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Effects of financial constraints and policy uncertainty on the economy with shifting trend inflation

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ABSTRACT
The primary purpose of this paper is to investigate macroeconomic, financial and welfare effects of financial constraints and policy uncertainty on the economy featuring shifting trend inflation. By developing a New Keynesian model incorporating trend inflation into staggered prices and staggered credit channel, we indicate three important findings. First, we report negligible welfare consequences of financial shocks, whereas policy uncertainty shocks dampen the economic welfare considerably. More importantly, financial frictions are a channel through which policy uncertainty stuns the economy more remarkably. Second, the welfare consequences and business cycles effects of shocks are greater in the high-trend-inflation economy, while the costs of exogenous variations in trend inflation are larger if there is policy uncertainty. Third, among staggered prices and staggered credit, the later plays a more vital role in transmitting adverse effects of shocks to trend inflation into the economy.

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1. Introduction
The recent global financial crisis in 2007 suggests that an introduction of financial sector into analysis of a standard model sheds vital new light on the sources of business cycle fluctuations (Bernanke et al., 1996; Caldara et al., 2013). Firms may face the financial constraints and frictions that are defined as the firm’s ability to borrow is completely subject to financial market’s capacity (Hoang, 2018; Jermann & Quadrini, 2012). Furthermore, economists and policy makers also realize that the U.S. economy are simultaneously buffeted by sustained rises in inflation and larger-than-usual uncertainty about future policy (Ha et al., 2020b). The term ‘uncertainty’, referred to as ‘objective uncertainty’ or ‘risk’ in the literature, is defined as the dispersion or spread of economic shocks distribution. It can be seen that the financial sector, the sustained rise in inflation, and policy uncertainty are three main features of the U.S. economy.
Trend inflation has recently gained attention of many scholars. For example, Chan et al. (2018) employ a bivariate model of inflation and survey-based long-run forecast of inflation to examine the nexus between trend inflation and the long-run forecast. By incorporating both staggered price and staggered wage contracts into the New Keynesian model, Ha et al. (2019) measure the welfare consequences of shifting trend inflation, while Ha et al. (2020a) develop the model with trend inflation capturing the characteristics of Vietnam’s economy to quantify the consequences of shifts in trend inflation. Adam and Weber (2019) use sticky price model incorporating heterogeneous firms and systematic firm-level productivity trend to predict the optimal trend inflation rate, while Kamber and Wong (2020) develop the model to study the influence of global factors in shifting trend inflation.

Moreover, evidence suggesting a close link between the trend inflation and the financial market emerges from the theoretical and empirical literature. Specifically, the efficiency of resource allocations in financial sector can be impacted by a sustained increase in inflation. Huybens and Smith (1998, 1999) discuss that credit market fictions with negative repercussions for financial sector performance can be adversely affected by a rise in inflation. More specifically, the increase in inflation rate leads to reduction in both real rate of money and assets, then signifies credit market frictions. This effect results in fewer loans, less efficient resource allocations, and a decline in intermediary activity and then capital investment. Both the long-run economic performance and equity market, as a consequence, are negatively affected (Choi et al., 1996. Boyd et al., 2001; Huybens & Smith, 1999) also argue that there is a nonlinear and negative relationship between inflation and banking sector development and equity market activity. Other authors also mention that the binding credit market frictions happen only when the inflation rate exceeds the threshold level (Azariadis & Smith, 1996; Choi et al., 1996). Therefore, the sustained inflation has adverse impacts on the financial sector.

Furthermore, an emergent literature has emphasized on financial market frictions as an additional channel through which policy uncertainty can stun the economy (Arellano et al., 2011; Christiano et al., 2014). With an imperfect financial market, higher uncertainty leads to a tighter credit constraint, then causes firms to reduce the size of their projects to avoid default, and leads to a more sizeable effect on firms’ output. In addition, Benk et al. (2005) and Jermann and Quadrini (2012) also regard financial shocks originating from the financial sector as a vital source of business cycles and propagating shocks’ propagation. Therefore, the model incorporating both financial frictions and policy uncertainty helps us to separately investigate their impacts on both the macroeconomic and financial market.

The aforementioned discussions inspire us to develop the model incorporating sustained rise in inflation, financial constraints and policy uncertainty. It is also important to provide reasons to explain why the economic welfare is the vital concern in this study. Aiyagari et al. (1998) develop the model establishing a link between money demand as well as the relative size of credit services sector and welfare costs of inflation. They also argue two different effects of inflation on the share of total output devoted to transaction services, and on the labor supply and investment decision. Money demand functions play a vital role in explaining welfare differences of
inflation rate. Regarding welfare of financial constraints, to our best knowledge, there are few theoretical studies on a relationship between financial frictions and welfare. Obiols-Homs (2011) exploit welfare effects of exogenous borrowing limits. He argues that tight borrowing limits might adversely affect the welfare. He also shows that the welfare is displayed by a bell shaped function of the borrowing limits. Regarding policy uncertainty, few papers exploit the welfare analysis but only consider one-sided movement of volatility (a decrease in the certain level of volatility of level shocks to zero), for example Lester et al. (2014) and Cho et al. (2015). The recent work by Xu (2017), and Bachmann et al. (2018) examines welfare consequences of time-varying volatility. Ha et al. (2020b) built the New Keynesian model to measure welfare consequences of the uncertainty about monetary policy in the economy with shifting trend inflation. However, there is no paper investigating interactions of trend inflation and financial constraint as well as policy uncertainty in term of welfare.

Therefore, the current literature has still remained some existing gaps for latter researchers to fill. These gaps can be listed here. (i) Authors have so far discussed the features of the U.S. economy, including the sustained rise in inflation, time-varying volatility and the financial frictions in isolation, while their interactions are expected to help us to exploit many important implications for both macroeconomic and financial dynamics. (ii) Regarding financial shocks6 and policy uncertainty shocks, previous studies have just focused on the aggregate impacts while little investigate their welfare effects. (iii) the financial constraints, which are potentially a channel through which both shifting trend inflation7 and policy uncertainty shocks transfer their adverse impacts on the economy, are not paid enough attention.

In this paper, we make at least three contributions to the literature. Firstly, we are the first to develop a New Keynesian model8 featuring time-varying trend inflation and financial frictions in the form of credit constraints, and policy uncertainty. The main goal of this paper is to investigate the macroeconomic, financial and welfare effects of financial constraints and policy uncertainty on the economy with shifts in trend inflation. To model a sustained increase of inflation, we use a highly persistent shock to trend inflation, regarded as the central bank’s slowly-moving implicit inflation targets as argued by Kozicki and Tinsley (2001), Ireland (2007), Cogley and Sbordone (2008) and Cogley et al. (2009). Therefore, trend inflation is not a constant value but a shock that follows a highly persistent AR(1) process. Regarding uncertainty, both structural shocks and time-varying volatility shocks jointly participate in the model. We concentrate on the uncertainty arising in the monetary policy shocks, the technology shocks, and the government spending shocks. The stochastic volatility shock is assumed to follow an AR(1) process as in Shephard (2008) and Fernandez-Villaverde et al. (2011). Moreover, we also consider the effects of financial shocks and frictions in the economy as Jermann and Quadrini (2012). Second, we investigate macroeconomic and financial effects of financial shocks and frictions under the working of exogenous constraint9 which the firm’s ability to borrow is completely subject to financial market’s capacity, instead of endogenous constraint discussed in the work of Kiyotaki and Moore (1997), Bernanke and Gertler (1989), Mendoza and Smith (2006) and Mendoza (2010).Third, the exogenous credit constraints include trend inflation, suggesting that any change in a central bank’s inflation targets also leads to
changes in the financial conditions. In particular, changes in inflation targets cause a
nominal price adjustment cost, and then credit constraints to fluctuate. These direct
effects of trend inflation on the financial sector are defined as a ‘staggered credit’
channel in this study. This paper follows Jermann and Quadrini (2012) to concentrate
on the period after 1984 since this time period is marked by major changes in the
U.S. financial market.

Some important findings should be emphasized. First, based on our baseline par-
parameter values, welfare costs of financial shocks were negligible, whereas policy uncer-
tainty shocks dampened the economic welfare more remarkably. Furthermore, we
also examined changes in welfare cost with respect to changes in relevant parameters.
We found that a large enough increase in the debt-to-output ratio, the substitution
level between debt and equity payout or a reduction in the tax advantage cause the
financial shocks to dampen economic welfare more greatly. The financial shocks’
properties, such as the persistence level and the volatility level also played a vital role
in explaining welfare consequences of financial shocks. Second, we provided empirical
evidences on interactions between trend inflation and financial shocks as well as pol-
icy uncertainty shocks. In term of welfare, welfare costs of these shocks became more
noticeable when the central banks set their inflation target to a higher level. In term
of responses of macroeconomic and financial variables to distinct shocks, these varia-
tles responded more to structural and volatility shocks in the high-trend-inflation
economy. These evidences suggest that the interaction between trend inflation and
these shocks were quantitatively important. Moreover, we also found that the finan-
cial friction is an additional channel though which policy uncertainty distorts
the economy.

The rest of this paper is organized as follows. Section 2 provides a description of
related works, while Section 3 presents the extended model. Section 4 explains the
method to compute welfare and welfare costs. Parameterization are presented in
Section 5 while Section 6 shows estimated results. Some conclusions are provided in
Section 7.

2. Literature review

This paper is mostly related to two strands of the literature. The first strand consists
of studies incorporating the financial shocks and frictions into an estimated DSGE
model, which has increasingly important to explain sources of fluctuations. Some first
work starts by Kiyotaki and Moore (1997), Bernanke et al. (1996), Mendoza and
Smith (2006) and Mendoza (2010) which firm’s ability to borrow is subjected to an
endogenous collateral constraint. In this regard, firm’s ability to borrow varies with
changes of profitability due to the business cycle. Moreover, firm can lose borrowing
constraints by over-accumulating capital and can partly determine the maximum
amount of debt to borrow. Jermann and Quadrini (2012) apply the same approach
but differ to what extent that they allow firms to use debt and equity payout to
finance investment, and they allow for negative values of equity payout that permit
firms to not limit to reinvest profits. Further, along with study of Benk et al. (2005),
they also consider the financial shock originating from the financial sector to be a
vital sources of business cycles and propagating other shocks. Their results indicate that the transmission mechanism of financial shocks on dynamics of real and financial variables is similar to the typical credit channel. The important role of the financial shock originating in the financial sector on the macroeconomic fluctuation is also emphasized by Christiano et al. (2008), Kiyotaki and Moore (2008) and Gilchrist et al. (2009). More recently, Hoang (2018) investigates employment and output influences of financial shocks. He develops a New Keynesian model incorporating financial frictions in the form of credit constraints and shows that the financial shock significantly affects output and employment variation. Ge et al. (2020) develop a DSGE model to uncover the transmission of diverse financial shocks. They show that there is an interaction between financial friction tied to banks and households over time. Furthermore, the financial shocks play a critical role on the dynamics of housing and macroeconomic variables. Furthermore, Kirchner (2020) employs financial frictions to capture the nonlinearities of the Great Financial Crisis. He considers the existence of shadow banking system as a type of these friction. In general, prior scholars have paid lots of attention to financial frictions and these frictions are captured in various forms.

