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Outlier identification and group satisfaction of rating experts: density-based spatial clustering of applications with noise based on multi-objective large-scale group decision-making evaluation

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ABSTRACT

Group satisfaction is a trending issue in large-scale group decision-making (LSGDM) but most existing studies maximize the group satisfaction of LSGDM from the perspective of consensus. However, the clustering algorithm in LSGDM also has an impact on group satisfaction. Hence, this paper proposes a density-based spatial clustering of applications with noise (DBSCAN)-based LSGDM approach in an intuitionistic fuzzy set (IFS) environment. The DBSCAN algorithm is used to identify experts with outlier ratings that can reduce the time consumption and iterations of the LSGDM process and maximize the satisfaction of the group decision. An easy-to-use function is then provided to estimate group satisfaction. Finally, a numerical example of data centre supplier evaluation and comparative analysis is constructed to validate the rationality and feasibility of the proposed DBSCAN-based LSGDM approach in an IFS environment. The results demonstrate that the proposed method can effectively identify outliers in expert ratings and improve group satisfaction in the LSGDM process.

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1. Introduction

In recent years, the study of large-scale group decision-making (LSGDM) has received extensive attention and many scholars have proposed and applied various decision-making methods to multiple aspects of economic and management sciences (Choi & Chen, 2021; Lu et al., 2022; Rodríguez et al., 2021; Li, 2022; Mardani et al., 2015). Generally, LSGDM refers to a problem wherein at least 20 experts participate in the decision-making process (Liu et al., 2014) and mainly includes four processes: clustering, weighting, consensus reaching, and alternative ranking (Ding et al., 2020; Li et al., 2021). Among them, the clustering process refers to clustering of experts with

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the same preference information into one subcluster; the weighting process refers to calculating the weights of subclusters, experts, and criteria; the consensus-reaching process focuses on adjusting the consensus degree of each subcluster to achieve the maximum consensus; and the alternatives-ranking process is performed by MULTIMOORA (Multi-Objective Optimization on the basis of a Ratio Analysis plus the full Multiplicative form) (Zhang et al., 2019; Brauers & Zavadskas, 2010), VIKOR (VIse Kriterijumska Optimizacija kompromisno Resenje) (Büyüközkan & Göçer, 2021), TOPSIS (Technique for Order Performance by Similarity to Ideal Solution) (Li & Chen, 2015; Kazancoglu et al., 2021), and other Multi-Criteria Decision Making (MCDM) methods to select optimal alternatives (Huang et al., 2021).

In LSGDM, clustering algorithms are mainly used to reduce the dimensions of decision makers (Ding et al., 2020) that improves the satisfaction and consensus of subclusters. Existing research on clustering algorithms for LSGDM includes three categories: partition-based (Tang et al., 2019; Wu & Xu, 2018; Petchrompo et al., 2021), hierarchical-based (Zhu et al., 2016; Hong et al., 2021), and model-based methods (Tang & Liao, 2021; Azadnia et al., 2012). For instance, Wu and Xu (2018) used the K-means method to group DMs with fuzzy preference relations; Li et al. (2019) used the fuzzy c-means (FCM) algorithm to cluster personalized individual semantics in linguistic LSGDM; Ozcalici and Bumin (2020) used a self-organized map (SOM) that is an artificial neural network-based method to cluster high-dimensional data in MCDM.

However, these clustering algorithms have some limitations in LSGDM with respect to the following: first, partition-based approaches identify the initial clustering centres subjectively, which means that different clustering centres may lead to diverse results. In addition, partition-based approaches are restricted to datasets with convexity, which presents limited application. Second, both the partition-based and hierarchical-based approaches are sensitive to outliers, which means that if outlier samples exist in the dataset, the clustering results may become biased and affect the stability of the subclusters. Third, model-based approaches such as fuzzy C-Means consider the membership between samples and subclusters but they are not very practical because of the tedious calculation process that may be time consuming for LSGDM. These problems may affect the consensus reaching of the group, leading to biased decision results and further reducing the group satisfaction of LSGDM. This means that it fails to yield optimal decisions. More objective and precise clustering algorithms with fewer parameters, reasonable clustering standards, and rigorous logic in the clustering process are required for LSGDM problems (Ding et al., 2020).

Additionally, the main purpose of LSGDM is to maximize group satisfaction; therefore, improving group satisfaction has received wide attention in the LSGDM problem research (Fu et al., 2020). Group satisfaction is the extent to which decision experts are satisfied with the decision process and results (Huang et al., 1999; Green & Taber, 1980). However, most existing research is based on group consensus, wherein many researchers consider group consensus to be equivalent to group satisfaction but there exists a difference between them. For example, if a decision process reaches a high degree of group consensus, it requires multiple rounds of iterative computation, resulting in a longer and time-consuming process. Furthermore, some decision experts may be dissatisfied, resulting in lower group satisfaction.

DBSCAN (density-based spatial clustering of applications with noise) is a density-based clustering algorithm that identifies noise contained in data sets with arbitrary numbers and cluster shapes. The kernel of the algorithm, without prior information, divides the regions with sufficient densities into subclusters based on the given global density parameters *MinPts* and ε (Zhu et al., 2021). Because the DBSCAN algorithm does not require a prespecified number of clusters and can detect arbitrarily-shaped clusters in a spatial database with noise (Hu et al., 2021), it has been widely used in different domains such as student behaviour pattern recognition and management (Li et al., 2021), heterogeneous text data detection (Nguyen & Shin, 2019), and industrial fault detection (Li et al., 2018).

In the decision-making process, expert preferences often contain a considerable amount of vague or uncertain information; therefore, IFS is widely used to express the preference of a decision maker for support, opposition, and hesitation towards alternatives by means of membership, non-membership, and hesitation (Pan & Deng, 2022). Because IFS can more delicately and flexibly describe the fuzziness of the objective world, it has become a trending research domain in route management (Hao et al., 2021), Industry 4.0 evaluation (Mahdiraji et al., 2020), and drug assessment (Xue & Deng, 2021).

Therefore, this paper proposes a DBSCAN-based LSGDM approach in an IFS environment. The DBSCAN clustering algorithm is used to cluster experts; the criteria importance through intercriteria correlation (CRITIC) weighting method is utilized to calculate the objective weight of the criteria. The MULTIMOORA approach, which contains the ratio system, reference point approach, and full multiplicative form, is used to rank the alternatives. The final ranking is determined by the dominance theory. Finally, an illustrative example is constructed for data centre supplier selection and a comparative analysis is conducted to verify the performance of the proposed DBSCAN-based LSGDM approach in an IFS environment.

The main contributions of this paper are summarized as follows:

1. The DBSCAN clustering algorithm is used to effectively identify the outliers of rating experts. It can not only maximize the group consensus and satisfaction but also provide new insight for clustering in the LSGDM process.
2. The CRITIC method is used for determining the objective weights of criteria. It incorporates both the contrast intensity of each criterion and the conflict between criteria to obtain the weights of the criteria (Diakoulaki et al., 1995).
3. An easy-to-use group satisfaction calculation function is provided to characterize the satisfaction of an expert with the complete LSGDM process based on the group consensus and iterations during the LSGDM process.

The remainder of this paper is organized as follows: Section 2 presents the preliminaries; Section 3 details the methodology proposed in this paper; Section 4 further demonstrates the methodology through a case illustration. Section 5 presents a comparative analysis. Section 6 concludes the paper with a summary of the results and provides an in-depth discussion of the subsequent research.

2. Preliminaries

2.1. DBSCAN clustering

DBSCAN clustering is a density-based algorithm proposed by Ester et al. (1996) that identifies noise contained in datasets having arbitrary numbers and shapes of clusters. The relevant concepts in DBSCAN include the following.

Definition 1 (Zhu et al., 2021). ε-neighbourhood: For $x_j \in D$, the ε-neighbourhood contains the samples from sample set D whose distance from x_j is not greater than ε, that is, $\left|N_\varepsilon(x_j)\right| = \left\{x_i \in D \middle| dist(x_i, x_j) \le \varepsilon\right\}$ and the number of samples in this sub-cluster is denoted as $\left|N_\varepsilon(x_j)\right|$.

Definition 2 (Zhu et al., 2021). Core point: For $x_j \in D$, x_j is defined as a core point if its ε-neighbourhood $N_\varepsilon(x_j)$ contains at least $MinPts$ samples, that is, $\left|N_\varepsilon(x_j)\right| \ge MinPts$.

Definition 3 (Zhu et al., 2021). Directly density-reachable: A sample x_i is directly density-reachable from x_j if it satisfies (1) $x_i \in N_\varepsilon(x_j)$ and (2) $\left|N_\varepsilon(x_j)\right| \ge MinPts$.

Definition 4 (Zhu et al., 2021). Density-reachable: A sample x_i is density-reachable from a sample x_j if there exists a sequence of samples $p_1, p_2, \ldots, p_t$ and $p_1 = x_i$, $p_t = x_j$, such that p_{i+1} is directly density-reachable from p_i.

Definition 5 (Zhu et al., 2021). Density connected: x_i and x_j are density-connected if there exists a core sample x_k, such that both x_i and x_j are reachable by the x_k density.

The specific procedure of the DBSCAN clustering algorithm is described in Algorithm 1.

Algorithm 1: DBSCAN Clustering Algorithm

INPUT: initial sample set $D = (x_1, x_2, \ldots, x_m)$, neighbourhood parameters (ξ, $MinPts$).

OUTPUT: k subclusters

BEGIN

1. Initialize the set of core points $\Omega = \emptyset$ the number of clusters $k = 0$, set of subclusters $C = \emptyset$.
2. For $j = 1, 2, \ldots, m$, calculate the ε-neighborhood subsample set $N_\varepsilon(x_j)$ of sample x_j.
3. If $\left|N_\varepsilon(x_j)\right| \ge MinPts$, add the sample x_j to the core object collection $\Omega = \Omega \cup \{x_j\}$.
4. If $\Omega = \emptyset$, end the algorithm; otherwise go to Step 5.
5. Calculate the pairwise distance between the core points and find the reachable density core points in Ω.
6. Cluster these core objects with their subsamples in the ε-neighbourhood to form a subcluster C_i and add them to the set of clusters $C = C \cup C_i$.
7. Iterate until no new clusters are created during Step 5.
8. Return k subclusters.

END

Source: Summarized based on previous studies

2.2. Intuitionistic fuzzy sets

The fuzzy sets (FS) theory was first proposed by Zadeh (1965) and utilized to characterize the fuzzy attitude of the decision maker in the decision-making process by means of membership. However, since it fails to portray the neutral state, Atanassov (1986) extended the FS and proposed the intuitionistic fuzzy sets (IFS) theory.

Definition 6 (Atanassov, 1986). Let $X = \{x_1, x_2, \ldots, x_n\}$ be a nonempty set; then the fuzzy set can be expressed as:

$$A = \{\langle x, \mu_A(x) \rangle | x \in X\}, \tag{1}$$

where $\mu_A(x)$ is the membership of the element x in X belonging to A, that is, $\mu_A : X \to [0, 1]$ and $0 \le \mu_A(x) \le 1, \ \forall x \in X$.

Definition 7 (Atanassov, 1986). Let $X = \{x_1, x_2, \ldots, x_n\}$ be a nonempty set; then the intuitionistic fuzzy set can be expressed as:

$$A = \{\langle x, \mu_A(x), \nu_A(x) \rangle | x \in X\}, \tag{2}$$

where $\mu_A(x)$ and $\nu_A(x)$ are the membership and nonmembership of element x in X belonging to A, respectively and $0 \le \mu_A(x) + \nu_A(x) \le 1, \ \forall x \in X$. Furthermore, for $\forall x \in X$,

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x), \tag{3}$$

where $\pi_A(x)$ represents the hesitation or uncertainty of element x in X belonging to A.

Definition 8 (Atanassov, 1986). Let $\alpha_1 = (\mu_{\alpha_1}, \nu_{\alpha_1})$ and $\alpha_2 = (\mu_{\alpha_2}, \nu_{\alpha_2})$ be any two intuitionistic fuzzy numbers; then the distance between them can be expressed as:

$$d_{IFD}(\alpha, \alpha) = \frac{1}{2}\left(|\mu_{\alpha_1} - \mu_{\alpha_2}| + |\nu_{\alpha_1} - \nu_{\alpha_2}|\right). \tag{4}$$

Further, let $A = (\alpha_1, \alpha_2, \ldots, \alpha_n)$ and $B = (\beta_1, \beta_2, \ldots, \beta_n)$ be intuitionistic fuzzy sets. Then, their weighted distance can be expressed as:

$$IFWD(A, B) = \sqrt{\left(\sum_{j=1}^{n} \omega_j \left(d_{IFD}(\alpha_j, \beta_j)\right)\right)\lambda}. \tag{5}$$

Definition 9 (Atanassov, 1986). Let $\alpha_j = (\mu_{\alpha_j}, \nu_{\alpha_j})$ be a set of intuitionistic fuzzy numbers and $IFWA : \Theta^n \to \Theta$. If

$$IFWA_\omega(\alpha_1, \alpha_2, \ldots, \alpha_n) = \omega_1\alpha_1 \oplus \omega_2\alpha_2 \oplus \cdots \oplus \omega_n\alpha_n, \tag{6}$$

then $IFWA$ is called an intuitionistic fuzzy weighted average operator. $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ is the exponential weight vector of α_j that satisfies $\omega_j \in [0, 1]$ and

$\sum_{j=1}^{n}\omega_j = 1$. It is worth noting that if $\omega = (1/n, 1/n, \ldots, 1/n)^T$, then the *IFWA* operator degenerates to the intuitionistic fuzzy average (*IFA*) operator as:

$$IFA_{\omega}(\alpha_1, \alpha_2, \ldots, \alpha_n) = (\alpha_1 \oplus \alpha_2 \oplus \cdots \oplus \alpha_n)^{1/n}. \tag{7}$$

Definition 10 (Liu & Wang, 2007). Given the intuitionistic fuzzy number $(\mu_A(x), \nu_A(x), \pi_A(x))$, the intuitionistic fuzzy score function can be defined as:

$$S(A) = \mu_A(x) + \mu_A(x)(1 - \mu_A(x) - \nu_A(x)). \tag{8}$$

The higher the value of $S(A)$, the better the corresponding alternative will meet the expectations of the decision-maker.

