

STUDY OF LEPTONIC AND RADIATIVE  $b$  QUARK DECAYS IN THE LIGHT OF  
UP-QUARK FLAVOUR CHANGING CONTRIBUTIONS: MULTIDOUBLET MODEL

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The effective  $bsZ$  vertex may be influenced by tree  $uu'Z$  vertex formed by a mixing with heavy exotic isosinglet up-type quarks. The electroweak penguin diagrams involving one insertion of the  $uu'Z$  vertex have been considered and we have calculated the contribution arising from those diagrams using the fourth generation Cabbibo-Kobayashi-Maskawa matrix elements; also the applicability of the generalized Glashow-Iliopoulos-Maiani mechanism is considered. The additional effects of the heavy isosinglets are compared with the effects of exotic heavy isodoublets appearing in multi-generational models. We see that in the effective vertex amplitude, the up-flavour-changing contribution interferes constructively and destructively with the one loop-penguin diagrams with and without any insertion of flavour changing coupling, respectively.

## 1. Introduction

It is an admitted fact that due to the unitarity of the Cabbibo-Kobayashi-Maskawa (CKM) matrix [1], in the flavour-changing neutral current (FCNC) processes in the Standard Model (SM) [2], the leading-order mass-independent term is strongly suppressed by the Glashow-Iliopoulos-Maiani (GIM) [3] cancellation mechanism. This is experimentally confirmed and it paves the way for investigating the new sources of FCNC. So, the study of virtual effects opened hydraheaded windows on electroweak symmetry breaking and physics beyond the SM. The examination of these indirect effects on new physics in

higher order processes yields a complementary approach to the search for direct production of new particles at high energy colliders.

To probe FCNC of radiative  $b$  decays, we see that apart from uncertainties, the CLEO data [4] are in good agreement with the leading-logarithmic QCD corrections, and partial calculations of next-to-leading logarithmic order is under way [5]. These new results opened the scope for investigations in various classes of models, namely: anomalous top-quark couplings [6], anomalous trilinear gauge couplings [7], fourth generation [8], two-Higgs-doublet model [9], three-Higgs-doublet model [10], supersymmetry [11], extended technicolour [12], leptoquarks [13] and left-right symmetric models [14].

Apart from these models, in the line of investigation conducted recently in LEP, we are contemplating the existence of a new U(1) gauge-boson coupling, predominantly to the third family, and it may have the consequence of enhancing the  $b$ -quark decay modes [15]. The sources of FCNC may also be coming from (i) the ratios between the masses of fermions involved in the flavour-changing (FC) transitions, or (ii) some new mass scale of the order of electroweak breaking scale, or it may be larger where it may arise from mixing between the light fermions and new heavy states with non-standard  $SU(2)_L$  assignments [16-19], or from multi-Higgs doublets model without natural flavour conservation [20-21], or by horizontal symmetries [22] in fermion mass hierarchy. Due to the fact that the fermion masses are small, the effects are naturally suppressed. But now, the appearance of the top quark, with a heavy mass of 180 GeV, has changed the scenario abruptly when the FC transition involves the  $t$  quark. Oflate investigation is going on at the phenomenological level [23-25] and also for model building [20-21].

Here we try to find out to what extent the effective  $bsZ$  vertex is modified by inserting a tree level  $uu'Z$  FCNC vertex, assuming a mixing between  $u, c, t$  quarks and new isosinglet heavy states of charge  $2/3$ .

## 2. General formulation

We assume the existence of  $n$  new  $Q = 2/3$  isosinglet L-handed quarks  $U_L^0$ . They can appear in vector like multiplets  $U_L^0, U_R^0$  and they are mixed with unknown up-type quarks  $u_L^0, u_R^0$ . The number  $n$  of  $U_L^0, U_R^0$  pairs is not that relevant for our formulation in general, and we keep it unspecified for the present.  $U_R^0$  and  $u_R^0$ , being both colour triplet  $Q = 2/3$  isosinglet states, have the same gauge quantum numbers, and, therefore, their couplings to the gauge bosons are unaffected by the mixing. This is not the case for the L-chirality states. The vector