The second strand is related to uncertainty or volatility shocks. The literature has so far mostly concentrated on the short-run real effects of volatility shocks. Unfortunately, scientific evidence on the aggregate effects of uncertainty is still inconclusive in the literature. Alexopoulos and Cohen (2009), Bloom (2009) and Bloom et al. (2012) illustrate a large impact of uncertainty about productivity on macroeconomy, whereas a little impact of this shock is reported by Bachmann and Bayer (2013), Bachmann et al. (2013), Bekkaert et al. (2013) and Born and Pfeifer (2014). Fasani (2017) shows that both output and inflation more likely decrease in response to uncertainty shocks. Bianchi et al. (2019) demonstrate a significant impacts of uncertainty on risk premia and business cycle fluctuations. More recently, Pellegrino et al. (2020) study consequences of uncertainty shocks during extreme events like the great recession and the Covid-19 outbreak. By using a nonlinear VAR framework, they document a large output loss caused by a financial uncertainty shock during the great recession. The Covid-19-induced uncertainty leads to more serious consequences as compared to that of great recession. It can also be seen that little efforts have devoted to the welfare effects of uncertainty or policy risk. Lucas (1987) shows a negligible gain from eliminating consumption fluctuations. Subsequent studies, namely Obstfeld (1994), Campbell and Cochrane (1999) and Krusell et al. (1999) find a larger welfare cost than Lucas (1987). More recently, Lester et al. (2014) and Cho et al. (2015) provide interesting results which welfare can be higher in a more volatile economy since agents can use uncertainty purposefully in their favour. However, these papers just concentrates on one-sided movement in volatility, which they examine welfare gain from decreasing volatility of shocks from a certain level to zero. This paper differs from the literature to the extent that it studies welfare costs of shocks to volatility understood as a two-sided movement. Ha et al. (2020b) study welfare costs of uncertainty about monetary policy. They develop a DSGE model featured by both monetary policy uncertainty and shifting trend inflation and show that the policy uncertainty distorts the economic welfare negligibly, but a rise in level of trend inflation cause the welfare consequence to be more serious.
3. Model

This section develops a New Keynesian model that is populated by four classes of agents, including households, final-goods producing firms, a continuum of intermediate-goods producing firms indexed by \( i \in [0, 1] \), and authorities.

3.1. Households

During each period, households receive the face value of bonds, \( (b_t) \), that purchase in the period \( (t-1) \) at the start of each period. They also receive real dividends, \( (d_t) \), from distinct intermediate-goods producing firms at the end of period, and they provide \( N_t(i) \) units of labor to each intermediate-goods producing firm \( i \) (\( i \in [0, 1] \)) during period \( t \) to earn \( W_tN_t \) where \( W_t \) is the nominal wage rate. In addition, a lump-sum tax, \( (T_t) \), imposed to finance the government spending, \( (G_t) \), impacts households’ budget. In each period, households buy consumption goods, \( (C_t) \), from the final-goods producing firms at the nominal price, \( (P_t) \). They also make a saving plan by purchasing \( b_{t+1} \) units of bonds in the period \( (t+1) \) issued by intermediate-goods producing firms at the price \( 1/R_t \), where \( R_t \) is the gross nominal interest rate between period \( t \) and \( t+1 \). To sum up, the flow budget constraint faced by households can be described as below

\[
P_tC_t + \frac{b_{t+1}}{R_t} \leq W_tN_t + b_t - T_t + P_td_t. \tag{1}
\]

Given this budget constraint, households maximize the expected discounted present value of future period utility

\[
\sum_{t=0}^{\infty} \beta^t (\ln(C_t-hC_{t-1}) - \frac{\omega}{1+\nu} N_t^{1+\nu})), \tag{2}
\]

where \( \beta \) and \( h \) denote the discount factor and the habit formation parameter and they are restricted as \( 0 < \beta < 1 \), \( 0 \leq h < 1 \). Other parameters, \( \omega \) and \( \nu \), measure the relative disutility labor effort, and the inverse of Frisch labor supply elasticity, respectively. We solve households’ problem by choosing \( C_t, N_t, \) and \( b_t \) such that maximize the utility described as in Equation (2) given the budget constraint described as in Equation (1) to yield

\[
\tilde{\lambda}_t = \frac{1}{C_t - hC_{t-1}} - \beta h \frac{1}{C_{t+1} - hC_t}, \tag{3}
\]

\[
\frac{W_t}{P_t} \tilde{\lambda}_t = \omega N_t^\nu, \tag{4}
\]

\[
\tilde{\lambda}_t = \beta R_t E_t \left( \frac{\tilde{\lambda}_{t+1}}{\pi_{t+1}} \right), \tag{5}
\]
where \( \hat{\lambda}_t \) is a non-negative Lagrangian multiplier on the budget constrains, \( \pi_t \) is the
gross inflation rate between \( t \) and \( t+1 \). Relations between the Lagrangian multiplier and
the intertemporal marginal utility of consumption goods, and relations between
the real wage and the substitution rate of leisure for consumption are presented in
the optimality condition (3) and (4), respectively. Equation (5) implies a link between
the real interest rate to the intertemporal marginal rate of substitution.

3.2. Final-goods producing firms

The final-goods producing firms are assumed to operate in a competitive environ-
ment. In order to manufacture \( Y_t \) units of final products, the firms employ \( Y_t(i) \) units
of intermediate goods and the constant-return-to-scale technology represented as
below

\[
\left[ \int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}} = Y_t, \tag{6}
\]

where \( \theta \) is the price elasticity of demand for intermediate goods. The profit maxi-
mization problem of final-goods producing firms is given as

\[
P_t \left[ \int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}} - \int_0^1 P_t(i)Y_t(i)di. \tag{7}
\]

The first order conditions for this problem are

\[
Y_t(i) = \left[ \frac{P_t(i)}{P_t} \right]^{-\theta} Y_t, \tag{8}
\]

\[
P_t = \left[ \int_0^1 P_t(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}}. \tag{9}
\]

Equation (9) is derived from an assumption of competitive environment so that
the firms have a zero profit in the equilibrium.

3.3. Intermediate-goods producing firms

There is a continuum of monopolistically competitive intermediate goods firm \( i, i \in [01] \). In order to produce \( Y_t \) units of differentiated intermediate goods \( (i) \), the in-
termediate-goods producing firms hire \( N_t(i) \) units of labor from households during the
period \( t \). The constant-return-to-scale technology of intermediate-goods producing
firms can be represented as
The logarithm of aggregate technology shock, \( A_t \), is described as a stationary stochastic process
\[
\ln (A_t) = \rho A \ln (A_{t-1}) + \sigma_A^2 \epsilon_A, \tag{11}
\]
where \( \epsilon_A \) is the serially uncorrelated innovation and \( \sigma_A^2 \) is a stochastic volatility shock that allows for the time-varying volatility of technology shock.

Intermediate-goods producers are assumed to set nominal prices as in staggered Rotemberg price fashion. We consider two scenarios: firms face and do not face the financial constraints.

### 3.3.1. Model without financial constraints

In the benchmark model, an intermediate-goods producing firm \((i)\), which does not face credit constraints, only selects the prices such that it maximizes the expected discounted future profits
\[
E_t \sum_{j=0}^{\infty} \beta^j \lambda P_{t+j} \left\{ \frac{P_{t+j}(i)}{P_{t+j} Y_{t+j}(i)} - \frac{W_t(i)}{P_{t+j}} N_{t+j}(i) - \frac{AC_{t+j}(i)}{P_{t+j}} \right\}, \tag{12}
\]
subject to the quadratic price adjustment cost, \( AC_t(i) \), that is given as
\[
AC_t(i) = \frac{\phi}{2} \left( \frac{P_t(i)}{(\pi^\mu_t \bar{\pi}_t^{1-\mu})^{\eta} P_{t-1}(i)} - 1 \right)^2 Y_t(i), \tag{13}
\]
where \( \phi \) captures the price adjustment cost, and \( \rho \) and \( \omega \) denote a degree of price indexation and weight on lagged inflation.

### 3.3.2. Model with financial constraints

In the second scenario, we develop the model consisting of an intermediate-goods producing firm \((i)\) with financial constraints. In particular, both equity and debt can be employed by firms as financial resources in this model. However, the debt is preferred to the equity as argued in the pecking order theory suggested by Myers (1984). Firms’ equity payouts, \((d_t)\), which are subject to a quadratic adjustment cost, are not perfectly substituted by debts, \((b_t)\). Accordingly, the actual cost, \( \Phi(d_{i,t}) \) given the equity payout, \( d_{i,t} \), is expressed as a sum of \( d_{i,t} \) and the quadratic adjustment cost as given
\[
\Phi(d_{i,t}) = d_{i,t} + \frac{\eta}{2} (d_{i,t} - d_t)^2, \tag{14}
\]
where \( \eta \geq 0 \) captures the degree of rigidities representing the substitution level between equity and debt. Equity payouts can take either negative or positive values. Negative equity payouts imply an issuance of equity. As argued by Jermann and Quadrini (2012), firms enjoy tax benefits from the government from issuing
one-period bonds. In particular, holders receive payments, \( b_{i,t} \), from the type-\( i \) firm. This firm, then, makes decision on new debts, \( (b_{i,t+1}) \) at the beginning of each period \( t \) to receive \( \left( \frac{b_{i,t+1}}{R_t} \right) \) from purchasers and \( \left( \frac{b_{i,t+1}}{R_t^e} - \frac{b_{i,t+1}}{R_t} \right) \) from the government, where \( R_t^e = R_t - \tau(R_t-1) \) is the effective gross interest rate for the firms. \( \tau \) is interpreted the tax benefit when issuing debt and \( R_t^e = R_t \) when there is no tax benefit (\( \tau = 0 \)).

