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Implicit government guarantees and bank risk

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ABSTRACT

We develop a model on bank risk and implicit government guarantees. This model concerns the willingness and capacity of implicit government guarantees. Using the Option Pricing Theory, we derive a mathematical formulation of maximizing the bank's net present value (NPV) with implicit government guarantees. Unlike previous work, both the loan portfolio and the bank's NPV are regarded as a combination of options underlying the risky project. We conduct comparative static analyses and numerical examples to examine how implicit government guarantees and capital control affect bank risk and its asset scale. The main insight of our analysis is that implicit government guarantees have some unintended consequences: (a) Inefficient and excessive risk taking (including bank's asset and overall risk); (b) Inefficient investment if there is no binding capacity constraint. We show that it is mainly due to the bank's excessive reliance on contingent assets. In addition, we demonstrate the ineffectiveness of capital constraint on risk control under certain circumstances. Therefore, we suggest that the gradual withdrawal of implicit government guarantees should be accompanied by multiple combinations of regulatory measures and proper institutional reform to avoid risk surges.

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1. Introduction

The global financial crisis that erupted in 2008 reignited the controversy over government guarantees due to the cost of bank failures and the deterioration of market discipline. As pointed out by Acharya et al. (2014), aggressive bailout packages have a short-term effect on the stabilization of the financial sector while ignoring the ultimate cost to taxpayers, which end up with a Pyrrhic victory. Therefore, regulators and academics have spared no effort in finding ways to eliminate too-big-to-fail subsidies and diminish the moral hazard associated with implicit and explicit government guarantees during the post-crisis period. US Dodd-Frank Acts and EU BRRD (the Bank Recovery and Resolution Directive) are cases on the relevant policies. The deposit

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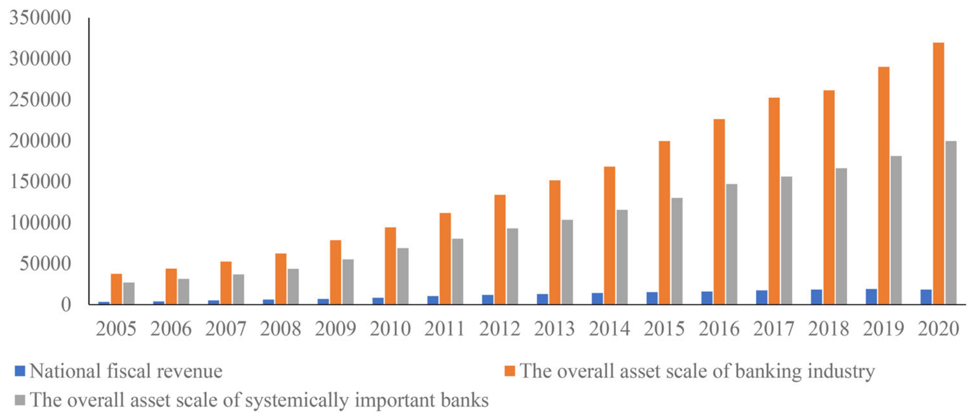

Figure 1. The overall asset scale of the banking industry and systemically important banks vs. national fiscal revenue in China (Unit: 1 billion RMB).
Source: Data on national fiscal revenue and the asset scale of systemically important banks comes from the Wind database. Data on the overall asset scale of the banking industry comes from China Banking and Insurance Regulatory Commission website.

insurance system (DIS) introduced earlier is also an important measure the regulatory authorities take to resolve the government's implicit guarantees to depositors. Not until 2015 did China officially launch DIS to make implicit subsidy explicit. Then, the promulgation of new Regulations of Asset Management in 2018 completely broke out the 'rigid repayment' of off-balance-sheet Wealth Management Products.

Implicit government guarantees for financial institutions (FIs) imply that once a FI faces a default crisis, even though the government does not provide legal direct guarantees (as explicit guarantees) for it, it may still be compensated through capital injection, financial subsidies, or direct takeover, etc. According to IMF's financial system stability assessment to China[1], there are widespread perceptions on implicit guarantees: banks often compensate retail investors for losses[2], while lenders assume that loss-making SOEs or FIs will be bailed out. A shift in the perceived value of implicit guarantees leading to a sharp increase in risk premia has become one of the biggest risks faced by China (IMF, 2017). In order to further deepen the structural reform of the financial supply-side and resolve financial risks, it is very important to evaluate the impact of implicit government guarantees on bank risks. Our research conclusions can provide some reference for the regulatory authorities to carefully plan implicit guarantee reforms to prevent and defuse financial risks.

