$$

where the hypercharge operator Y is independent of $\beta = 0, 1$.

It is obvious that since $K_i(0) = K_i(1)\cot^2\theta_W$, the only way the expression for $Y/2$, relation (3.2b), can be satisfied simultaneously for both, the Glashow and Weinberg–Salam branches, is that $T_i(0)$ and $T_i(1)$ are related as

$$T_i(1) = \frac{K_i(0)}{K_i(1)}T_i(0) = T_i(0)\cot^2\theta_W, \ \ i = 2, ..., n. \tag{3.2d}$$

But relation (3.2d) is nothing more than rescalling of generators; as the non–Abelian generators are fixed by normalization condition this can be done only with the Abelian U(1) generators Y_i. Thus, we conclude that higher energy scale partial true unification is simultaneously describable in both branches only if

$$\mathrm{G} = \mathrm{U}_2(1) \times \cdots \times \mathrm{U}_n(1), \ \ T_i(\beta) \equiv Y_i(\beta), \ \ i > 2, \ \ \beta = 0, 1. \tag{3.3}$$

What kind of higher energy scale partial unifications are allowed separately in the two branches?

First, suppose that the non–Abelian generators $T_i(\beta)$ are involved. Group theory invariably gives for the corresponding $K_i(\beta) = 1$. Only $\beta = 0$, the Glashow branch, can accomodate here; namely, with a suitable choice of angles $\theta_2(0), \theta_3(0)$,..., every

$K_i(0)$, $i = 2, ..., n$, except one, can be scaled to 1. For one $K_i(0)$ that cannot be scaled to 1, one must be able to rescale the corresponding $T_i(0)$ to accomodate the relation for Y; at least one generator $T_i(0)$ has to be the Abelian generator. Hence, for the Glashow branch we write

$$\mathrm{G}(\beta = 0) = \mathrm{G}' \times \mathrm{U}'(1);$$

an example for $n = 3$ being $\mathrm{G}' = \mathrm{SU}(2)_R$, $\mathrm{U}'(1)$=$\mathrm{U}(1)_{B-L}$.

For $\beta = 1$, relations (2.11) indicate that $K_i(1) < 1$, $i = 2, ..., n$, do not depend on θ_W; no $K_i(1)$ can be rescaled to 1. Hence, every generator $T_i(1)$ of $\mathrm{G}(\beta = 1)$ must be rescalable in order to accomodate the expression for $Y/2$. That means that $T_i(1)$ is the Abelian generator of $\mathrm{U}_i(1)$. Equivalently, we have for the Weinberg–Salam branch that

$$\mathrm{G}(\beta = 1) = \mathrm{U}(1)_2 \times \cdots \times U(1)_n.$$

Why these two branches are so different at higher energy scales is indeed puzzling. Different gauge groups that go with these two branches are addressed elsewhere [3]. Here we discuss only the gauge groups that are admitted by both of these two branches.

3.1. Uniqueness of interactions

It is very reasonable to require that the physics be the same regardless of the branches that are used for the description, meaning that the interactions be the same (unique) when described in either of the branches. This is the essence of the uniqueness of the partial true unification of interactions. The uniqueness requirement restricts G to just $\mathrm{U}(1)_2 \times \cdots \times \mathrm{U}(1)_n$ at an energy scale that corresponds to $n > 2$. Let us point out that relation (3.2d) which now explicitly reads as

$$K_i(0)Y_i(0) = K_i(1)Y_i(1) \equiv K_i(\beta)Y_i(\beta), \;\; i = 2, ..., n; \;\; \beta = 0, 1, \qquad (3.4)$$

is consistent with the equality of gauge couplings $Y_i(0)g_i(0) = Y_i(1)g_i(1)$, $i \geq 2$.

Clearly, these facts assure the uniquenesses of the photon, Standard Model neutral vector boson, as well as of neutral current interactions at any energy scale, regardless of whether this gauge group is viewed in the Glashow or the Weinberg–Salam branch.