Moreover, the intermediate goods are traded in the monopolistically competitive market, thus they are differentiated and are not perfect substitutions for another to produce the final goods. Therefore, the intermediate-goods producing firms set their own prices such that their demands are met at their predetermined price. And the firms’ objectives are the same as households since they are owned by households. To pursue these objectives, the type-\( i \) firm makes a decision on the selling price, \( (p_{i,t}) \), labor demand, \( (N_{i,t}) \), equity payout, \( (d_{i,t}) \), and new debts, \( (b_{i,t+1}) \), subject to a quadratic adjustment cost at the beginning of period. The nominal price adjustment cost is presented as

\[
\chi(P_{i,t}, P_{i,t-1}) = \frac{\phi}{2} \left[ \frac{P_{i,t}}{(\bar{\pi}_t^{o} \bar{\pi}_t^{1-o})^p P_{i,t-1}} - 1 \right]^2 Y_t, \tag{15}
\]

where \( \phi \) denotes the degree of price adjustment cost and \( \bar{\pi}_t \) is trend inflation interpreted as central bank’s implicit inflation target and private sector’s long-run inflation expectation. We also assume that firms finance the total cost, including the wage bills, \( (W_tN_t) \), the actual cost of equity payout, \( (\Phi(d_{i,t})) \), matured intertemporal debts, \( (b_{i,t}) \), and the cost of nominal price adjustment, \( (\chi(P_{i,t}, P_{i,t-1})) \) at the beginning of each period. Therefore, the exogenous credit constraint faced by firms can be written as

\[
f_t \geq \frac{W_tN_{i,t} + b_{i,t} - \frac{b_{i,t+1}}{R_t^e}}{P_t} + \Phi(d_{i,t}) + \chi(P_{i,t}, P_{i,t-1}) + \frac{b_{i,t+1}}{P_t R_t}, \tag{16}
\]

where \( f_t \) denotes the financial market condition. Notice that \( f_t \) behaves in the model as a shock due to randomness in the financial market’s condition. These financial shocks follow a stationary stochastic process

\[
\ln (f_t) = (1-\rho_f) \ln (f) + \rho_f \ln (f_{t-1}) + \epsilon_{f,t}, \tag{17}
\]

where \( \rho_f \in [0, 1) \), and \( f \) capture the shock persistence value and the steady-state value of the financial shock, respectively. \( \epsilon_{f,t} \) is the serially uncorrelated innovation, which has a normal distribution with zero mean and standard deviation \( \sigma_f \).

Moreover, we expand the model with an assumption that trend inflation \( (\bar{\pi}_t) \) participates in the model as a shock rather than a simple steady-state value. The evolution of trend inflation can be described as a AR(1) process to model the sustained rise in inflation as follow

\[
\ln (\bar{\pi}_t) = (1-\rho_{\pi}) \ln (\bar{\pi}) + \rho_{\pi} \ln (\bar{\pi}_{t-1}) + \epsilon_{\pi,t}, \tag{18}
\]
where $\rho_x \in [0, 1)$, and $\pi^*$ are the shock persistence value and trend inflation, respectively. $\varepsilon_{\pi_i}$ is a standard normal and independent of time. Equation (15) indicates that as long as there are changes in the inflation targets, the nominal price adjustment cost, and then the credit constraint fluctuates accordingly.

To sum up, the maximization problem of type-$i$ intermediate-goods producing firm is expressed as

$$\max_{b_{i,t+1}, d_{i,t}, N_{i,t}, P_{t, t}} E_0 \sum_{t=0}^{\infty} \beta^t \gamma_t d_{i,t},$$

subject to

$$Y_t(i) = A_t N_{i,t},$$
$$Y_t(i) = \frac{P_t(i)}{P_t} Y_t,$$
$$\Phi(d_{i,t}) + \chi(P_{i,t}, P_{i,t-1}) + \frac{W_t N_{i,t} + b_{i,t}}{P_t} \leq \frac{P_t Y_{i,t} + \frac{b_{i,t+1}}{P_t R_t}}{P_t},$$
$$\Phi(d_{i,t}) = d_{i,t} + \frac{\eta}{2} (d_{i,t} - d_t)^2,$$
$$\chi(P_{i,t}, P_{i,t-1}) = \frac{\Phi}{2} \left[ \frac{P_t}{\left( \pi_{t-1} \pi_t^{1-\omega} P_t \right)^{P_{i,t-1}}} - 1 \right] Y_t,$$
$$f_t \geq \frac{W_t N_{i,t} + b_{i,t} - \frac{b_{i,t+1}}{P_t R_t}}{P_t} + \Phi(d_{i,t}) + \chi(P_{i,t}, P_{i,t-1}) \frac{b_{i,t+1}}{P_t R_t}.$$
where $K_{i,t}$ and $\mu_{i,t}$ are the non-negative Lagrangian multipliers regarding the budget and borrowing constraints, respectively. Solving the profit maximization problem for firms with respect to price, debt, and equity payouts, we obtain Equations (e1), (e2), and (e3), respectively. The optimization condition for labor demands, Equation (e4), reflects a constraint for firm which the demand and supply for intermediate-goods must be equal. The exogenous credit constraint and the associated Kuhn-Tucker condition are presented in Equations (e5) and (e6).

For the sake of simplicity, we impose an assumption of symmetric equilibrium that the the intermediate-goods producing firms make identical decisions. In other words, we have $b_{i,t} = b_t, d_{i,t} = d_t, d_i = d, N_{i,t} = N_t, P_{i,t} = P_t, \Lambda_{i,t} = \Lambda_t$ and $\mu_{i,t} = \mu_t$. Therefore, the first order conditions can be reexpressed in the symmetric equilibrium as follows

\[
A_t^{-1}0_t \frac{W_t}{P_t} (\theta_t-1) \left[ \frac{\mu_t-L_t}{\mu_t} - \phi \left( \frac{P_t}{(\pi_{t-1}^{1-\omega})^{\beta} P_{t-1}} - 1 \right) \right] \frac{P_t}{(\pi_t^{1-\omega})^{\beta} P_{t-1}} = 0, \quad (e1)
\]

\[
\beta \phi E_t \left\{ \frac{\mu_{t+1}}{\mu_t} - \frac{P_t}{P_{t+1}} \right\} = \Lambda_t \frac{1}{R_t}, \quad (e2)
\]

\[
\mu_t \left[ 1 + \eta(d_t - d) \right] = \lambda_t, \quad (e3)
\]

\[
Y_t = A_t N_t, \quad (e4)
\]

\[
d_t + \eta(d_t - d)^2 + \frac{\phi}{2} \left( \frac{P_t}{(\pi_{t-1}^{1-\omega})^{\beta} P_{t-1}} - 1 \right)^2 Y_t + \frac{W_t N_t + b_t}{P_t} = Y_t + \frac{b_{t+1}}{R_t P_t}, \quad (e5)
\]

\[
\Lambda_t \left[ (Y_t + \frac{b_{t+1}}{P_t R_t} - f_t) \right] = 0. \quad (e6)
\]

**Proposition 1.** In the model with constant trend inflation, the financial constraint binds in the steady-state if there is a tax benefit ($\tau > 0$) from issuing new debt.

**Proof.** In the steady-state, we have (the detail about the steady-state model are discussed in Appendix C)

\[
\Lambda(\bar{\pi}) = \left\{ \frac{R(\bar{\pi})}{R^e(\bar{\pi})} - 1 \right\} \mu(\bar{\pi}).
\]

Because the budget of firm in the equilibrium is binding, it implies $(\mu(\bar{\pi}) > 0)$. We also have $(\frac{R(\bar{\pi})}{R^e(\bar{\pi})} - 1) > 0$ due to the positive tax benefit ($\tau > 0$). Accordingly, $\Lambda(\bar{\pi})$ also takes a positive value or there is a binding financial constraint.
Proposition 2. If trend inflation varies over time and debts are not perfectly substituted by equity payout, the shifting trend inflation can lead to changes in financial conditions, then a more sizeable impacts on consumption and employment, and finally on the welfare.\textsuperscript{11}

Proof. The financial constraint in the study can be written as

\[
f_t \geq \frac{W_t N_{i,t} + b_{i,t} - \frac{b_{i,t+1}}{P_t}}{P_t} + \Phi(d_{i,t}) + \chi(P_{i,t}, P_{i,t-1}) + \frac{b_{i,t+1}}{P_t R_t}.
\]  

(19)

The property of time-varying trend inflation causes the nominal price adjustment, \((\chi(P_{i,t}, P_{i,t-1}))\), to change accordingly, then financial conditions which leads to changes in consumption and employment. In particular, negative financial shocks (lower \(f_t\)) require firms to decrease their equity payouts \((d_t)\). In the case that firms cannot decrease \(d_t\), they must cut their employment. Hence, the changes in employment due to financial shocks depend on the flexibility between debts and equity payout. Because debts and equity payouts cannot be perfectly substituted \((\eta > 0)\), changes in firms equity cannot accommodate a debt adjustment triggered by the financial shocks.

3.4. Authority’s policy

3.4.1. Monetary policy

We modify the standard Taylor rule (1993) as bellows

\[
\frac{R_t}{R_t} = (R_{t-1}\bar{R}^\rho) \left[\frac{\pi_t^{\phi^e} (y_t \bar{y})^{\phi_y}}{\pi_t^e}\right]^{1-\rho_x} \exp \left(\sigma_t^R \epsilon_{R_t}\right),
\]

(20)

where \(y_t = \frac{Y_t}{Z_t}\), \(\bar{R}, \bar{y}\) are the steady state of \(R_t\) and \(Y_t\), respectively. The parameter \(\rho_R\) illustrates the degree of interest rate smoothing, and \(\phi^e\) and \(\phi_y\) are respectively Taylor coefficient on inflation and output gap. \(\epsilon_{R_t}\) is an i.i.d monetary policy shock. \(\sigma_t^R\) is a volatility shock that allows for the time-varying volatility of a policy shock.

3.4.2. Fiscal policy

The government finances its expenditures, \((G_t)\), purchased final goods at the nominal price, \((P_t)\), and to subsidize the intermediate-goods producing firm by using a lump-sum tax collected from the household. Hence the government’s budget constraint is written as

\[
P_t G_t + b_{t+1} \left(\frac{1}{R_t^e} - \frac{1}{R_t}\right) = T_t.
\]

(21)

Let \(g_t\) denote the government spending growth, and then the government spending is a fraction of aggregate output
\[ G_t = \left( 1 - \frac{1}{g_t} \right) Y_t, \]  

where the logarithm of \( g_t \) participates in the model as an AR(1) process

\[ \ln (g_{t+1}) = (1-\rho_g) \ln (\bar{g}) + \rho_g \ln (g_t) + \sigma_g \epsilon_{g,t}, \]  

where \( (1-\bar{g}) \) is the value of government spending relative to output in the steady state, \( \rho_g \) is the government shock persistence. \( \epsilon_{g,t} \) is the government spending shock with zero mean and standard deviation \( \sigma_g \).