$$\mathcal{U}_{uL}^0 = \begin{pmatrix} u^0 \\ U^0 \end{pmatrix}_L$$

of the doublet ( $u^0$ ) and the singlet ( $U^0$ ) gauge eigenstates is related to the corresponding vector of the "light" ( $u$ ) and "heavy" ( $U$ ) mass eigenstates

$$\mathcal{U}_{uL} = \begin{pmatrix} u \\ U \end{pmatrix}_L$$

through a unitary matrix  $\Pi$  such that

$$\begin{pmatrix} u^0 \\ U^0 \end{pmatrix}_L = \Pi \begin{pmatrix} u \\ U \end{pmatrix} \quad (1)$$

$$\Pi = \begin{bmatrix} P & Q \\ R & S \end{bmatrix} \quad (2)$$

Here  $U = [t_1, t_2, \dots, t_n]^T$  and  $u = [u, c, t]^T$ . Yet,  $\Pi$  is unitary  $P$  and  $R$  are not themselves unitary. In the weak basis, the charged fermion neutral current shall contain  $P^\dagger P$  and  $R^\dagger R$  which are not necessarily diagonal, and thus the mixing in general induces FCNC's among the light particles. In order to avoid this problems, the assumptions [18] are made that each ordinary left- and right-handed fermion mixes with its own exotic partner. In this case  $P^\dagger P$  and  $R^\dagger R$  are diagonal, and thus eliminate FCNC's. With this assumption, we can write  $(P_a^\dagger P_a)_{ij} = (c_a^i)^2 \delta_{ij}$  and  $(R_a^\dagger R_a)_{ij} = (s_a^i)^2 \delta_{ij}$ , where  $a = \text{Left, Right}$ . Here  $(s_a^i)^2 \equiv 1 - (c_a^i)^2 \equiv \sin^2 \theta_a^i$ , where  $\theta_{\text{Left, (Right)}}^i$  is mixing angle in the  $i$ th Left-handed (Right-handed) ordinary fermion and its exotic partner. The unitarity of  $\Pi$  implies  $\Pi^\dagger \Pi = \Pi \Pi^\dagger = \text{unit matrix}$ , and so

$$P^\dagger P + R^\dagger R = PP^\dagger + QQ^\dagger \equiv I_{3 \times 3} \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (3)$$

We now introduce a unitary matrix  $\Delta$  for the L-handed down type quarks, so that

$$\begin{aligned} d_L^0 &= \Delta d_L \\ \Delta \Delta^\dagger &= \Delta^\dagger \Delta = I_{3 \times 3} \end{aligned} \quad (4)$$

We also introduce a  $(3+n) \times 3$  matrix  $X$ , given by

$$X = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ \vdots & & \\ 0 & 0 & 0 \end{pmatrix} \quad (5)$$

and with the help of (1) we write the charged current (CC) coupled to the  $W^\pm$  bosons as

$$\begin{aligned} \frac{1}{2} J_\mu^W &= \overline{\mathcal{U}}_{uL}^0 \gamma_\mu X d_L^0 \\ &= \overline{\mathcal{U}}_{uL} \gamma_\mu \Pi^\dagger X d_L \end{aligned} \quad (6)$$

From the above, we see that we can define a  $(3+n) \times 3$  mixing matrix  $V$  given by

$$V = \Pi^\dagger X \Delta \equiv \begin{pmatrix} P^\dagger & \Delta \\ Q^\dagger & \Delta \end{pmatrix} \quad (7)$$

We identify  $P^\dagger \Delta$  as the  $3 \times 3$  Cabbibo–Kobayashi–Maskawa (CKM) matrix [1] for the light states  $u$  and we see that it is not unitary. We see from (3) and (4)

$$\begin{aligned} V^\dagger V &= \Delta^\dagger (PP^\dagger + QQ^\dagger) \Delta \\ &= I_{3 \times 3} \end{aligned} \quad (8)$$

Thus  $V$  is analogous to the unitary CKM matrix.