As a consequence, one may simply write for the neutral current projection operators for either of the branches ($\beta = 0, 1$) the equivalent expressions:

$$(NC(0))_i = (NC(1))_i = (NC(\beta))_i \equiv (NC)_i, \;\; \beta = 0, 1, \;\; i = 2, ..., n. \qquad (3.5)$$

3.2. The second neutral current interaction

Having established that it is $\mathrm{G} = \mathrm{U}(1)_2 \times \cdots \times \mathrm{U}(1)_n$ (with $n \geq 2$) that corresponds to some unification scale, the question is: what kind of neutral current interaction one finds just beyond the Standard Model? To answer this question, one simply has to look at $(NC)_2$. As relation (3.5) indicates, it makes no difference in which branch we look; let us settle for the Glashow branch, $\beta = 0$. Specifying $\alpha = 2$ in (2.12b) and taking into account relations (3.1c), one obtains

$$(NC(0))_2 = \frac{\xi_e e}{c_W} \left\{ Y_f \cot\theta_2 - \frac{K_2(0) Y_2^f(0)}{s_2 c_2} \right\},$$

which, with relations

$$\frac{Y}{2} = \sum_{i=2}^{n} K_i(\beta) T_i(\beta), \quad \beta = 0, 1, \tag{3.6}$$

becomes

$$(NC)_2 = \frac{\xi_e e}{c_W} \left\{ Y_f \cot\theta_2 - \frac{K_2(\beta) Y_2^f(\beta)}{s_2 c_2} \right\}, \quad \beta = 0, 1. \tag{3.7}$$

Here, when "evaluating" the right side, one still has to specify β. Of course, the branch with $\beta = 1$ also could have been derived directly by starting with Eq. (2.13b). In any case, the next level neutral current interaction requires the knowledge of an additional quantum number $Y_2^f(\beta)$, $\beta = 0$ or 1.

4. *Mass relations and restrictions on angles*

In either of the branches, $\beta = 0, 1$, there are two mass terms, the Cartesian and the polar mass terms; the polar mass term is the result of diagonalization of the Cartesian mass term:

$$\mathcal{L}(\text{mass}) = -\frac{1}{2} \sum_{i,j=1}^{n} A^i_\mu M^2_{ij} A^\mu_j, \tag{4.1a}$$

$$= -\frac{1}{2} \sum_{\alpha,\beta=0}^{n-1} Z^{\theta_\alpha}_\mu M^2_{\theta_\alpha\theta_\beta} Z^\mu_{\theta_\beta}, \tag{4.1b}$$

where Z_{θ_α} denotes the physical free neutral vector boson field. Utilizing the transformation properties of these fields, relations (2.5) and (2.4), one has:

$$M^2_{\theta_\alpha\theta_\beta} = \delta_{\alpha\beta} \sum_{i,j=1}^{n} \xi_i \xi_j C_{i\theta_\alpha} M^2_{ij} C_{i\theta_\alpha} = \delta_{\alpha\beta} M^2_{\theta_\alpha}, \tag{4.2a,b}$$

$$M^2_{ij} = \xi_i \xi_j \sum_{\alpha,\beta=0}^{n-1} C_{i\theta_\alpha} M^2_{\theta_\alpha\theta_\beta} C_{j\theta_\beta} = \xi_i \xi_j \sum_{\alpha=0}^{n-1} C_{i\theta_\alpha} M^2_{\theta_\alpha} C_{j\theta_\alpha} = M^2_{ji}. \tag{4.3a,b,c}$$

As a consequence, one also has these generic relations:

$$M_{ii}^2 = \sum_{\alpha=0}^{n-1} (C_{i\theta_\alpha})^2 M_{\theta_\alpha}^2, \quad \sum_{i=1}^{n} M_{ii}^2 = \sum_{\alpha=0}^{n-1} M_{\theta_\alpha}^2. \tag{4.4a, b}$$

Relations (4.1) to (4.4) are generic; valid for either of the branches, no matter what gauge group G is in either of these branches.