### 3.5. Market clearing condition

The market clearing condition in the labor market can be expressed as

\[ N_t = \int N_t(i)di. \]  

The condition in model is given

\[ Y_t = C_t + \frac{\Phi}{2} \left[ \frac{P_{l,t}}{(\pi_{t-1}^{\omega} \pi_t^{1-\omega})^p P_{l,t-1}} - 1 \right]^2 Y_t + G_t, \]  

where the second term is the aggregate price adjustment cost. Finally, the zero net supply of bond is described as

\[ b_t = 0. \]

### 3.6. Policy risks

The standard deviation \( \sigma^k_t \) is assumed to follow an AR(1) stochastic volatility process as argued by Fernandez-Villaverde et al. (2011) and Shephard (2008) as given

\[ \sigma^k_t = (1-\rho_{\sigma^k}) \sigma^k + \rho_{\sigma^k} \sigma^k_{t-1} + \eta_{k,t}^k, \epsilon_t^k \sim N(0,1), \]  

where \( \sigma^k \) is the unconditional mean of \( \sigma^k_t \) and \( k \) represents the time-varying volatility shocks in the model, \( k \) could be the productivity, monetary and fiscal policy shock (\( k \in \{R,A,G\} \)). The shock to the volatility, \( \epsilon_t^k \), is an i.i.d process that is assumed to be independent of the level shock, \( \epsilon_{k,t} \). \( \eta^k \) represents one-standard deviation of uncertainty. It is worth noticing that a one-standard deviation of uncertainty shocks increases the volatility of the respective shock processes by \( \exp (\eta^k) - 1 \times 100 \).


4. Welfare’s issues computation

We follow Ha et al. (2020b) to employ the perturbation method to obtain the approximation to the policy functions around the deterministic steady-state. Then we can use them to measure the welfare. The third order Taylor expansion of the household’s utility function around the deterministic steady-state can be used to decompose the welfare into the different components as follows:

\[
E\left[ \sum_{t=0}^{\infty} \beta^t u(x_t) \right] \approx \sum_{t=0}^{\infty} \beta^t u(\bar{x}) + \sum_{t=0}^{\infty} \beta^t Mu(\bar{x})E[(x_t - \bar{x})] + \sum_{t=0}^{\infty} \beta^t Nu(\bar{x})E[(x_t - \bar{x}) \otimes (x_t - \bar{x})]
\]

\[
+ \sum_{t=0}^{\infty} \beta^t Ku(\bar{x})E[(x_t - \bar{x}) \otimes (x_t - \bar{x}) \otimes (x_t - \bar{x})],
\]

where \(x_t = [C_t, \sigma_{t-1}, H_t]\); and \(Mu(\bar{x}), Nu(\bar{x})\) and \(Ku(\bar{x})\) are vector which contain the first, second and third derivative of \(u(\cdot)\) evaluated at \(\bar{x}\) which are the deterministic steady state of \(x_t\). In our study, we follow Nakata’s approach using the pruning algorithm suggested by Kim et al. (2008) to compute the welfare and welfare costs.

Like the similar spirit of the literature, the compensating variation in consumption that enhances the welfare of a typical household in one economy to make them as better-off as others in another economy, can be defined as welfare cost (\(wc\)). Mathematically, \(wc\) can be represented as

\[
E\left[ \sum_{t=0}^{\infty} \beta^t u\left(1 + \frac{wc}{100}\right)C_{A,t}, H_{A,t} \right] = E\left[ \sum_{t=0}^{\infty} \beta^t u(C_{B,t}, H_{B,t}) \right],
\]

where \(C_{A,t}, H_{A,t}\) are consumption and labor supply in the economy with \(\sigma_k > 0\) and \(C_{B,t}, H_{B,t}\), are in economy with \(\sigma_k = 0\). Here, \(k\) could be the shock to trend inflation, financial shock or uncertainty shocks (the volatility shocks arising in the technology, government spending growth and monetary policy shock.).

5. Parameterization

The parameter values, which we use to quantify the welfare costs of shifting trend inflation, financial constraint and policy uncertainty in the next step, are reported in Table B1. We split model parameters into two subsets. The first subset includes parameters that we can directly compute them without solving the model or whose values are standard in the literature. Regarding the time-varying stochastic volatility process arising in the technology and government spending growth and monetary policy shocks, the article is based on Born and Pfeifer (2014) to select their parameter value. According to Born and Pfeifer (2014), a joint estimation of all parameters in the model with time-varying volatility is computationally difficult. They, therefore, estimate these parameter separately from the New Keynesian model. The unavailability of data and complicated procedure to compute these parameters hinder us from measuring them. Furthermore, calculating these parameters is not a purpose of this
paper. In particular, the variance and persistence level of volatility shock arising in monetary policy shocks are 0.363 and 0.921, respectively. The paper also reports the moderate evidence of volatility shock arising in the technology and government spending growth shock with a high level of variance (0.312 and 0.308) and persistence (0.632, 0.655).

Furthermore, some parameter values are calibrated using the standard calibration technique based on the steady-state values. In particular, the steady-state inflation ($\pi^*$), the steady-state share of government expenditure ($1 - \tilde{g}$), the steady-state debt-output ratio are, respectively 1.006, $\frac{1}{1-0.34}$, and 0.41. Some parameters are taken from the literature. For example, the discount factor, $\beta$, the habit information, $h$, and the inverse Frisch elasticity, $v$, are set to 0.99, 0.8 and 1.00, respectively. As in Jermann and Quadrini (2012), we set the tax advantage, $\tau$, to 0.35.

Moreover, we follow Justiniano and Primiceri (2008) to set the values for parameters related to persistence level and standard deviation of structural shocks. Accordingly, the autoregressive parameters of the government expenditure shock are set to 0.98. The degree of price indexation, $\rho$, and the elasticity of substitution, $\tilde{\theta}$, are set to 0 and 10, respectively. The estimations of Cogley and Sbordone (2008) are consistent with those in the literature on constant non-zero trend inflation and imperfect indexation. Regarding the shock to trend inflation process, we set its persistence level, $\rho_t$, and standard deviation, $\sigma_t$, to 0.995 and 0.0008 as in Cogley et al. (2009). We also follow Hoang (2018) to set parameter values related to financial shocks.

Table 1 compares the moments generated by the parameterized model with moments computed by the data. The reported volatility and correlation statistics are for the HP-filtered U.S. data during 1984Q1–2015Q1 period. Table 1 shows that the
model does a good job for matching the volatility of consumption, output, labor, debt, and the correlation between consumption, labor, and output. In sum, the key features of the data are captured reasonably well by the calibrated model. Therefore, the calibrated model can provide an appropriate laboratory for the subsequent welfare analysis.

6. Results

The present study first quantifies welfare costs of financial constraint shocks, and policy uncertainty shocks in the economy with distinct levels of trend inflation. In each exercise, we also perform additional sensitivity analysis to observe changes in welfare costs with respect to changes in relevant parameter values. Subsequently, we conduct the simulation to investigate the cyclical effects of trend inflation on structural shocks.

6.1. Welfare costs of financial constraint

6.1.1. Main results

In the first analysis, we measure welfare costs of financial constraint shocks. Table 2 reports these costs and properties of an economy corresponding to the cases that a central bank, in turn, sets 2-percent annualized and 4-percent annualized trend inflation. It can be seen that welfare is smaller in the economy with unexpected changes in financial conditions. Using our baseline parameter values, welfare costs are nearly 0.008 percent in the 2-annualized-percent economy. The variance of financial constraint shocks reduces the welfare and they do so mainly through their effects on consumption and leisure. The properties of economy provide more intuitions to explain this welfare distinction. There are decreasing trends in the mean consumption and leisure, while their volatility tends to increase. However, their effects are not significant, thus welfare costs of financial constraint shock are modest. Moreover, we observe an opposite trend in mean value of debt and equity. In particular, there is a rise in the mean value of debt, whereas those of equity payout decreases due to financial constraint shocks. The financial constraint shocks also cause the financial market to be more volatile that are reflected by a simultaneous growth of standard deviation values of debt and equity payout.

To observe impacts of trend inflation on welfare consequences of financial shocks, we assume that the central bank sets trend inflation to 4 annualized percent and then quantify welfare costs of financial shocks. With a higher level of trend inflation, welfare costs slightly improve from 0.008 percent to 0.009 percent. The findings of this analysis imply that the higher level of trend inflation signifies adverse impacts of financial shocks to cause more serious problems. To observe this point more clearly,
we let the trend inflation level take distinct levels, then quantify corresponding welfare costs. Figure 1 illustrates results of this exercise. Two important points should be conveyed. First, a higher level of trend inflation leads to a greater welfare cost. Second, Figure 1 exhibits a non-linear relationship between these costs and levels of trend inflation. In words, a given amount of change in trend inflation leads to a more severe consequence of financial shocks when trend inflation takes a high value.

In brevity, welfare costs of financial shocks using our baseline parameters are not significant. However, we find interactions between these costs and levels of trend inflation. With an increase in trend inflation, these shocks distort the economic welfare more substantially.

### 6.1.2. Sensitivity analysis

In addition to an analyze of variance of welfare costs to changes in trend inflation level, we also conduct further exercises to investigate how these costs respond to different relevant parameter values. In this exercise, we consider fluctuations in the

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**Table 2. Welfare costs of financial constraint shocks.**

<table>
<thead>
<tr>
<th></th>
<th>( \pi^\prime = 1.02^{0.25} )</th>
<th>( \pi^\prime = 1.04^{0.25} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \sigma_f = 0 )</td>
<td>( \sigma_f &gt; 0 )</td>
</tr>
<tr>
<td>Welfare cost</td>
<td>-81.60</td>
<td>-81.60</td>
</tr>
<tr>
<td>E(C)(*)</td>
<td>102.27</td>
<td>102.28</td>
</tr>
<tr>
<td>E(H)(*)</td>
<td>-6.45</td>
<td>-1.97</td>
</tr>
<tr>
<td>E(d)(*)</td>
<td>-67.32</td>
<td>-67.49</td>
</tr>
<tr>
<td>( \sigma_C )</td>
<td>0.7051</td>
<td>0.7052</td>
</tr>
<tr>
<td>( \sigma_H )</td>
<td>1.38</td>
<td>1.39</td>
</tr>
<tr>
<td>( \sigma_D )</td>
<td>0.05</td>
<td>3.89</td>
</tr>
<tr>
<td>( \sigma_d )</td>
<td>2.21</td>
<td>2.25</td>
</tr>
</tbody>
</table>

*Note: (*) expressed as percentage deviation from the deterministic steady-state. Source: An author's calculations.*
financial market by adjusting the debt-to-output level, the tax advantage, the substitution level between debt and equity payout as in the top panel of Figure 2, and the financial shock properties as in the bottom panel of the same figure.

Regarding the debt-to-output ratio, an increase in this ratio may create more financial pressures, then signify welfare costs of financial shocks. From the top left panel of Figure 2, these costs rocket-up with respect to a jump in the ratio of debt to output. We continue to let the tax advantage values change and observe movements of welfare costs. In this study, tax advantage can be considered as benefits from issuing debts. When there is no tax advantage, financial shocks have no welfare consequence. This conclusion is aligned with Proposition 2 in this paper and in the study of Jermann and Quadrini (2012). However, our model predicts that as long as there is a tax advantage from issuing debts, the enforcement is not always binding due to uncertainty. Only if the tax advantage is sufficiently large (say greater than 0.5), welfare costs of financial shocks tend to diminish. Otherwise, these costs will rise. Accordingly, welfare costs follow a bell shaped curve when the level of tax advantage increases.