2.3. MULTIMOORA

The MULTIMOORA approach, which is based on MOORA, was proposed by Brauers and Zavadskas (2010). This method considers the additive utility function, multiplicative utility function, and reference point method, which means that the MULTIMOORA method has the advantages of several MCDM methods simultaneously. It is assumed that there are m alternatives $A_i (i = 1, 2, \ldots, m)$ and n criteria $C_j (j = 1, 2, \ldots, n)$. $F = (x_{ij})_{m \times n}$ represents the rating of the decision maker on alternative A_i with respect to criteria C_j. The ratio system, reference point approach, and full multiplicative form in the MULTIMOORA approach are elaborated upon in the following subsections.

2.3.1. Ratio system

The ratio system defines a standardized rating matrix $F^* = (x^*_{ij})_{m \times n}$, wherein the standardized rating x^*_{ij} of each alternative A_i with respect to each criterion C_j is calculated as follows:

$$x^*_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}}. \tag{9}$$

Considering two types of criteria, namely benefit-based and cost-based, the standardized formula for assessing value is

$$y^*_i = \sum_{j=1}^{g} x^*_{ij} - \sum_{j=g+1}^{n} x^*_{ij}, \tag{10}$$

where $C_j (j = 1, 2, \ldots, g)$ is the benefit criterion, $C_j (j = g + 1, g + 2, \ldots, n)$ is the cost criterion, and y^*_i represents the standardized rating of alternative A_i for all criteria. The final preferences for the alternatives are obtained by ranking y^*_i. The larger the value of y^*_i, the higher is the ranking of A_i.

2.3.2. Reference point approach

The preference point r_j is obtained based on the standardized matrix $F^* = (x^*_{ij})_{m \times n}$. If C_j is a benefit criterion, then $r_j = \max_i x^*_{ij}$; and if C_j is a cost criterion, then $r_j = \min_i x^*_{ij}$. The final ranking is obtained by calculating the deviation of the standard value relative to the reference point by

$$P_i = \max_j \left| r_j - x^*_{ij} \right|, \tag{11}$$

where P denotes the maximum deviation of the standard value of the alternative A_i under all criteria with respect to the reference point. Therefore, the smaller the value of P, the lower is the ranking of alternative A_i.

2.3.3. Full multiplicative form

The utility function of the full multiplicative form for alternative ranking is

$$U_i = \frac{\prod_{j=1}^{g} x^*_{ij}}{\prod_{j=g+1}^{n} x^*_{ij}}, \tag{12}$$

where $C_j (j = 1, 2, \ldots, g)$ is the benefit criterion, $C_j (j = g+1, g+2, \ldots, n)$ is the cost criterion, and U_i is the utility value of A_i. The final alternative preference is obtained by sorting the U_i. Therefore, the larger the U_i, the higher is the ranking of A_i.

2.4. CRITIC

CRITIC is an objective weighting method first proposed by Diakoulaki et al. (1995). This weighting method determines the objective weights through both the contrast strength and conflicting nature of the indicators. The contrast strength refers to the difference between the values of different samples on the same indicator that can be expressed by calculating the standard deviation; the larger the standard deviation, the greater is the strength of contrast. The conflict of indicators is expressed by the correlation between different indicators; the stronger the correlation, the smaller is the conflict of indicators (Krishnan et al., 2021). The weight coefficients of each evaluation index are determined by combining the comparative strengths and conflicting aspects of each index. The CRITIC method is calculated using the following formula:

$$c_j = \sigma_j \sum_{t=1}^{n} (1 - r_{ij}), \tag{13}$$

where c_j is the value of the weight coefficient and σ_j is the standard deviation of the$j$th evaluation index, respectively, and r_{ij} represents the correlation coefficient between the two indices. $\sum_{t=1}^{n} (1 - r_{ij})$ indicates the conflicting nature of the jth evaluation indicator and other evaluation indicators. Therefore, the greater the value of σ_j, the higher is the value of c_j. The weights corresponding to each indicator are obtained by normalizing c_j as:

$$W_j = \frac{c_j}{\sum_{j=1}^{n} c_j}. \tag{14}$$

3. DBSCAN-based LSGDM method in an IFS environment

3.1. Framework of the proposed DBSCAN-based LSGDM approach

To solve the applicability problem of clustering algorithms in the LSGDM process, this paper proposes a DBSCAN clustering-based LSGDM method in an IFS environment. This method includes the following four components, shown in Figure 1.

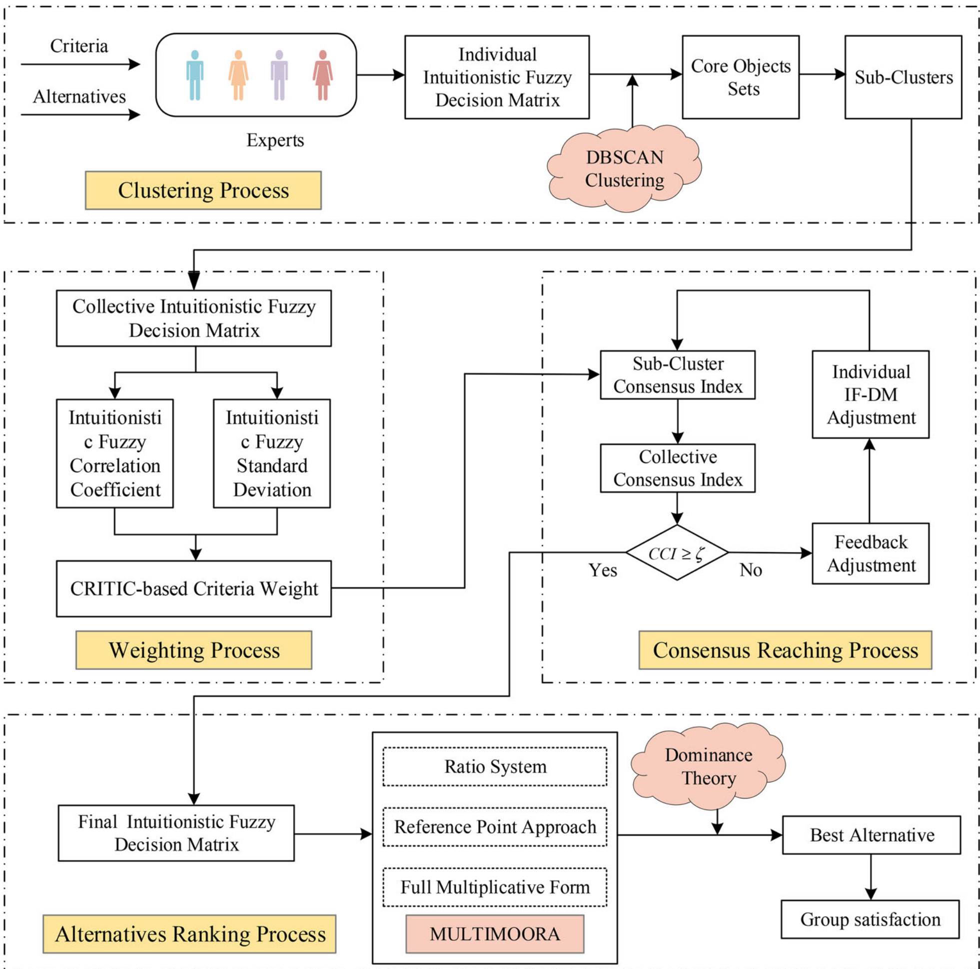

Figure 1. Framework of the proposed LSGDM model.
Source: Self-formulated.

1. *Clustering process*. The DBSCAN clustering algorithm is used to cluster the experts. The advantage of this algorithm is that it can automatically identify outliers among experts, thus maximizing the degree of consensus and satisfaction of the subclusters.
2. *Weighting process*. Considering that the weight of the criterion is unknown in LSGDM, the CRITIC method is applied to calculate the criterion weights. This assignment method determines the objective weights mainly through two aspects: comparison intensity and conflicting nature of the indicators.
3. *Consensus-reaching process*. The degree of consensus within the subclusters as well as the overall degree of consensus is measured, such that the result of the group decision is maximal consensus.
4. *Alternative ranking process*. The MULTIMOORA method is used to rank alternatives. It consists of three parts: ratio system, reference point approach, and full multiplication form, each of which yields the ranking results of the alternatives. The final ranking of the alternatives is combined with the dominance theory to determine the best group decision alternative.

3.2. Proposed DBSCAN-based LSGDM approach in an IFS environment

Let $X = (x_1, x_2, \ldots, x_m)$ be the set of alternatives, $E = (e_1, e_2, \ldots, e_t)$ the set of experts, and $C = (c_1, c_2, \ldots, c_n)$ the set of criteria. The vectors of criteria weights $W = (w_1, w_2, \ldots, w_n)$ and expert weights $\Lambda = (\lambda_1, \lambda_2, \ldots, \lambda_t)$ remain unknown but they satisfy $\sum_{j=1}^{n} w_j = 1$ and $\sum_{k=1}^{t} \lambda_k = 1$. The intuitionistic fuzzy decision matrix R_{ij}^k of the kth expert for the set of alternatives can be expressed as:

$$R_{ij}^k = \begin{bmatrix} (\mu_{11}^k, \nu_{11}^k, \pi_{11}^k) & (\mu_{12}^k, \nu_{12}^k, \pi_{12}^k) & \cdots & (\mu_{1n}^k, \nu_{1n}^k, \pi_{1n}^k) \\ (\mu_{21}^k, \nu_{21}^k, \pi_{21}^k) & (\mu_{22}^k, \nu_{22}^k, \pi_{22}^k) & \cdots & (\mu_{2n}^k, \nu_{2n}^k, \pi_{2n}^k) \\ \vdots & \vdots & \ddots & \vdots \\ (\mu_{m1}^k, \nu_{m1}^k, \pi_{m1}^k) & (\mu_{m2}^k, \nu_{m2}^k, \pi_{m2}^k) & \cdots & (\mu_{mn}^k, \nu_{mn}^k, \pi_{mn}^k) \end{bmatrix}, \tag{15}$$

where $(\mu_{ij}^k, \nu_{ij}^k, \pi_{ij}^k)$ is the intuitionistic fuzzy number of expert e^k under the jth criterion of the ith alternative. The calculation steps of the proposed DBSCAN-based LSGDM approach in the IFS environment in this study are elaborated upon in the following subsections.

3.2.1. Clustering process

The DBSCAN algorithm is used to cluster the experts. The advantage of DBSCAN is that it can automatically identify outliers using simple calculations and fewer iterative processes. Subsequently, the decision preference information of the group is integrated using the agglomerative operator; thus, the intuitionistic fuzzy decision preference matrix of the group can be obtained.

Step 1. Construction of an intuitionistic fuzzy distance matrix. According to Eq. (5), the intuitionistic fuzzy distances between experts e^k and e^s $(k, s = 1, 2, \ldots, t)$ regarding alternative i can be obtained as

$$d_i^{ks} = \sqrt{\frac{1}{3n}\sum_{j=1}^{n}\left[\left(\mu_{ij}^k - \mu_{ij}^s\right)^2 + \left(\nu_{ij}^k - \nu_{ij}^s\right)^2 + \left(\pi_{ij}^k - \pi_{ij}^s\right)^2\right]}, \tag{16}$$

where $(\mu_{ij}^k, \nu_{ij}^k, \pi_{ij}^k)$ and $(\mu_{ij}^s, \nu_{ij}^s, \pi_{ij}^s)$ are the intuitionistic fuzzy ratings of experts e^k and e^s, respectively, with respect to alternative i under the criterion j. Based on this, the intuitionistic fuzzy distance matrix can be obtained as:

$$d^{ks} = \begin{bmatrix} d_{11}^{ks} & d_{12}^{ks} & \cdots & d_{1m}^{ks} \\ d_{21}^{ks} & d_{22}^{ks} & \cdots & d_{2m}^{ks} \\ \vdots & \vdots & \ddots & \vdots \\ d_{m1}^{ks} & d_{m2}^{ks} & \cdots & d_{mm}^{ks} \end{bmatrix}. \tag{17}$$

Step 2. Clustering of experts. According to Algorithm 1, the DBSCAN algorithm is used to cluster the experts, given the clustering initialization parameters ξ and *MinPts*. Thereafter, the experts can be divided into g subclusters, namely $G_1, G_2, \ldots, G_g$.

Step 3. Calculation of the weights of experts and subclusters. The weight of expert e^k in the rth subcluster G_r can be expressed as:

$$\lambda_r^k = \frac{1}{\#L_r}. \tag{18}$$

Further, the weights of subcluster G_r can be represented by:

$$\lambda_r = \frac{\#L_r}{\sum_{r=1}^{g}\#L_r}, \tag{19}$$

where $\#L_r$ denotes the number of experts in the subcluster $G_r (r = 1, 2, ..., g)$.

