Economic Research-Ekonomska Istraživanja

ISSN: (Print) (Online) Journal homepage: https://www.tandfonline.com/loi/rero20

A hesitant fuzzy SMART method based on a new score function for information literacy assessment of teachers

Yong Qin, Zeshui Xu & Xinxin Wang

To cite this article: Yong Qin, Zeshui Xu & Xinxin Wang (2023) A hesitant fuzzy SMART method based on a new score function for information literacy assessment of teachers, Economic Research-Ekonomska Istraživanja, 36:1, 357-382, DOI: 10.1080/1331677X.2022.2076712

To link to this article: https://doi.org/10.1080/1331677X.2022.2076712

Published online: 28 May 2022.

ECONOMIC RESEARCH-EKONOMSKA ISTRAŽIVANJA
2023, VOL. 36, NO. 1, 357–382
https://doi.org/10.1080/1331677X.2022.2076712

A hesitant fuzzy SMART method based on a new score function for information literacy assessment of teachers

Yong Qin, Zeshui Xu and Xinxin Wang

Business School, Sichuan University, Chengdu, China

ABSTRACT
As two powerful and flexible tools for decision-makers (DMs) to model the complex cognition, the hesitant fuzzy set (HFS) and hesitant fuzzy linguistic term set (HFLTS) allow DMs to express their opinions with several possible membership values or linguistic terms on the objects over each criterion. The aim of this article is to develop a novel score function of the HFS and HFLTS including hesitant degree and fuzzy degree information. For this purpose, the notion of fuzzy degree of the hesitant fuzzy element (HFE) and hesitant fuzzy linguistic element (HFLE) is introduced first. Then, considering both the hesitant degree and fuzzy degree information in expressions, the new score function, namely the Score-H&FD, is designed. Based on which, we extend the classical SMART (simple multi-attribute rating technique) method to the hesitant fuzzy environment. As a result, the hesitant fuzzy SMART (HF-SMART) method is developed in this article. Afterwards, we apply our proposed approach to assess and rank several teachers concerning information literacy. Finally, sensitive analysis and comparative analysis are carried out. The results show that the proposed method in this article has substantial advantages and applicability.

ARTICLE HISTORY
Received 25 October 2021
Accepted 4 May 2022

KEYWORDS
Hesitant fuzzy set; hesitant fuzzy linguistic term set; fuzzy degree; Score-H&FD; HF-SMART method.

JEL CODES
A20; C44; D81

1. Introduction

In our real life, multi-criteria decision-making (MCDM) problems are one of the most common types of activities that human beings face. On a regular basis, a set of alternatives and several corresponding criteria need to be determined in advance. Then, the evaluation information of each alternative over different criteria is given by DMs. Finally, the performance of each alternative is integrated by the selected MCDM technique. As a result, the alternatives are ranked and the optimal one is identified. At present, many MCDM techniques have been widely used and then applied to many fields (Luo et al., 2021; Qin et al., 2022). However, owing to the inherent limitations and ambiguities of DMs' cognition, evaluation information or preferences on alternatives cannot always be expressed in precise numbers. What is

CONTACT Zeshui Xu xuzeshui@263.net

more, the traditional decision-making methods and steps based on the real numbers are no longer suitable for complex and changing decision-making environments (Lourenzutti & Krohling, 2013). To characterise and ulteriorly model the knowledge and cognition of a DM more comprehensively under uncertain environment, Zadeh (1965) offered a flexible and pragmatic tool, i.e., fuzzy set (FS), to deal with such a situation. Since the first proposal of FS, the fuzzy theory has been attracting a lot of interest, and thereby many extensions have been developed.

Specifically, hesitant fuzzy set (HFS), was proposed by Torra and Narukawa (2009), which allows DMs to give a set of possible values in terms of the membership function. In fact, it is in line with the scenes in the real world, and such cases are everywhere in our lives. For example, we are likely to struggle with how much to rate the quality of a product, and 7, 8 or even 9 points will appear in our mind at the same time (full marks are 10 points). Under this circumstance, the problem of DMs hesitating between several degrees of membership is well resolved. Concurrently, it is also a useful and practical means to better depict and handle the vague and hesitant information in the process of MCDM. Later, the concept of the hesitant fuzzy element (HFE), essential component of HFS, is introduced by Xia and Xu (2011). On this basis, its score function, fundamental aggregation operators, distance and similarity measures are also developed (Xu & Xia, 2011a). Among them, the score function plays an indispensable role in ranking HFEs, especially for the HFE score-based MCDM method.

To acquire the reasonable score values and better serve to decision-making problems, a variety of score functions of HFE are investigated and proposed from many perspectives by researchers. In light of the variance values in the memberships, Liao and Xu (2013) developed a score function for comparing HFEs, namely the score value-variance. Similarly, a generalised score function was put forward by Zhang and Xu (2014). Considering the support degree among the grades of memberships in a HFE, a power average-based score function was developed (Liao et al., 2018). In addition, from the perspective of deviation degree, the HFE deviation score function was introduced (Wang et al., 2019). Although many forms of score functions for HFEs have been investigated, the hesitant information, an intrinsic feature of the HFE, is generally ignored. However, this is a significant issue that needs to be addressed. To this end, the quantitative expression of hesitant degree with respect to HFEs was pioneered by Liao et al. (2015b) and Li et al. (2015). Since then, the hesitant degree information is considered in solving hesitant fuzzy decision-making problems. For example, taking the hesitant degree of HFEs into consideration, a novel similarity measure between HFEs was defined and applied for pattern recognition (Zeng et al., 2016).

For many situations, it is not easy for DMs to give their opinions or evaluations concerning qualitative criteria in a quantitative form. They are more inclined to express their cognitions in linguistic terms, such as *'beautiful'* scenery, *'poor'* quality and *'bad'* service. In this case, Zadeh (1975) put forward a fuzzy linguistic approach that enables DMs to express their qualitative opinions in linguistic variables. The emergence of linguistic variables makes the decision-making models under fuzzy environment more flexible and applicable. In the traditional linguistic models, the

DMs merely permitted to use a single linguistic term to represent their qualitative thinking and reasoning. Whereas, the characteristic of hesitant and irresolute perceptions for DMs makes it challenge to do this. To eliminate the defects of the linguistic approach, the notion of hesitant fuzzy linguistic term set (HFLTS) was introduced by Rodriguez et al. (2012). With the help of the HFLTS, the qualitative judgments of DMs are reflected more reasonably and comprehensively. Afterwards, the mathematical expression of HFLTS and its basic components, namely the hesitant fuzzy linguistic element (HFLE), were developed by Liao et al. (2015). In the same vein, once a new fuzzy theory or model is introduced, it is not evitable to carry out a comparison between the basic components. As a result, the score function of the HFLE was first designed by Liao et al. (2015a). As mentioned previously, the hesitant information is a unique property in hesitant fuzzy related theory. Also, the hesitant degree should be taken into consideration on operations in terms of the HFLE. Thereby, the different definitions on hesitant degree of the HFLE were introduced (Liao et al., 2020; Liao et al., 2019; Wei et al., 2018). On this basis, Liao et al. (2019) developed a novel hesitant degree-based score function of the HFLE, i.e., the Score-HeDLiSF.

From the above analysis, we can see that there is a growing body of literature that realises the importance of hesitant information in the hesitant fuzzy MCDM problems. In particular, some scholars have attempted to incorporate hesitant information into the construction of the corresponding score functions for the HFE or HFLE. However, another crucial feature about the HFE or HFLE, namely fuzzy degree information, is rarely considered and removed in the existing research. Thus, in this study, we are devoted to filling this gap and introducing the fuzzy degree functions for the HFE and HFLE. More to the point, considering both the hesitant degree and fuzzy degree information in expressions, a new score function of the HFE and HFLE, namely the score function base on the hesitant degree and fuzzy degree (named Score-H&FD), is designed.

The SMART (simple multi-attribute rating technique) method is a convenient and pragmatic model to deal with MCDM problems. Due to its ease of use, it has been widely used in various fields. In the traditional SMART approach, the minimum and maximum values over each criterion's performance need to be predefined by DMs. After that, the evaluation information is given in the predefined interval. Furthermore, the criteria are usually assessed in a single linguistic term form so as to obtain the importance for each criterion. As the MCDM problems have become more complex, especially under hesitant fuzzy environment, the traditional SMART method is no longer suitable and applicable. With regard to this, we propose a hesitant fuzzy SMART (HF-SMART) approach in this article to deal with the MCDM problems with hesitant fuzzy information. However, there is nearly no work to investigate this.

Based on the above review and analysis, this article proposes a hesitant fuzzy SMART method combining the novel score function with the hesitant degree and fuzzy degree. Then, we employ the HF-SMART approach to solve a case of teachers' information literacy evaluation with hesitant fuzzy information. In conclusion, our contributions can be outlined as follows:

1. We first introduce the concept of the fuzzy degree of the HFE and HFLE. According to this concept, an approach is offered to compute the fuzzy degree of the HFE and HFLE.
2. Considering both the hesitant degree and fuzzy degree information in expressions, we design a novel score function of the HFE and HFLE, named as the Score-H&FD. The Score-H&FD contains more information than the previous traditional score functions.
3. Based on the Score-H&FD, a HFE and HFLE score-based MCDM technique, namely the HF-SMART method, is put forward in this article. Using this approach, we conduct an evaluation on teachers' information literacy. As a result, its effectiveness and robustness are validated.

We organise the rest of this article as follows: Section 2 reviews the basic knowledge of the HFS, HFLTS and classical SMART method. The novel score functions of the HFS and HFLTS are introduced in Section 3 based on the hesitant degree and fuzzy degree information. Section 4 illustrates the detailed procedure of the HF-SMART method. In Section 5, an illustrative example is presented and related discussions are conducted. Finally, some conclusions and remarks are given in Section 6.