Now for  $\theta_W$ , the “Weinberg weak mixing angle”,  $s = \sin \theta_W$ ,  $c = \cos \theta_W$ , and we define a  $(3+n) \times (3+n)$  matrix

$$I_3 = X \times X^\dagger \equiv \begin{pmatrix} I_{3 \times 3} & 0 \\ 0 & 0 \end{pmatrix}, \quad (9)$$

the projector operator acting on the L-handed  $1/2$  isospin doublet states. And we can write the neutral current (NC) coupled to the  $Z$  boson in terms of the mass eigenstates, as

$$\frac{1}{2} J_\mu^W = \frac{1}{2} \overline{\mathcal{U}}_{uL} \gamma_\mu \Delta^\dagger I_3 \Delta \mathcal{U}_{uL} - s^2 \overline{\mathcal{U}}_u \gamma_\mu E \mathcal{U}_u \quad (10)$$

In (10), the second term remains flavour diagonal since the matrix of electric charge  $E$  is proportional to the “Identity”, i.e.,  $E = (2/3) \times \text{Identity}$ . But when we consider the current matrix, the isospin part  $\frac{1}{2} I_3$  is not proportional to the Identity, and, therefore, the corresponding isospin couplings are flavour changing.

Let us now define the NC-mixing matrix

$$\mathcal{N} = \Pi^\dagger I_3 \Pi = \begin{pmatrix} P^\dagger P & P^\dagger Q \\ Q^\dagger P & Q^\dagger Q \end{pmatrix}. \quad (11)$$

We see that  $\mathcal{N}$  is not unitary.

But from (4), (7) and (11), we get

$$\begin{aligned} \mathcal{N} &= VV^\dagger, \\ \mathcal{N}\mathcal{N}^\dagger &= \mathcal{N}^2 = \mathcal{N}, \quad \text{and} \\ \mathcal{N}V &= V. \end{aligned} \quad (12)$$

We see that  $\mathcal{N}$  is idempotent, since from (11) we can interpret  $\mathcal{N}$  as the projection operator on the L-doublets written on the basis of mass eigenstates. On actual calculation, we obtain that  $\mathcal{N}_{uu} = 0.9999$ , and  $\mathcal{N}_{cc} = 0.9974$ , i.e., all are approximately equal 1. Now, we may

note further that  $\mathcal{N}_{uu} \approx \mathcal{N}_{cc} \approx 1$  because of the experimental bounds on the left handed up and charm quarks which are flavour diagonal. We note again that, as

$$\sum_{a=1}^n \mathcal{N}_{aa} = \text{Tr}(V^\dagger V) = 3, \quad (13)$$

we get  $\sum_{a=3}^n \mathcal{N}_{aa} \approx 1$ . Further, we may note that  $\mathcal{N}$  is not symmetric, i.e.,  $\mathcal{N}_{uu} \neq \mathcal{N}'_{uu}$ .

Now,  $V^\dagger V = I_{3 \times 3}$  and  $\mathcal{N}V = V$  imply  $V^\dagger \mathcal{N}V = V^\dagger V = I_{3 \times 3}$ , and so all the mass independent terms in the new penguin diagrams which carry structure  $V^\dagger \mathcal{N}V$  are cancelled off in spite of the presence of the FC couplings. Actually, from (8) and (12), we can write

$$\sum_{jk} V_{js}^* (\mathcal{N}_{jk} - \delta_{jk}) V_{kb} = 0 \quad (14)$$

Following [26], we get the usual SM L-handed and R-handed chiral couplings of up type quarks as

$$\begin{aligned} g_L^u &= \frac{1}{2} - \frac{2}{3}s^2, & \text{and} \\ g_R^u &= -\frac{2}{3}s^2 \end{aligned} \quad (15)$$

From (9), it is evident that L-handed up quark couplings changed for mixing with the new isosinglets, and introduce an FC term. For the sake of generality, we write  $\mathcal{U}_{u_i} \mathcal{U}_{u_j} Z$  coupling as

$$\begin{aligned} g_L^{ij} &= \frac{1}{2} \mathcal{N}_{ij} - \frac{2}{3}s^2 \\ &= g_L^u \delta_{ij} + \frac{1}{2} (\mathcal{N}_{ij} - \delta_{ij}), \quad \text{where} \\ i, j &= u, c, t, t_1, t_2, \dots, t_n \end{aligned} \quad (16)$$

The term  $g_L^u$  represents the extension of SM to  $3+n$  L-handed doublets with no tree level FCNC, and the term  $\frac{1}{2} (\mathcal{N}_{ij} - \delta_{ij})$  shows that the new  $n$  states are isosinglets.