However, we proceed with gauge group G = $U(1)_2 \times \cdots \times U(1)_n$ as the one that is allowed if the interactions are to be unique in either of the branches, $\beta = 0, 1$. The energy scale just beyond the Standard Model is already reached with $n = 3$. Then, remembering that $c_\alpha(\beta = 0, 1) = c_\alpha$, $s_\alpha(\beta = 0, 1) = s_\alpha$, $\alpha \geq 2$; $s_1(0) = c_W$, $c_1(1) = c_W$, one finds that for Glashow branch ($\beta = 0$), for example, relation (2.4e) gives:

$$C_{1\theta_0}(0) = s_W, \quad C_{1\theta_1}(0) = c_W, \quad C_{1\theta_2}(0) = 0,$$

$$C_{2\theta_0}(0) = c_W c_2, \quad C_{2\theta_1}(0) = s_W c_2, \quad C_{2\theta_2}(0) = -s_2,$$

$$C_{3\theta_0}(0) = c_W s_2, \quad C_{3\theta_1}(0) = s_W s_2, \quad C_{3\theta_2}(0) = c_2.$$

These, in turn, when substituted into (4.3b) yield,

$$M_W^2 = s_W^2 M_\gamma^2 + c_W^2 M_Z^2,$$

$$M_{22}^2 = c_W^2 c_2^2 M_\gamma^2 + s_W^2 c_2^2 M_Z^2 + s_2^2 M_{\theta_2}^2,$$

$$M_{33}^2 = c_W^2 s_2^2 M_\gamma^2 + s_W^2 s_2^2 M_Z^2 + c_2^2 M_{\theta_2}^2, \tag{4.5a, b, c}$$

where $M_{\theta_0} = M_\gamma = 0$, $M_{\theta_1} = M_Z$, $M_{\theta_2} = M_{Z'}$, and $M_{11} = M_W$ ($M_{22} = M_B$). Here M_{11} is the Cartesian mass of $A_1^\mu = W_3^\mu$ and, as such, of the whole $SU(2)_L$ triplet $\vec{W}^\mu$. Relation (4.5a) is the Weinberg mass relation. Exactly the same mass relations follow from the Weinberg–Salam branch ($\beta = 1$). Furthermore, from relations (4.5b,c) one also has,

$$M_{22}^2 - s_W^2 M_Z^2 = s_2^2 (M_{Z'}^2 - s_W^2 M_Z^2), \tag{4.6a}$$

$$M_{33}^2 - s_W^2 M_Z^2 = c_2^2 (M_{Z'}^2 - s_W^2 M_Z^2), \tag{4.6b}$$

from which one has at once:

$$\tan^2 \theta_2 = \frac{M_{22}^2 - s_W^2 M_Z^2}{M_{33}^2 - s_W^2 M_Z^2}. \tag{4.6c}$$

These relations hold for $n = 3$; that is, they hold at the energy scale just beyond the Standard Model. In fact, it is easily seen that s_2 is energy scale dependent.

Namely, for the Standard Model ($n = 2$), we have $M_{22} \equiv M_B = s_W M_Z$ and from (4.6a) we have $s_2 = 0$; but now relation (4.5c) gives $M_{33} = M_{Z'}$. At the energy scale just beyond the Standard Model ($n = 3$) we expect $M_{Z'} > M_Z$, $M_W, M_B(= M_{22})$. Then it should also be expected that $M_{33}^2 > M_{22}^2 > M_Z^2$. Hence, we have that,

$$\tan\theta_2 < 1 \to 0 < s_2^2 < \frac{1}{2}, \quad r = \frac{M_Z}{M_{Z'}} < 1, \qquad (4.6d, e)$$

that is, s_2 is similar to s_W.

5. *Reducing the second neutral current interaction to the fermion hypercharge current interaction*

The second neutral current interaction, relation (3.7), generally requires the knowledge of the additional quantum number Y_2. However, as is customary, one demands that under gauge group $\mathrm{U}(1)_2 \times \cdots \times \mathrm{U}(1)_n$ fermions behave in the same way as in the Standard Model [4]. Here this would mean that for fermions $\mathrm{U}(1)_2^f = \mathrm{U}(1)_Y$ and, as a consequence, the fermions have no quantum numbers associated with $\mathrm{U}(1)_i$, $i = 3, ..., n$; $Y_i^f(\beta = 0, 1) = 0$ for $i \geq 3$. Then, according to relation (3.6),