By contrast, welfare costs of financial shocks positively correlate to the substitution level between equity and debt. Intuitively, the parameter $\eta$ plays a vital role in determining impacts of financial frictions. Our model predicts that if $\eta$ is equal to zero, implying a frictionless economy, there is no welfare consequence. This conclusion stems from the fact that changes in firms’ equity can quickly accommodate debt adjustment triggered by financial shocks. As long as $\eta$ increases, firms readjust fund sources slowly because the substitution between equity and debt becomes costly. Therefore, financial shocks have more significant impacts on the economy. Welfare costs of financial shocks in this situation increase dramatically when $\eta$ reaches higher values as in the top middle panel of Figure 2. This finding is aligned with Jermann and Quadrini (2012).

Finally, we also analyse changes in welfare consequences of financial shocks due to financial shock properties. Welfare costs behave as expected to the rise in persistence

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**Figure 2.** Welfare costs of financial shocks. Source: An author’s calculations.
level and volatility level of financial shocks. Moreover, these values cause welfare costs to move non-linearly. This evidence implies that a given amount of changes in the persistence level and volatility level can bring about more serious problems when they take a high level.

6.2. Welfare costs of policy uncertainty

6.2.1. Main results

In this analysis, we concentrate on welfare costs of policy uncertainty arising in monetary policy shocks, technology shocks and government spending shocks in two scenarios that the central bank sets its inflation target to 2 annualized percent and 4 annualized percent. Table 3 reports results of this exercise. Our focus firstly lies on 2-percent-trend-inflation economy. Some important facts should be listed here. First, the volatility shocks distort the economic welfare significantly. Welfare costs are 1.34 percent. These sizable welfare costs stem from a reduction in mean value of consumption and leisure (from $-0.07$ and $0.07$ to $-0.14$ and $0.14$, respectively) and a rise in their volatility (from $0.11$ and $0.09$ to $0.16$ and $0.13$, respectively). Analysing changes in business cycle properties provides more intuitions to explain a mechanism that policy uncertainty shocks distort the economic welfare. In addition to effects on macroeconomic business cycle, we also observe changes in the financial market due to policy uncertainty shocks. A reduction in both mean value of debt (from $0.09$ to $0.25$) and equity payout (from $0.80$ to $1.66$) as well as a rise in their volatility level are reported in Table 3.

More importantly, Table 3 reports changes in welfare costs and economic properties when the central bank sets its inflation targets to higher levels. Welfare costs of policy uncertainty shocks in the 4-percent-trend-inflation economy are more sizable (2.87 percent) as compared to those in the 2-percent-trend-inflation economy (1.34 percent). The result implies that welfare consequences of policy uncertainty can be signified by a rise in central bank’s inflation targets.

In brevity, we provide empirical evidences on interactions between constant positive level of trend inflation and policy uncertainty shocks in term of welfare costs. Welfare costs of policy uncertainty are not negligible and a higher level trend inflation signifies these costs to distort the economy more remarkably.

<table>
<thead>
<tr>
<th>Welfare cost of uncertainties.</th>
<th>( \pi^* = 2% )</th>
<th>( \pi^* = 4% )</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Uncertainty ((\eta^k = 0))</td>
<td>PR ((\eta^k &gt; 0))</td>
<td>No Uncertainty ((\eta^k = 0))</td>
</tr>
<tr>
<td>Welfare cost</td>
<td>1.34%</td>
<td>2.87%</td>
</tr>
<tr>
<td>(E(C)(*))</td>
<td>-0.07</td>
<td>-0.14</td>
</tr>
<tr>
<td>(E(H)(*))</td>
<td>1.15</td>
<td>2.38</td>
</tr>
<tr>
<td>(E(D)(*))</td>
<td>-0.14</td>
<td>-0.29</td>
</tr>
<tr>
<td>(\sigma_B)</td>
<td>0.11</td>
<td>0.16</td>
</tr>
<tr>
<td>(\sigma_B)</td>
<td>0.09</td>
<td>0.13</td>
</tr>
<tr>
<td>(\sigma_B)</td>
<td>0.06</td>
<td>0.07</td>
</tr>
<tr>
<td>(\sigma_B)</td>
<td>0.14</td>
<td>0.21</td>
</tr>
</tbody>
</table>

Note: PR is the economy with the policy rate risk. (*) expressed as percentage deviation from the deterministic steady-state. \(\eta^k\) is the variance of policy uncertainty shocks, where \(k \in \{R,A,G\}\).

Source: An author’s calculations.
6.2.2. Sensitivity analysis

Figure 3 plots changes in welfare costs of policy uncertainty shocks against trend inflation levels and relevant parameters governing the pricing environment, such as the degree of price indexation and the rigidities of price adjustment. Two important points should be conveyed. First, an increase in welfare costs of policy uncertainty stems from either a rise in trend inflation and rigidities of price adjustment or a reduction in degree of price indexation. Intuitively, a higher chance that non-optimizing firms are able to catch up with price changes by optimizing firms (a greater price indexation level) makes the cost of uncertainty less severe. A larger value of $\phi$, implying less frequent price adjustment generates large welfare costs. The changes in welfare costs due to changes in relevant parameters are consistent with previous studies. Second, Figure 3 exhibits a non-linear relationship between welfare costs and the degree of price indexation and rigidities of price adjustment. It suggests that when these parameters reach a high value, a given amount of their changes might leads to a more sizeable distinction in welfare costs.

In the subsequent analysis, we examine responses of welfare costs of policy uncertainty to relevant parameters belonging to the financial sector. Figure 4 shows that welfare costs of policy uncertainty rise if there is a reduction in tax advantage or a growth in the substitution level equity and debt and the volatility level of respective shocks. The result implies that either a lower benefit from issuing debts or a higher cost when substituting between equity and debt signifies welfare costs of policy uncertainty.

**Figure 3.** Welfare costs of policy uncertainty shocks.
Source: An author’s calculations.

**Figure 4.** Welfare costs of policy uncertainty shocks.
Source: An author’s calculations.

6.2.2. Sensitivity analysis

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In the subsequent analysis, we examine responses of welfare costs of policy uncertainty to relevant parameters belonging to the financial sector. Figure 4 shows that welfare costs of policy uncertainty rise if there is a reduction in tax advantage or a growth in the substitution level equity and debt and the volatility level of respective shocks. The result implies that either a lower benefit from issuing debts or a higher cost when substituting between equity and debt signifies welfare costs of policy uncertainty.
We also conduct another analysis to investigate whether financial frictions are a channel through which uncertainty distorts the economy as argued by Arellano et al. (2011) and Christiano et al. (2014). The procedure of experiment is presented as follows. First, we set up two scenarios that firms do and do not face the financial constraint. The former is similar to those of Nakata (2014) or Ha et al. (2019), while the latter is the model developed in this study. Regarding the model without financial constraint, we employ parameter values as in Ha et al. (2019). Table 4 reports welfare costs of policy uncertainty in these two scenarios. It can be seen that welfare costs in the model featuring financial frictions are more sizable as compared to those without. We doubt that welfare cost difference in two scenarios may stem from parameter differences. We then let these parameters change to check that. After some further exercises, we realize that our conclusion still holds. The mechanism to explain for this finding is as follows. Monetary policy uncertainty as presented in Equation (20) causes fluctuation in policy interest rate. These fluctuation in the policy interest rate then influences firms’ decision on new debts at the beginning of period since they receive \( b_{i,t+1} \) from purchasers and \( b_{i,t+1} - b_{i,t+1} \) from the government where \( R_t^c = R_t - \tau(R_t - 1) \) is the effective gross interest for the firms. From Equation (16), we also see that the exogenous credit constraints are conditional on both \( R_t \) and \( R_t^c \), hence dynamics of policy interest rate due to policy uncertainty influence the exogenous credit constraints, then the firms’ outcome. The effort of this paper is to distinguish changes in the exogenous credit constraints that stem from the financial market condition or from policy uncertainty.

### 6.3. The cycle effects of trend inflation

This section analyses impacts of constant positive trend inflation on the impulse response functions of the shocks. We focus on impulse responses of key macroeconomic variables and financial variables to two types of shocks: the normal structural shocks and time-varying volatility shocks in the model for distinct levels of trend inflation. Our concentrate firstly lies on interactions of trend inflation and structural shocks, including shocks to trend inflation and financial shocks, the subsequent section is devoted to volatility shocks arising in the monetary policy shocks, technology shocks, and the government spending shocks.

#### 6.3.1. Trend inflation and structural shocks

Figures 5 and 6 plot impulse response functions of each shock. The top panels of each figure represent impulse responses of consumption and labor supply, while the bottom panels are variables in the financial market, including debt and equity. The
Figure 5. Trend inflation and impulse responses of shocks to trend inflation. 
Source: An author’s calculations.

Figure 6. Trend inflation and impulse responses of financial shocks. 
Source: An author’s calculations.
black line, blue line and red line correspond to the economy with 0 percent annualized trend inflation, 2 percent annualized trend inflation and 4 percent annualized trend inflation, respectively.

The top panel of Figure 5 plots impulse responses of consumption and labor to shocks to trend inflation. These shocks cause consumption and labor supply to increase in the short-run. With a higher level of trend inflation, the response level to these shocks of consumption is substantially smaller, while those in labor supply increases more sizeably. As a consequence, an increase in trend inflation impacts impulse responses of consumption and labor supply to shocks to trend inflation. The similar evidence happens for debt and equity payout as illustrated in the bottom panel of Figure 5. Debt and equity payouts decrease due to shocks to trend inflation and both variables respond more to these shocks for higher levels of trend inflation.

Figure 6 plots responses of variables to financial shocks. Similarly, we also find interactions between trend inflation levels and responses of variables to financial shocks. In particular, consumption and labor supply increase in response to a one-standard-deviation rise in financial shocks. These results are aligned with Jermann and Quadrini (2012) and Hoang (2018). Higher levels of trend inflation signify impacts of shocks on consumption but lower magnitude of effects on labor supply. Furthermore, increases in debt and equity are attributed to the financial shocks. These financial variables response more to these shocks when the central bank sets inflation targets to higher levels.

6.3.2. Trend inflation and volatility shocks
In the following exercise, we explore interactions between trend inflation levels and policy uncertainty shocks. Figures 7–9 depict response of variables to policy uncertainty arising in monetary policy shocks, technology shocks and government spending shocks, respectively. These uncertainty shocks cause consumption and labor supply to decrease. Moreover, these effects tend to last in the long-run. Our results are consistent with Born and Pfeifer (2014). Regarding debt and equity, both immediately decrease in the short-run but increase right after that. These effects are more noticeable in the economy with higher levels of trend inflation. An increase in trend inflation level signifies these responses to the policy uncertainty shocks more significantly. The finding implies that there are interactions between trend inflation and responses of variables to policy uncertainty shocks and these interactions are quantitatively important.