Step 4. Construct the intuitionistic fuzzy decision matrix of the group. Let $R_{ij,r}^k = (r_{ij,r}^k)_{m\times n}$ be the intuitionistic fuzzy decision matrix of experts e^k in the subcluster G_r, and $\lambda_r = \left\{\lambda_r^1, \lambda_r^2, \ldots, \lambda_r^k\right\}$ is the weight vector in subcluster G_r. By combining the *IFWA* operator, the intuitionistic fuzzy number of alternative i in subcluster G_r under criterion j is obtained as follows:

$$\begin{aligned} r_{ij,r} &= IFWA_\lambda\left(r_{ij}^1, r_{ij}^2, ..., r_{ij}^{\#L_r}\right). \\ &= \lambda_r^1 r_{ij}^1 \oplus \lambda_r^2 r_{ij}^2 \oplus \cdots \oplus \lambda_r^{\#L_r} r_{ij}^{\#L_r} \\ &= \left(1 - \prod_{k=1}^{\#L_r}\left(1-\mu_{ij}^k\right)^{\lambda_r^k}, \prod_{k=1}^{\#L_r}\left(\nu_{ij}^k\right)^{\lambda_r^k}, \prod_{k=1}^{\#L_r}\left(1-\mu_{ij}^k\right)^{\lambda_r^k} - \prod_{k=1}^{\#L_r}\left(\nu_{ij}^k\right)^{\lambda_r^k}\right) \end{aligned} \tag{20}$$

Therefore, the intuitionistic fuzzy decision matrix of the subcluster G_r after integration can be obtained as:

$$R_{ij,r}=\begin{bmatrix} r_{11,r} & r_{12,r} & \cdots & r_{1n,r} \\ r_{21,r} & r_{22,r} & \cdots & r_{2n,r} \\ \vdots & \vdots & \ddots & \vdots \\ r_{m1,r} & r_{m2,r} & \cdots & r_{mn,r} \end{bmatrix}, \tag{21}$$

where $r_{ij,r}=(\mu_{ij,r},\nu_{ij,r},\pi_{ij,r})$ denotes the intuitionistic fuzzy number of subcluster G_r.

Further, based on Eq. (20), the collective intuitionistic fuzzy decision matrix is obtained by combining the weights of the subcluster λ_r:

$$R_{ij}=\begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ r_{m1} & r_{m2} & \cdots & r_{mn} \end{bmatrix}. \tag{22}$$

3.2.2. Weighting process

The weights of the criteria are calculated using the IF-CRITIC method that combines both comparative strength and conflicting nature of each criterion to calculate its weight.

Step 5. Calculation of the correlation coefficients of the criteria. Based on the collective intuitionistic fuzzy decision matrix R_{ij}, the intuitionistic fuzzy correlation coefficients of each criterion can be further calculated to obtain $IFCC_{jt}$:

$$IFCC_{jt}=\frac{\sum_{i=1}^{m}\left(S\left(r_{ij}^{N}\right)-S\left(r_{j}^{N}\right)\right)\left(S\left(r_{it}^{N}\right)-S\left(r_{t}^{N}\right)\right)}{\sqrt{\sum_{i=1}^{m}\left(S\left(r_{ij}^{N}\right)-S\left(r_{j}^{N}\right)\right)^{2}}\sqrt{\sum_{i=1}^{m}\left(S\left(r_{it}^{N}\right)-S\left(r_{t}^{N}\right)\right)^{2}}}, \tag{23}$$

where $S(r_j^N)=1/m\sum_{i=1}^{m}S(r_{ij}^N)$ denotes the mean value of the score function after normalization of the criterion j, $S(r_{ij}^N)$ denotes the score function of the alternative i under the criterion j, $S(r_t^N)=1/m\sum_{i=1}^{m}S(r_{it}^N)$ represents the mean value of the score function after normalization of the criterion t, and $S(r_{it}^N)$ denotes the score function of the alternative i under the criterion t.

Step 6. Calculation of the standard deviation of the criterion. According to the score function of each alternative under each criterion, the intuitionistic fuzzy standard deviation of criterion j can be calculated as follows:

$$IFSD_j=\sqrt{\frac{1}{m-1}\sum_{i=1}^{m}\left(S\left(s_{ij}^{N}\right)-S\left(s_{j}^{N}\right)\right)^{2}}, \tag{24}$$

where $S(s_j^N)=1/m\sum_{i=1}^{m}S(s_{ij}^N)$ and $S(s_t^N)=1/m\sum_{i=1}^{m}S(s_{it}^N)$.

Step 7. Calculation of the criteria weights. Based on each criterion, let the intuitionistic fuzzy correlation coefficient be $IFCC_{jt}$ and the intuitionistic fuzzy standard deviation be $IFSD_j$, then the weight w_j of each criterion can be obtained as follows:

$$w_j = \frac{IFSD_j \sum_{t=1}^{n}(1 - IFCC_{jt})}{\sum_{j=1}^{n}\left(IFSD_j \sum_{t=1}^{n}(1 - IFCC_{jt})\right)}, \quad j = 1, 2..., n, \tag{25}$$

where $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$.

3.2.3. Consensus reaching process (CRP)

Consensus metrics are powerful tools for evaluating the consistency of individual and group preference views, and consensus measures can demonstrate the degree of preference consistency among decision makers. Generally, CRP consists of two parts: a consensus degree calculation and feedback adjustment. If the calculated consensus degree is higher than a given threshold, it indicates a high degree of consistency in group preferences; conversely, it is necessary to adjust individual preferences to satisfy the given consensus threshold.

Step 8. Calculation of the subcluster consensus index ($SCCI$). Calculate the consensus index of subcluster r with respect to alternative i :

$$SCCI_{i,r} = \sum_{k=1}^{\#L_r} \lambda_r^k \sum_{j=1}^{m} w_j \left(1 - \left|S\left(r_{ij,r}^k\right) - S(r_{ij,r})\right|\right), \tag{26}$$

where $S(r_{ij,r}^k)$ is the intuitionistic fuzzy score function of expert e^k in subcluster G_r with respect to alternative i under criterion j, $S(r_{ij,r}^k)$ is the collective intuitionistic fuzzy score function of subcluster G_r under criterion j with respect to alternative i, w_j is the weight of the criterion j, λ_r^k is the weight of the experts e^k in the subcluster G_r, and $\#L_r$ is the number of experts in the subcluster G_r.

Further, the degree of consensus for all subclusters with respect to alternative i can be calculated as:

$$SCCI_i = \sum_{r=1}^{g} \lambda_r \sum_{j=1}^{m} w_j \left(1 - \left|S(r_{ij,r}) - S(r_{ij})\right|\right), \tag{27}$$

where $S(r_{ij,r})$ is the collective intuitionistic fuzzy score function for the rth subcluster under the criterion j with respect to alternative i. $S(r_{ij})$ is the collective intuitionistic fuzzy score function on scheme i under criterion j, where λ_r is the weight of the rth subcluster.

Step 9. Calculation of the collective consensus index (CCI). CCI is calculated based on the subcluster consensus index ($SCCI_i$) of alternative i as:

$$CCI = \min_i \{SCCI_i\}. \tag{28}$$

Given a consensus degree threshold ξ, if $CCI \geq \xi$ and $SCCI_i \geq \xi$, then a consensus is reached within each subcluster; otherwise, further adjustment is required. The consensus adjustment process is described in Step 12.

Step 10. Feedback adjustment. The collective average intuitionistic fuzzy preference is used to adjust the expert ratings, thereby improving the group consensus. This preference is calculated using:

$$\overline{r_{ij}} = \sum_{i=1}^{m}\sum_{j=1}^{n}\left[\min\left(r_{ij,r}, r_{ij}\right), \max\left(r_{ij,r}, r_{ij}\right)\right], \tag{29}$$

where $\overline{r_{ij}}$ is the average intuitionistic fuzzy number of all experts in the alternative i under the criterion j.

3.2.4. Alternatives ranking process

IF-MULTIMOORA is used to rank the alternatives. This approach first normalizes the rating of the expert to eliminate the differences between the various criteria owing to the different dimensions. Then, the final ranking of the schemes is determined by calculating the evaluation values through the IF-ratio system, IF-reference point approach, and IF-full multiplicative form. The result is obtained based on the dominance theory.

Step 11. IF-ratio system. According to the formula of the IF-ratio System, based on the reference (Zhang et al., 2019), the Eq. (30) is used to rank the combined assessment values of the alternatives:

$$U_i^{RS} = IFAWA(\beta_{ij}|j = 1, 2, \ldots, n; w) = \left(1 - \prod_{j=1}^{n}\left(1 - \mu_{ij}\right)^{w_j}, \prod_{j=1}^{n}{\nu_{ij}}^{w_j}\right). \tag{30}$$

Furthermore, the score function of U_i^{RS} is calculated; the higher the value of U_i^{RS}, the higher is the ranking of the solution.

Step 12. IF-reference point approach. The positive ideal alternative $r_j^+ = (\max_i \mu_{ij}, \min_i \nu_{ij})$ is calculated for each criterion. The distance between each alternative and the positive ideal alternative r_j^+ can be obtained by:

$$U_i^{RP} = \min_i d\left(r_{ij}, r_{ij}^+\right) = \min_i \sqrt{\frac{1}{2}\sum_{j=1}^{n}\left\{w_j\left[\left(\mu_{ij} - \mu_{ij}^+\right)^2 + \left(\nu_{ij} - \nu_{ij}^+\right)^2 + \left(\pi_{ij} - \pi_{ij}^+\right)^2\right]\right\}}. \tag{31}$$

The closer the alternative is to the positive ideal solution, the higher is the ranking.

Step 13. IF-Full multiplicative form. The *IFGWA* operator is used to assemble the intuitionistic fuzzy scores of each criterion according to the formula of the IF-full multiplicative form, thus ranking the combined assessment values of the alternatives:

$$U_i^{FMF} = IFGWA(\beta_{ij}|j = 1, 2, \ldots, n; w) = \left(\prod_{j=1}^{n}\mu_{ij}{}^{w_j}, 1 - \prod_{j=1}^{n}(1 - \nu_{ij})^{w_j}\right). \tag{32}$$

Step 14. Determination of the final alternative. Combined with the dominance theory, a pairwise comparison of the generalized dominance relationships in a ternary array of three sets of results obtained by IF-MULTIMOORA is performed to determine the final ranking results of each alternative and further determine the optimal alternative among them.

Step 15. Calculation of group satisfaction. The group satisfaction *Sa* for the complete LSGDM process is calculated as follows:

$$Sa = 1/\left(1 + e^{-\left(\frac{CCI}{Iter}\right)}\right), \tag{33}$$

where CCI is the collective consensus index, $Iter = Iter_{cl} + Iter_{co}$ is the number of iterations in the LSGDM process, $Iter_{cl}$ is the number of iterations in the clustering process, and $Iter_{co}$ is the number of iterations in the CRP. The higher the value of CCI, the lower is the value of $Iter$ and the higher is the value of Sa.

4. Numerical example

Owing to global digital construction, the digital economy has gradually become an important engine of national economic development. According to the China Academy of Information and Communications Technology (2021), the digital economy in 47 countries reached $32.6 trillion in 2020, with a nominal growth of 3.0% year-on-year, accounting for 43.7% of their total GDP. A data centre, which is the core infrastructure of digital construction, is an important carrier for the development of the digital economy. Therefore, it is crucial to select an appropriate data centre provider for the stable development of the digital economy of an enterprise.

Table 1. Final clustering results.

Cluster ID	Instance	Weight	Cluster ID	Instance	Weight
G_1	$e_1, e_2, e_3, e_4, e_5, e_6$	0.3	G_4	e_7	0.05
G_2	$e_8, e_{13}, e_{14}, e_{15}, e_{16}$	0.25	G_5	e_9	0.05
G_3	$e_{10}, e_{11}, e_{12}, e_{18}, e_{19}, e_{20}$	0.3	G_6	e_{17}	0.05

Source: Self-calculated.

In this chapter, a numerical example is provided to select the ideal data centre suppliers using the proposed DBSCAN-based LSGDM problem in an IFS environment. It is assumed that there are 20 experts evaluating five data centre suppliers $x_i(i = 1, 2, \ldots, 5)$, each with the aforementioned five criteria. The results of the intuitionistic fuzzy ratings for each supplier by each expert and criterion are presented in Appendix A. For example, the rating of expert e_1 on data centre supplier x_1 is

$$e_{11} = \left[\langle 0.65, 0.23, 0.12\rangle \quad \langle 0.71, 0.12, 0.17\rangle \quad \langle 0.86, 0.06, 0.08\rangle \quad \langle 0.77, 0.14, 0.09\rangle \quad \langle 0.81, 0.12, 0.07\rangle \right].$$

4.1. Clustering process

The pairwise matrix of the intuitionistic fuzzy distance matrices between each expert can be obtained using Eqs. (16) and (17) and is listed in Appendix B. Let $\varepsilon = 0.1$ and$MinPts = 3$; then, based on Eqs. (2)–(4) and the intuitionistic fuzzy distance matrix d, the set of core objects can be obtained as:

$$\Omega = \left\{ (e_1, e_2, e_3, e_5, e_6), (e_8, e_{13}, e_{14}, e_{15}, e_{16}), (e_{10}, e_{11}, e_{12}, e_{18}, e_{19}, e_{20}) \right\}.$$

Further, the 20 experts are clustered into 3 subclusters and 3 outliers after DBSCAN clustering, where the three subclusters are $G_1 = \{e_1, e_2, e_3, e_4, e_5, e_6\}$, $G_2 = \{e_8, e_{13}, e_{14}, e_{15}, e_{16}\}$, and $G_3 = \{e_{10}, e_{11}, e_{12}, e_{18}, e_{19}, e_{20}\}$, and the experts e_7, e_9, and e_{17} do not belong to any subcluster; consequently, they are labelled as outliers and grouped into separate clusters, namely G_4, G_5 and G_6. The weight vector of the six subclusters is calculated using Eq. (19), and the final clustering results are detailed in Table 1.