The first term gives us the scope to compare results for the isosinglets case with those of a multigenerational model. Further, from (16), we see that the calculation of effective  $bsZ$  vertex in the presence of the tree level FC couplings can be done by calculating SM contribution [27] extended to  $3+n$  generations, and by computing two additional diagrams given in Fig. 1 which arise from the second term in (16).

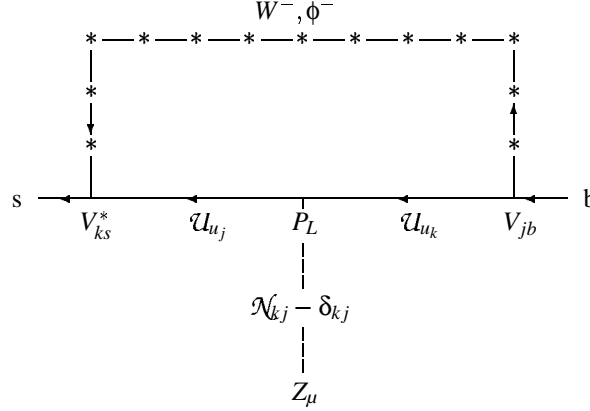


Fig. 1. Electroweak penguin diagrams with  $W$  boson and scalar  $\phi$  which include flavour changing vertex  $\mathcal{N}_{k,j,j \neq k}$ . The relevant mixing matrices appearing at the vertices are shown explicitly, and  $P_L = \frac{1}{2}(1 - \gamma_5)$  is the  $L$ -handed chiral projector.

### 3. Calculation of amplitude with and without FC

The amplitude for the sum of the one loop penguin diagrams which do not contain any insertion of the FC couplings in the 't Hooft–Feynman gauge can be written with the help of [27] with  $x_u = m_u^2/m_W^2$  and  $\xi_i = V_{is}^* V_{ib}$  as

$$\mathcal{A}_{eff}^{withoutFC} = \frac{g^3}{(4\pi)^2 c} \left( \sum_{u=c,t} \xi_u F_1(x_u) \right) (\bar{s}_L \gamma_\mu b_L), \quad (17)$$

where  $F_1(x)$  an Inami and Lim function given by

$$F_1(x) = \frac{x^2 - x - 5}{4(x-1)} + \frac{3x^2 + 2x}{4(x-1)^2} \ln x \quad (18)$$

Due to the relation (8), we have  $\sum_i \xi_i = 0$ , and thus the mass independent terms are cancelled out and divergences are eliminated like in the GIM mechanism.

We see that (17) is not gauge invariant by itself. To make it gauge invariant, we have to add box diagrams for the processes  $b \rightarrow s\bar{l}l$ , with  $l = \nu, l^\pm$ .

Introducing the Inami and Lim function

$$F_2(x) = \frac{2}{5} \left[ \frac{1}{x-1} - \frac{x \ln x}{(x-1)^2} \right] \quad (19)$$

we have the physical gauge invariant quantity for their decay amplitude

$$\mathcal{A}_{eff}^{withoutFC} = \frac{g^3}{(4\pi)^2 c} \left( \sum_{u=u,c,t} \xi_u (F_1(x_u) + F_2(x_u)) \right) \times (\overline{s_L} \gamma_\mu b_L) \quad (20)$$

Now we turn our attention to the second term in (16), namely,  $\frac{1}{2}(\mathcal{N}_{ij} - \delta_{ij})$ . We have considered new states to be isosinglet, so there shall be no extra contributions from the FC couplings. The diagram corresponding to this term, as stated earlier, is given in Fig. 1. The loops at  $W$  boson or the scalar  $\phi$  are logarithmically divergent. But the chiral projection operator  $P_L = \frac{1}{2}(1 - \gamma_5)$  reduces the degree of divergence by a factor of 2, and thus the divergence is eliminated.