In short, our study provides empirical evidences on responses of variables in both the macroeconomic and financial market to distinct structural shocks and policy uncertainty shocks. We clearly distinguish between financial shocks and policy uncertainty shocks. More importantly, we find interactions of the levels of trend inflation and responses of variables to shocks, and these interactions are quantitatively important.

6.4. Welfare costs of constant and shifting trend inflation
6.4.1. Main results
Tables 5 and 6 report welfare costs of constant and shifting trend inflation in the Rotermberg model, respectively. First, an increase in trend inflation leads to a rise in
Figure 7. Trend inflation and impulse responses of monetary policy uncertainty shocks. Source: An author’s calculations.

Figure 8. Trend inflation and impulse responses of technology uncertainty shocks. Source: An author’s calculations.
Figure 9. Trend inflation and impulse responses of government spending uncertainty shocks.
Source: An author's calculations.

Table 5. Welfare cost of constant trend inflation: Rotemberg model.

<table>
<thead>
<tr>
<th></th>
<th>W/O Uncertainty ((\eta^R = 0))</th>
<th>Policy Risks (PR) ((\eta^R &gt; 0))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(\pi^* = 0)%</td>
<td>(\pi^* = 6)%</td>
</tr>
<tr>
<td>Welfare cost</td>
<td>0.83%</td>
<td>1.02%</td>
</tr>
<tr>
<td>Steady-state C</td>
<td>0.821</td>
<td>0.813</td>
</tr>
<tr>
<td>Steady-state H</td>
<td>1.066</td>
<td>1.067</td>
</tr>
<tr>
<td>E(C)((^*))</td>
<td>-0.47</td>
<td>-0.79</td>
</tr>
<tr>
<td>E(H)((^*))</td>
<td>0.04</td>
<td>0.05</td>
</tr>
<tr>
<td>100(\sigma_C)</td>
<td>1.31</td>
<td>1.46</td>
</tr>
<tr>
<td>100(\sigma_H)</td>
<td>1.77</td>
<td>2.53</td>
</tr>
</tbody>
</table>

\[\text{Note: (*) expressed as percentage deviation from the deterministic steady-state.}
\]
\[\text{Source: An author's calculations.}\]

Table 6. Welfare cost of shifting trend inflation: Rotemberg model.

<table>
<thead>
<tr>
<th></th>
<th>W/O uncertainty</th>
<th>With uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sigma^R = 0, \eta^R = 0)</td>
<td>(\sigma^R &gt; 0, \eta^R = 0)</td>
<td>(\sigma^R &gt; 0, \eta^R &gt; 0)</td>
</tr>
<tr>
<td>WC</td>
<td>0.17%</td>
<td>0.38%</td>
</tr>
<tr>
<td>Steady-state C</td>
<td>0.82</td>
<td>0.82</td>
</tr>
<tr>
<td>Steady-state H</td>
<td>1.07</td>
<td>1.07</td>
</tr>
<tr>
<td>E(C)((^*))</td>
<td>-0.32</td>
<td>-0.55</td>
</tr>
<tr>
<td>E(H)((^*))</td>
<td>0.03</td>
<td>0.05</td>
</tr>
<tr>
<td>100(\sigma_C)</td>
<td>1.26</td>
<td>1.29</td>
</tr>
<tr>
<td>100(\sigma_H)</td>
<td>1.90</td>
<td>1.90</td>
</tr>
</tbody>
</table>

\[\text{Note: (*) expressed as percentage deviation from the deterministic steady-state.}
\]
\[\text{Source: An author's calculations.}\]
the price adjustment costs, then an asymmetric change in consumption and labor supply decision. Without policy uncertainty, welfare costs of constant trend inflation are 0.83 percent. However, the presence of monetary policy uncertainty lead to more sizable welfare costs (1.02 percent).

Our model exhibits a non-linear relationship between trend inflation and steady-state variables. The top panel of these figures plots changes in the deterministic steady-state levels of consumption and labor supply, while the bottom panel shows the deterministic steady-state levels of debt, equity and financial condition against the level of trend inflation. In term of welfare computation, for a given value of $p, \phi$ and boy, a given amount of increase in trend inflation leads to greater reduction in the steady-state level of consumption and leisure, especially when a central bank sets a higher inflation target. These results provide more intuition to explain changes in welfare costs due to constant trend inflation. Regarding the financial market, trend inflation causes the steady-state level of equity payout and financial condition to decrease but leads to an increase in the steady-state level of debt. In other words, a higher level of trend inflation might dampen the financial market more significantly. More importantly, by comparing welfare costs of constant and shifting trend inflation in the model with staggered price solely as Nakata (2014) and Ha et al. (2019) and the model with both staggered prices credits as we did in our study, we indicate that welfare costs are greater if the staggered credit present. The results imply that staggered credits play a vital role in transmitting adverse impacts of constant and shifting trend inflation to the economy.

The following exercise compares welfare costs of exogenous alternations in trend inflation in three economies, including the economy without shifting trend inflation and policy uncertainty ($\sigma_\pi = 0, \eta^R = 0$), the economy without policy uncertainty but with shifting trend inflation ($\sigma_\pi > 0, \eta^R = 0$), and the economy with shifting trend inflation and monetary policy uncertainty ($\sigma_\pi > 0, \eta^R > 0$). The steady-state inflation is set to 2 annualized percent. Changes in welfare costs of shifting trend inflation and dynamic properties of an economy are reported in Table 6. Several striking points should be emphasized here. First, the model without policy uncertainty shows how shifting trend inflation distorts the economic welfare. Welfare costs of shifting trend inflation are 0.17 percent. Changes in mean values of consumption and leisure provide evidence to prove our argument. The shifting trend inflation leads to a decline in mean consumption and an increase in the mean hours worked. Moreover, the volatility level of consumption and hours worked enlarge with respect to the shocks. These changes in mean values and volatility values of consumption and hours worked provide intuitions to explain welfare costs of shifting trend inflation. The time-varying volatility shocks then magnify these changes to produce higher welfare costs.

Since policy functions for consumption and labor supply are respectively concave and convex functions of trend inflation, a mean-preserving spread in the shock distribution lowers mean consumption and increases mean labor supply. As argued by Nakata (2014), a shock to trend inflation can be interpreted as taking the economy to a new steady-state with various levels of trend inflation if trend inflation is highly persistent. The concave policy function for consumption implies that a positive shock to trend inflation leads to reduction in consumption by an amount greater than its
increase caused by a negative shock. The convex policy function for labor supply implies that a positive shock to trend inflation leads to an increase in labor supply by an amount greater than its reduction caused by a negative shock. Hence, consumption declines and labor supply rises due to a higher variance of shock to trend inflation.

6.4.2. Sensitivity analysis

Subsequently, we examine how welfare costs of shifting trend inflation are sensitive to changes in relevant parameters. The top panel of Figure 10 depicts that sensitivity with respect to parameters controlling pricing environment, such as the degree of price indexation and rigidities of price adjustment. An increase in welfare costs is attributed to either a fall in the degree of price indexation or a rise in the rigidities of price adjustment. The bottom panel shows how welfare costs of shifting trend inflation respond to changes in shock properties (the persistence and volatility level of shocks) and the trend inflation levels. These parameters cause welfare costs to increase. The striking point is that all relationships are non-linear, implying that a given amount of increase in these parameters leads to a larger change in welfare when these parameters reach to a higher point.

7. Remarked conclusions

We develop a New Keynesian model featuring time-varying trend inflation, financial frictions in the form of credit constraints, and policy uncertainty. Our main objectives are to measure the macroeconomic, financial and welfare effects of financial constraints and policy uncertainty on the economy with shifting trend inflation. The main contributions of this paper are listed as follows. First, this paper was the first attempt to develop a New Keynesian model featuring time-varying trend inflation and financial frictions in the form of credit constraints, and policy uncertainty.
Second, under the working of exogenous constraint, this paper examined the macro-
economic and financial effects of financial shocks and frictions. Third, we added the
'staggered credit’ channel to measure the welfare consequences and other consequen-
ces of shifting trend inflation. Lastly, these analyses also help us distinguish the
impacts of financial shocks and policy uncertainty shocks on the economy. We use a
highly persistent shock to trend inflation regarded as the central bank’s slowly-mov-
ing implicit inflation target to model a sustained increase in trend inflation. Regard-
ing policy uncertainty, we jointly use both structural shocks and time-varying
volatility shocks and we concentrate on the uncertainty arising in the technology
shocks, monetary policy shocks and the government spending shocks.

Our study revealed important findings. First, the financial shocks affect the welfare
of economy negligibly, while there is a remarkable welfare consequence due to the
policy uncertainty shocks. For the sensitivity analysis, we allow changes in relevant
parameters and measure their effects on welfare consequences of shocks. The results
show that the financial shocks and uncertainty shocks dampen the welfare of econ-
omy more significantly if there is either a enough rise in the debt-to-output ratio, the
substitution level between debt and equity payout or a reduction in the tax advantage.
Second, our study provides empirical evidence on trend inflation and financial shocks
and policy uncertainty shocks in terms of welfare consequences and macroeconomic
and financial effects and these interactions are quantitatively important. Regarding
welfare consequences, welfare costs of financial, policy uncertainty shocks and shocks
to trend inflation become more significant in the high-trend-inflation economy.
Regarding macroeconomic and financial effects, the macroeconomic and financial
variables respond more to structural and volatility shocks if there is a rise in trend
inflation. Third, the results provide compelling evidence that staggered credits are an
important channel, through which the authorities would control the impacts of both
shifting trend inflation and policy uncertainty more effectively.

The findings of this study could be interpreted in light of limitations. First, the
study still focused on the case of an advanced economy, while the inconsistent policy
implementations are more evident in the case of developing countries. It is critical to
design a similar model to measure the consequences of policy implementation in
these regions. However, the model set up as we did in this paper requires lots of
information in both the macroeconomic and financial market, which is not available
in the case of developing countries. Hence, applying this model to the case of devel-
oping countries is more likely to produce imprecise results. Therefore, future research
will follow with the incoming flow of more appropriate, sophisticated, and updated
prior information. Second, the model is, itself, limited as the designed capital market
is quite simple. This capital or credit system may not be strong enough to capture
the flexibility and fluctuations of the financial market. Modifying the financial market
in this way will make the whole system more complicated. Future studies should be
developed in this direction.

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responsibility for this article.
Disclosure statement

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Notes

1. Bernanke et al. (1996) argue that introducing credit frictions into a standard model has a long tradition due to its advantages such as an improvement of ability to explain cyclical fluctuations or a broader class of important cyclical phenomena such as changes in credit extension and spread between safe and risky interest rate.

2. To model a sustained increase of inflation, we use a highly persistent shock to trend inflation, regarded as the central bank’s slowly-moving implicit inflation targets as argued by Kozicki and Tinsley (2001), Ireland (2007), Cogley and Sbordone (2008) and Cogley et al. (2009). The literature calls it shifting trend inflation.