2. Preliminaries

In this section, some knowledge and concepts about the HFS, HFLTS and traditional SMART method are introduced, which are essential parts in the next sections.

2.1. Hesitant fuzzy set

Hesitant fuzzy set (Torra, 2010), as an extension of fuzzy set, a collection of possible membership values is allowed to occur simultaneously in a set for each element, which can adequately characterise the situations where the DMs are hesitant in providing decision information on objects.

Suppose that X is a fixed set, a hesitant fuzzy set (HFS) A on X is defined in terms of a function $h_A(x)$ when applied to X returns a subset of [0,1] (Torra, 2010; Xia & Xu, 2011), i.e.,

$$A = \{\langle x, h_A(x)\rangle | x \in X\}, \tag{1}$$

where $h_A(x)$ is a finite subset of [0,1], indicating the union of the possible membership values of the element $x \in X$ to the set A. As a matter of convenience, $h_A(x)$ is usually called a hesitant fuzzy element (HFE), which is the fundamental composition of HFS (Xu & Xia, 2011b).

To rank the HFEs, Xia and Xu (2011) defined the score function of HFE $h(x) = \{\gamma_1, \gamma_2, \cdots, \gamma_n\}$ with the length n, i.e.,

$$S(h(x)) = \frac{1}{n}\sum_{\gamma\in h(x)} \gamma \tag{2}$$

Afterwards, some scholars pointed out that the hesitant degree is a significant and essential feature of HFE and should be taken into consideration. Li et al. (2015) introduced the concept of hesitant degree from the perspective of the length of the HFE, denoted as:

$$HD(h(x)) = 1 - \frac{1}{n} \tag{3}$$

It is evident that the longer the length of the HFE, the higher hesitation it is. This result is consistent with our cognition.

The values in the HFE are usually out of order, which is not conducive to the comparison between two HFEs. For the sake of simplicity, the values for the HFE are arranged in ascending or descending order. At the same time, another challenge arises since the number of values in each HFE may turn out differently. In order to measure the distance between two HFEs efficiently, the shorter one should be extended until the number of values is the same as another (Xu & Xia, 2011c; Xu & Zhang, 2013).

Additionally, some basic operations on HFEs are defined (Xu & Xia, 2011b). Given that three HFEs h, h_1 and h_2, $\lambda > 0$, then

1. $h^{\lambda} = \cup_{\gamma \in h}\{\gamma^{\lambda}\}$;
2. $\lambda h = \cup_{\gamma \in h}\{1 - (1-\gamma)^{\lambda}\}$;
3. $h_1 \oplus h_2 = \cup_{\substack{\gamma_1 \in h_1 \\ \gamma_2 \in h_2}} \{\gamma_1 + \gamma_2 - \gamma_1\gamma_2\}$;
4. $h_1 \otimes h_2 = \cup_{\substack{\gamma_1 \in h_1 \\ \gamma_2 \in h_2}} \{\gamma_1\gamma_2\}$.

In particular, when a set of membership values are between the open-interval (0, 1) in HFEs, the algebraic division and algebraic subtraction operations on the HFEs are considered as (Farhadinia, 2015):

1. $h_1 \oslash h_2 = \cup_{\substack{\gamma_1 \in h_1 \\ \gamma_2 \in h_2}} \oslash\{\gamma_1, \gamma_2\} = \cup_{\substack{\gamma_1 \in h_1 \\ \gamma_2 \in h_2}} min\left\{1, \frac{\gamma_1}{\gamma_2}\right\}$;
2. $h_1 \ominus h_2 = \cup_{\substack{\gamma_1 \in h_1 \\ \gamma_2 \in h_2}} \ominus\{\gamma_1, \gamma_2\} = \cup_{\substack{\gamma_1 \in h_1 \\ \gamma_2 \in h_2}} max\left\{0, \frac{\gamma_1 - \gamma_2}{1-\gamma_2}\right\}$.

2.2. Hesitant fuzzy linguistic term set

Motivated by the concept of HFS (Hai et al., 2018; Zadeh, 1975), the hesitant fuzzy linguistic term set (HFLTS) was defined by Rodriguez et al. (2012) to fully respond to all possible linguistic cognition from experts (Liao et al., 2018). Furthermore, the mathematical expression of HFLTS is given and refined by Liao et al. (2015), i.e.,

Let $x_i \in X$ $(i = 1, 2, \cdots, n)$ be fixed and $S = \{s_t | t = -\tau, \cdots, -1, 0, 1, \cdots, \tau\}$ be a linguistic term set. The HFLTS H_S on X can be shown as:

$$H_S = \{\langle x_i, h_s(x_i)\rangle | x_i \in X\}, \tag{4}$$

where $h_s(x_i)$ represents some possible values in the linguistic term set S and is usually expressed as $h_s(x_i) = \{s_{\phi l}(x_i) | s_{\phi l}(x_i) \in S;\ l = 1, \cdots, L;\ \phi_l \in \{-\tau, \cdots, -1, 0, 1, \cdots, \tau\}\}$, L is the number of linguistic terms for $h_s(x_i)$. The $h_s(x_i)$ indicates the possible member degrees of the linguistic variable x_i to the linguistic term set S. For convenience's sake, $h_s(x_i)$ is usually called hesitant fuzzy linguistic element (HFLE).

Although several linguistic values can be presented for a linguistic variable through HFLE, it's still not like the human way of thinking and reasoning. Thus, a context-free grammar that is more similar to human beings' expressions was proposed by Rodriguez et al. (2012), based on which, simple but elaborated linguistic expressions are generated. The definition of context-free grammar G_H is as follows (Liao et al., 2015a):

Let G_H be a context-free grammar, and $S = \{s_t | t = -\tau, \cdots, -1, 0, 1, \cdots, \tau\}$ be a linguistic term set. The elements of $G_H = (V_N, V_T, I, P)$ are defined as follows:

$$V_N = \{\langle primary\ term\rangle, \langle composite\ term\rangle, \langle unary\ relation\rangle, \langle binary\ relation\rangle, \langle conjunction\rangle\}$$

$$V_T = \{lower\ than,\ greater\ than,\ at\ least,\ at\ most,\ between,\ and,\ s_{-\tau}, \cdots, \\ s_{-1}, s_0, s_1, \cdots, s_\tau\}, I \in V_N$$

$$P = \{I ::= \langle primary\ term\rangle | \langle composite\ term\rangle \langle composite\ term\rangle ::= \langle unary\ relation\rangle \\ \langle primary\ term\rangle | \langle binary\ relation\rangle \langle conjunction\rangle \langle primary\ term\rangle \\ \langle primary\ term\rangle ::= s_{-\tau} | \cdots | s_{-1} | s_0 | s_1 | \cdots | s_\tau \\ \langle unary\ relation\rangle ::= lower\ than | greater\ than \langle binary\ relation\rangle ::= between \\ \langle conjunction\rangle ::= and\}.$$

where V_N refers to a set of nonterminal symbols, V_T depicts a set of terminals' symbols, I represents the starting symbol and P indicates the production rules.

In order to translate the linguistic expressions ll from the DMs into the HFLE, a translation function E_{G_H} was introduced (Liao et al., 2018), i.e.,

$$E_{G_H} : ll \to H_S, \tag{5}$$

where S represents the LTS utilised by G_H.

According to the production rules, most types of linguistic expressions can be processed by means of the following transformations.

1. $E_{G_H}(s_\alpha) = \{s_\alpha | s_\alpha \in S\}$;
2. $E_{G_H}(at\ most\ s_\beta) = \{s_\alpha | s_\alpha \in S \text{ and } s_\alpha \le s_\beta\}$;
3. $E_{G_H}(less\ than\ s_\beta) = \{s_\alpha | s_\alpha \in S \text{ and } s_\alpha < s_\beta\}$;
4. $E_{G_H}(at\ least\ s_\beta) = \{s_\alpha | s_\alpha \in S \text{ and } s_\alpha \ge s_\beta\}$;
5. $E_{G_H}(more\ than\ s_\beta) = \{s_\alpha | s_\alpha \in S \text{ and } s_\alpha > s_\beta\}$;
6. $E_{G_H}(between\ s_\beta\ and\ s_{\beta'}) = \{s_\alpha | s_\alpha \in S \text{ and } s_\beta \le s_\alpha \le s_{\beta'}\}$.

Similarly, in order to distinct and rank between two HFLEs, the comparison operation is necessary to define. Therefore, motivated by the score function of HFS, the score function of the HFLE is given by Liao et al. (2015a).

Assume a HFLE $H_S = \cup_{S_{\delta l} \in H_S}\{s_{\delta l}|l = 1, \cdots, L\}$, then the score function of H_S is considered as:

$$\gamma(H_S) = \frac{1}{L}\sum_{S_{\delta l} \in H_S} s_{\delta l} = s_{\frac{1}{L}}\sum_{l=1}^{L} \delta l, \tag{6}$$

where L is the number of linguistic terms for H_S and the score of the HFLE is still a linguistic term.

It is noted that when a DM gives his/her linguistic expression as $H_S{}^1$ = '*a little high*' $\{s_1\}$, at the same time, another DM gives his/her linguistic expression as $H_S{}^2$ = '*between medium and high*' $\{s_0, s_1, s_2\}$. Using Eq. (6) to calculate the score of $H_S{}^1$ and $H_S{}^2$ separately, we can easily know that $\gamma(H_S{}^1) = \gamma(H_S{}^2) = s_1$. Such a result seems reasonable, but not very consistent with human's cognition and thinking. To circumvent this defect, the concept of hesitant degree was introduced to depict the degree of hesitation for DMs when giving the linguistic evaluation (Wei et al., 2018). Afterwards, a general approach to measure the hesitant degree of the HFLE was proposed by Liao et al. (2019), i.e.,

$$HD(H_S) = \frac{L(H_S)ln(L(H_S))}{(2\tau + 1)ln(2\tau + 1)}, \tag{7}$$

where $HD(H_S)$ denotes the hesitant degree function of H_S.