From (14), we see that when we sum over all the  $u$  and  $U$ , all terms independent of  $u$  and  $U$  masses, and in particular the poles at  $D = 4$ , are wiped out like the GIM cancellation law and thus lead to a finite contribution from the diagram involving  $W$  boson loop.

Now we write the amplitude obtained from the second term in (16) as

$$\mathcal{A}_{eff}^{withFC} = \mathcal{A}_W + \mathcal{A}_\phi = \frac{g^3}{(4\pi)^2 c} \left( \sum_{kj} V_{ks}^* (\mathcal{N}_{kj} - \delta_{kj}) V_{jb} F_1(x_k, x_j) \right) (\overline{s_L} \gamma_\mu b_L), \quad (21)$$

where

$$F(x, y) = \frac{1}{4(x-y)} \left( \frac{y-1}{x-1} x^2 \ln x - \frac{x-1}{y-1} y^2 \ln y \right). \quad (22)$$

Here  $k, j$  run over  $u, c, t, t_1, t_2, \dots, t_n$ . We may note that (21) is also gauge invariant.

Amplitude for the effective  $bsZ$  vertex now can be written as

$$\begin{aligned} \mathcal{A}^{bsZ} &= \mathcal{A}_{eff}^{withoutFC} + \mathcal{A}_{eff}^{withFC} \\ &= \frac{g^3}{(4\pi)^2 c} \left( \sum_u \xi_u (F_1(x_u) + F_2(x_u)) \right) (\overline{s_L} \gamma_\mu b_L) \\ &+ \frac{g^3}{(4\pi)^2 c} \left( \sum_{u'd'} V_{us}^* (\mathcal{N}_{u'd'} - \delta_{u'd'}) V_{d'b} F(x_u, x_{u'}) \right) (\overline{s_L} \gamma_\mu b_L) \end{aligned} \quad (23)$$

We see that as  $x \rightarrow y$ ,

$$F(x, y) \rightarrow \frac{x}{4} - \frac{x \ln x}{2(x-1)} \equiv F_3(x) \quad (24)$$

which is included in  $F_1$ .

Then we can write the amplitude as

$$\mathcal{A}^{bsZ} = \frac{g^3}{(4\pi)^2 c} \left( \sum_u \xi_u (F_1(x_u) + F_2(x_u)) \right) (\overline{s_L} \gamma_\mu b_L)$$

$$\begin{aligned}
 & + \frac{g^3}{(4\pi)^2 c} \left( \sum_u^{U_n} V_{us}^* (\mathcal{N}_{uu} - 1) V_{ub} F_3(x_u) \right) (\overline{s_L} \gamma_\mu b_L) \\
 & + \frac{g^3}{(4\pi)^2 c} \left( \sum_{u,u'}^{u \neq u'} V_{us}^* (\mathcal{N}_{uu'} V_{u'b}) F_1(x_u, x_{u'}) \right) \times (\overline{s_L} \gamma_\mu b_L) \quad (25)
 \end{aligned}$$

Thus from the note below Eq. (12), we see that for the second sum in the above expression the first two terms are not contributing, and contributions are coming from the top quark and  $t_1, t_2, \dots$

#### 4. Results and discussion

We take  $m_u = 0.007$  GeV,  $m_c = 1.5$  GeV,  $m_t = 180$  GeV, and  $m_W = 88.22$  GeV. The first term of (25) is without FC.

TABLE 1. Functions  $F_1(x), F_2(x)$  and their sum for different values of the parameter  $x = m_U^2/m_W^2$ .

$m_U$	0.007	1.50	180	200	250
$F_1(x)$	1.2499	1.2491	3.0864	3.4691	4.1066
$F_2(x)$	-2.4999	-2.4939	-0.6301	-0.5643	-0.4403
Total	-1.2501	-1.2448	2.4563	2.9048	3.6663

We see that the dominant contribution is coming from the top quark.

Next, we look at the second term. The function  $F_3(x)$  is calculated for different values of mass of the up type quark, for  $m_t = 180$  GeV and above, viz.; 200, 250, 300, 400 and 500 GeV, and is given in the Table 2.