Based on Eqs. (20) and (21), the intuitionistic fuzzy decision matrix of the six subclusters can be calculated, as shown in Appendix C. Furthermore, based on the weight vectors of the subclusters, the collective group intuitionistic fuzzy decision matrix is obtained by combining it with Eq. (22) as:

$$R_{ij} = \begin{bmatrix} \langle 0.73, 0.16, 0.11\rangle & \langle 0.77, 0.11, 0.12\rangle & \langle 0.72, 0.14, 0.14\rangle & \langle 0.74, 0.09, 0.17\rangle & \langle 0.76, 0.14, 0.10\rangle \\ \langle 0.72, 0.08, 0.20\rangle & \langle 0.79, 0.05, 0.17\rangle & \langle 0.78, 0.06, 0.16\rangle & \langle 0.81, 0.10, 0.08\rangle & \langle 0.76, 0.03, 0.21\rangle \\ \langle 0.79, 0.06, 0.15\rangle & \langle 0.74, 0.12, 0.14\rangle & \langle 0.66, 0.12, 0.22\rangle & \langle 0.65, 0.14, 0.21\rangle & \langle 0.70, 0.07, 0.23\rangle \\ \langle 0.73, 0.09, 0.18\rangle & \langle 0.83, 0.11, 0.07\rangle & \langle 0.73, 0.05, 0.21\rangle & \langle 0.75, 0.06, 0.18\rangle & \langle 0.68, 0.17, 0.16\rangle \\ \langle 0.89, 0.00, 0.11\rangle & \langle 0.70, 0.13, 0.17\rangle & \langle 0.75, 0.14, 0.11\rangle & \langle 0.78, 0.06, 0.16\rangle & \langle 0.72, 0.15, 0.13\rangle \end{bmatrix}.$$

Based on the group intuition fuzzy decision matrix, the score function for each alternative under each criterion can be obtained as follows:

$$S_{ij} = \begin{bmatrix} 0.8143 & 0.8610 & 0.8176 & 0.8609 & 0.8393 \\ 0.8603 & 0.9178 & 0.9070 & 0.8797 & 0.9197 \\ 0.9109 & 0.8407 & 0.8036 & 0.7857 & 0.8654 \\ 0.8646 & 0.8815 & 0.8887 & 0.8943 & 0.7822 \\ 0.9878 & 0.8162 & 0.8326 & 0.9014 & 0.8102 \end{bmatrix}.$$

4.2. Weighting process

In the weighting session of the criteria, according to Eqs. (23) and (24), the intuition fuzzy correlation coefficient matrix $IFCC_{ij}$ and intuitionistic fuzzy standard deviation $IFSD_j$ between the criteria can be obtained as follows:

$$IFCC_{ij} = \begin{bmatrix} 0.3195 & 0.2963 & 0.3172 & 0.3090 & 0.3045 \\ 0.2963 & 0.2751 & 0.2944 & 0.2865 & 0.2822 \\ 0.3172 & 0.2944 & 0.3164 & 0.3066 & 0.3035 \\ 0.3090 & 0.2865 & 0.3066 & 0.2997 & 0.2952 \\ 0.3045 & 0.2822 & 0.3035 & 0.2952 & 0.2929 \end{bmatrix},$$

$$IFSD_j = (0.8846, 0.9532, 0.8888, 0.9133, 0.9240).$$

Therefore, based on Eq. (25), the criterion weight is $w = (0.1911, 0.2126, 0.1925, 0.2002, 0.2036)$.

4.3. Consensus-reaching process

In the CRP session, the consensus degree matrices within the subclusters G_1, G_2 andG_3 are obtained according to Eqs. (26) and (27); the results are listed in Table 2. It can be concluded from these results that the consensus degree of subcluster G_1 for each alternative is $SCCI_{i1} = (0.9542, 0.9411, 0.9548, 0.9544, 0.9346)$; the consensus degree of subcluster G_2 for each alternative is $SCCI_{i2} = (0.9777, 0.9786, 0.9587, 0.9856, 0.9772)$; and the consensus degree of subcluster G_3 for each alternative is $SCCI_{i3} = (0.9835, 0.9937, 0.9808, 0.9795, 0.9686)$. Further, the consensus degree of all experts for each alternative can be obtained as $SCCI_i = (0.9718, 0.9711, 0.9648, 0.9732, 0.9601)$. Among them, all experts achieved the highest consensus degree of 0.9732 for alternative x_4, and the lowest consensus degree of 0.9602 for alternative x_5.

Further, based on Eq. (28), it can be calculated that $CCI = 0.9601$. Given a consensus threshold of $\xi = 0.75$, it is evident that consensus has been reached both

Table 2. *SSCI* for each alternative based on FCM.

Alternatives	$SCCI_{i1}$	$SCCI_{i2}$	$SCCI_{i3}$	$SCCI_i$
x_1	0.9542	0.9777	0.9835	0.9718
x_2	0.9411	0.9786	0.9937	0.9711
x_3	0.9548	0.9587	0.9808	0.9648
x_4	0.9544	0.9856	0.9795	0.9732
x_5	0.9346	0.9772	0.9686	0.9601

Source: Self-calculated.

within and between subclusters, thereby indicating that the results of group decision-making have reached a consensus.

4.4. Ranking-alternative process

In the IF-MULTIMOORA method, according to the ratio system, the collective intuitionistic fuzzy number for the five alternatives can be obtained as:

$$\left[\langle 0.75, 0.12, 0.13\rangle \quad \langle 0.78, 0.06, 0.17\rangle \quad \langle 0.71, 0.10, 0.20\rangle \quad \langle 0.75, 0.09, 0.16\rangle \quad \langle 0.78, 0.00, 0.22\rangle\right]^T.$$

Therefore, the intuitionistic fuzzy score function of the five alternatives is $(0.8413, 0.9037, 0.8492, 0.8716, 0.9501)^T$. Furthermore, the final ranking result of the alternatives is $x_5 \succ x_2 \succ x_4 \succ x_3 \succ x_1$.

According to the IF-reference point system, the intuitionistic fuzzy number of the ideal alternative can be obtained as:

$$r_j^+ = \left[\langle 0.89, 0.00, 0.11\rangle \quad \langle 0.83, 0.05, 0.12\rangle \quad \langle 0.78, 0.05, 0.17\rangle \quad \langle 0.81, 0.06, 0.13\rangle \quad \langle 0.76, 0.03, 0.21\rangle\right]^T.$$

Combining this with Eq. (31), the intuitionistic fuzzy distance between each alternative and the ideal alternative is calculated as (0.0994, 0.0701, 0.0983, 0.0917, 0.0787). The smaller the distance, the closer an alternative is to the ideal alternative. Therefore, according to the reference point system, the final ranking results of the five alternatives can be obtained as $x_2 \succ x_5 \succ x_4 \succ x_3 \succ x_1$.

According to Eq. (32) of the IF full multiplicative model, the intuitionistic fuzzy numbers of the five alternatives can be obtained as:

$$\left[\langle 0.75, 0.12, 0.13\rangle \quad \langle 0.77, 0.07, 0.16\rangle \quad \langle 0.71, 0.10, 0.20\rangle \quad \langle 0.74, 0.10, 0.16\rangle \quad \langle 0.76, 0.10, 0.14\rangle\right]^T.$$

Furthermore, the intuitionistic fuzzy score function of the five alternatives can be obtained by $(0.8893, 0.8980, 0.8412, 0.8623, 0.8665)^T$. Therefore, the final ranking result of the alternatives can be obtained as $x_2 \succ x_5 \succ x_4 \succ x_3 \succ x_1$.

According to the dominance theory, the final ranking of the alternatives is $x_2 \succ x_5 \succ x_4 \succ x_3 \succ x_1$. Therefore, according to the IF-MULTIMOORA ranking results, the 20 experts have the highest group preference for alternative x_2; therefore, the optimal alternative x_2 is chosen for this case. The results are listed in Table 3.

Furthermore, since $CCI = 0.9601$ and the number of iterations is 1, the group satisfaction Sa for the entire decision process can be calculated using Eq. (33) as:

$$Sa = \frac{1}{1 + \exp(0.9601/1)} = 0.7231.$$

From this, we can conclude that group satisfaction is 0.7231, which indicates a high level of satisfaction with the entire LSGDM process.

Table 3. Ranking results of DBSCAN based LSGDM approach.

Alternatives	IF-RS	IF-RP	IF-FMF	IF-MULTIMOORA
x_1	5	5	5	5
x_2	2	1	1	1
x_3	4	4	4	4
x_4	3	3	3	3
x_5	1	2	2	2

Source: Self-calculated.

Table 4. Information on FCM clustering parameters.

Algorithms	Initializing the clustering centre	Number of clusters	Number of iterations	Clustering thresholds	Membership threshold
FCM	$\{e_2, e_8, e_{14}\}$	3	6	$\theta = 0.05$	$\lambda = 0.4$

Source: Self-calculated.

5. Comparative analysis

To further validate the effectiveness of the proposed DBSCAN-based LSGDM approach, we compared it with the FCM-based LSGDM approach (Xu & Wu, 2010) and K-means based LSGDM approach (Tang et al., 2019) by constructing a set of comparison experiments based on the previously-introduced simulation dataset. This means that the datasets of these comparative analyses remain consistent but a difference exists in the clustering approaches.

5.1. Comparison with FCM-based LSGDM approach

Referring to Xu and Wu (2010), we used the fuzzy C-means algorithm to cluster the intuitionistic fuzzy preference information of the experts. Given the initialized clustering centres $\{e_2, e_8, e_{14}\}$, the number of clusters is $k = 3$. After six iterations, the distance between the cluster centres of the 6th iteration and 5th iteration is obtained as 0.049, which is less than the given distance threshold $\theta = 0.05$. Therefore, the clustering results obtained at this point are robust. The results are listed in Table 4.

Furthermore, information on the membership of each expert in each subcluster is obtained. Given a membership threshold $\lambda = 0.4$, an expert is considered to belong to a subcluster if his or her membership in that subcluster is greater than 0.4. For example, the membership of an expert e_1 to subclusters G_1, G_2, and G_3 are 0.2192, 0.6389, and 0.1418, respectively; therefore, the expert e_1 can be considered to belong to the subcluster G_2. The membership of the 20 experts in each subcluster is listed in Table 5.

After aggregating the membership results for the experts, the 20 experts were divided into three subclusters: $G_1 = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_9, e_{17}\}$, $G_2 = \{e_8, e_{13}, e_{14}, e_{15}, e_{16}\}$, and $G_3 = \{e_{10}, e_{11}, e_{12}, e_{18}, e_{19}, e_{20}\}$. The components of subclusters G_2 and G_3 in fuzzy C-means clustering are identical to those of the proposed DBSCAN-based LSGDM approach. The main difference between the two methods is that the components of the subcluster G_1 vary and the subcluster G_1 obtained by the FCM-based method contains three experts: e_7, e_9, and e_{17} with outlier ratings. The results are listed in Table 6.

Table 5. Membership of each expert for each subcluster based on FCM.

Experts	G_1	G_2	G_3	Experts	G_1	G_2	G_3
e_1	0.2192	0.6389	0.1418	e_{11}	0.9269	0.0438	0.0293
e_2	0.1555	0.7016	0.1429	e_{12}	0.9519	0.0283	0.0198
e_3	0.1558	0.6573	0.1869	e_{13}	0.0716	0.0949	0.8335
e_4	0.1497	0.6339	0.2164	e_{14}	0.0300	0.0588	0.9113
e_5	0.1574	0.6435	0.1991	e_{15}	0.0242	0.0543	0.9215
e_6	0.1740	0.6431	0.1830	e_{16}	0.0485	0.0843	0.8672
e_7	0.2531	0.4101	0.3368	e_{17}	0.2540	0.4011	0.3449
e_8	0.0685	0.1068	0.8246	e_{18}	0.9188	0.0468	0.0343
e_9	0.2409	0.4406	0.3185	e_{19}	0.9465	0.0327	0.0208
e_{10}	0.8720	0.0741	0.0539	e_{20}	0.8847	0.0661	0.0492

Source: Self-calculated.

Table 6. Clustering results based on FCM.

Cluster ID	Instance
G_1	$e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_9, e_{17}$
G_2	$e_8, e_{13}, e_{14}, e_{15}, e_{16}$
G_3	$e_{10}, e_{11}, e_{12}, e_{18}, e_{19}, e_{20}$

Source: Self-calculated.

Table 7. *SSCI* for each alternative based on FCM.

Alternatives	$SCCI_{i1}$	$SCCI_{i2}$	$SCCI_{i3}$	$SCCI_i$
x_1	0.9047	0.9778	0.9832	0.9552
x_2	0.9153	0.9777	0.9937	0.9622
x_3	0.9290	0.9587	0.9811	0.9563
x_4	0.9153	0.9858	0.9794	0.9602
x_5	0.9033	0.9776	0.9688	0.9499

Source: Self-calculated.