Remark 1 (Liao et al., 2019). The hesitant degree function $HD(H_S)$ belongs to [0, 1] and satisfies the following properties:

1. When $HD(H_S) = 0$, which represents there is only one linguistic term in H_S, i.e., $H_S = \{s_\delta\}$, $\delta \in \{-\tau, \tau\}$;
2. When $HD(H_S) = 1$, which indicates that the HFLE H_S contains all the linguistic terms, i.e., $H_S = \{s_\delta|\delta = -\tau, \cdots, \tau\}$;
3. If $L(H_S{}^1) \leq L(H_S{}^2)$, then $HD(H_S{}^1) \leq HD(H_S{}^2)$.

According to the previous analysis, a score function of HFLE considering the hesitant degree is defined (Liao et al., 2019). Suppose that $S = \{s_t|t = -\tau, \cdots, -1, 0, 1, \cdots, \tau\}$ is a LTS. Then the score of the HFLE $H_s(x_i) = \{s_{\phi l}(x_i)|s_{\phi l}(x_i) \in S;\ l = 1, \cdots, L;\ \phi_l \in \{-\tau, \cdots, -1, 0, 1, \cdots, \tau\}\}$ is considered as:

$$\rho(H_S) = (1 - HD(H_S)) \times \left(\frac{1}{L}\sum_{l=1}^{L} \phi l\right), \tag{8}$$

where $HD(H_S)$ is the aforementioned hesitant degree function of H_S. It is noted that the score function $\rho(H_S)$ is a crisp number. Besides, the obtained score with the hesitant degree information can characterise the performance of the HFLE more delicately.

2.3. The bird's eye of the SMART method

The Simple Multi-Attribute Rating Technique (SMART), initially proposed by Winterfeldt and Edwards (Edwards & Barron, 1994; Lootsma, 1996), is a practical and effective MCDM method based on linear value functions to assess and rank alternatives (Kangas et al., 2010; Oyetunji & Anderson, 2006). Both quantitative and qualitative criteria can be applied to this method, and it is considered one of the compensatory techniques. Due to its ease of use, it has been widely used in critical facility vulnerability assessment (Akgun et al., 2010), operational environment of forest bioenergy production evaluation (Malovrh et al., 2016), students' evaluation (Borissova & Keremedchiev, 2019) and external walls selection in hot and humid climates (Boostani & Hancer, 2018). Let $A = \{A_1, A_2, \cdots, A_n\}$ be a set of optional alternatives, where A_i represents the *ith* alternative and $i = 1, 2, \cdots, n$; $C = \{C_1, C_2, \cdots, C_m\}$ be a set of criteria of alternatives, where C_j represents the *jth* criterion and $j = 1, 2, \cdots, m$. As a result, the implementation steps of the classical SMART method are further detailed as follows:

Step 1. The decision matrix $D = (r_{ij})_{n\times m}$ of each alternative A_i under different criteria $C_j(j = 1, 2, \cdots, m)$ is established based on the evaluation information from the DMs, shown as:

$$D = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_n \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_m \end{array} \\ \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1m} \\ r_{21} & r_{22} & \cdots & r_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ r_{n1} & r_{n2} & \cdots & r_{nm} \end{bmatrix} \end{array} \quad i = 1, 2, \cdots, n; j = 1, 2, \cdots, m \tag{9}$$

Step 2. For the given criteria, they need to be ranked as shown in Table 1.

Step 3. Rating the criteria. In this step, the maximum ρ_{max} and the minimum ρ_{min} for each criterion are determined by the DMs. In other words, the evaluation information for each criterion is between intervals ρ_{min} and ρ_{max}. Afterwards, the whole decision-making interval is divided into several equidistant sub-intervals φ from Eq. (10).

$$\rho_{min}, \ \rho_{min} + \varphi_0, \ \rho_{min} + \varphi_1, \ \cdots \tag{10}$$

Equation (11) is usually utilised to obtain φ.

$$\varphi_\delta - \varphi_{\delta-1} = \eta\varphi_{\delta-1} \tag{11}$$

Then based on Eq. (12), the geometric progression is obtained.

Table 1. The seven ranking of qualitative criteria.

Poor	Fairly weak	Medium	Fairly good	Good	Very good	Excellent
4	5	6	7	8	9	10

Source: The Authors.

$$\varphi_\delta = (1+\eta)\varphi_{\delta-1} = (1+\eta)^2\varphi_{\delta-2} = (1+\eta)^\delta\varphi_0 \tag{12}$$

As a result, Eq. (13) can be derived as follows:

$$\rho_{max} = \varphi_\delta + \rho_{min} \tag{13}$$

Step 4. Determining the effective performance of alternatives. For the qualitative criteria, they are processed according to Table 1. For the quantitative criteria, they are processed by Eq. (14).

$$\xi = log_2{}^{\frac{\rho_\vartheta - \rho_{min}}{\rho_{max}-\rho_{min}}\times 64}, \tag{14}$$

where ρ_ϑ denotes the given value of the criteria related to each alternative. At the same time, for the positive criteria, the value of ξ needs to be summed with the number 4 to match the criteria in Table 1, i.e., $Q_{ij} = 4 + \xi$; for the negative criteria, the value of ξ needs to be subtracted from 10 to match the criteria in Table 1, i.e., $Q_{ij} = 10-\xi$.

Step 5. Weights normalisation. Let g_j be the rank related to the criteria C_j given by the DMs. Then the denormalised weights are calculated by Eq. (15).

$$\varpi_j = \left(\sqrt{2}\right)^{g_j},\ j = 1, \cdots, m \tag{15}$$

Later, the normalised value of each criterion is obtained by Eq. (16).

$$\omega_j = \frac{\left(\sqrt{2}\right)^{g_j}}{\sum_{j=1}^{m}\sqrt{2}^{g_j}} \tag{16}$$

Step 6. Ranking the alternatives. Based on Eq. (17), the final score ψ_i is calculated as follows:

$$\psi_i = \sum_{j=1}^{m}\omega_j \cdot Q_{ij},\ i = 1, \cdots, n \tag{17}$$

As a result, the best alternative is selected in terms of the final score ψ_i.

3. A New score function of HFS and HFLTS

As mentioned above, the HFE or HFLE is eventually transformed into a numerical number by score function for comparison and ranking. In the existing research, it has been considered to remove its uncertainty to obtain more accurate results. The most concerned in current work is the hesitation information contained in it. As a result, the concept of the hesitant degree is proposed by relevant scholars. After eliminating the hesitation information, the average performance of the score function related to the HFE or HFLE has been dramatically improved. However, the fuzzy uncertainty information in the HFE or HFLE is ignored. Take the HFE as an example, one expert gives his/her evaluation information as $h_1 = \{0.1, 0.9\}$, and another expert provides his/her evaluation information as $h_2 = \{0.4, 0.6\}$. Obviously, after calculating by Eq. (2) or Eq. (3), they have the same scores and hesitant degrees. We can find that the membership values 0.1 and 0.9 in h_1 display more specific thinking and reasoning than the membership values 0.4 and 0.6 in h_2. Therefore, the above calculation result is inconsistent with our cognition. It is necessary to consider the fuzzy uncertainty information in operations on the HFE or HFLE. In human's actual perception, the more extreme the evaluation is (the worse or better), the less fuzziness it contains. Conversely, if the assessment is given in the middle (that is, neither good nor bad), the fuzziness is usually higher. Motivated by the idea of information entropy (Shannon, 1948), this article defines the fuzzy degree of the HFE and HFLE. Furthermore, considering hesitant and fuzzy information, in this article, we propose a new score function of HFS and HFLTS.

Let $p = \{p_1, p_2, \cdots, p_n\}$ be a set of discrete probabilities, then the information entropy is defined as:

$$E = -\sum_{i=1}^{n} p_i log_2 p_i, \tag{18}$$

where $p_i (i = 1, 2, \cdots, n)$ denotes the probabilities of occurrence. If there are only two sources of information p and q, the general information entropy will be transformed into binary information entropy, in particular we have the following

$$E^2 = -plog_2 p - qlog_2 q, \tag{19}$$

where $p \in [0, 1]$, $q \in [0, 1]$ and $p + q = 1$, the function graph is shown in Figure 1.

In Figure 1, the curve is symmetric about $p = 0.5$ and achieves a maximum of 1 at $p = 0.5$. Meanwhile, with $p = 0.5$ as the centre, the closer the value is to 0 or 1, the smaller the information entropy is. When $p = 0$ or $p = 1$, the value of the information entropy is reduced to 0. It is very consistent with human cognitive habits. Similar to the idea of binary information entropy, the concept of fuzzy degree related to membership values in HFE and HFLE is defined.

Definition 1. For a reference set X, let $h(x) = \{\gamma_1, \gamma_2, \cdots, \gamma_n\}$ be a HFE with length n, where $\gamma_i (i = 1, \cdots, n)$ denotes the possible membership values of $x \in X$ and $\gamma_i \in [0, 1]$, then the mapping function from membership value γ_i to the fuzzy degree of HFE $FD(h(x))^{HFE}$ is represented as:

$$FD(h(x))^{HFE} = \frac{1}{n}\sum_{i=1}^{n} \left(-\gamma_i log_2 \gamma_i - (1-\gamma_i) log_2 (1-\gamma_i)\right) \tag{20}$$

In analogous to the definition of the HFE, the fuzzy degree function of the HFLE is defined.