3. In particular, Born and Pfeifer (2014) argue that the U.S data were plagued by high shock volatilities since the 1970s. The evidence for shifts on the variance of innovations is also provided by Bernanke et al. (1996), Cogley and Sargent (2005), Primiceri (2005) and Sims and Zha (2006). Moreover, Fernandez-Villaverde et al. (2015) also argue that the presence of both stochastic volatility and parameter drifting successfully models the U.S. economy. Shifting trend inflation is a type of parameter drifting, in which the steady-state inflation drifts over time.

4. The evidence on the relation between macroeconomic and financial market is provide in Appendix A. Ha et al. (2020b) also provide empirical evidence on the existence of trend inflation and policy uncertainty.

5. This argument is aligned with Stockman (1981) and Cooley and Hansen (1989, 1991).


7. Nakata (2014) considers the staggered prices solely, while Ha et al. (2019) incorporate both the staggered prices and staggered wages.

8. There are plausible reasons for using the New Keynesian model. First, this model allow us to study optimizing behaviour of various agents without making an assumption of zero trend inflation that are sometime restrictive as argued in the literature (Ha et al., 2019). Second, we are able to add more channels to the available New Keynesian incorporating shifting trend inflation and policy uncertainty to examine whether the effects of shifting trend inflation and policy uncertainty change in the model with and without such channels. There is no paper using alternative models to study these factors on the literature. Third, the New Keynesian model allows me to employ the perturbation method to jointly quantify welfare consequences of structural and time-varying volatility shocks. We are also able to distinguish between the effects of financial shocks and policy uncertainty shocks in this New Keynesian model.

9. The endogenous constraint in these studies, however, still needs to be discussed. First, Jermann and Quadrini (2012) show that the value of capital, the discounted value of the interperiod debt and financial shocks determine the value of the intraperiod loan firms. Therefore, the credit constraint can be loosed or tightened by financial shocks or others that affect the value of capital, the discounted value of the interperiod debt. Further, Kocherlakota (2000) suggests that the effects of financial shocks could be potentially magnified when considering an endogenous credit constraint, whereas the exogenous credit constraint does not.
10. The detailed description of variables is reported in Appendix B.

11. Trend inflation, by itself, distorts the economic welfare by lowering consumption and leisure.

12. ’Standard’ means that they are widely accepted in the literature of New Keynesian model.

13. Since many economists, such as Ascari et al. (2018), Blanchard et al. (2010) and Krugman (2014) advocate the proposal that increases the inflation target to 4 percent. If such proposal are implemented for a long enough period, it would lead to a higher trend inflation.

14. The codes and results are provided by the author upon request.

15. Notice that a greater level of price indexation implies that prices are less dispersed at any trend inflation rate because this index allows non-optimizing firms to catch up with prices set by optimizing firms. Therefore, both consumption and labor supply are less impacted by the different levels of trend inflation, thus a higher degree of price indexation produces lower welfare costs of shifting trend inflation.

16. The correlations between GDP growth and credit growth in the whole period and in the 1984-2015 period are 0.64 and 0.86, respectively.

17. There were major changes in the financial markets during the post-1984 period as argued by Jermann and Quadrini (2012).

References


Appendix

A. Evidence on the interactions between macroeconomic and financial market

Prior scholars, such as Jermann and Quadrini (2012) and Hoang (2018) have indicated empirical evidence on interactions between macroeconomic and financial market. By using the empirical data, we also find such relationship. In particular, the historical data in Figure 11 shows a pro-cyclical relations between the dynamics of credit growth rate and the growth rate of real output and hours worked. Specifically, a credit contraction negatively affects the output and hours worked growth, while an expansion of credit leads to a growth in output and hours worked. An interaction between two markets become stronger during the period after 1984. During the period 1984–1990, as there is a dramatic reduction in credit growth both hours worked growth and GDP growth have a tendency to decline. From 1990 to 1999, a substantial increase in credit growth drive the macroeconomy up. The pro-cyclical association between credit growth and output and hours worked growth continues to remain till the end of research period.

To observe effects of sustained inflation on the financial market, we plot a figure representing relationships between three time series data, including inflation, the equity payout and debt in the non-financial business sector. The data are extracted by HP filter to get the cyclical

Figure 11. Interactions between macroeconomic and financial markets in U.S. (1954–2015).

Source: Federal Reserve Bank of St. Louis.
components as in Figure 12. Equity payout is defined as total dividends paid minus equity issues of non-financial corporate business, and debt include only liabilities related to credit market transactions. Some facts can be listed here. First, in the post-1984 period, a cyclical component of inflation negatively correlates with those of debt and equity in the period (1984–2001) and positively correlate after that. Second, the cyclical component of debt and equity payout are positively correlated, implying that to some extent they may be substituted. For example, if firms have to reduce debts, they tend to use equity as a financing source, thus equity payouts diminish.

B. Parameters

Table B1. Calibration.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Calibrated value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
<td>0.9678</td>
</tr>
<tr>
<td>$h$</td>
<td>Consumption habit</td>
<td>0.8</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Labor supply disutility</td>
<td>1.0</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Inverse Frisch elasticity of labor supply</td>
<td>1.0</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Substitution level between equity and debt</td>
<td>0.91</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Tax advantage</td>
<td>0.35</td>
</tr>
<tr>
<td>$boy$</td>
<td>Debt-output ratio</td>
<td>0.41</td>
</tr>
<tr>
<td>$1-g^{-1}$</td>
<td>Steady state share of Government expenditure</td>
<td>$1/(1-0.3410)$</td>
</tr>
<tr>
<td>$\rho_A$</td>
<td>AR(1) coefficient for technology shock</td>
<td>0.9</td>
</tr>
<tr>
<td>$\rho_g$</td>
<td>AR(1) coefficient for government spending shock</td>
<td>0.98</td>
</tr>
<tr>
<td>$\rho_f$</td>
<td>AR(1) coefficient for financial shock</td>
<td>0.9</td>
</tr>
<tr>
<td>$100\sigma_A$</td>
<td>Standard deviation of technology shock</td>
<td>1.10</td>
</tr>
<tr>
<td>$100\sigma_g$</td>
<td>Standard deviation of government spending</td>
<td>0.55</td>
</tr>
<tr>
<td>$100\sigma_f$</td>
<td>Standard deviation of financial shock</td>
<td>1.00</td>
</tr>
<tr>
<td>Rotemberg price setting</td>
<td>Price elasticity</td>
<td>10.0</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Degree of price adjustment cost</td>
<td>$[33,50,70,100]$</td>
</tr>
</tbody>
</table>

(continued)
### C. The non-linear, steady-state and linearized system

#### The non-linear system

\[
\bar{\lambda}_t = \frac{1}{C_t - h \bar{C}_{t-1}} - \beta h \frac{1}{C_{t+1} - C_t} \quad \text{(NL1)}
\]

\[
\bar{w}_t \bar{\lambda}_t = \chi_t N_t' \quad \text{(NL2)}
\]

\[
\bar{\lambda}_t = \beta R_t E_t \left( \frac{\bar{\lambda}_{t+1}}{\pi_{t+1}} \right) \quad \text{(NL3)}
\]

\[
A_t^{-1} \frac{W_t}{P_t} - (\theta_t - 1) \frac{P_{t+1}}{P_t} \mu_t = \frac{1}{P_t} \phi \left[ \frac{P_{t+1}}{(1 - \psi_{t+1}) P_t} - 1 \right] \frac{P_t}{(1 - \psi_t) P_{t-1}} \quad \text{(NL4)}
\]

\[
\beta \phi E_t \left[ \frac{\bar{\mu}_{t+1}}{\mu_t} \right] \left( \frac{P_{t+1}}{(1 - \psi_{t+1}) P_t} - 1 \right) \left( \frac{P_{t+1}}{(1 - \psi_t) P_{t-1}} \right) \left( \frac{Y_{t+1}}{Y_t} \right) = 0
\]

\[
\frac{\mu_t}{R_t} - \beta E_t \left[ \frac{P_{t+1}}{P_{t+1}} \right] = \Lambda_t \frac{1}{R_t} \quad \text{(NL5)}
\]

\[
\mu_t [1 + \eta(d_t - d)] = \lambda_t \quad \text{(NL6)}
\]

\[
Y_t = A_t N_t \quad \text{(NL7)}
\]

\[
d_t + \eta(d_t - d)^2 + \frac{1}{2} \left[ \frac{P_t}{(1 - \psi_t) P_{t-1}} - 1 \right]^2 Y_t + \frac{W_t N_t + b_t}{P_t} = Y_t + \frac{b_{t+1}}{R_t P_t} \quad \text{(NL8)}
\]

#### Table B1. Continued.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Calibrated value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\rho)</td>
<td>Degree of price indexation</td>
<td>[0.0,0.33,0.67,1.0]</td>
</tr>
<tr>
<td>(\omega)</td>
<td>Weight on lagged inflation</td>
<td>1.00</td>
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<tr>
<td>Monetary policy</td>
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<tr>
<td>(\phi_x)</td>
<td>Taylor coefficient on the inflation gap</td>
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</tr>
<tr>
<td>(\phi_y)</td>
<td>Taylor coefficient on the output gap</td>
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<tr>
<td>(\rho_R)</td>
<td>AR(1) coefficient for monetary shock</td>
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</tr>
<tr>
<td>100(\sigma_R)</td>
<td>Standard deviation of monetary shock</td>
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</tr>
<tr>
<td>Shifting trend inflation</td>
<td></td>
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</tr>
<tr>
<td>(\pi^*)</td>
<td>Steady-state level of trend inflation</td>
<td>[1.00^0.25 \ldots 1.06^0.25]</td>
</tr>
<tr>
<td>(\rho\pi^*)</td>
<td>Persistence level of shocks to trend inflation</td>
<td>[0.99 \ldots 0.9995 \ldots 0.9999]</td>
</tr>
<tr>
<td>100(\sigma\pi^*)</td>
<td>Standard deviation level of shocks to trend inflation</td>
<td>[0.1,0.075,0.05,0.025,0]</td>
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<tr>
<td>Stochastic volatility shocks</td>
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<td>(\rho_v)</td>
<td>Persistence coefficient of volatility shocks</td>
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<td>(\sigma_v)</td>
<td>Conditional mean of volatility shocks</td>
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<td>(\eta_v)</td>
<td>One-standard deviation volatility shocks</td>
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<tr>
<td>(\rho_{\sigma v})</td>
<td>Persistence coefficient of volatility shocks</td>
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<td>Conditional mean of shock</td>
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<td>(\eta_{\sigma})</td>
<td>One-standard deviation uncertainty</td>
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<td>(\rho_{\sigma^2})</td>
<td>Persistence coefficient of volatility shocks</td>
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<td>Conditional mean of shock</td>
<td>-4.983</td>
</tr>
<tr>
<td>(\eta_{\sigma^2})</td>
<td>One-standard deviation uncertainty</td>
<td>0.308</td>
</tr>
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</table>
\[ f_t \geq Y_t + \frac{b_{t+1}}{P_t R_t} \] (NL9)