Table 7 presents the consensus degree of each subcluster with respect to each alternative according to the FCM-based approach. From this, it can be concluded that the consensus degree of subcluster G_1 for each alternative is $SCCI_{i1} = (0.9047,\ 0.9153,\ 0.9290,\ 0.9153,\ 0.9033)$; the consensus degree of subcluster G_2 for each alternative is $SCCI_{i2} = (0.9778,\ 0.9777,\ 0.9587,\ 0.9858,\ 0.9776)$; and the consensus degree of subcluster G_3 for each alternative is $SCCI_{i3} = (0.9832,\ 0.9937,\ 0.9811,\ 0.9794,\ 0.9688)$. Furthermore, the consensus degree of all experts for each alternative can be obtained as $SCCI_i = (0.9552,\ 0.9622,\ 0.9563,\ 0.9602,\ 0.9499)$. The collective consensus index $CCI = 0.9499$, based on fuzzy C-means clustering, is obtained using Eq. (28).

Similarly, the CRITIC method is used to calculate the objective weights of the criteria, which gives the final group decision intuitionistic fuzzy preference matrix. The final consensus vector for the group decision is (0.9172, 0.9176, 0.9229, 0.9194, 0.8943), given a consensus threshold $\xi = 0.75$, which shows that the final group decision has reached a consensus. Furthermore, by combining the MULTIMOORA approach, the final ranking result can be obtained as $x_2 \succ x_5 \succ x_4 \succ x_3 \succ x_1$. This shows that the FCM-based LSGDM approach gives the highest preference to alternative x_2 among the 20 experts, which is also consistent with the ranking results obtained in this study. The results are listed in Table 8.

Table 8. Ranking results of the FCM-based method.

Alternatives	IF-RS	IF-RP	IF-FMF	IF-MULTIMOORA
x_1	5	5	5	5
x_2	2	1	1	1
x_3	4	4	4	4
x_4	3	3	3	3
x_5	1	2	2	2

Source: Self-calculated.

Furthermore, because $CCI = 0.9499$ and the number of iterations is 6, the group satisfaction Sa for the entire decision process can be calculated using Eq. (33). From this, we can conclude that group satisfaction is 0.5395, which indicates a high level of satisfaction for the entire LSGDM process.

$$Sa = \frac{1}{1 + \exp(0.9499/6)} = 0.5395.$$

In summary, comparing the DBSCAN-based and FCM-based approaches, it can be concluded that although the final ranking results of the alternatives remain consistent, differences exist in the clustering results, group consensus, and group satisfaction. The group consensus and group satisfaction obtained using the DBSCAN-based approach were higher than those obtained using the FCM-based approach. This is primarily because the DBSCAN-based method can effectively identify the outliers among the rating experts and ensure that experts within each subcluster have consistent preference, thereby improving group consensus and group satisfaction. However, the FCM-based approach fails to identify the outliers of the rating expert, thus allowing several to be included in subcluster G_1, which reduces the consensus degree of G_1 as well as the collective consensus index. In addition, six iterative rounds of the clustering process are performed using the FCM-based method, which is time consuming and computationally intensive. This further reduces satisfaction in the final group.

Moreover, it is worth noting that the fuzzy C-means-based clustering algorithm requires initialization of the clustering centres and number of clusters. In addition, the selection of clustering and membership thresholds may also have a significant impact on the final clustering results. In contrast, the DBSCAN-based clustering algorithm is simpler, requiring only the initialization of the parameters ε and $MinPts$ to complete the clustering process and can automatically identify outliers in the sample.

5.2. Comparison with K-means based LSGDM approach

Following Tang et al. (2019), the K-means algorithm was used to cluster the intuitionistic fuzzy preference information of experts. Given the initialized clustering centres, $\{e_1, e_{10}, e_{18}\}$, the number of clusters is $k = 3$. After two iterations, the distance between the samples and cluster centres was minimized, indicating that the clustering process was complete. Therefore, the clustering results obtained at this point are robust. The results are listed in Table 9.

Table 9. Information on K-means clustering parameters.

Algorithms	Initializing the clustering centre	Number of clusters	Number of iterations	Clustering thresholds
K-means	$\{e_1, e_{10}, e_{18}\}$	3	2	$\theta = 0.05$

Source: Self-calculated.

Furthermore, it can be inferred that the 20 experts are divided into three subclusters, namely $G_1 = \{e_1, e_2, e_3, e_4, e_5, e_6\}$, $G_2 = \{e_7, e_8, e_{13}, e_{14}, e_{15}, e_{16}, e_{17}\}$, and $G_3 = \{e_{10}, e_{11}, e_{12}, e_{18}, e_{19}, e_{20}\}$, based on the principle of maximum membership. Compared with the DBSCAN-based approach, the K-means based method partitions experts into three subclusters and each sample is assigned to a specific subcluster. The clustering results are listed in Table 10.

Furthermore, the weight vector of the subclusters can be obtained as (0.35, 0.35, 0.3). Combining it with Eq. (22), the intuitionistic fuzzy decision matrix for the population can be assembled as:

$$R_{ij} = \begin{bmatrix} \langle 0.70, 0.18, 0.12\rangle & \langle 0.73, 0.14, 0.13\rangle & \langle 0.67, 0.18, 0.15\rangle & \langle 0.71, 0.11, 0.18\rangle & \langle 0.73, 0.17, 0.11\rangle \\ \langle 0.69, 0.11, 0.21\rangle & \langle 0.76, 0.06, 0.18\rangle & \langle 0.75, 0.08, 0.17\rangle & \langle 0.78, 0.13, 0.09\rangle & \langle 0.74, 0.04, 0.22\rangle \\ \langle 0.76, 0.07, 0.17\rangle & \langle 0.70, 0.15, 0.15\rangle & \langle 0.62, 0.15, 0.23\rangle & \langle 0.61, 0.17, 0.22\rangle & \langle 0.67, 0.08, 0.25\rangle \\ \langle 0.70, 0.11, 0.19\rangle & \langle 0.80, 0.13, 0.07\rangle & \langle 0.70, 0.07, 0.23\rangle & \langle 0.72, 0.07, 0.21\rangle & \langle 0.64, 0.20, 0.16\rangle \\ \langle 0.86, 0.00, 0.14\rangle & \langle 0.67, 0.16, 0.17\rangle & \langle 0.71, 0.17, 0.12\rangle & \langle 0.73, 0.09, 0.18\rangle & \langle 0.69, 0.18, 0.13\rangle \end{bmatrix}.$$

Comparing the group decision matrix obtained by the DBSCAN-based clustering algorithm with that obtained by the K-means based algorithm, it can be observed that there are some differences between the two. The primary reason is that the size, membership and weights of the subclusters obtained by the two clustering algorithms are different; therefore, the final group decision matrix is not the same.

Table 11 presents the consensus degree of each subcluster with respect to each alternative according to the K-means based approach. From this, it can be concluded that the consensus degree of subcluster G_1 for each alternative is $SCCI_{i1} = (0.9334, 0.9310, 0.9406, 0.9439, 0.9168)$, the consensus degree of subcluster G_2 for each alternative is $SCCI_{i2} = (0.9396, 0.9177, 0.9437, 0.9259, 0.9322)$, and the consensus degree of subcluster G_3 for each alternative is $SCCI_{i3} = (0.9832, 0.9937, 0.9811, 0.9794, 0.9688)$. Furthermore, the consensus degree of all experts for each alternative can be obtained as $SCCI_i = (0.9521, 0.9475, 0.9551, 0.9497, 0.9393)$. The collective consensus index $CCI = 0.9393$, based on fuzzy C-means clustering, is obtained using Eq. (28).

According to the collective decision matrix, the coefficient of the weight vector of each criterion is (0.2132, 0.2010, 0.1923, 0.2007, 0.1927). Therefore, the final ranking results of the group decisions can be obtained, as listed in Table 12. From Table 12, it can be seen that according to K-means based IF-MULTIMOORA, the final ranking result of the alternatives is $x_2 \succ x_5 \succ x_4 \succ x_3 \succ x_1$, which is consistent with the decision results of the proposed DBSCAN-based IF-MULTIMOORA as well as the proposed DBSCAN-based LSGDM. This is because, although there are differences in the group decision matrix and criterion weight vector, alternative x_2 has a clear advantage over the other alternatives, followed by alternativex_5, and then by alternativesx_4, x_3 and x_1, respectively, regardless of the method used to calculate the decision result.

Table 10. Final clustering results.

Cluster ID	Instance
G_1	$e_1, e_2, e_3, e_4, e_5, e_6, e_9$
G_2	$e_7, e_8, e_{13}, e_{14}, e_{15}, e_{16}, e_{17}$
G_3	$e_{10}, e_{11}, e_{12}, e_{18}, e_{19}, e_{20}$

Source: Self-calculated.

Table 11. *SSCI* for each alternative for the K-means based approach.

Alternatives	$SCCI_{i1}$	$SCCI_{i2}$	$SCCI_{i3}$	$SCCI_i$
x_1	0.9334	0.9396	0.9832	0.9521
x_2	0.9310	0.9177	0.9937	0.9475
x_3	0.9406	0.9437	0.9811	0.9551
x_4	0.9439	0.9259	0.9794	0.9497
x_5	0.9168	0.9322	0.9688	0.9393

Source: Self-calculated.

Table 12. Ranking results for the K-means based method.

Alternatives	IF-RS	IF-RP	IF-FMF	IF-MULTIMOORA
x_1	5	5	5	5
x_2	2	1	1	1
x_3	4	4	4	4
x_4	3	3	3	3
x_5	1	2	2	2

Source: Self-calculated.

Furthermore, because $CCI = 0.9393$ and the number of iterations is 2, the group satisfaction Sa for the entire decision process can be calculated using Eq. (33). From this, we can conclude that the group satisfaction is 0.6153, which indicates a high level of satisfaction for the entire LSGDM process.

$$Sa = \frac{1}{1 + \exp(0.9393/2)} = 0.6153.$$

In summary, comparing the DBSCAN-based and K-means based approaches, it can be concluded that although the final ranking results of the alternatives remain consistent, differences exist in the clustering results, group consensus, and group satisfaction. The group consensus and group satisfaction obtained by the DBSCAN-based approach were higher than those obtained by the K-means based approach. The primary reason for this is that K-means clustering is a hard clustering method through which each sample can only belong to one subcluster; therefore, it fails to identify outlier rating experts. Thus, subclusters G_1 and G_2 also contain outlier rating experts, which reduces the subcluster consensus indices $SCCI_1$ and $SCCI_2$ as well as the collective consensus index CCI. In addition, the K-means based method must first initialize the clustering centres and then generate subclusters by iterative computation, which further reduces the group satisfaction.

Moreover, comparing the K-means based and FCM-based methods, it can be concluded that the group consensus obtained by the FCM-based method is higher than that of the K-means based method but the group satisfaction obtained by the FCM-based method is lower than that obtained by the K-means based method because there exists no significant difference in group consensus between the two approaches. This is because the FCM-based method obtains the membership of subclusters by iterative calculation and then determines the experts of each subcluster according to the maximum membership. A total of 6 rounds of iterative calculations are performed in the clustering process, which is time-consuming and results in a low final group satisfaction. The K-means based method has fewer iterations in the clustering process, which results in higher final group satisfaction.

6. Conclusion and future work

In this study, we explored the DBSCAN clustering-based LSGDM approach in an IFS environment. First, we used the DBSCAN clustering algorithm for clustering decision experts, which has the benefit of automatically identifying outliers among experts. Second, we used the CRITIC method to calculate the objective weights of the criteria, which incorporates both contrast intensity of each criterion and the conflict between criteria to obtain the weights of the criteria. Furthermore, the MULTIMOORA approach was utilized to rank the alternatives and an easy-to-use group satisfaction calculation function was proposed to characterize the satisfaction of the expert with the complete LSGDM process based on the group consensus and iterations during the LSGDM process. Finally, the effectiveness of the proposed algorithm was verified by conducting a comparison experiment.

By comparing the DBSCAN-based, FCM-based, and K-means based approaches, it can be inferred that although the ranking results obtained by these three methods remained the same, the group consensus and group satisfaction varied. The group consensus obtained using the DBSCAN-based approach was the highest, followed by that obtained using the FCM-based approach and the lowest being the one obtained using the K-means based approach. In terms of group satisfaction, the group consensus obtained using the DBSCAN-based approach was the highest, followed by that obtained using the K-means based approach and the lowest being the one obtained using the FCM-based method. The is because the DBSCAN-based method can effectively identify the outliers among the rating experts and ensure that experts within each subcluster have consistent preference, thereby, improving the group consensus and group satisfaction. The FCM-based method obtains the membership of subclusters by iterative calculation, which is time-consuming and results in a lower group satisfaction. Although the K-means based clustering algorithm is a hard clustering algorithm wherein each sample can only belong to one subcluster, it requires fewer iterations, leading to a higher group satisfaction.

The shortcoming of this study is that a simulated study was implemented to illustrate the proposed approach, which may not be very convincing; however, the main purpose was to provide a feasible way to improve group satisfaction in the LSGDM. In future research, we will further investigate the weighting approach by fusing online reviews into the LSGDM process to maximize group satisfaction.