Definition 2. Let $S = \{s_\delta | \delta = -\tau, \cdots, -1, 0, 1, \cdots, \tau\}$ be a LTS. For a HFLE $H_S = \cup_{S_{\delta l} \in H_S} \{s_{\delta l} | l = 1, \cdots, L\}$, where L is the number of linguistic terms for H_S, then the

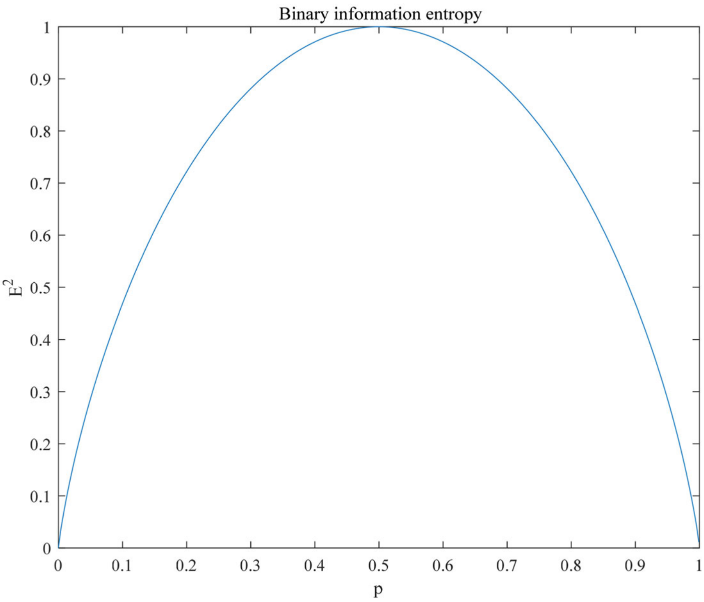

Figure 1. The function graph of binary information entropy.
Source: The Authors.

mapping function from linguistic term $s_{\delta l}(l = 1, \cdots, L)$ to the fuzzy degree of HFLE $FD(H_S)^{HFLE}$ is defined as:

$$FD(H_S)^{HFLE} = \frac{1}{L}\sum_{l=1}^{L}\left(-\ell(s_{\delta l})log_2\ell(s_{\delta l}) - (1-\ell(s_{\delta l}))log_2(1-\ell(s_{\delta l}))\right), \tag{21}$$

where the semantics of linguistic terms are uniformly distributed, i.e., $\ell(s_{\delta l}) = (\delta l + \tau)/2\tau$.

In what follows, we define the new score function of HFE and HFLE considering both the hesitant degree and fuzzy degree.

Definition 3. For a reference set X, let $h(x) = \{\gamma_1, \gamma_2, \cdots, \gamma_n\}$ be a HFE with the length n, where $\gamma_i(i = 1, \cdots, n)$ denotes the possible membership values of $x \in X$, then the score of a HFE is represent as:

$$\Omega(h(x))^{HFE} = (1-HD(h(x)))^{\lambda} \times (1-FD(h(x))^{HFE})^{1-\lambda} \times \left(\frac{1}{n}\sum_{i=1}^{n}\gamma_i\right), \tag{22}$$

where $HD(h(x))$ is the hesitant degree of $h(x)$, $FD(h(x))^{HFE}$ is the fuzzy degree of $h(x)$ and $\lambda \in [0, 1]$ is the adjustment coefficient. In this article, $HD(h(x))$ and $FD(h(x))^{HFE}$ are considered equally important, so the value of λ is 0.5.

Remark 2. The new score function of the HFE defined in this article has the following properties:

1. If $\gamma_1 = \gamma_2 = \cdots = \gamma_n = 0.5$, $FD(h(x))^{HFE} = 1$. Hence, $\Omega(h(x))^{HFE} = 0$, that is to say, in the case of all membership values is 0.5, this HFE does not provide any information for decision-making.
2. If $\gamma_1 = \gamma_2 = \cdots = \gamma_n = 0$ or $\gamma_1 = \gamma_2 = \cdots = \gamma_n = 1$, then $HD(h(x)) = 0$ and $FD(h(x))^{HFE} = 0$. In this scenario, the HFE does not contain any hesitation and fuzziness information.

Definition 4. Let $S = \{s_\delta | \delta = -\tau, \cdots, -1, 0, 1, \cdots, \tau\}$ be a LTS. The score of a HELE $H_S = \cup_{s_{\delta l} \in H_S} \{s_{\delta l} | l = 1, \cdots, L\}$ is represent as:

$$\Omega(H_S)^{HFLE} = (1 - HD(H_S))^{\beta} \times (1 - FD(H_S)^{HFLE})^{1-\beta} \times \left(\frac{1}{L} \sum_{l=1}^{L} ((\delta l + \tau)/2\tau) \right), \tag{23}$$

where $HD(H_S)$ is the hesitant degree of H_S, $FD(H_S)^{HFLE}$ is the fuzzy degree of H_S and $\beta \in [0, 1]$ is the adjustment coefficient. In this article, $HD(H_S)$ and $FD(H_S)^{HFLE}$ are considered equally important, so the value of β is 0.5.

Remark 3. The new score function of the HFLE defined in this article has the following properties:

1. If $s_{\delta l} | l = 1, \cdots, L = s_0$, $FD(H_S)^{HFLE} = 1$. Hence, $\Omega(H_S)^{HFLE} = 0$, that is to say, in the case of all membership values is s_0, this HFLE does not provide any information for decision-making.
2. If $s_{\delta l} | l = 1, \cdots, L = s_{-\tau}$ or $s_{\delta l} | l = 1, \cdots, L = s_\tau$, then $HD(H_S) = 0$ and $FD(H_S)^{HFLE} = 0$. In this situation, the HFLE does not contain any hesitation and fuzziness information.
3. If $L = 2\tau + 1$, then $HD(H_S) = 1$. Thus, $\Omega(H_S)^{HFLE} = 0$, under such situation, this HFLE is meaningless.

For short, the new score function of HFE and HFLE based on the hesitant and fuzzy degree is called Score-H&FD. To further verify the feasibility of the Score-H&FD, taking the Score-H&FD of the HFE as an example, a simulation experiment is implemented, in which, the adjustment coefficient λ is set to 0.5, n represents the quantity of randomly generated membership values, and the number of simulations is uniformly adjusted to 2000. As illustrated in Figure 2, the overall density distribution of the four simulations is similar, and the score values all belong to [0,1] as expected. A closer look at the score values and overall scores decrease in conjunction with the increase in n. It can be interpreted as the larger the value of n, the higher the hesitation and fuzziness of the DMs, and thereby the score will decrease accordingly.

4. A Novel SMART method with hesitant fuzzy information

According to the previous analysis, this part proposes a new procedure of the novel SMART method with hesitant fuzzy information based on the idea of the Score-H&FD of HFS and HFLTS. The flow chart of the proposed method for MCDM is given in Figure 3.

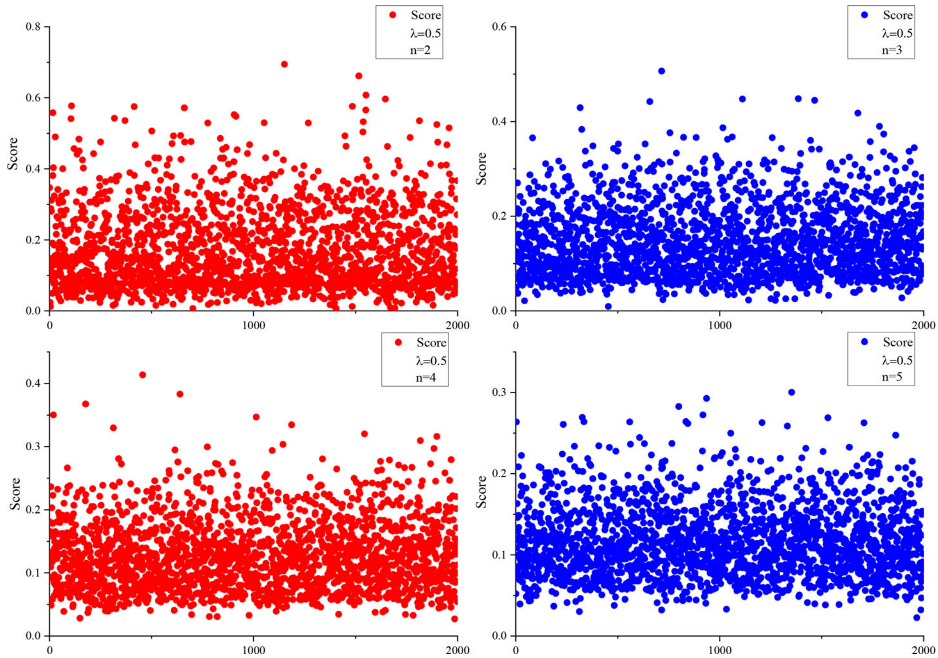

Figure 2. The simulation experiment of the Score-H&FD of HFE.
Source: The Authors.

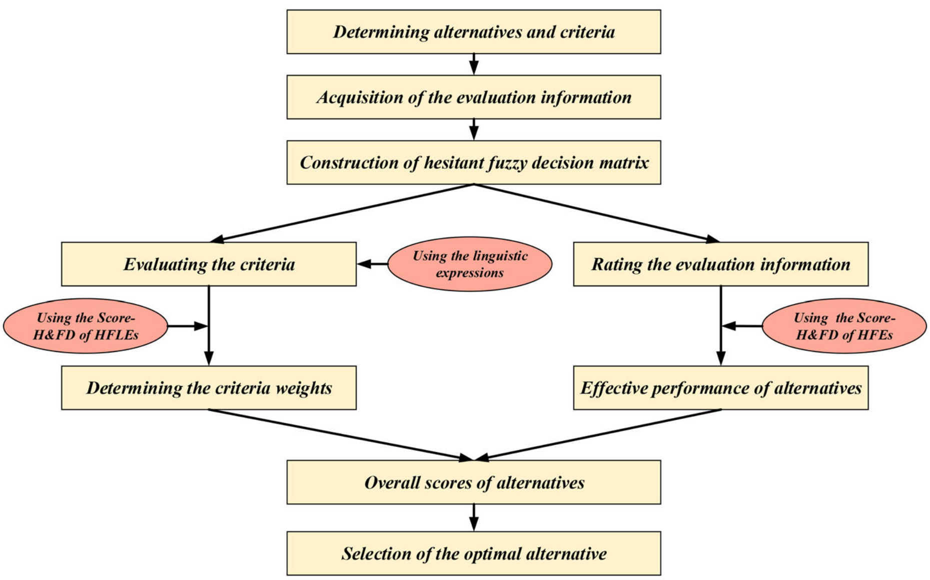

Figure 3. The flow chart of the proposed HF-SMART method.
Source: The Authors.