TABLE 2. Function  $F_3(x)$  for different values of the parameter  $x = m_U^2/m_W^2$ .

$m_U$	180	200	250	300	400	500
$F_3(x)$	0.250	0.465	1.160	2.076	4.524	7.834

We see that the second term, which is also flavour diagonal, gives dominant contribution for large masses of  $U$ . Of course, mixing with the isosinglets reduces this contribution to some extent because of what we have noted below Eq. (25).

For the third term, which is the additional effect of the FC vertices, we note the following:

(i)  $F(x, y)$  is a symmetric function on exchange of  $x$  and  $y$ .

(ii) For small values of quark masses, its value is negligibly small:  $F(x_u, x_c) = -0.0006959$ . Even for intergenerational mixing like  $(x_t, x_u)$  or  $(x_t, x_c)$ , the value is negative but small,  $F(x_t, x_u) = -0.0504244$  and  $F(x_t, x_c) = -0.504103$ , resulting in reduction



of the contribution as compared to values in Table 1. The FC generational mixing of top quark interferes destructively upon the contributions without FC and, consequently, the strength of the effective  $bsZ$  vertex is weakened with respect to the doublet case. Thus, we may expect reduction in  $b \rightarrow sl^+l^-$  decay rates.

(iii) As  $F(x, y) = F(y, x)$ , keeping  $m_t = 180$  GeV fixed, we vary the masses of exotic  $U$ 's, taking the values 200, 250, 300, 400, 500 and 600 GeV; the results are given in Table 3. We see that  $F(x, y)$  is a monotonic function of exotic quark masses  $U_i$ .

TABLE 3. Function  $F(x_t, x_U)$  with  $m_t = 180$  GeV for different values of the parameter  $x = m_U^2/m_W^2$ .

$m_U$	180	200	250	300	400	500	600
$F(x_t, x_U)$	0.2502	0.3472	0.0591	0.7946	1.1580	1.5110	1.8020

For heavy masses, the values of  $F(x, y)$  are positive and increase with the mass:

$$\begin{aligned} F(U_{300}, U_{400}) &= 3.0508, & F(U_{400}, U_{500}) &= 5.9296, \\ F(U_{300}, U_{500}) &= 3.9335, & F(U_{400}, U_{600}) &= 7.2116 \\ F(U_{300}, U_{600}) &= 4.7279. \end{aligned}$$

The mixing of the top quark with the heavy masses appreciates the rates, and the same is true for the case of mixing of any pair of the exotic massive states  $(U_i, U_j)$ .

(iv) The mixing of higher generational masses of 500 Gev and 600 GeV with  $u$  quark and  $c$  quarks yield virtually the same results:

$$\begin{aligned} F(x_{U=500}, x_u) &= -0.9390, & F(x_{U=600}, x_u) &= -1.02439 \\ F(x_{U=500}, x_c) &= -0.93877, & F(x_{U=600}, x_c) &= -1.02439 \end{aligned}$$

Thus, the effects of massive singlets' FC mixing with up and charm quark are same as those noted in (ii), and so the effective  $bsZ$  vertex is further weakened.

Now we turn to the matrices  $V$  and  $\mathcal{N}$ . As stated earlier,  $V_L$  is not unitary, and following [18], the elements can be written as

$$V_{Lij} = c_L^{u_i} c_L^{d_j} \hat{V}_{Lij} \quad (26)$$

where  $\hat{V}_L$  is the usual unitary CKM matrix. The values of  $c_L^{u_i}$  and  $c_L^{d_j}$  were calculated from the values of  $s$ 's collected from [28] and are given below:

$$\begin{aligned} (s_L^d)^2 &= 0.0023 & (s_R^d)^2 &= 0.019 \\ (s_L^s)^2 &= 0.0036 & (s_R^s)^2 &= 0.021 \\ (s_L^c)^2 &= 0.0042 & (s_R^c)^2 &= 0.010 \\ (s_L^b)^2 &= 0.0020 & (s_R^b)^2 &= 0.010 \end{aligned}$$

Now we calculate the amplitude  $\mathcal{A}^{bsZ}$  given in Eq. (24), term by term, in units of  $\frac{g^3}{(4\pi)^2 c} (\overline{s_L} \gamma_\mu b_L)$ . We also take the CKM matrix elements from the fourth generation calculated in Ref. 29.