Disclosure statement

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Appendix A

$$e_1 = \begin{bmatrix} \langle 0.65, 0.23, 0.12\rangle & \langle 0.71, 0.12, 0.17\rangle & \langle 0.86, 0.06, 0.08\rangle & \langle 0.77, 0.14, 0.09\rangle & \langle 0.81, 0.12, 0.07\rangle \\ \langle 0.68, 0.12, 0.20\rangle & \langle 0.64, 0.23, 0.13\rangle & \langle 0.71, 0.25, 0.04\rangle & \langle 0.69, 0.17, 0.14\rangle & \langle 0.78, 0.09, 0.13\rangle \\ \langle 0.72, 0.16, 0.12\rangle & \langle 0.66, 0.11, 0.23\rangle & \langle 0.75, 0.15, 0.10\rangle & \langle 0.83, 0.05, 0.12\rangle & \langle 0.67, 0.28, 0.05\rangle \\ \langle 0.75, 0.07, 0.18\rangle & \langle 0.76, 0.13, 0.11\rangle & \langle 0.77, 0.08, 0.15\rangle & \langle 0.75, 0.21, 0.04\rangle & \langle 0.79, 0.15, 0.06\rangle \\ \langle 0.82, 0.04, 0.14\rangle & \langle 0.67, 0.13, 0.20\rangle & \langle 0.71, 0.26, 0.03\rangle & \langle 0.92, 0.02, 0.08\rangle & \langle 0.60, 0.30, 0.10\rangle \end{bmatrix}$$

$$e_2 = \begin{bmatrix} \langle 0.76, 0.13, 0.11\rangle & \langle 0.68, 0.14, 0.18\rangle & \langle 0.81, 0.09, 0.10\rangle & \langle 0.64, 0.21, 0.15\rangle & \langle 0.69, 0.08, 0.23\rangle \\ \langle 0.67, 0.24, 0.09\rangle & \langle 0.71, 0.18, 0.11\rangle & \langle 0.76, 0.16, 0.08\rangle & \langle 0.58, 0.28, 0.14\rangle & \langle 0.64, 0.17, 0.19\rangle \\ \langle 0.73, 0.15, 0.12\rangle & \langle 0.66, 0.27, 0.07\rangle & \langle 0.72, 0.08, 0.20\rangle & \langle 0.63, 0.13, 0.24\rangle & \langle 0.77, 0.05, 0.18\rangle \\ \langle 0.80, 0.07, 0.13\rangle & \langle 0.83, 0.11, 0.06\rangle & \langle 0.75, 0.12, 0.13\rangle & \langle 0.91, 0.03, 0.06\rangle & \langle 0.71, 0.22, 0.07\rangle \\ \langle 0.77, 0.06, 0.17\rangle & \langle 0.74, 0.11, 0.15\rangle & \langle 0.73, 0.21, 0.06\rangle & \langle 0.83, 0.03, 0.14\rangle & \langle 0.68, 0.25, 0.07\rangle \end{bmatrix}$$

$$e_3 = \begin{bmatrix} \langle 0.64, 0.13, 0.23\rangle & \langle 0.55, 0.23, 0.22\rangle & \langle 0.71, 0.11, 0.18\rangle & \langle 0.62, 0.23, 0.15\rangle & \langle 0.75, 0.14, 0.11\rangle \\ \langle 0.59, 0.27, 0.14\rangle & \langle 0.66, 0.15, 0.19\rangle & \langle 0.73, 0.11, 0.16\rangle & \langle 0.64, 0.24, 0.12\rangle & \langle 0.63, 0.27, 0.10\rangle \\ \langle 0.71, 0.16, 0.13\rangle & \langle 0.67, 0.26, 0.07\rangle & \langle 0.88, 0.04, 0.08\rangle & \langle 0.72, 0.07, 0.21\rangle & \langle 0.71, 0.15, 0.14\rangle \\ \langle 0.69, 0.04, 0.27\rangle & \langle 0.77, 0.14, 0.09\rangle & \langle 0.73, 0.12, 0.15\rangle & \langle 0.65, 0.24, 0.11\rangle & \langle 0.78, 0.05, 0.17\rangle \\ \langle 0.74, 0.06, 0.20\rangle & \langle 0.68, 0.28, 0.04\rangle & \langle 0.69, 0.13, 0.18\rangle & \langle 0.65, 0.34, 0.01\rangle & \langle 0.71, 0.08, 0.21\rangle \end{bmatrix}$$

$$e_4 = \begin{bmatrix} \langle 0.72, 0.14, 0.14\rangle & \langle 0.69, 0.15, 0.16\rangle & \langle 0.59, 0.23, 0.18\rangle & \langle 0.67, 0.16, 0.17\rangle & \langle 0.74, 0.04, 0.22\rangle \\ \langle 0.77, 0.12, 0.11\rangle & \langle 0.72, 0.09, 0.19\rangle & \langle 0.63, 0.17, 0.20\rangle & \langle 0.81, 0.11, 0.08\rangle & \langle 0.71, 0.23, 0.06\rangle \\ \langle 0.86, 0.07, 0.07\rangle & \langle 0.75, 0.13, 0.12\rangle & \langle 0.83, 0.15, 0.02\rangle & \langle 0.75, 0.18, 0.07\rangle & \langle 0.66, 0.05, 0.29\rangle \\ \langle 0.79, 0.08, 0.13\rangle & \langle 0.69, 0.12, 0.19\rangle & \langle 0.87, 0.06, 0.07\rangle & \langle 0.91, 0.07, 0.02\rangle & \langle 0.63, 0.22, 0.15\rangle \\ \langle 0.79, 0.17, 0.04\rangle & \langle 0.68, 0.03, 0.29\rangle & \langle 0.72, 0.21, 0.07\rangle & \langle 0.64, 0.03, 0.33\rangle & \langle 0.75, 0.19, 0.06\rangle \end{bmatrix}$$

$$e_5 = \begin{bmatrix} \langle 0.68, 0.29, 0.03\rangle & \langle 0.73, 0.17, 0.10\rangle & \langle 0.82, 0.11, 0.07\rangle & \langle 0.76, 0.22, 0.02\rangle & \langle 0.65, 0.26, 0.09\rangle \\ \langle 0.71, 0.19, 0.10\rangle & \langle 0.66, 0.23, 0.11\rangle & \langle 0.91, 0.04, 0.05\rangle & \langle 0.72, 0.07, 0.21\rangle & \langle 0.67, 0.12, 0.21\rangle \\ \langle 0.78, 0.14, 0.08\rangle & \langle 0.71, 0.13, 0.16\rangle & \langle 0.65, 0.21, 0.14\rangle & \langle 0.71, 0.06, 0.23\rangle & \langle 0.70, 0.15, 0.15\rangle \\ \langle 0.79, 0.08, 0.13\rangle & \langle 0.79, 0.09, 0.12\rangle & \langle 0.67, 0.16, 0.17\rangle & \langle 0.72, 0.06, 0.22\rangle & \langle 0.86, 0.10, 0.04\rangle \\ \langle 0.83, 0.05, 0.12\rangle & \langle 0.75, 0.19, 0.06\rangle & \langle 0.66, 0.03, 0.31\rangle & \langle 0.74, 0.18, 0.08\rangle & \langle 0.72, 0.21, 0.07\rangle \end{bmatrix}$$

$$e_6 = \begin{bmatrix} \langle 0.69, 0.28, 0.03\rangle & \langle 0.73, 0.17, 0.10\rangle & \langle 0.77, 0.17, 0.10\rangle & \langle 0.64, 0.11, 0.25\rangle & \langle 0.66, 0.23, 0.11\rangle \\ \langle 0.87, 0.07, 0.06\rangle & \langle 0.67, 0.14, 0.19\rangle & \langle 0.71, 0.16, 0.13\rangle & \langle 0.83, 0.05, 0.12\rangle & \langle 0.68, 0.17, 0.15\rangle \\ \langle 0.77, 0.15, 0.08\rangle & \langle 0.67, 0.14, 0.19\rangle & \langle 0.71, 0.16, 0.13\rangle & \langle 0.75, 0.12, 0.13\rangle & \langle 0.69, 0.09, 0.22\rangle \\ \langle 0.76, 0.06, 0.18\rangle & \langle 0.68, 0.21, 0.11\rangle & \langle 0.82, 0.11, 0.07\rangle & \langle 0.74, 0.25, 0.01\rangle & \langle 0.78, 0.08, 0.14\rangle \\ \langle 0.81, 0.13, 0.06\rangle & \langle 0.58, 0.33, 0.09\rangle & \langle 0.77, 0.08, 0.15\rangle & \langle 0.69, 0.10, 0.21\rangle & \langle 0.79, 0.04, 0.17\rangle \end{bmatrix}$$

$$e_7 = \begin{bmatrix} \langle 0.67, 0.24, 0.09\rangle & \langle 0.75, 0.07, 0.18\rangle & \langle 0.43, 0.39, 0.17\rangle & \langle 0.98, 0.01, 0.01\rangle & \langle 0.40, 0.32, 0.28\rangle \\ \langle 0.62, 0.26, 0.12\rangle & \langle 0.75, 0.16, 0.10\rangle & \langle 0.90, 0.06, 0.04\rangle & \langle 0.60, 0.26, 0.14\rangle & \langle 0.57, 0.25, 0.18\rangle \\ \langle 0.68, 0.02, 0.30\rangle & \langle 0.53, 0.33, 0.14\rangle & \langle 0.56, 0.41, 0.03\rangle & \langle 0.64, 0.17, 0.20\rangle & \langle 0.74, 0.20, 0.06\rangle \\ \langle 0.53, 0.31, 0.16\rangle & \langle 0.73, 0.09, 0.18\rangle & \langle 0.55, 0.26, 0.19\rangle & \langle 0.59, 0.39, 0.02\rangle & \langle 0.54, 0.36, 0.10\rangle \\ \langle 0.58, 0.34, 0.09\rangle & \langle 0.53, 0.36, 0.11\rangle & \langle 0.47, 0.35, 0.18\rangle & \langle 0.53, 0.17, 0.31\rangle & \langle 0.62, 0.32, 0.06\rangle \end{bmatrix}$$

$$e_8 = \begin{bmatrix} \langle 0.71, 0.26, 0.03\rangle & \langle 0.61, 0.17, 0.22\rangle & \langle 0.56, 0.25, 0.18\rangle & \langle 0.74, 0.10, 0.16\rangle & \langle 0.53, 0.34, 0.13\rangle \\ \langle 0.82, 0.12, 0.07\rangle & \langle 0.88, 0.02, 0.10\rangle & \langle 0.62, 0.27, 0.12\rangle & \langle 0.81, 0.12, 0.08\rangle & \langle 0.84, 0.02, 0.15\rangle \\ \langle 0.68, 0.15, 0.18\rangle & \langle 0.71, 0.10, 0.19\rangle & \langle 0.49, 0.27, 0.23\rangle & \langle 0.40, 0.29, 0.31\rangle & \langle 0.75, 0.11, 0.14\rangle \\ \langle 0.61, 0.12, 0.28\rangle & \langle 0.73, 0.20, 0.07\rangle & \langle 0.74, 0.13, 0.13\rangle & \langle 0.62, 0.02, 0.37\rangle & \langle 0.63, 0.22, 0.16\rangle \\ \langle 0.72, 0.13, 0.16\rangle & \langle 0.74, 0.10, 0.16\rangle & \langle 0.63, 0.22, 0.15\rangle & \langle 0.50, 0.27, 0.23\rangle & \langle 0.72, 0.16, 0.12\rangle \end{bmatrix}$$

$$e_9 = \begin{bmatrix} \langle 0.93, 0.01, 0.06\rangle & \langle 0.83, 0.13, 0.04\rangle & \langle 0.57, 0.29, 0.14\rangle & \langle 0.93, 0.02, 0.05\rangle & \langle 0.89, 0.09, 0.02\rangle \\ \langle 0.52, 0.26, 0.22\rangle & \langle 0.59, 0.06, 0.35\rangle & \langle 0.67, 0.01, 0.32\rangle & \langle 0.53, 0.12, 0.36\rangle & \langle 0.87, 0.04, 0.09\rangle \\ \langle 0.70, 0.12, 0.19\rangle & \langle 0.61, 0.35, 0.04\rangle & \langle 0.60, 0.36, 0.03\rangle & \langle 0.49, 0.17, 0.33\rangle & \langle 0.88, 0.02, 0.10\rangle \\ \langle 0.70, 0.21, 0.09\rangle & \langle 0.87, 0.06, 0.07\rangle & \langle 0.45, 0.10, 0.45\rangle & \langle 0.98, 0.01, 0.01\rangle & \langle 0.64, 0.01, 0.35\rangle \\ \langle 0.38, 0.28, 0.34\rangle & \langle 0.90, 0.07, 0.02\rangle & \langle 0.61, 0.27, 0.12\rangle & \langle 0.54, 0.05, 0.41\rangle & \langle 0.97, 0.03, 0.00\rangle \end{bmatrix}$$