Let $A = \{A_1, A_2, \cdots, A_n\}$ be a set of alternatives, $C = \{C_1, C_2, \cdots, C_m\}$ be the criteria. Similar to the traditional SMART method described in Subsection 2.3, the procedure of the HF-SMART method is further detailed as follows:

Table 2. The linguistic expressions of criteria.

Criteria	C_1	C_2	$\cdots$	C_j
Evaluations	ll_1	ll_2	$\cdots$	ll_j

Source: The Authors.

Step 1. Acquiring the evaluation information for the alternatives over several criteria from the DMs. Furthermore, it is usually expressed as the decision matrix $H = (h_{ij})_{n\times m}$, in which, the evaluation information satisfies the characteristic of the HFE.

$$H = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_n \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_m \end{array} \\ \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1m} \\ h_{21} & h_{22} & \cdots & h_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ h_{n1} & h_{n2} & \cdots & h_{nm} \end{bmatrix} \end{array} = (h_{ij})_{n\times m} \quad i = 1, 2, \cdots, n; j = 1, 2, \cdots, m' \tag{24}$$

where h_{ij} denotes some possible membership values of the ith alternative A_i over the jth criterion C_j.

Step 2. Evaluating the criteria with the linguistic expressions ll_j. It is generated by the context-free grammar described in Subsection 2.2, can be written as shown in Table 2.

Then, the above linguistic judgments for each criterion are transformed into the HFLE through the transformation function E_{G_H}, shown as in Table 3.

Step 3. Rating the criteria. Similarly, the maximum h_{max} and minimum h_{min} of each criterion are given in advance by the DMs. What is more, the whole hesitant fuzzy decision-making interval is divided into several equidistant sub-intervals $\imath$ from Eq. (25).

$$h_{min}, \ \ h_{min} + \imath_0, \ \ h_{min} + \imath_1, \ \cdots \tag{25}$$

In the same way, Eq. (26) will be derived as follows:

$$h_{max} = \imath_\varepsilon + h_{min} \tag{26}$$

Step 4. Determining the effective performance of alternatives. It should be noted that when the evaluation information is given by experts, the types of the criteria have been considered. The larger the values, the better the performance of criteria. Therefore, there is no need to conduct operations on negative criteria here. Meanwhile, to ensure the non-negativity of the effective performance, Eq. (27) is adjusted as follows:

$$\phi_{ij} = log_2\left(\Omega(\frac{h_{\vartheta_{ij}} - h_{min}}{h_{max} - h_{min}})^{HFE} + 1\right), \tag{27}$$

Table 3. The HFLE judgment information for criteria.

Criteria	C_1	C_2	$\cdots$	C_j
Evaluations	H_{S_1}	H_{S_2}	$\cdots$	H_{S_j}

Source: The Authors.

where $h_{\vartheta_{ij}}$ depicts the evaluation of the ith alternative against the jth criterion and Ω refers to the Score-H&FD of the HFE.

Step 5. Weights normalization. Let $\Omega(H_S{}^j)^{HFLE}$ be the Score-H&FD of the HFLE judgment information over the criterion C_j given by the DMs. Then, the denormalised weights are calculated by Eq. (28).

$$\varpi_j = \left(\sqrt{2}\right)^{\Omega(H_S{}^j)^{HFLE}}, \ j = 1, \cdots, m \tag{28}$$

Afterwards, the normalised value of each criterion is obtained by Eq. (29).

$$\omega_j = \frac{\left(\sqrt{2}\right)^{\Omega(H_S{}^j)^{HFLE}}}{\sum_{j=1}^{m} \sqrt{2}^{\Omega(H_S{}^j)^{HFLE}}} \tag{29}$$

Step 6. Ranking the alternatives. Based on Eq. (30), the final score χ_i is calculated as follows:

$$\chi_i = \sum_{j=1}^{m} \omega_j \cdot \phi_{ij}, \ \ i = 1, \cdots, n \tag{30}$$

Ultimately, the alternatives are ranked on the basis of the final scores $\chi_i(\ i = 1, \cdots, n)$.

5. Illustrative example

In this section, a teacher's information literacy evaluation case is presented with the HF-SMART method. As a result, the description of the case will be started first, then the solution process and results in Subsection 5.2 are given. Subsection 5.3 illustrates some comparisons with sensitivity analysis and discussions.

5.1. Case description

In recent years, with the rapid development of the Internet, big data, artificial intelligence and other information technologies, the transformation of the industrial society into information society has been much promoted. The information has become the most active and significant element in all fields of society at present. Needless to say, information literacy has gradually become the core skill of people to adapt to the modern information society. Nowadays, it has not only become an essential indicator for evaluating the overall quality of talents, but also has become a necessary survival, work and learning ability for everyone in the information era. Against this background, cultivating information awareness and carrying out information education have become an inevitable trend in today's world education reform. It has also become an essential direction for school education innovation.

Teachers, as the primary resource for educational development, are an important guarantee for advancing education reform and modernization. In a modern society where online learning resources are highly developed, whether teachers can effectively acquire and use the required educational information resources has become one of the essential qualities of teachers in the future. Concurrently, teachers must take the initiative to adapt the changes in education informatisation brought about by information technology and make reasonable use of information technology and resources to conduct diverse educational teaching. For example, an interactive learning environment with both graphics and text can be created via information technology. Moreover, abundant network learning resources could be fully utilised for online teaching, further promoting students' understanding and absorption of knowledge. In particular, under the impact of the COVID-19 pandemic, it has spawned the widespread use of online education. What is more, the Ministry of Education of the People's Republic of China has proposed that online education should become the normalization in the future, which also puts forward direct requirements for teachers' information literacy. Therefore, the evaluation and assessment of teachers' information literacy have become an important measure for college staff management.

Business School of Sichuan University has a strong faculty. Following the education philosophy of 'Aspire Morality, Inherit Culture, Advocate Science, Pursue Truth', Business School has always had extremely high requirements for teachers' basic literacy. Keeping pace with the times, information literacy evaluation is rightfully included in the performance assessment of teachers. According to the instructions of the college leaders, the four criteria of information awareness (C_1), information acquisition (C_2), information application (C_3) and information security (C_4) are adopted for the evaluation. Now, a year-end information literacy assessment is conducted for five young teachers: A_1, A_2, A_3, A_4, A_5. In light of this, Business School invites several relevant experts to make a fair and reasonable evaluation as much as possible over the five young teachers over the above four criteria. Due to the uncertainty, fuzziness and vagueness in assessing these five teachers, the experts employ the HFS and HFLTS to evaluate alternatives over criteria.

5.2. Solution based on the HF-SMART method

Using the HF-SMART method proposed in this article to tackle the teacher's information literacy evaluation problem under this section. As a result, the detailed decision-making process is given as follows:

Step 1. The evaluation information for the five teachers over the four criteria are given by the experts. Thus, the decision-making matrix is listed in Table 4.

Table 4. Hesitant fuzzy decision matrix.

	C_1	C_2	C_3	C_4
A_1	$\{0.3, 0.4\}$	$\{0.15, 0.2\}$	$\{0.3, 0.35\}$	$\{0.25, 0.4, 0.7\}$
A_2	$\{0.25, 0.45\}$	$\{0.2, 0.4, 0.6\}$	$\{0.4, 0.6\}$	$\{0.3, 0.8\}$
A_3	$\{0.3, 0.4, 0.55\}$	$\{0.25, 0.3, 0.7\}$	$\{0.55, 0.6, 0.8\}$	$\{0.4, 0.6, 0.8\}$
A_4	$\{0.3, 0.35, 0.6\}$	$\{0.3, 0.7\}$	$\{0.7, 0.85\}$	$\{0.35, 0.4, 0.75\}$
A_5	$\{0.25, 0.4, 0.6\}$	$\{0.4, 0.5, 0.6\}$	$\{0.3, 0.85\}$	$\{0.4, 0.65, 0.75\}$

Source: The Authors.

Step 2. Suppose that the LTS used in this case for evaluating the teachers with respect to the four criteria is defined as follows:

$$S = \{s_{-3} = Fair\ unimportant(FU), s_{-2} = Unimportant(U), s_{-1} = A\ little\ unimportant(ALU), s_0 = Medium(M), s_1 = A\ little\ important(ALI), s_2 = Important(I), s_3 = Very\ important(VI)\}$$

Then, each criterion is evaluated with linguistic evaluation shown in Table 5. What is more, the hesitant fuzzy linguistic decision matrix is generated as listed in Table 6.

Step 3. The maximum and minimum membership values in each hesitant fuzzy decision-making element over each criterion are discussed and given by the experts, namely, $C_1 \in [0.2_{min}, 0.7_{max}]$, $C_2 \in [0.1_{min}, 0.75_{max}]$, $C_3 \in [0.3_{min}, 0.9_{max}]$ and $C_4 \in [0.25_{min}, 0.85_{max}]$. Afterwards, the rating of criteria is computed as shown in Table 7.

