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DISSIPATIVE ENVIRONMENTAL EFFECTS AND THE QUANTUM EVOLUTION OF A SPIN SYSTEM

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By considering a retarded discrete time version of the Schrödinger equation, we discuss how such an equation can be used to describe the environmentally induced decay of a spin system when the environment has the general properties of having a continuous unbounded energy spectrum.

1. Introduction

In the past decade, a new approach has been filtering into the general structure of the quantum theory that is motivated by the problem of the interaction of a quantum system with a discrete spectrum with an environment that has an unbounded continuous spectrum. The theories of quantum friction [1], the Schrödinger-Langevin approach [2,3], the quantum evolution of Brownian particle interacting with a thermal environment [4–6] (giving rise to a non-linear Schrödinger equation) have all suggested that dissipative effects on quantum systems leave real measurable signatures that can be described by a few parameters connecting the system to the environment. Also Nanopolous has expounded on the profundity of the ”Procrustean Principle” where in all quantum systems are unavoidably open systems due to the interaction of the particle (local string modes) with the delocalized (non-local) truncated string modes of string theory [7,8]. These studies have lead to corrections to the time evolution of K_L, K_S system which generate small CPT violations and in a cosmological setting give rise to an arrow of time [9,10]. There is also another approach to dissipative quantum systems originally pioneered by Caldirola et. al., suggesting that the dissipative effects of the environment can be understood through the retarded Schrödinger evolution of a modified Schrödinger equation [11,12]. Here the discrete time parameter (τ) expresses the interaction of the system with the environment and, in a sense, can be viewed as a "relaxation time" expressing how the system's wave function responds to a loss of energy to the environment. The environment may have a complicated quantum structure with all of its quantum correlations at a single time, but the evolution of these correlations and the effect they have on a single quantum system may be expressible through a single discrete time parameter τ. In previous studies, we have discussed the non-dissipative Markov environmental effects on a spin system [13–16] wherein the quantum particle has a dynamical evolution driven by the Schrödinger equation, supplemented by Markov environmental effects. The basic motivation for such a study is that small discrete time jumps might be unobservable over long time intervals, but if short enough time intervals are studied through spin polarization procession, chaotic Markov effects might be observable. These studies also suggest that there might be two time scales in quantum physics, one "scaled" by Schrödinger evolution, the second scaled by non-local Markov environmental effects. In what follows, we discuss the time evolution of a dissipative quantum spin system wherein the effect of the environment is expressed in the form of discrete time difference retarded Schrödinger equation. Our study gives definite signatures for the precession frequency in an external magnetic field of a spin system as well as the decay rate of the different components of the x spin-polarization generated by environmental effects. It is hoped that these studies further encourage investigations of quantum systems interacting with a random field of photons or other particles [17] (the environment) and shed light on such questions on the "arrow of time" and the existence of other time scales in physics.

2. Dissipative Quantum Evolution of a Spin System

We begin by considering the retarded discrete time difference version of the Schrödinger equation studied in Ref. 11.

$$H\Psi = i\hbar\left(\frac{\Psi(t)-\Psi(t-\tau)}{\tau}\right), \tag{1}$$

where τ is the discrete time interval.

If H does not depend on t, we may write Eq. (1) as

$$H\Psi = i\hbar\frac{\left(1-\mathrm{e}^{-\tau\partial/\partial t}\right)\Psi(t)}{\tau}.$$

Letting $\Psi = \exp(-\alpha t)\Psi(0)$, where α is a function of H, we have

$$H\mathrm{e}^{-\alpha t}\Psi(0) = i\hbar\left(\frac{1-\mathrm{e}^{\tau\alpha}}{\tau}\right)\mathrm{e}^{-\alpha t}\Psi(0),$$

or

$$-\frac{\mathrm{i}\tau H}{\hbar} = 1 - \mathrm{e}^{\alpha\tau},$$

giving

$$\alpha = \frac{1}{\tau}\ln_{\mathrm{e}}\left(1 + \frac{\mathrm{i}\tau H}{\hbar}\right). \tag{2}$$