$$\frac{Y_f}{2} = K_2(\beta) Y_2^f(\beta), \quad \beta = 0, 1,$$

where it is worthwhile remembering that for $n \geq 3$, $K_2(1) = c_2^2$, $K_3(1) = (s_2 c_2)^2$, etc. ($K_2(0) = (\cot\theta_W c_2)^2$, $K_3(0) = (\cot\theta_W s_2 c_2)^2$, etc.). Specifically, for $n = 3$, $s_2 \neq 0$, $s_3 = s_4 = ... = 0$. Hence, for $n \geq 3$ there still may be non–elementary fermion particles allowed which, in addition to $Y_2(\beta) \neq 0$, also may have $Y_3(\beta)$, $Y_4(\beta)$, etc. $\neq 0$. An example of such a non–elementary fermion is, of course, a scalar Higgs particle. Its existence with $K_3(\beta = 0, 1) \neq 0$ would assure that $s_2 \neq 0$.

Coming back to the fermions, it is not difficult to see that from (3.7) one has:

$$n \text{ arbitrary}, \quad Y_i^f(\beta = 0, 1) = 0, \quad i \geq 3:$$

$$(NC)_2 \equiv (NC)_Y = -\frac{\xi_e e \tan\theta_2}{c_W} \left(\frac{Y_f}{2}\right). \qquad (5.1a)$$

Substituting this into (2.6b), we obtain

$$\mathcal{L}(NC; \theta_2) \equiv \mathcal{L}_Y = g_Y \overline{\Psi} \gamma_\mu \frac{Y_f}{2} \Psi Z'^\mu, \quad g_Y = -\frac{\xi_e e \tan\theta_2}{c_W}, \quad Z'^\mu \equiv Z_{\theta_2}^\mu, \quad \xi_e^2 = 1. \qquad (5.1b)$$

The Lagrangian density of all fermion neutral current interactions is obtained from relations (2.12) (or (2.13)) and (5.1):

$$\mathcal{L}(NC) = e\xi_e \overline{\Psi}\gamma_\mu Q_f \Psi A^\mu + g_{NC}\overline{\Psi}\gamma_\mu (T_{3L} - s_W^2 Q_f)\Psi Z^\mu + g_Y \overline{\Psi}\gamma_\mu \frac{Y_f}{2}\Psi Z'^\mu, \qquad (5.2a,b,c)$$

$$g_{NC} = -\frac{\xi_e e}{s_W c_W}, \quad g_Y = -\frac{\xi_e e t_2}{c_W} = s_W t_2 g_{NC}, \qquad (5.2d)$$

where $t_2 = \tan\theta_2$. To these we add the fermion charge changing Lagrangian density [5,6]:

$$\mathcal{L}(CC) = \frac{g}{\sqrt{2}}(\overline{\Psi}\gamma_\mu T_{+L}\Psi W_+^\mu + \overline{\Psi}\gamma_\mu T_{-L}\Psi W_-^\mu), \qquad (5.2e)$$

$$T_{\pm L} = T_{1L} \pm \mathrm{i}T_{2L}, \quad W_\pm^\mu = \frac{1}{\sqrt{2}}(W_1^\mu \mp \mathrm{i}W_2^\mu),$$

where $W_\pm$ represent positive and negative charged vector bosons whose common mass M_W we already encountered in mass relations; W_i, $i = 1,2,3$, is $\mathrm{SU}(2)_L$ triplet (see the discussion after relation (2.2)).

The effect of the fermion hypercharge interaction (5.2c) is expected to be small; namely $|g_Y| = 0.26|g_{NC}| = 0.62e$ for $s_2 \approx s_W$, for example. Furthermore, since effective coupling constant g_Y is small, there is no a priori reason for $M_{Z'}$ to be very large.