Step 4. Calculating the effective performance of each alternative against each criterion by Eq. (27). Here, the value of λ is set to 0.5. For instance, the effective performance of evaluation information for the teacher A_1 over the criterion C_1 is as follows:

Firstly, the normalised $h_{11}^0 = \frac{h_{\vartheta_{ij}} - h_{min}}{h_{max} - h_{min}} = \frac{h_{11} - h_{min}}{h_{max} - h_{min}} = \frac{\{0.3, 0.4\} - \{0.208\}}{\{0.700\} - \{0.208\}} = \frac{\left\{\frac{0.092}{0.792}, \frac{0.192}{0.792}\right\}}{\left\{\frac{0.492}{0.792}\right\}} = \left\{\frac{0.092}{0.492}, \frac{0.192}{0.492}\right\}$ is computed. Then, the corresponding hesitant degree and fuzzy degree of it are calculated as follows:

$$HD(h_{11}^0) = 1 - \frac{1}{2} = \frac{1}{2}$$

$$FD(h_{11}^0)^{HFE} = \frac{1}{2}\left(\begin{array}{c} -\frac{0.092}{0.492} log_2 \frac{0.092}{0.492} - \left(1 - \frac{0.092}{0.492}\right) log_2\left(1 - \frac{0.092}{0.492}\right) - \\ \frac{0.192}{0.492} log_2 \frac{0.192}{0.492} - \left(1 - \frac{0.192}{0.492}\right) log_2\left(1 - \frac{0.192}{0.492}\right) \end{array}\right) = 0.830$$

Afterwards, the Score-H&FD of h_{11}^0 is given, i.e.,

$$\Omega(h_{11}^0)^{HFE} = (1 - \frac{1}{2})^{0.5} \times (1 - 0.830)^{1-0.5} \times \left(\frac{1}{2} \times \left(\frac{0.092}{0.492} + \frac{0.192}{0.492}\right)\right) = 0.084$$

Finally, the effective performance of evaluation information is obtained.

Table 5. Linguistic evaluations on the four criteria.

Criteria	C_1	C_2	C_3	C_4
Evaluations	At least *I*	Between *ALU* and *M*	Between *M* and *ALI*	Between *ALI* and *I*

Source: The Authors.

Table 6. Hesitant fuzzy linguistic decision matrix on the four criteria.

Criteria	C_1	C_2	C_3	C_4
Evaluations	$\{s_2, s_3\}$	$\{s_{-1}, s_0\}$	$\{s_0, s_1\}$	$\{s_1, s_2\}$

Source: The Authors.

Table 7. Rating the criteria.

Performance	C_1	C_2	C_3	C_4
FU	0.208	0.111	0.310	0.260
U	0.216	0.121	0.319	0.269
ALU	0.232	0.141	0.338	0.288
M	0.263	0.182	0.375	0.325
ALI	0.325	0.263	0.450	0.400
I	0.450	0.425	0.600	0.550
VI	0.700	0.750	0.900	0.850

Source: The Authors.

$$\phi_{11} = log_2\left(\Omega\left(h_{11}^0\right)^{HFE} + 1\right) = log_2(0.084 + 1) = 0.117$$

Therefore, the corresponding normalised hesitant fuzzy decision matrix $H^0 = (h_{ij})^0_{5\times4}$, hesitant degree, fuzzy degree, Score-H&FD and effective performance are shown in Tables 8–12.

Step 5. Computing the normalised weight for each criterion. Similarly, the value of β is set to 0.5. For example, regarding the criterion C_1, we calculate its normalised weight, shown as:

Firstly, the hesitant degree of the hesitant fuzzy linguistic evaluation information ${H_S}^1$ is computed by Eq. (7).

$$HD({H_S}^1) = \frac{2 \times ln2}{(2 \times 3 + 1)ln(2 \times 3 + 1)} = 0.102$$

Then, the fuzzy degree of the hesitant fuzzy linguistic evaluation information ${H_S}^1$ is obtained by Eq. (21).

$$FD({H_S}^1)^{HFLE} = \frac{1}{2}\left(-\frac{5}{6} \times log_2\frac{5}{6} - \left(1 - \frac{5}{6}\right) \times log_2\left(1 - \frac{5}{6}\right)\right) = 0.325$$

Afterwards, the Score-H&FD of $\Omega({H_S}^1)^{HFLE}$ is calculated, i.e.,

$$\Omega({H_S}^1)^{HFLE} = (1 - 0.102)^{0.5} \times (1 - 0.325)^{1-0.5} \times \left(\frac{1}{2} \times \left(\frac{2+3}{6} + \frac{3+3}{6}\right)\right) = 0.714$$

Table 8. Normalised hesitant fuzzy decision matrix.

	C_1	C_2	C_3	C_4
A_1	{0.187,0.390}	{0.061,0.139}	{0.068}	{0.237,0.746}
A_2	{0.085,0.492}	{0.139,0.452,0.765}	{0.153,0.492}	{0.068,0.916}
A_3	{0.187,0.390,0.695}	{0.217,0.296,0.921}	{0.407,0.492,0.831}	{0.237,0.576,0.916}
A_4	{0.187,0.289,0.797}	{0.296,0.921}	{0.661,0.915}	{0.153,0.237,0.831}
A_5	{0.085,0.390,0797}	{0.452,0.609,0.765}	{0.915}	{0.237,0.661,0.831}

Source: The Authors.

Table 9. Hesitant degree of the normalised hesitant fuzzy decision matrix.

	C_1	C_2	C_3	C_4
A_1	0.500	0.500	0	0.500
A_2	0.500	0.667	0.500	0.500
A_3	0.667	0.667	0.667	0.667
A_4	0.667	0.500	0.500	0.667
A_5	0.667	0.667	0	0.667

Source: The Authors.

Table 10. Fuzzy degree of the normalised hesitant fuzzy decision matrix.

	C_1	C_2	C_3	C_4
A_1	0.830	0.457	0.358	0.804
A_2	0.710	0.787	0.808	0.388
A_3	0.849	0.676	0.877	0.730
A_4	0.763	0.636	0.671	0.688
A_5	0.705	0.915	0.418	0.790

Source: The Authors.

Table 11. Score-H&FD of the normalised hesitant fuzzy decision matrix.

	C_1	C_2	C_3	C_4
A_1	0.084	0.052	0.054	0.154
A_2	0.110	0.120	0.100	0.272
A_3	0.095	0.157	0.117	0.173
A_4	0.119	0.259	0.320	0.131
A_5	0.133	0.102	0.698	0.152

Source: The Authors.

Table 12. Effective performance of the normalised hesitant fuzzy decision matrix.

	C_1	C_2	C_3	C_4
A_1	0.117	0.073	0.076	0.207
A_2	0.150	0.164	0.137	0.347
A_3	0.131	0.210	0.159	0.230
A_4	0.162	0.333	0.400	0.178
A_5	0.180	0.140	0.764	0.205

Source: The Authors.

So, the denormalised weight of it is obtained by Eq. (28).

$$\varpi_1 = \left(\sqrt{2}\right)^{\Omega\left(H_S{}^1\right)^{HFLE}} = \left(\sqrt{2}\right)^{0.714} = 1.281$$

As a result, the hesitant degree, fuzzy degree, Score-H&FD and corresponding denormalised weight of each hesitant fuzzy linguistic evaluation over each criterion are listed in Table 13.

Table 13. Hesitant degree, fuzzy degree, Score-H&FD and denormalised weight of the hesitant fuzzy linguistic evaluations.

Criteria	C_1	C_2	C_3	C_4
Hesitant degree	0.102	0.102	0.102	0.102
Fuzzy degree	0.325	0.959	0.959	0.784
Score-H&FD	0.714	0.080	0.112	0.330
Denormalised weights	1.281	1.028	1.039	1.121

Source: The Authors.

Table 14. Normalised weight of each criterion.

Criteria	C_1	C_2	C_3	C_4
Normalised weights	0.287	0.230	0.233	0.251

Source: The Authors.

Finally, the normalised weight of each criterion is obtained by Eq. (29), and the results are shown in Table 14.

Step 6. The ranking of alternatives is determined based on Eq. (30), i.e.,

$$\chi_1 = (0.117 \times 0.287) + (0.073 \times 0.230) + (0.076 \times 0.233) + (0.207 \times 0.251) = 0.120$$

$$\chi_2 = (0.150 \times 0.287) + (0.164 \times 0.230) + (0.137 \times 0.233) + (0.347 \times 0.251) = 0.200$$

$$\chi_3 = (0.131 \times 0.287) + (0.210 \times 0.230) + (0.159 \times 0.233) + (0.230 \times 0.251) = 0.181$$

$$\chi_4 = (0.162 \times 0.287) + (0.333 \times 0.230) + (0.400 \times 0.233) + (0.178 \times 0.251) = 0.261$$

$$\chi_5 = (0.180 \times 0.287) + (0.140 \times 0.230) + (0.764 \times 0.233) + (0.205 \times 0.251) = 0.313$$

Hence, the final ranking is

$$A_5 \succ A_4 \succ A_2 \succ A_3 \succ A_1$$

In other words, for the five young teachers, the information literacy of A_5 is the highest.

5.3. Discussions

In this subsection, a sensitivity analysis is conducted by changing the parameters α and β. Afterwards, the HF-SMART method proposed in this article and the HF-SMART methods without considering hesitant degree or fuzzy degree are compared.

5.3.1. Sensitivity analysis

In our proposed method, two parameters α and β are uncertain. Different values of α and β indicate the respective importance of the hesitant degree and fuzzy degree. Therefore, the fluctuations of these two parameters may cause the final ranking to

Table 15. Sensitivity analysis.