\[ Y_t = C_t + \frac{\phi}{2} \left[ \frac{P_t}{\left( \left( \frac{P_{t-1}^{\alpha} \bar{p}_{t-1}^{1-\alpha} \right) P_{t-1} \right) - 1 \right)} \right]^2 Y_t + G_t \] (NL10)

\[ r_t = \frac{(r_{t-1})^{\rho}}{r_t} \left[ \frac{\bar{p}_t}{\bar{y}_t} \right]^{1-\rho} \sigma_t e^t \] (NL11)

\[ P_t G_t + b_{t+1} \left( \frac{1}{R_t^e - \frac{1}{R_t}} \right) = T_t \] (NL12)

\[ \ln(\theta_t) = (1-\rho_0) \ln(\theta) + \rho_0 \ln(\theta_{t-1}) + \epsilon_{\theta_t} \] (NL13)

\[ \ln(A_t) = (1-\rho_A) \ln(A) + \rho_A \ln(A_{t-1}) + \epsilon_{A_t} \] (NL14)

\[ \ln(g_t) = (1-\rho_g) \ln(g) + \rho_g \ln(g_{t-1}) + \epsilon_{g_t} \] (NL15)

\[ \ln(f_t) = (1-\rho_f) \ln(f) + \rho_f \ln(f_{t-1}) + \epsilon_{f_t} \] (NL16)

\[ \ln(er_t) = (1-\rho_{er}) \ln(er) + \rho_{er} \ln(er_{t-1}) + \epsilon_{er_t} \] (NL17)

\[ \ln(\pi_t) = (1-\rho_\pi) \ln(\pi^*) + \rho_\pi \ln(\pi_{t-1}) + \epsilon_{\pi_t} \] (NL18)

\[ g_{b,t+1} = \frac{b_{t+1}}{b_t} \] (NL19)

\[ g_{c,t} = \frac{C_t}{C_{t-1}} \] (NL20)

\[ g_{N,t} = \frac{N_t}{N_{t-1}} \] (NL21)

\[ g_{Y,t} = \frac{Y_t}{Y_{t-1}} \] (NL22)

Let define \[ \bar{b}_t = \frac{b_t}{P_t R_t}, \bar{w}_t = \frac{W_t}{P_t R_t}, \bar{T}_t = \frac{T_t}{P_t R_t}, \bar{\pi}_t = \frac{\pi_t}{P_t R_t}, g_{b,t+1} = \frac{b_{t+1}}{b_t}, g_{c,t} = \frac{C_t}{C_{t-1}}, g_{N,t} = \frac{N_t}{N_{t-1}}, g_{Y,t} = \frac{Y_t}{Y_{t-1}} \]
The implementable equilibriums

\[ \tilde{\lambda}_t = \frac{1}{C_t - hC_{t-1}} - \beta h \frac{1}{C_{t+1} - C_t} \]  \hspace{1cm} (NL1)

\[ \tilde{w}_t \tilde{\lambda}_t = \chi \gamma \pi_t^{\nu} \]  \hspace{1cm} (NL2)

\[ \tilde{\lambda}_t = \beta R_t E_t \left( \frac{\tilde{\lambda}_{t+1}}{\pi_{t+1}} \right) \]  \hspace{1cm} (NL3)

\[ A_t^{-1} \theta_t \tilde{w}_t - (0_t - 1) \frac{\mu_t - \Lambda_t}{\mu_t} - \phi \left[ \frac{\pi_t}{(\pi_{t-1}^{\mu} \pi_t^{\nu} \pi_t^{\rho_0})^{1-\omega}} - 1 \right] \frac{\pi_t}{(\pi_{t-1}^{\mu} \pi_t^{\nu} \pi_t^{\rho_0})^{1-\omega}} \]

\[ + \frac{\beta \phi E_t \left( \frac{\mu_t+1}{\mu_t} \right)}{(\pi_{t+1}^{\mu} \pi_t^{\nu} \pi_t^{\rho_0})^{1-\omega}} - 1 \left( \frac{\pi_t}{(\pi_{t+1}^{\mu} \pi_t^{\nu} \pi_t^{\rho_0})^{1-\omega}} \right) \left( \frac{Y_{t+1}}{Y_t} \right) = 0 \]  \hspace{1cm} (NL4)

\[ \frac{\mu_t}{R_t^2} - \beta E_t \left[ \frac{\mu_{t+1}}{\pi_{t+1}} \right] = \frac{\Lambda_t}{R_t} \]  \hspace{1cm} (NL5)

\[ \mu_t \left[ 1 + \eta (d_t - d) \right] = \lambda_t \]  \hspace{1cm} (NL6)

\[ Y_t = A_t N_t \]  \hspace{1cm} (NL7)

\[ d_t + \frac{\eta}{2} (d_t - d)^2 + \phi \left[ \frac{\pi_t}{(\pi_{t-1}^{\mu} \pi_t^{\nu} \pi_t^{\rho_0})^{1-\omega}} - 1 \right]^2 Y_t + \left( 1 - \tau_{N,t} \right) \tilde{w}_t N_t + \frac{\tilde{b}_t}{\pi_t} = Y_t + \frac{\tilde{b}_{t+1}}{R_t^2} \]  \hspace{1cm} (NL8)

\[ f_t \geq Y_t + \frac{\tilde{b}_{t+1}}{R_t} \]  \hspace{1cm} (NL9)

\[ Y_t = C_t + \frac{\phi}{2} \left[ \frac{\pi_t}{(\pi_{t-1}^{\mu} \pi_t^{\nu} \pi_t^{\rho_0})^{1-\omega}} - 1 \right]^2 Y_t + \left( 1 - \frac{1}{g_t} \right) Y_t \]  \hspace{1cm} (NL10)

\[ \frac{r_t}{R_t} = \left( \frac{r_{t-1}}{R_t} \right) \left[ \left( \frac{\pi_t}{R_t} \right) \left( \frac{Y_t}{Y_t} \right) \right]^{1-\rho} \sigma_t e^{r_t} \]  \hspace{1cm} (NL11)

\[ G_t + \tilde{b}_{t+1} \left( \frac{1}{R_t^2} - \frac{1}{R_t} \right) = \bar{T}_t \]  \hspace{1cm} (NL12)

\[ \ln (\theta_t) = (1 - \rho_0) \ln (\theta) + \rho_0 \ln (\theta_{t-1}) + \epsilon_{\theta_t} \]  \hspace{1cm} (NL13)

\[ \ln (A_t) = (1 - \rho_A) \ln (A) + \rho_A \ln (A_{t-1}) + \epsilon_{A_t} \]  \hspace{1cm} (NL14)

\[ \ln (g_t) = (1 - \rho_g) \ln (g) + \rho_g \ln (g_{t-1}) + \epsilon_{g_t} \]  \hspace{1cm} (NL15)
\[
\ln \left( f_i \right) = (1 - \rho_f) \ln \left( f \right) + \rho_f \ln \left( f_{i-1} \right) + \epsilon_{fi} \quad \text{(NL16)}
\]
\[
\ln \left( er_i \right) = (1 - \rho_{er}) \ln \left( er \right) + \rho_{er} \ln \left( er_{i-1} \right) + \epsilon_{er_i} \quad \text{(NL17)}
\]
\[
\ln \left( \pi_j \right) = (1 - \rho_a) \ln \left( \pi^* \right) + \rho_a \ln \left( \pi_{j-1} \right) + \epsilon_{\pi_j} \quad \text{(NL18)}
\]
\[
g_{b, t+1} = \frac{b_{t+1}}{b_t} \quad \text{(NL19)}
\]
\[
g_c, t = \frac{C_c}{C_{c,t-1}} \quad \text{(NL20)}
\]
\[
g_{N, t} = \frac{N_t}{N_{t-1}} \quad \text{(NL21)}
\]
\[
g_{Y, t} = \frac{Y_t}{Y_{t-1}} \quad \text{(NL22)}
\]
\[
R^* = R_t - \tau(R_t - 1) \quad \text{(NL23)}
\]

**The steady state system**

\[
R(\bar{\pi}) = \frac{\bar{\pi}}{\bar{\beta}} \quad \text{(Rss)}
\]
\[
R^c(\bar{\pi}) = R(\bar{\pi}) - \tau(R(\bar{\pi}) - 1) \quad \text{(Ress)}
\]
\[
W(\bar{\pi}) = \frac{Z}{\theta} \left\{ (\theta - 1) \left( 2 - \frac{R(\bar{\pi})}{R^c(\bar{\pi})} \right) + \phi(1 - \beta) \bar{\pi}^{1-\rho} (\bar{\pi}^{1-\rho} - 1) \right\} \quad \text{(Wss)}
\]
\[
Y(\bar{\pi}) = \left\{ \left( 1 - \frac{1}{\theta} \right)^{\nu_v} \left( 1 - \frac{1-h\beta}{1-h} \right) \frac{w(\bar{\pi})}{1 - \frac{\phi}{2} \left( \bar{\pi}^{1-\rho} - 1 \right)^2 - \left( 1 - \frac{1}{\theta} \right)} \right\}^{\frac{1}{1-\nu_v}} \quad \text{(Yss)}
\]
\[
N(\bar{\pi}) = Y(\bar{\pi}) \quad \text{(Nss)}
\]
\[
C(\bar{\pi}) = \left[ 1 - \frac{\phi}{2} \left( \bar{\pi}^{1-\rho} - 1 \right)^2 - \left( 1 - \frac{1}{\theta} \right) \right] Y(\bar{\pi}) \quad \text{(Css)}
\]
\[
\lambda(\bar{\pi}) = \frac{1}{C(\bar{\pi})} \frac{1-h\beta}{1-h} \quad \text{(lambdass)}
\]
\[
\mu(\bar{\pi}) = \lambda(\bar{\pi}) \quad \text{(muss)}
\]
\[ \Lambda(\bar{\pi}) = \left( \frac{R(\bar{\pi})}{R^e(\bar{\pi})} - 1 \right) \mu(\bar{\pi}) \]  
(Lambdass)

\[ T(\bar{\pi}) = \left\{ \left( 1 - \frac{1}{g} \right) + \frac{b}{Y} \left( \frac{1}{Re(\bar{\pi})} - \frac{1}{R(\bar{\pi})} \right) \right\} Y(\bar{\pi}) \]  
(Tss)

\[ d(\bar{\pi}) = \left\{ 1 - \frac{\phi}{2} (\bar{\pi}^{1-\rho} - 1)^2 \right\} - \frac{W(\bar{\pi})N(\bar{\pi})}{Y(\bar{\pi})} - \left( 1 - \frac{Y}{R^e(\bar{\pi})} \right) Y(\bar{\pi}) \]  
(dss)

\[ f(\bar{\pi}) = \left\{ 1 + \frac{b}{Y R(\bar{\pi})} \right\} Y(\bar{\pi}) \]  
(dss)