For purely leptonic processes involving neutrinos, we shall assume that $q^2 \ll M_Z^2, M_{Z'}^2, M_W^2$, where q^μ is the neutrino–electron covariant momentum transfer. We denote with s the total energy squared in the center of mass system and $s = 2m_e E_\nu$ in the laboratory frame. The cross–sections are valid for $m_e^2 \ll s$. The contributions to reactions $\nu_e e \to \nu_e e$ and $e^+e^- \to \nu_e\overline{\nu}_e$ come from (5.2b), (5.2c) and (5.2e) (with Fierz rearrangements); possible contributions from exchanges of Higgs particles are ignored under the assumption that their masses are extremely large. The results for the Born approximation cross–sections are ($G_F^2/\sqrt{2} = g^2/(8M_W^2)$):

$$\sigma(\nu_e e^- \to \nu_e e^-) = \frac{G_F^2 s}{4\pi}(1+A+B), \quad \sigma(\overline{\nu}_e e^- \to \overline{\nu}_e e^-) = \frac{G_F^2 s}{12\pi}(1+A+C), \qquad (5.3a,b)$$

$$s = 2m_e E_\nu, \qquad (5.3c)$$

$$\sigma(e^+e^- \to \nu_\mu\overline{\nu}_\mu \ \text{ or } \ \nu_\tau\overline{\nu}_\tau) = \frac{G_F^2 s}{24\pi}\left(1 - A + \frac{3B+C}{4}\right), \qquad (5.4a)$$

$$\sigma(e^+e^- \to \nu_e\overline{\nu}_e) = \frac{G_F^2 s}{24\pi}\left(1 + A + \frac{3B+C}{4}\right), \quad s = (p_{e^-} + p_{e^+})^2, \qquad (5.4b,c)$$

where

$$A = s_W^2 \left[4 + (t_2 r)^2\right], \quad B = \frac{s_W^4}{3} \left[16 + 20(t_2 r)^2 + 7(t_2 r)^4\right], \tag{5.5}$$

$$C = s_W^4 \left[16 + 28(t_2 r)^2 + 13(t_2 r)^4\right].$$

In order to analyse these cross–sections, we have to settle for value of s_W. We determine it from the mass relation (4.5a),

$$s_W^2 = 1 - \frac{M_W^2}{M_Z^2} \approx 0.226, \tag{5.6}$$

where the values of $M_W \approx 80.22$ GeV and $M_Z \approx 91.173$ GeV [7] were used. Incidentally, according to Sirlin, relation (5.6) represents the "on-shell scheme" definition of s_W [8].

The data on $\nu_e e$ scattering are available with a good precision [9]:

$$\sigma_{\nu_e e}(\exp) = (2.30 \pm 0.42) \frac{G_F^2 s}{4\pi}. \tag{5.7a}$$

Setting $t_2 = 0$ and $s_W^2 = 0.226$ into relation (5.3c), the Born approximation Standard Model value for the cross–section is obtained:

$$\sigma_{\nu_e e}(\mathrm{SM}) = 2.18 \frac{G_F^2 s}{4\pi}. \tag{5.7b}$$

The difference between the two cross–sections we attribute to the contribution from the fermion hypercharge interaction. Hence, equating the difference between (5.7a) and (5.7b) with the difference between (5.3c) and (5.7b), and after solving the quadratic equation for $(t_2 r)^2$ (with $s_W^2 = 0.226$), we obtain

$$t_2 r = 0.45: \quad 0.45 < t_2 < 1 \to 0.17 < s_2^2 < 0.5,$$

$$0.45 < r < 1 \to M_Z < M_{Z'} < 2.2 M_Z. \tag{5.7c}$$

The experimental values for the $\overline{\nu}_e e$ scattering cross–sections are given for two ranges of recoil electron kinetic energy T_e [10]:

$$\sigma_{\overline{\nu}_e e}(\exp) = (2.76 \pm 0.15) \frac{G_F^2 s}{4\pi}, \quad 1.5 < T_e < 3.0 \text{ MeV}, \tag{5.8a}$$

$$\sigma_{\overline{\nu}_e e}(\exp) = (2.82 \pm 0.12) \frac{G_F^2 s}{4\pi}, \quad 3.0 < T_e < 4.5 \text{ MeV}. \tag{5.8b}$$

On the other hand, with $t_2 = 0$ and $s_W^2 = 0.226$, relation (5.3d) yields the Born approximation Standard Model value

$$\sigma_{\overline{\nu}_e e}(\mathrm{SM}) = 2.721 \frac{G_F^2 s}{12\pi}. \tag{5.8c}$$