Parameters		A_1	A_2	A_3	A_4	A_5	Ranking
$\alpha = 0$	$\beta = 0$	0.085	0.178	0.148	0.227	0.250	$A_5 \succ A_4 \succ A_2 \succ A_3 \succ A_1$
$\alpha = 0.1$	$\beta = 0.1$	0.090	0.181	0.154	0.233	0.261	$A_5 \succ A_4 \succ A_2 \succ A_3 \succ A_1$
$\alpha = 0.2$	$\beta = 0.2$	0.097	0.185	0.160	0.240	0.273	$A_5 \succ A_4 \succ A_2 \succ A_3 \succ A_1$
$\alpha = 0.3$	$\beta = 0.3$	0.104	0.190	0.166	0.247	0.286	$A_5 \succ A_4 \succ A_2 \succ A_3 \succ A_1$
$\alpha = 0.4$	$\beta = 0.4$	0.111	0.195	0.173	0.254	0.299	$A_5 \succ A_4 \succ A_2 \succ A_3 \succ A_1$
$\alpha = 0.5$	$\beta = 0.5$	0.120	0.200	0.181	0.261	0.313	$A_5 \succ A_4 \succ A_2 \succ A_3 \succ A_1$
$\alpha = 0.6$	$\beta = 0.6$	0.129	0.205	0.189	0.268	0.328	$A_5 \succ A_4 \succ A_2 \succ A_3 \succ A_1$
$\alpha = 0.7$	$\beta = 0.7$	0.139	0.212	0.197	0.276	0.345	$A_5 \succ A_4 \succ A_2 \succ A_3 \succ A_1$
$\alpha = 0.8$	$\beta = 0.8$	0.150	0.218	0.206	0.285	0.363	$A_5 \succ A_4 \succ A_2 \succ A_3 \succ A_1$
$\alpha = 0.9$	$\beta = 0.9$	0.161	0.225	0.216	0.294	0.383	$A_5 \succ A_4 \succ A_2 \succ A_3 \succ A_1$
$\alpha = 1$	$\beta = 1$	0.173	0.233	0.227	0.303	0.406	$A_5 \succ A_4 \succ A_2 \succ A_3 \succ A_1$

Source: The Authors.

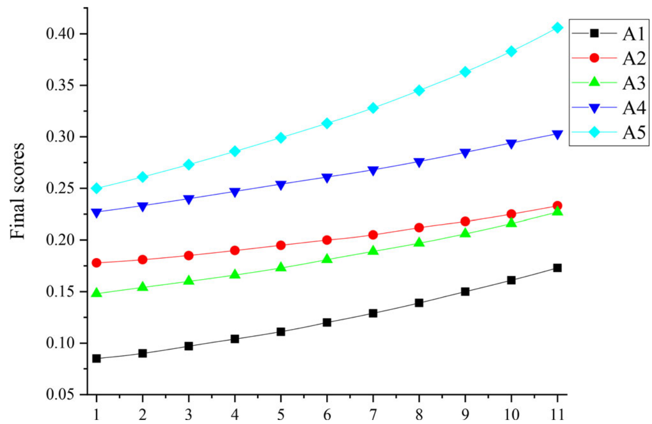

Figure 4. Sensitivity of final scores.
Source: The Authors.

change. Table 15 reports the final scores and ranking results when α and β fluctuate between 0 and 1.

It is not difficult to find that no matter how the values of α and β fluctuate, the final ranking results remain unchanged. At the same time, with the increasing values of the parameters α and β, which means that the fuzzy degree becomes more important, the final score of each alternative also increases accordingly. However, their respective growth rates are different. As illustrated in Figure 4, the ranking position of A_5 always takes the top spot and grows fastest. In addition, A_1 and A_4 display almost parallel growth. More importantly, as the values of α and β increase, the numerical gap between A_2 and A_3 is getting smaller and smaller, reaching the minimum when $\alpha = \beta = 1$. In other words, the fuzzy degree is not taken into account in terms of the HF-SMART method proposed in this article under this situation.

Taken together, the values of α and β do not change the ranking results of the alternatives. As a result, the HF-SMART assessment model has strong robustness and stability.

5.3.2. Comparative analysis

To further validate the feasibility and superiority of the HF-SMART model in this article, four scenarios are designed. Situation 1: Neither hesitant degree nor fuzzy

Table 16. Comparative analysis under different situations.

Situations	A_1	A_2	A_3	A_4	A_5	Ranking
1	0.302	0.468	0.596	0.625	0.692	$A_5 \succ A_4 \succ A_3 \succ A_2 \succ A_1$
2	0.173	0.233	0.227	0.303	0.406	$A_5 \succ A_4 \succ A_2 \succ A_3 \succ A_1$
3	0.085	0.178	0.148	0.227	0.250	$A_5 \succ A_4 \succ A_2 \succ A_3 \succ A_1$
4	0.120	0.200	0.181	0.261	0.313	$A_5 \succ A_4 \succ A_2 \succ A_3 \succ A_1$

Source: The Authors.

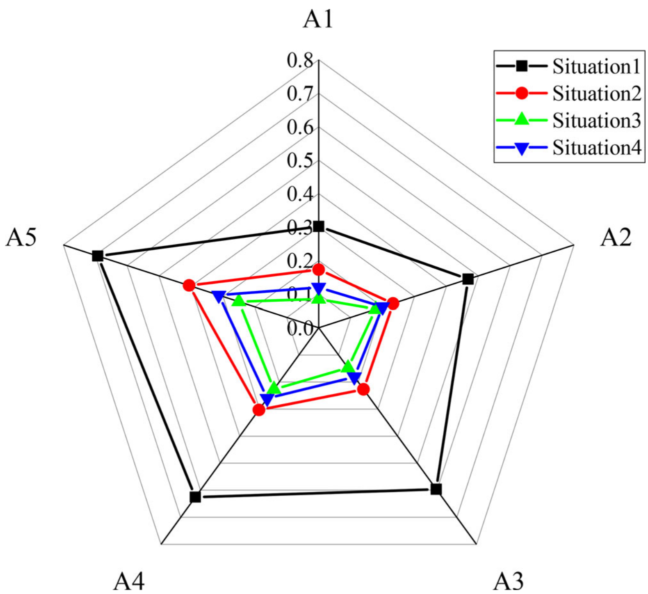

Figure 5. The final scores under different situations.
Source: The Authors.

degree information is considered. Situation 2: Only the hesitant degree information is considered. Situation 3: Only the fuzzy degree information is considered. Situation 4: Both the hesitant degree and fuzzy degree information are considered. Based on the above process, we calculate the decision results of situation 1, 2, 3 and 4. As a result, Table 16 presents the final scores and their respective ranking under different situations.

In Table 16, A_5 is always the best. However, if we take the hesitant degree or fuzzy degree into consideration (i.e., the situation 2, 3 or 4), then the ranking is $A_5 \succ A_4 \succ A_2 \succ A_3 \succ A_1$. If not, namely the situation 1, then the ranking is $A_5 \succ A_4 \succ A_3 \succ A_2 \succ A_1$. Obviously, the ranking positions of A_2 and A_3 take a change. The ranking position of the alternative A_2 goes back, and the alternative A_3 goes forward. Thus, we can clearly know that the final ranking results are affected by the hesitant degree or fuzzy degree information. In other words, the hesitant degree and fuzzy degree information could interfere with the accuracy of the final ranking, so it is necessary to remove them.

Furthermore, Figure 5 presents the final scores for alternatives under different situations, which commendably illustrates the hesitant degree and fuzzy degree information included in the original decision-making matrix. As a result, the final scores in situation 1 are significantly higher than in the other three situations. When the hesitant degree or fuzzy degree information is considered and removed, there is a significant reduction in the final score with respect to each alternative, as shown in the situations 2 and 3 of Figure 5. Moreover, it is worth noting that when both the hesitant degree and fuzzy degree information are considered and removed (i.e., situation 4), the final score for each alternative should be the lowest. However, the actual final scores are between the situation 2 and the situation 3. This is because we set the adjustment coefficients α and β, which balances the effects of hesitant degree and fuzzy degree information.

From all the discussions above, the classical SMART method is extended to the hesitant fuzzy environment. As a result, we put forward the HF-SMART method in this article, which has a wider range of application scenarios. In addition, we remove the hesitant fuzzy information in the original decision matrix (i.e., the hesitant degree and fuzzy degree information), which makes the final ranking result more accurate. Meanwhile, a sensitivity analysis through adjusting parameters α and β is conducted, and the final ranking result does not change. Therefore, a strong robustness is reflected in our proposed method. More importantly, through a comparative analysis under different situations, we can see that the final ranking derived from our method is more real and objective. Whereas, one source of weakness in this method which may have affected the ranking results is the setting of the parameters α and β, although the ranking position for five alternatives does not change in this case study. As for the determination of parameters α and β, it generally needs to be based on the actual decision situation or the preference of the DMs.

6. Conclusions

SMART is a practical tool to tackle the MCDM problems with crisp numbers, whereas it is not able to solve similar issues under the hesitant fuzzy environment. Nowadays, HFS and HFLTS are two effective tools to characterise human beings' hesitancy and fuzziness. To this end, we broaden the traditional SMART method to handle hesitant fuzzy decision-making problems by means of HFS and HFLTS, and thus the HF-SMART method is proposed in this article. On this basis, we first define the concept of the fuzzy degree of HFE and HFLE, and then put forward a new score function to compare the HFEs and HFLEs, which involves the hesitant and fuzzy degree information. In contrast to the existing score functions, the uncertain information is removed and the scores are expressed as precise numerical values as well. As a result, it enables the HFEs and HFLEs to be more comparable. With the help of the HF-SMART method, an information literacy evaluation case for five young teachers is carried out. Judging from the evaluation results, we can see that A_5 has the highest information literacy. Finally, we perform a sensitivity analysis and comparative analysis to validate our proposed method's rationality and practicality.

In future research, the reliability and applicability of the proposed fuzzy degree measure for DMs, and the new score functions for HFEs and HFLEs will be further explored. In addition, we will also work on applying the method proposed in this article to a broader range of fuzzy environments.

Disclosure Statement

No potential conflict of interest was reported by the author(s).

Funding

This work was supported by the National Natural Science Foundation of China (Nos. 71571123 and 71771155).