The first term, considering the sum up to the  $t$  quark, has the contributions:  $+ 0.00021533$ ,  $- 0.019014597$  and  $+ 0.160212211$ , from the  $u, c$  and  $t$  quarks, respectively, yielding the total value of  $0.141412944$ .

For the second term, the contribution from  $u$  is only positive and all other contributions are negative. To have an estimate, we calculated the terms up to  $t_2$ , and assuming  $t_1 = 400$  GeV and  $t_2 = 500$  GeV. The contributions coming from  $u, c, t, t_1$ , and  $t_2$  are of the order  $10^{-7}, 10^{-4}, 10^{-3}$  and  $8 \times 10^{-2}$ , respectively. The total contribution of the second term up to  $t_2$  is  $- 0.163019137$ . We see that the second term acts destructively upon the first term. Thus, the penguin diagram for the effective  $bsZ$  vertex is reduced to  $- 0.021$ , which is less than  $+ 0.14$  in absolute value, i.e.,

$$|\text{Sum of the first and second term}| < |\text{First term}|$$

Hence, the experimental upper limit of  $b \rightarrow s\bar{l}(l = \nu, l^\pm)$  does not put any restriction for mixing produced by the  $uu'Z$  vertex.

The third term has a chequered character. The  $u$ -block with a total of four cross-values of  $F(x,y)$  gives the value  $+ 0.000104866$ , the  $c$ -block with a total of three cross-values gives  $- 0.009075973$ , the  $t$ -block with two cross-value gives  $+ 0.044967028$  and the  $t_1$ -block has only one term and that is  $+ 0.010810562$ . As the sum of the first two terms is  $- 0.021606193$ , the addition of the  $u$ -block does not matter much, while the addition of the  $c$ -block reduces the value to  $- 0.0305773$ . But the addition of the  $t$ -block is substantial, as it rather restores the value of  $\mathcal{A}$  to its positive value, i.e., to  $+ 0.0143$  and then  $t_1$ -block adds and finally taking all terms upto  $t_2$ , we get the value of  $\mathcal{A}^{bsZ}$  of  $+ 0.02520029$ . We expect progressively increasing values from further terms. So, the introduction of exotic up type heavy singlets slowly augments the decay rate and we get a clear testing ground to investigate the presence of the fourth generation.

## 5. Conclusion

The contribution to the effective  $bsZ$  vertex of the new penguin diagrams induced by a  $uu'Z$  vertex is bound to be smaller than the SM result, and interferes destructively with it. Hence, the rate for the FC decay is lowered by this effect, and the experimental upper limits on  $b \rightarrow sl^+l^-$  and on  $b \rightarrow s\nu\bar{\nu}$  do not imply any constraint on a mixing induced  $uu'Z$  vertex, but it may help the investigation of the existence of the fourth generation.

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#### MULTIDUBLETNI MODEL ZA LEPTONSKE I RADIJATIVNE RASPADE $b$ KVARKA U SVJETLU DOPRINOSA OD PROMJENE OKUSA $u$ -KVARKA

Na efektivan vrh  $bsZ$  može utjecati vrh  $uu'Z$  krošnje koji je nastao miješanjem teških egzotičnih kvarkova tipa  $u$ . Razmatraju se elektroslabi pingvinski dijagrami s jednim ubacivanjem vrha  $uu'Z$  i izračunavaju doprinosi koji nastaju zbog tih dijagrama uz primjenu Cabbibo-Kobayashi-Maskawinih matričnih elemenata. Razmatra se također primjenjivost poopćenog Glashow-Iliopoulos-Maiani mehanizma. Dodatni se učinci teških izosingleta uspoređuju s učincima egzotičnih teških izodubleta koji se javljaju u višegeneracijskim modelima. Nalazi se da u efektivnoj vršnoj amplitudi doprinos od promjene  $u$ -okusa interferira konstruktivno i destruktivno s pingvinskim dijagramima s jednom petljom sa odnosno bez umetanja vezanja koje mijenja okus.