$$e_{10} = \begin{bmatrix} \langle 0.60, 0.32, 0.08\rangle & \langle 0.85, 0.10, 0.05\rangle & \langle 0.72, 0.12, 0.16\rangle & \langle 0.43, 0.18, 0.40\rangle & \langle 0.80, 0.10, 0.10\rangle \\ \langle 0.49, 0.04, 0.47\rangle & \langle 0.67, 0.04, 0.29\rangle & \langle 0.82, 0.01, 0.17\rangle & \langle 0.75, 0.15, 0.10\rangle & \langle 0.57, 0.01, 0.42\rangle \\ \langle 0.82, 0.06, 0.12\rangle & \langle 0.83, 0.17, 0.00\rangle & \langle 0.49, 0.05, 0.45\rangle & \langle 0.60, 0.18, 0.23\rangle & \langle 0.42, 0.37, 0.21\rangle \\ \langle 0.48, 0.25, 0.27\rangle & \langle 0.85, 0.13, 0.02\rangle & \langle 0.59, 0.00, 0.41\rangle & \langle 0.58, 0.23, 0.20\rangle & \langle 0.45, 0.40, 0.15\rangle \\ \langle 1.00, 0.00, 0.00\rangle & \langle 0.42, 0.39, 0.19\rangle & \langle 0.84, 0.14, 0.02\rangle & \langle 0.90, 0.02, 0.08\rangle & \langle 0.39, 0.31, 0.31\rangle \end{bmatrix}$$

$$e_{11} = \begin{bmatrix} \langle 0.62, 0.29, 0.10\rangle & \langle 0.83, 0.11, 0.07\rangle & \langle 0.71, 0.11, 0.18\rangle & \langle 0.47, 0.16, 0.36\rangle & \langle 0.79, 0.14, 0.08\rangle \\ \langle 0.51, 0.10, 0.39\rangle & \langle 0.69, 0.10, 0.21\rangle & \langle 0.83, 0.02, 0.16\rangle & \langle 0.73, 0.14, 0.14\rangle & \langle 0.59, 0.01, 0.40\rangle \\ \langle 0.83, 0.09, 0.08\rangle & \langle 0.81, 0.14, 0.05\rangle & \langle 0.51, 0.07, 0.42\rangle & \langle 0.61, 0.17, 0.22\rangle & \langle 0.43, 0.35, 0.22\rangle \\ \langle 0.50, 0.24, 0.26\rangle & \langle 0.84, 0.13, 0.04\rangle & \langle 0.59, 0.06, 0.35\rangle & \langle 0.59, 0.21, 0.20\rangle & \langle 0.48, 0.38, 0.14\rangle \\ \langle 0.95, 0.01, 0.04\rangle & \langle 0.47, 0.31, 0.22\rangle & \langle 0.81, 0.19, 0.00\rangle & \langle 0.87, 0.03, 0.11\rangle & \langle 0.40, 0.28, 0.33\rangle \end{bmatrix}$$

$$e_{12} = \begin{bmatrix} \langle 0.61, 0.29, 0.10\rangle & \langle 0.84, 0.11, 0.05\rangle & \langle 0.73, 0.13, 0.14\rangle & \langle 0.47, 0.16, 0.37\rangle & \langle 0.80, 0.12, 0.08\rangle \\ \langle 0.51, 0.05, 0.44\rangle & \langle 0.69, 0.08, 0.23\rangle & \langle 0.80, 0.02, 0.18\rangle & \langle 0.77, 0.17, 0.06\rangle & \langle 0.58, 0.01, 0.41\rangle \\ \langle 0.80, 0.05, 0.15\rangle & \langle 0.82, 0.14, 0.04\rangle & \langle 0.51, 0.08, 0.41\rangle & \langle 0.61, 0.22, 0.17\rangle & \langle 0.47, 0.28, 0.24\rangle \\ \langle 0.49, 0.21, 0.30\rangle & \langle 0.81, 0.11, 0.08\rangle & \langle 0.60, 0.02, 0.38\rangle & \langle 0.60, 0.22, 0.19\rangle & \langle 0.47, 0.31, 0.22\rangle \\ \langle 0.96, 0.03, 0.01\rangle & \langle 0.47, 0.34, 0.19\rangle & \langle 0.83, 0.16, 0.01\rangle & \langle 0.88, 0.04, 0.08\rangle & \langle 0.43, 0.29, 0.28\rangle \end{bmatrix}$$

$$e_{13} = \begin{bmatrix} \langle 0.72, 0.28, 0.00\rangle & \langle 0.61, 0.19, 0.20\rangle & \langle 0.55, 0.26, 0.19\rangle & \langle 0.73, 0.10, 0.18\rangle & \langle 0.52, 0.36, 0.12\rangle \\ \langle 0.83, 0.13, 0.05\rangle & \langle 0.90, 0.01, 0.10\rangle & \langle 0.60, 0.28, 0.12\rangle & \langle 0.81, 0.10, 0.09\rangle & \langle 0.84, 0.01, 0.15\rangle \\ \langle 0.67, 0.14, 0.19\rangle & \langle 0.71, 0.09, 0.20\rangle & \langle 0.47, 0.23, 0.30\rangle & \langle 0.39, 0.14, 0.47\rangle & \langle 0.70, 0.01, 0.29\rangle \\ \langle 0.61, 0.12, 0.27\rangle & \langle 0.71, 0.24, 0.05\rangle & \langle 0.70, 0.12, 0.18\rangle & \langle 0.61, 0.01, 0.38\rangle & \langle 0.62, 0.30, 0.08\rangle \\ \langle 0.70, 0.10, 0.20\rangle & \langle 0.78, 0.09, 0.14\rangle & \langle 0.61, 0.24, 0.15\rangle & \langle 0.48, 0.32, 0.20\rangle & \langle 0.73, 0.14, 0.13\rangle \end{bmatrix}$$

$$e_{14} = \begin{bmatrix} \langle 0.70, 0.24, 0.06\rangle & \langle 0.64, 0.16, 0.20\rangle & \langle 0.50, 0.33, 0.17\rangle & \langle 0.72, 0.11, 0.17\rangle & \langle 0.61, 0.32, 0.07\rangle \\ \langle 0.83, 0.01, 0.16\rangle & \langle 0.91, 0.01, 0.08\rangle & \langle 0.62, 0.32, 0.06\rangle & \langle 0.81, 0.18, 0.01\rangle & \langle 0.86, 0.02, 0.12\rangle \\ \langle 0.70, 0.02, 0.28\rangle & \langle 0.51, 0.10, 0.39\rangle & \langle 0.48, 0.32, 0.20\rangle & \langle 0.43, 0.25, 0.33\rangle & \langle 0.73, 0.02, 0.26\rangle \\ \langle 0.64, 0.18, 0.18\rangle & \langle 0.70, 0.25, 0.05\rangle & \langle 0.79, 0.13, 0.08\rangle & \langle 0.58, 0.01, 0.40\rangle & \langle 0.63, 0.31, 0.06\rangle \\ \langle 0.70, 0.11, 0.19\rangle & \langle 0.78, 0.08, 0.14\rangle & \langle 0.60, 0.27, 0.13\rangle & \langle 0.45, 0.37, 0.18\rangle & \langle 0.73, 0.27, 0.00\rangle \end{bmatrix}$$

$$e_{15} = \begin{bmatrix} \langle 0.72, 0.22, 0.08\rangle & \langle 0.63, 0.16, 0.21\rangle & \langle 0.51, 0.28, 0.21\rangle & \langle 0.75, 0.12, 0.14\rangle & \langle 0.65, 0.26, 0.09\rangle \\ \langle 0.83, 0.14, 0.03\rangle & \langle 0.92, 0.01, 0.07\rangle & \langle 0.61, 0.36, 0.03\rangle & \langle 0.82, 0.12, 0.07\rangle & \langle 0.86, 0.01, 0.13\rangle \\ \langle 0.72, 0.01, 0.27\rangle & \langle 0.48, 0.12, 0.40\rangle & \langle 0.50, 0.29, 0.21\rangle & \langle 0.39, 0.34, 0.27\rangle & \langle 0.72, 0.02, 0.26\rangle \\ \langle 0.64, 0.14, 0.22\rangle & \langle 0.69, 0.24, 0.07\rangle & \langle 0.79, 0.14, 0.07\rangle & \langle 0.58, 0.01, 0.41\rangle & \langle 0.62, 0.33, 0.05\rangle \\ \langle 0.72, 0.10, 0.18\rangle & \langle 0.76, 0.09, 0.15\rangle & \langle 0.62, 0.22, 0.16\rangle & \langle 0.48, 0.38, 0.14\rangle & \langle 0.72, 0.22, 0.06\rangle \end{bmatrix}$$

$$e_{16} = \begin{bmatrix} \langle 0.77, 0.23, 0.00\rangle & \langle 0.62, 0.19, 0.19\rangle & \langle 0.45, 0.36, 0.19\rangle & \langle 0.74, 0.09, 0.18\rangle & \langle 0.64, 0.36, 0.01\rangle \\ \langle 0.84, 0.11, 0.05\rangle & \langle 0.90, 0.00, 0.10\rangle & \langle 0.60, 0.39, 0.02\rangle & \langle 0.80, 0.19, 0.01\rangle & \langle 0.86, 0.01, 0.13\rangle \\ \langle 0.70, 0.01, 0.29\rangle & \langle 0.46, 0.09, 0.45\rangle & \langle 0.44, 0.31, 0.25\rangle & \langle 0.36, 0.34, 0.30\rangle & \langle 0.71, 0.00, 0.29\rangle \\ \langle 0.63, 0.15, 0.22\rangle & \langle 0.68, 0.27, 0.05\rangle & \langle 0.77, 0.15, 0.08\rangle & \langle 0.56, 0.01, 0.43\rangle & \langle 0.61, 0.37, 0.03\rangle \\ \langle 0.71, 0.10, 0.19\rangle & \langle 0.79, 0.06, 0.15\rangle & \langle 0.57, 0.26, 0.16\rangle & \langle 0.43, 0.39, 0.18\rangle & \langle 0.71, 0.27, 0.02\rangle \end{bmatrix}$$

$$e_{17} = \begin{bmatrix} \langle 0.70, 0.03, 0.27\rangle & \langle 0.62, 0.09, 0.29\rangle & \langle 0.42, 0.28, 0.30\rangle & \langle 0.50, 0.36, 0.14\rangle & \langle 0.74, 0.24, 0.02\rangle \\ \langle 0.44, 0.20, 0.35\rangle & \langle 0.46, 0.33, 0.22\rangle & \langle 0.49, 0.34, 0.22\rangle & \langle 0.99, 0.01, 0.00\rangle & \langle 0.84, 0.13, 0.03\rangle \\ \langle 0.75, 0.23, 0.02\rangle & \langle 0.54, 0.11, 0.36\rangle & \langle 0.61, 0.17, 0.22\rangle & \langle 0.56, 0.40, 0.05\rangle & \langle 0.78, 0.05, 0.17\rangle \\ \langle 0.99, 0.00, 0.01\rangle & \langle 0.97, 0.02, 0.01\rangle & \langle 0.51, 0.05, 0.44\rangle & \langle 0.50, 0.11, 0.40\rangle & \langle 0.63, 0.26, 0.11\rangle \\ \langle 0.92, 0.01, 0.07\rangle & \langle 0.63, 0.22, 0.15\rangle & \langle 0.55, 0.42, 0.03\rangle & \langle 0.56, 0.11, 0.33\rangle & \langle 0.79, 0.07, 0.14\rangle \end{bmatrix}$$

$$e_{18} = \begin{bmatrix} \langle 0.61, 0.23, 0.15\rangle & \langle 0.80, 0.12, 0.08\rangle & \langle 0.73, 0.17, 0.10\rangle & \langle 0.51, 0.13, 0.36\rangle & \langle 0.79, 0.14, 0.08\rangle \\ \langle 0.53, 0.09, 0.38\rangle & \langle 0.69, 0.08, 0.23\rangle & \langle 0.83, 0.03, 0.14\rangle & \langle 0.79, 0.18, 0.03\rangle & \langle 0.59, 0.03, 0.38\rangle \\ \langle 0.82, 0.07, 0.11\rangle & \langle 0.80, 0.15, 0.06\rangle & \langle 0.53, 0.10, 0.37\rangle & \langle 0.62, 0.23, 0.15\rangle & \langle 0.50, 0.27, 0.23\rangle \\ \langle 0.50, 0.27, 0.23\rangle & \langle 0.83, 0.16, 0.02\rangle & \langle 0.63, 0.13, 0.25\rangle & \langle 0.64, 0.27, 0.09\rangle & \langle 0.53, 0.28, 0.19\rangle \\ \langle 0.92, 0.04, 0.04\rangle & \langle 0.50, 0.24, 0.26\rangle & \langle 0.79, 0.12, 0.09\rangle & \langle 0.83, 0.04, 0.14\rangle & \langle 0.53, 0.23, 0.24\rangle \end{bmatrix}$$

$$e_{19} = \begin{bmatrix} \langle 0.63, 0.22, 0.15\rangle & \langle 0.82, 0.16, 0.02\rangle & \langle 0.75, 0.14, 0.11\rangle & \langle 0.49, 0.13, 0.38\rangle & \langle 0.80, 0.12, 0.08\rangle \\ \langle 0.52, 0.07, 0.41\rangle & \langle 0.72, 0.10, 0.18\rangle & \langle 0.82, 0.02, 0.16\rangle & \langle 0.79, 0.17, 0.03\rangle & \langle 0.61, 0.02, 0.37\rangle \\ \langle 0.81, 0.10, 0.09\rangle & \langle 0.79, 0.12, 0.09\rangle & \langle 0.52, 0.09, 0.39\rangle & \langle 0.61, 0.23, 0.16\rangle & \langle 0.49, 0.21, 0.30\rangle \\ \langle 0.50, 0.22, 0.28\rangle & \langle 0.79, 0.12, 0.09\rangle & \langle 0.63, 0.02, 0.34\rangle & \langle 0.60, 0.20, 0.20\rangle & \langle 0.50, 0.30, 0.20\rangle \\ \langle 0.94, 0.06, 0.00\rangle & \langle 0.49, 0.24, 0.27\rangle & \langle 0.81, 0.12, 0.07\rangle & \langle 0.87, 0.02, 0.11\rangle & \langle 0.42, 0.27, 0.31\rangle \end{bmatrix}$$

$$e_{20} = \begin{bmatrix} \langle 0.59, 0.27, 0.14\rangle & \langle 0.83, 0.11, 0.06\rangle & \langle 0.75, 0.12, 0.13\rangle & \langle 0.49, 0.15, 0.36\rangle & \langle 0.82, 0.15, 0.04\rangle \\ \langle 0.53, 0.08, 0.39\rangle & \langle 0.71, 0.12, 0.17\rangle & \langle 0.81, 0.02, 0.17\rangle & \langle 0.78, 0.16, 0.06\rangle & \langle 0.60, 0.08, 0.32\rangle \\ \langle 0.79, 0.08, 0.12\rangle & \langle 0.81, 0.18, 0.01\rangle & \langle 0.54, 0.11, 0.35\rangle & \langle 0.64, 0.21, 0.15\rangle & \langle 0.50, 0.13, 0.38\rangle \\ \langle 0.51, 0.12, 0.37\rangle & \langle 0.81, 0.17, 0.03\rangle & \langle 0.62, 0.03, 0.35\rangle & \langle 0.60, 0.18, 0.22\rangle & \langle 0.49, 0.28, 0.23\rangle \\ \langle 0.92, 0.04, 0.04\rangle & \langle 0.50, 0.25, 0.25\rangle & \langle 0.80, 0.06, 0.14\rangle & \langle 0.85, 0.07, 0.08\rangle & \langle 0.49, 0.27, 0.24\rangle \end{bmatrix}$$