ORCID

Yong Qin http://orcid.org/0000-0002-4966-7899
Zeshui Xu http://orcid.org/0000-0003-3547-2908

References

Akgun, I., Kandakoglu, A., & Ozok, A. F. (2010). Fuzzy integrated vulnerability assessment model for critical facilities in combating the terrorism. *Expert Systems with Applications*, *37*(5), 3561–3573. https://doi.org/10.1016/j.eswa.2009.10.035

Boostani, H., & Hancer, P. (2018). A model for external walls selection in hot and humid climates. *Sustainability*, *11*(1), 100. https://doi.org/10.3390/su11010100

Borissova, D., & Keremedchiev, D. (2019). Group decision making in evaluation and ranking of students by extended simple multi-attribute rating technique. *Cybernetics and Information Technologies*, *19*(3), 45–56. https://doi.org/10.2478/cait-2019-0025

Edwards, W., & Barron, F. H. (1994). SMARTS and SMARTER: Improved simple methods for multiattribute utility measurement. *Organizational Behavior and Human Decision Processes*, *60*(3), 306–325. https://doi.org/10.1006/obhd.1994.1087

Farhadinia, B. (2015). Study on division and subtraction operations for hesitant fuzzy sets, interval-valued hesitant fuzzy sets and typical dual hesitant fuzzy sets. *Journal of Intelligent & Fuzzy Systems*, *28*(3), 1393–1402. https://doi.org/10.3233/IFS-141423

Hai, W., Xu, Z., & Zeng, X.-J. (2018). Hesitant fuzzy linguistic term sets for linguistic decision making: Current developments, issues and challenges. *Information Fusion*, *43*, 1–12. https://doi.org/10.1016/j.inffus.2017.11.010

Kangas, A., Saarinen, N., Saarikoski, H., Leskinen, L. A., Hujala, T., & Tikkanen, J. (2010). Stakeholder perspectives about proper participation for Regional Forest Programmes in Finland. *Forest Policy and Economics*, *12*(3), 213–222. https://doi.org/10.1016/j.forpol.2009.10.006

Li, D., Zeng, W., & Li, J. (2015). New distance and similarity measures on hesitant fuzzy sets and their applications in multiple criteria decision making. *Engineering Applications of Artificial Intelligence*, *40*, 11–16. https://doi.org/10.1016/j.engappai.2014.12.012

Liao, H., Gou, X., Xu, Z., Zeng, X.-J., & Herrera, F. (2020). Hesitancy degree-based correlation measures for hesitant fuzzy linguistic term sets and their applications in multiple criteria decision making. *Information Sciences*, *508*, 275–292. https://doi.org/10.1016/j.ins.2019.08.068

Liao, H., Qin, R., Gao, C., Wu, X., Hafezalkotob, A., & Herrera, F. (2019). Score-HeDLiSF: A score function of hesitant fuzzy linguistic term set based on hesitant degrees and linguistic scale functions: An application to unbalanced hesitant fuzzy linguistic MULTIMOORA. *Information Fusion*, *48*, 39–54. https://doi.org/10.1016/j.inffus.2018.08.006

Liao, H., Wu, X., Keikha, A., & Hafezalkotob, A. (2018). Power average-based score function and extension rule of hesitant fuzzy set and the hesitant power average operators. *Journal of Intelligent & Fuzzy Systems*, *35*(3), 3873–3882. https://doi.org/10.3233/JIFS-18794

Liao, H., & Xu, Z. (2013). A VIKOR-based method for hesitant fuzzy multi-criteria decision making. *Fuzzy Optimization and Decision Making*, *12*(4), 373–392. https://doi.org/10.1007/s10700-013-9162-0

Liao, H., Xu, Z., Herrera-Viedma, E., & Herrera, F. (2018). Hesitant fuzzy linguistic term set and its application in decision making: A state-of-the-art survey. *International Journal of Fuzzy Systems*, *20*(7), 2084–2110. https://doi.org/10.1007/s40815-017-0432-9

Liao, H., Xu, Z., & Zeng, X.-J. (2015a). Hesitant fuzzy linguistic VIKOR method and its application in qualitative multiple criteria decision making. *IEEE Transactions on Fuzzy Systems*, *23*(5), 1343–1355. https://doi.org/10.1109/TFUZZ.2014.2360556

Liao, H., Xu, Z., & Zeng, X.-J. (2015b). Novel correlation coefficients between hesitant fuzzy sets and their application in decision making. *Knowledge-Based Systems*, *82*, 115–127. https://doi.org/10.1016/j.knosys.2015.02.020

Liao, H., Xu, Z., Zeng, X.-J., & Merigo, J. M. (2015). Qualitative decision making with correlation coefficients of hesitant fuzzy linguistic term sets. *Knowledge-Based Systems*, *76*, 127–138. https://doi.org/10.1016/j.knosys.2014.12.009

Lootsma, F. A. (1996). A model for the relative importance of the criteria in the Multiplicative AHP and SMART. *European Journal of Operational Research*, *94*(3), 467–476. https://doi.org/10.1016/0377-2217(95)00129-8

Lourenzutti, R., & Krohling, R. A. (2013). A study of TODIM in a intuitionistic fuzzy and random environment. *Expert Systems with Applications*, *40*(16), 6459–6468. https://doi.org/10.1016/j.eswa.2013.05.070

Luo, Y., Zhang, X., Qin, Y., Yang, Z., & Liang, Y. (2021). Tourism attraction selection with sentiment analysis of online reviews based on probabilistic linguistic term sets and the IDOCRIW-COCOSO model. *International Journal of Fuzzy Systems*, *23*(1), 295–308. https://doi.org/10.1007/s40815-020-00969-9

Malovrh, S. P., Kurttila, M., Hujala, T., Karkkainen, L., Leban, V., Lindstad, B. H., Peters, D. M., Rhodius, R., Solberg, B., Wirth, K., Stirn, L. Z., & Krc, J. (2016). Decision support framework for evaluating the operational environment of forest bioenergy production and use: Case of four European countries. *Journal of Environmental Management*, *180*, 68–81. https://doi.org/10.1016/j.jenvman.2016.05.021

Oyetunji, A. A., & Anderson, S. D. (2006). Relative effectiveness of project delivery and contract strategies. *Journal of Construction Engineering and Management*, *132*(1), 3–13. https://doi.org/10.1061/(ASCE)0733-9364(2006)132:1(3)

Qin, Y., Wang, X. X., & Xu, Z. S. (2022). Ranking tourist attractions through online reviews: A novel method with intuitionistic and hesitant fuzzy information based on sentiment analysis. *International Journal of Fuzzy Systems*, *24*(2), 755–777. https://doi.org/10.1007/s40815-021-01131-9

Rodriguez, R. M., Martinez, L., & Herrera, F. (2012). Hesitant fuzzy linguistic term sets for decision making. *IEEE Transactions on Fuzzy Systems*, *20*(1), 109–119. https://doi.org/10.1109/TFUZZ.2011.2170076

Shannon, C. E. (1948). A mathematical theory of communication. *Bell System Technical Journal*, *27*(3), 379–423. https://doi.org/10.1002/j.1538-7305.1948.tb01338.x

Torra, V. (2010). Hesitant fuzzy sets. *International Journal of Intelligent Systems*, *25*(6), 529–539. https://doi.org/10.1002/int.20418

Torra, V., & Narukawa, Y. (2009). *On hesitant fuzzy sets and decision* [2009 IEEE international conference on fuzzy systems, vols 1-3] [Paper presentation].18th IEEE International Conference on Fuzzy Systems, Jeju Isl, South Korea.

Wang, B., Liang, J., & Pang, J. (2019). Deviation degree: A perspective on score functions in hesitant fuzzy sets. *International Journal of Fuzzy Systems*, *21*(7), 2299–2317. https://doi.org/10.1007/s40815-019-00722-x

Wei, C., Rodriguez, R. M., & Martinez, L. (2018). Uncertainty measures of extended hesitant fuzzy linguistic term sets. *IEEE Transactions on Fuzzy Systems*, *26*(3), 1763–1768. https://doi.org/10.1109/TFUZZ.2017.2724023

Xia, M., & Xu, Z. (2011). Hesitant fuzzy information aggregation in decision making. *International Journal of Approximate Reasoning*, *52*(3), 395–407. https://doi.org/10.1016/j.ijar.2010.09.002

Xu, Z., & Xia, M. (2011a). Distance and similarity measures for hesitant fuzzy sets. *Information Sciences*, *181*(11), 2128–2138. https://doi.org/10.1016/j.ins.2011.01.028

Xu, Z., & Xia, M. (2011b). Induced generalized intuitionistic fuzzy operators. *Knowledge-Based Systems*, *24*(2), 197–209. https://doi.org/10.1016/j.knosys.2010.04.010

Xu, Z., & Xia, M. (2011c). On distance and correlation measures of hesitant fuzzy information. *International Journal of Intelligent Systems*, *26*(5), 410–425. https://doi.org/10.1002/int.20474

Xu, Z., & Zhang, X. (2013). Hesitant fuzzy multi-attribute decision making based on TOPSIS with incomplete weight information. *Knowledge-Based Systems*, *52*, 53–64. https://doi.org/10.1016/j.knosys.2013.05.011

Zadeh, L. A. (1965). Fuzzy sets. *Information and Control*, *8*(3), 338–353. https://doi.org/10.1016/S0019-9958(65)90241-X

Zadeh, L. A. (1975). The concept of a linguistic variable and its application to approximate reasoning-III. *Information Sciences*, *9*(1), 43–80. https://doi.org/10.1016/0020-0255(75)90017-1

Zeng, W., Li, D., & Yin, Q. (2016). Distance and similarity measures between hesitant fuzzy sets and their application in pattern recognition. *Pattern Recognition Letters*, *84*, 267–271. https://doi.org/10.1016/j.patrec.2016.11.001

Zhang, X., & Xu, Z. (2014). The TODIM analysis approach based on novel measured functions under hesitant fuzzy environment. *Knowledge-Based Systems*, *61*, 48–58. https://doi.org/10.1016/j.knosys.2014.02.006