We now consider a spin-1 gauge boson of charge -e, in a z component magnetic field. The hamiltonian of the boson is

$$H = \frac{e}{m} S_z B$$

(B is the z-component of the magnetic field), where

$$S_z = \hbar \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \tag{3}$$

As pointed out in Ref. 11, if we apply H to Eq. (1), the ground state will decay along with the excited states. In order to stabilize the ground state, we write

$$(H - H_0)\,\Psi = \mathrm{i}\hbar \left(\frac{\Psi(t) - \Psi(t-\tau)}{\tau}\right), \tag{4}$$

where H_0 is the ground state eigenvalue. From Eqs. (3) and (4),

$$H_0 = -\frac{e\hbar B}{m} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix},$$

and

$$H - H_0 = \frac{e\hbar B}{m} \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{5}$$

The application of Eq. (5) to Eq. (4) will not change any of the dynamic properties of the spin-1 system, such as precession frequencies, since they depend on the difference of the eigenvalues. It will, however, render the ground state stable. Suppose we consider a spin-1 gauge boson in the initial state

$$\Psi(0) = \begin{pmatrix} 1/2 \\ 1/\sqrt{2} \\ 1/2 \end{pmatrix}, \tag{6}$$

with

$$< S_x >= \Psi^+ S_x \Psi = \Psi^+ \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \Psi = \hbar.$$

Using Eq. (2) with $H \to H - H_0$, we have ($\Psi(t) = \mathrm{e}^{-\alpha t}\Psi(0)$),

$$\Psi(t) = \exp\left\{ -\frac{t}{\tau} \ln_\mathrm{e} \left(1 + \frac{\mathrm{i}\tau}{\hbar}(H - H_0) \right) \right\} \Psi(0). \tag{7}$$

Upon substituting Eq. (6) into Eq. (7), we find

$$\Psi(t) = \begin{pmatrix} \frac{1}{2} \exp\left[-\frac{1}{\tau} \ln_\mathrm{e} \left(1 + \frac{\mathrm{i}\tau}{\hbar} \left(\frac{eB2\hbar}{m} \right) \right) t \right] \\ \frac{1}{\sqrt{2}} \exp\left[-\frac{1}{\tau} \ln_\mathrm{e} \left(1 + \frac{\mathrm{i}\tau}{\hbar} \left(\frac{eB\hbar}{m} \right) \right) t \right] \\ \frac{1}{2} \end{pmatrix}. \tag{8}$$

If we write Eq. (8) as

$$\Psi(t) = \begin{pmatrix} \frac{1}{2}\mathrm{e}^{-\alpha_1 t} \\ \frac{1}{\sqrt{2}}\mathrm{e}^{-\alpha_2 t} \\ \frac{1}{2} \end{pmatrix}, \tag{9}$$

we find at time t

$$< S_x >= \Psi^+ S_x \Psi = \frac{\hbar}{4} \left(\mathrm{e}^{-(\alpha_1 + \alpha_2^*)t} + \mathrm{e}^{-(\alpha_1^* + \alpha_2)t} + \mathrm{e}^{-\alpha_2 t} + \mathrm{e}^{-\alpha_2^* t} \right). \tag{10}$$

Upon expanding the natural log in Eq. (8) ($\left(\ln_e(1+x) \approx x - \frac{1}{2}x^2 + \frac{1}{3}x^3\right)$) to the third power in $(H - H_0)$, α_1 and α_2 are given by

$$\alpha_1 = \mathrm{i} \left(\frac{2eB}{m} - \frac{8}{3} \left(\frac{eB}{m} \right)^3 \tau^2 \right) + 2\tau \left(\frac{eB}{m} \right)^2$$

$$\alpha_2 = \mathrm{i} \left(\frac{eB}{m} - \frac{1}{3} \left(\frac{eB}{m} \right)^3 \tau^2 \right) + \frac{1}{2}\tau \left(\frac{eB}{m} \right)^2. \tag{11}$$

When we substitute Eq. (11) into Eq. (10) and use $\mathrm{e}^{\mathrm{i}x} + \mathrm{e}^{-\mathrm{i}x} = 2\cos x$, we obtain

$$< S_x >= \frac{\hbar}{2} \exp\left(-\frac{5}{2} \left(\frac{eB}{m} \right)^2 \tau t \right) \cos\left[\left(\frac{eB}{m} - \frac{7}{3} \left(\frac{eB}{m} \right)^3 \tau^2 \right) t \right]$$

$$+\frac{\hbar}{2}\exp\left(-\frac{1}{2}\left(\frac{eB}{m}\right)^2\tau t\right)\cos\left[\left(\frac{eB}{m}-\frac{1}{3}\left(\frac{eB}{m}\right)^3\tau^2\right)t\right]. \qquad (12)$$