Equating the difference between (5.3d) and (5.8c) to the differences between (5.8a,b) and (5.8c), and using $s_W^2 = 0.226$, we obtain, respectively,

$$1.5 < T_e < 3.0 \text{ MeV}, \;\; t_2 r = 0.14, \;\; 0.14 < t_2 < 1 \to 0.02 < s_2^2 < 0.5,$$

$$0.14 < r < 1 \to M_Z < M_{Z'} < 7M_Z, \tag{5.8d}$$

$$3.0 < T_e < 4.5 \text{ MeV}, \;\; t_2 r = 0.245, \;\; 0.245 < t_2 < 1 \to 0.006 < s_2^2 < 0.5,$$

$$0.245 < r < 1 \to M_Z < M_{Z'} < 4M_Z. \tag{5.8e}$$

The pair processes $e^+e^- \to \nu_e\overline{\nu}_e$, $\nu_\mu\overline{\nu}_\mu$ and $\nu_\tau\overline{\nu}_\tau$ are important in stellar matter at high temperatures (10^8 to 10^9 K) and moderate densities [9]. The quantity of most interest is Q, the energy loss per cubic centimeter per second which, in addition to $\sigma(e^+e^- \to \nu\overline{\nu}\text{'s})$, also depends on the available densities of e^- and e^+. Unfortunately, the calculations utilize rough approximations making these processes, at present, unsuitable for setting the limits on s_2 and $M_{Z'}$.

6. *Discussion and conclusion*

The partial true unification of gauge group $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{G}$, where G is the generalization of $\mathrm{U}(1)_Y$, can be made through two branches, the Glashow and the Weinberg–Salam branches. At the lower energy scale, these two branches are equivalent. At higher energy scales, while the Glashow branch can accommodate many gauge models, the Weinberg–Salam branch can accommodate only $\mathrm{G} = \mathrm{U}(1)_2 \times \cdots \times \mathrm{U}(1)_n$. Hence, these two branches are equivalent at the higher energy scale only when $\mathrm{G} = \mathrm{U}(1)_2 \times \cdots \times \mathrm{U}(1)_n$. When the elementary fermions behave as Standard Model fermions with respect to $\mathrm{U}(1)_2$ and are invariant under the rest of the U(1)'s, and if there is at least one non–elementary fermion particle $s(x)$ with at least $\mathrm{U}(1)_{2,3}$ quantum numbers different from zero, there will exist a new fermion hypercharge current interaction. If $s(x)$ is a new Higgs boson, it can easily generate the mass $M_{Z'}$ of Z′ [4].

Analysis of purely leptonic processes indicates that mass $M_{Z'}$ of the new neutral vector boson Z′, that mediates the fermion hypercharge neutral current interaction, is in the range accessible to experimental scrutiny in the near future. In this analysis, we cannot use the definition for θ_W according to $\tan\theta_W = g'/g (\equiv g_2(1)/g_1(1))$ at the energy scale just beyond the Standard Model [11]. In fact for $n > 2$ we have explicitly that $g_2(0)/g_1(0) = c_2 \cot\theta_W$ (Glashow branch) and $g_2(1)/g_1(1) = c_2 \tan\theta_W$ (Weinberg–Salam branch).

The fermion hypercharge interaction changes cross–sections $\sigma(\nu_e e)$ and $\sigma(\overline{\nu}_e e)$ by 5% and 4%, respectively, from their Standard Model values. These changes are significant and overall very close to a typical 2% change that one finds, for example, in $\sigma(\nu_e e)$ due to electroweak corrections in the Standard Model [12]. As such, those electroweak corrections also depend on the choice of at least one mass, the Higgs boson mass (accepting that the top quark mass is about 200 GeV/c^2) as our hypercharge interaction corrections depend on the mass $M_{Z'}$.

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OGRANIČENJE UNIFIKACIJSKIH BAŽDARNIH GRUPA OPISOM INTERAKCIJA NEUTRALNIH STRUJA U PROSTORU KONSTANTI VEZANJA

Formalizam polarnih konstanti vezanja, primijenjen na interakcije slabih struja, pokazuje da se parcijalno pravo ujedinjenje razmatranih interakcija može načiniti metodom Glashowa ili Weinberg–Salama. Raspravljaju se rezultati tih pristupa.