Appendix B

$$d = \begin{pmatrix}
0 & 0.08 & 0.10 & 0.11 & 0.09 & 0.09 & 0.16 & 0.14 & 0.18 & 0.16 & 0.14 & 0.14 & 0.15 & 0.16 & 0.15 & 0.17 & 0.17 & 0.12 & 0.13 & 0.13 \\
 & 0 & 0.09 & 0.09 & 0.09 & 0.10 & 0.15 & 0.13 & 0.16 & 0.17 & 0.15 & 0.15 & 0.14 & 0.15 & 0.15 & 0.16 & 0.17 & 0.13 & 0.14 & 0.13 \\
 & & 0 & 0.11 & 0.10 & 0.10 & 0.15 & 0.13 & 0.17 & 0.17 & 0.16 & 0.16 & 0.14 & 0.15 & 0.15 & 0.17 & 0.16 & 0.14 & 0.15 & 0.14 \\
 & & & 0 & 0.12 & 0.09 & 0.16 & 0.12 & 0.17 & 0.18 & 0.17 & 0.16 & 0.14 & 0.14 & 0.14 & 0.16 & 0.16 & 0.14 & 0.15 & 0.15 \\
 & & & & 0 & 0.09 & 0.15 & 0.12 & 0.17 & 0.17 & 0.15 & 0.15 & 0.13 & 0.14 & 0.14 & 0.16 & 0.17 & 0.14 & 0.15 & 0.14 \\
 & & & & & 0 & 0.16 & 0.12 & 0.18 & 0.17 & 0.15 & 0.15 & 0.13 & 0.14 & 0.14 & 0.16 & 0.17 & 0.13 & 0.14 & 0.13 \\
 & & & & & & 0 & 0.14 & 0.17 & 0.20 & 0.18 & 0.19 & 0.16 & 0.15 & 0.16 & 0.17 & 0.20 & 0.16 & 0.18 & 0.18 \\
 & & & & & & & 0 & 0.17 & 0.18 & 0.16 & 0.16 & 0.03 & 0.06 & 0.06 & 0.07 & 0.16 & 0.15 & 0.16 & 0.15 \\
 & & & & & & & & 0 & 0.23 & 0.22 & 0.22 & 0.18 & 0.19 & 0.19 & 0.20 & 0.20 & 0.20 & 0.21 & 0.21 \\
 & & & & & & & & & 0 & 0.03 & 0.03 & 0.19 & 0.20 & 0.20 & 0.21 & 0.20 & 0.06 & 0.05 & 0.07 \\
 & & & & & & & & & & 0 & 0.03 & 0.17 & 0.19 & 0.19 & 0.20 & 0.19 & 0.05 & 0.04 & 0.05 \\
 & & & & & & & & & & & 0 & 0.17 & 0.19 & 0.19 & 0.20 & 0.19 & 0.04 & 0.03 & 0.04 \\
 & & & & & & & & & & & & 0 & 0.06 & 0.06 & 0.07 & 0.17 & 0.16 & 0.16 & 0.16 \\
 & & & & & & & & & & & & & 0 & 0.03 & 0.03 & 0.16 & 0.17 & 0.18 & 0.17 \\
 & & & & & & & & & & & & & & 0 & 0.03 & 0.16 & 0.17 & 0.18 & 0.17 \\
 & & & & & & & & & & & & & & & 0 & 0.17 & 0.18 & 0.19 & 0.19 \\
 & & & & & & & & & & & & & & & & 0 & 0.18 & 0.18 & 0.18 \\
 & & & & & & & & & & & & & & & & & 0 & 0.04 & 0.05 \\
 & & & & & & & & & & & & & & & & & & 0 & 0.04 \\
 & & & & & & & & & & & & & & & & & & & 0
\end{pmatrix}$$

Appendix C

$$G_1 = \begin{bmatrix} \langle 0.69,0.19,0.12\rangle & \langle 0.69,0.16,0.15\rangle & \langle 0.77,0.11,0.12\rangle & \langle 0.69,0.17,0.14\rangle & \langle 0.72,0.12,0.16\rangle \\ \langle 0.73,0.15,0.12\rangle & \langle 0.68,0.16,0.16\rangle & \langle 0.76,0.13,0.11\rangle & \langle 0.73,0.13,0.15\rangle & \langle 0.69,0.16,0.15\rangle \\ \langle 0.77,0.13,0.10\rangle & \langle 0.69,0.16,0.15\rangle & \langle 0.77,0.12,0.11\rangle & \langle 0.74,0.09,0.17\rangle & \langle 0.70,0.11,0.19\rangle \\ \langle 0.77,0.06,0.17\rangle & \langle 0.76,0.13,0.11\rangle & \langle 0.78,0.10,0.12\rangle & \langle 0.81,0.11,0.08\rangle & \langle 0.77,0.12,0.11\rangle \\ \langle 0.80,0.07,0.13\rangle & \langle 0.69,0.14,0.17\rangle & \langle 0.72,0.12,0.16\rangle & \langle 0.77,0.07,0.16\rangle & \langle 0.71,0.15,0.14\rangle \end{bmatrix}$$

$$G_2 = \begin{bmatrix} \langle 0.72,0.24,0.03\rangle & \langle 0.62,0.17,0.20\rangle & \langle 0.52,0.29,0.19\rangle & \langle 0.73,0.10,0.16\rangle & \langle 0.59,0.33,0.08\rangle \\ \langle 0.83,0.07,0.10\rangle & \langle 0.90,0.01,0.09\rangle & \langle 0.61,0.32,0.07\rangle & \langle 0.81,0.14,0.05\rangle & \langle 0.85,0.02,0.13\rangle \\ \langle 0.69,0.03,0.27\rangle & \langle 0.59,0.10,0.31\rangle & \langle 0.48,0.28,0.24\rangle & \langle 0.39,0.26,0.35\rangle & \langle 0.72,0.02,0.26\rangle \\ \langle 0.63,0.14,0.23\rangle & \langle 0.70,0.24,0.06\rangle & \langle 0.76,0.13,0.11\rangle & \langle 0.59,0.01,0.40\rangle & \langle 0.62,0.30,0.08\rangle \\ \langle 0.71,0.11,0.18\rangle & \langle 0.77,0.08,0.15\rangle & \langle 0.61,0.24,0.15\rangle & \langle 0.47,0.34,0.19\rangle & \langle 0.72,0.20,0.08\rangle \end{bmatrix}$$

$$G_3 = \begin{bmatrix} \langle 0.61,0.27,0.12\rangle & \langle 0.83,0.12,0.06\rangle & \langle 0.73,0.13,0.14\rangle & \langle 0.48,0.15,0.37\rangle & \langle 0.80,0.13,0.08\rangle \\ \langle 0.51,0.07,0.42\rangle & \langle 0.70,0.08,0.22\rangle & \langle 0.82,0.02,0.16\rangle & \langle 0.77,0.16,0.07\rangle & \langle 0.59,0.02,0.39\rangle \\ \langle 0.81,0.07,0.12\rangle & \langle 0.81,0.15,0.04\rangle & \langle 0.52,0.08,0.40\rangle & \langle 0.61,0.20,0.19\rangle & \langle 0.47,0.25,0.28\rangle \\ \langle 0.50,0.21,0.29\rangle & \langle 0.82,0.13,0.05\rangle & \langle 0.61,0.03,0.36\rangle & \langle 0.60,0.22,0.18\rangle & \langle 0.49,0.32,0.19\rangle \\ \langle 0.96,0.00,0.04\rangle & \langle 0.48,0.29,0.23\rangle & \langle 0.81,0.13,0.06\rangle & \langle 0.87,0.03,0.10\rangle & \langle 0.44,0.27,0.29\rangle \end{bmatrix}$$

$$G_4 = \begin{bmatrix} \langle 0.67,0.24,0.09\rangle & \langle 0.75,0.07,0.18\rangle & \langle 0.43,0.39,0.17\rangle & \langle 0.98,0.01,0.01\rangle & \langle 0.40,0.32,0.28\rangle \\ \langle 0.62,0.26,0.12\rangle & \langle 0.75,0.16,0.10\rangle & \langle 0.90,0.06,0.04\rangle & \langle 0.60,0.26,0.14\rangle & \langle 0.57,0.25,0.18\rangle \\ \langle 0.68,0.02,0.30\rangle & \langle 0.53,0.33,0.14\rangle & \langle 0.56,0.41,0.03\rangle & \langle 0.64,0.17,0.20\rangle & \langle 0.74,0.20,0.06\rangle \\ \langle 0.53,0.31,0.16\rangle & \langle 0.73,0.09,0.18\rangle & \langle 0.55,0.26,0.19\rangle & \langle 0.59,0.39,0.02\rangle & \langle 0.54,0.36,0.10\rangle \\ \langle 0.58,0.34,0.09\rangle & \langle 0.53,0.36,0.11\rangle & \langle 0.47,0.35,0.18\rangle & \langle 0.53,0.17,0.31\rangle & \langle 0.62,0.32,0.06\rangle \end{bmatrix}$$

$$G_5 = \begin{bmatrix} \langle 0.93,0.01,0.06\rangle & \langle 0.83,0.13,0.04\rangle & \langle 0.57,0.29,0.14\rangle & \langle 0.93,0.02,0.05\rangle & \langle 0.89,0.09,0.02\rangle \\ \langle 0.52,0.26,0.22\rangle & \langle 0.59,0.06,0.35\rangle & \langle 0.67,0.01,0.32\rangle & \langle 0.53,0.12,0.36\rangle & \langle 0.87,0.04,0.09\rangle \\ \langle 0.70,0.12,0.19\rangle & \langle 0.61,0.35,0.04\rangle & \langle 0.60,0.36,0.03\rangle & \langle 0.49,0.17,0.33\rangle & \langle 0.88,0.02,0.10\rangle \\ \langle 0.70,0.21,0.09\rangle & \langle 0.87,0.06,0.07\rangle & \langle 0.45,0.10,0.45\rangle & \langle 0.98,0.01,0.01\rangle & \langle 0.64,0.01,0.35\rangle \\ \langle 0.38,0.28,0.34\rangle & \langle 0.90,0.07,0.02\rangle & \langle 0.61,0.27,0.12\rangle & \langle 0.54,0.05,0.41\rangle & \langle 0.97,0.03,0.00\rangle \end{bmatrix}$$

$$G_6 = \begin{bmatrix} \langle 0.70,0.03,0.27\rangle & \langle 0.62,0.09,0.29\rangle & \langle 0.42,0.28,0.30\rangle & \langle 0.50,0.36,0.14\rangle & \langle 0.74,0.24,0.02\rangle \\ \langle 0.44,0.20,0.35\rangle & \langle 0.46,0.33,0.22\rangle & \langle 0.49,0.34,0.22\rangle & \langle 0.99,0.01,0.00\rangle & \langle 0.84,0.13,0.03\rangle \\ \langle 0.75,0.23,0.02\rangle & \langle 0.54,0.11,0.36\rangle & \langle 0.61,0.17,0.22\rangle & \langle 0.56,0.40,0.05\rangle & \langle 0.78,0.05,0.17\rangle \\ \langle 0.99,0.00,0.01\rangle & \langle 0.97,0.02,0.01\rangle & \langle 0.51,0.05,0.44\rangle & \langle 0.50,0.11,0.40\rangle & \langle 0.63,0.26,0.11\rangle \\ \langle 0.92,0.01,0.07\rangle & \langle 0.63,0.22,0.15\rangle & \langle 0.55,0.42,0.03\rangle & \langle 0.56,0.11,0.33\rangle & \langle 0.79,0.07,0.14\rangle \end{bmatrix}$$