Since the states $S_z = 1, 0$ decay to the ground state, the time evolution according to Eq. (4) follows the proportion of particles in $S_z = 1, 0, -1$, but not the total number, since Eq. (4) does not conserve probability. To take into account the fact that the total number of spinning particles is conserved, we may divide Eq. (12) by $\Psi^+\Psi$ and thus normalize the probability. We may thus write Eq. (12) as

$$< S_x >= \frac{\frac{\hbar}{2}\exp\left(-\frac{5}{2}\left(\frac{eB}{m}\right)^2\tau t\right)\cos\omega_1 t + \frac{\hbar}{2}\exp\left(-\frac{1}{2}\left(\frac{eB}{m}\right)^2\tau t\right)\cos\omega_2 t}{\frac{1}{4}\exp\left(-4\left(\frac{eB}{m}\right)^2\tau t\right)+\frac{1}{2}\exp\left(-\left(\frac{eB}{m}\right)^2\tau t\right)+\frac{1}{4}}, \qquad (13)$$

where

$$\omega_1 = \frac{eB}{m} - \frac{7}{3}\left(\frac{eB}{m}\right)^3\tau^2$$

$$\omega_2 = \frac{eB}{m} - \frac{1}{3}\left(\frac{eB}{m}\right)^3\tau^2.$$

3. Conclusion

We can see from Eq. (13) that when τ is finite, there are two distinct frequencies with the respective components of $< S_x >$ damping at different rates. A plot of $< S_x >$ versus t would yield a Doppler like [18] plot versus time with the two components having different damping factors. The first frequency $\omega_1 = (eB/m) - (7/3)\ (eB/m)^2\tau^2$ will have damping rate with a damping factor $\exp(-\gamma_1 t)$, with $\gamma_1 = (5/2)\ (eB/m)^2\tau$, and the second frequency $\omega_2 = (eB/m) - (1/3)\ (eB/m)^2\tau^2$ will have a damping factor $\exp(-\gamma_2 t)$, with $\gamma_2 = (1/2)\ (eB/m)^2\tau$.

The ratio of the two amplitudes of $< S_x >$ will vary as

$$\frac{< S_x >_1}{< S_x >_2} = \exp\left(-2\left(\frac{eB}{m}\right)^2\tau t\right). \qquad (14)$$

Thus a plot of $< S_x >_1 / < S_x >_2$ versus time can be used to find the discrete time interval τ. Estimates of the discrete time interval have been previously quoted as $\tau \leq 10^{-26}$s [19]. Assuming a field of $B = 100$ T (1 T= 1 W/m^2), for heavy gauge bosons, $< S_x >_1 / < S_x >_2$ will damp to a value of 1/e in a time (see Eq. (14)):

$$\left(\frac{1.6\cdot 10^{-19}\times 100}{2\times 10^{-26}}\right)^2 \times (10^{-26})\ t = 1, \qquad (15)$$

giving $t = 10^{10}$ s. For $B = 10^6$ T, $t = 10^2$ s. Stronger fields cause smaller time intervals that satisfy the condition $< S_x >_1 / < S_x >_2 = 1/\mathrm{e}$. Since gauge bosons are known to decay in intervals characteristic of the weak interaction time ($t < 10^{-8}$ s), we would need a field of $B \approx 10^{11}$ T to generate a decay time that can compete with the weak interaction time scale. Such fields could only be found in the atmosphere of a pulsar [20]. Also, it might be possible to apply the above analysis to heavy ions in situations where the precession time can be made commensurate with the collision time. Here the environment would also be well defined. The fundamantal question in the above analysis is just what properties of the environment (random photons or other particles) determine the discrete time interval. Certainly the temperature [21] and most likely the composition of the environment will play a vital role in determining the discrete time interval τ [22,23]. We may look at τ as a relaxation time determined by the collective properties of the environment which arises when the spin system is immersed in the environment. The fact that the discrete quantum system has a specific property determined by one parameter τ is an amazing result that opens up a new frontier in studying the properties of quantum systems interacting with the environment.

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DISIPATIVNI UČINCI OKOLIŠA I KVANTAN RAZVOJ SPINSKOG SUSTAVA

Raspravlja se primjena Schrödingerove jednadžbe s diskretnim vremenom i s kašnjenjem za opis raspada spinskog sustava, kada okoliš ima neprekidan i neograničen energijski spektar.