According to Merton (1977) and Gray et al. (2007), the value of government guarantees is related to the value of bank assets. Whereas there is a big disparity between the government's financial condition and the asset scale of the banking industry. From Figure 1, both the overall asset scale of China's banking industry and systemically important banks[3] are much larger than the national fiscal revenue. Moreover, guarantees—even if not triggered—are not costless, they can be considered as the guarantor's contingent liabilities, which may become a significant burden. Therefore, there should exist an upper limit of implicit government guarantees. However, most previous literature concentrated on the broad public guarantee, without distinguishing the capacity and willingness of guarantees. And it is often assumed that government is of full capacity to fund public subsidies (Diamond & Dybvig, 1983; Cooper & Ross, 2002; Tsafack et al., 2021). It is inappropriate and deviating from the reality to consider only the willingness of guarantees or assume that the government has the full guarantee capacity when measuring the impact of implicit government guarantees on bank risk. Considering the guarantee capacity is of great importance and necessity in research.

Bank's NPV maximization is expanded to include its loan portfolios and implicit government guarantees in our analysis. Using Option Pricing Theory, the constraint of guarantee capacity is incorporated into our model. The effects of both willingness and capacity of government implicit guarantees on bank risks (asset risk and overall risk) are examined. We address these questions in the following steps. First, we develop a new model on loan portfolios, then under the no-guarantee condition, examining the effect of tightening capital constraint on bank asset risk and scale. Second, implicit government guarantees' value is added to the loan model to establish the bank's NPV model (hereinafter referred to as the 'value maximization model'), which aims to maximize the sum of these two parts. We assume that the explicit guarantees provided to banks must be paid at the present value of their expected returns which means that they do not have any effect on banks' NPV, while implicit guarantees are provided free of charge that increase banks' NPV. Through comparative static analysis, we reveal how bank risk and asset scale vary with the capacity and willingness of government in maximization. Finally, we present some numerical examples to illustrate the identified effects and reveal other significant effects that are not recognized in analytical computations.

Some important results have been concluded from our analysis. First, implicit government guarantees contribute not only to banks' inefficient and excessive risk taking including both asset and overall risk but also to inefficient investment and deterioration in asset quality when there is no binding capacity constraint. Second, when guarantee capacity constraint is binding, the effects of willingness on bank risk and investment are opposite to capacity. At this time, an increasing willingness seen like an empty promise can no longer trigger banks to take risks. But an increase in capacity will make guarantees credible, thereby stimulating banks to take risks. Third, an important reason for the previous two conclusions is that banks tend to overly rely on implicit government guarantees. Finally, we found that capital constraint was not always effective in controlling bank risks.

The novelty of this article is that we develop a new loan value model using Option Pricing Theory, in which a loan portfolio is considered as a combination of call options underlying a risky project and gets fixed income at best. The previous literature that assumed the dynamics of bank asset value is geometric Brownian motion, which grows in a geometric trend and can be infinite (Merton, 1977; Gray et al., 2007; Tsafack et al., 2021). But in reality, the main asset of a bank is loan portfolios, and its income is limited. Bank's NPV can further be modelled as a combination of call (and put) options underlying the value of the risky project. In addition, our work introduces guarantee capacity constraint into the theoretical model, enabling us to examine bank risks under the dislocation of willingness and capacity, which is of greater practical significance.

The rest of this article is organized as follows. Section 2 provides some reviews of the related literature. Section 3 establishes a model of loan portfolios. Section 4 presents the basic model of bank value maximization. Section 5 considers the capacity of implicit guarantees. Section 6 presents some numerical examples. Section 7 concludes.

2. Literature review

Previous literature suggested that government guarantees had two different effects on bank risk-taking: moral hazard effect and franchise value effect. If the moral hazard effect dominates, government guarantees may exacerbate market discipline (Baron, 2020) which leads to banks' inefficient reckless behavior—they become less concerned about their risk-taking and are motivated to invest in higher-risk projects to get higher returns if possible (Kareken, 1986). Under this circumstance, the creditors have no incentive to claim risk premia for the observed higher bank risk, which means that implicit government guarantees have distorted debt financing of banks (Dong et al., 2021), or to supervise banks whose losses are expected to be bailed out (Flannery, 1998; Sironi, 2003; Gropp et al., 2006). Thus, the moral hazard arising from government guarantees contributes substantially to excessive risk-taking in the banking industry (Gietl & Kassner, 2020; Viva et al., 2021). On the other hand, if the franchise value effect dominates, banks are less likely to take risks (Keeley, 1990). In this case, once bankruptcy occurs, banks will lose future monopoly rents that stem from continuing operations. This effect can curb the exposures of protected banks and benefit them from lower financing costs. Luong et al. (2020) used Australian data and found strong causal evidence to indicate that government guarantees helped deposit-taking institutions to reduce their funding costs and encouraged them to convert their loan portfolios into housing loans thereby reducing their riskiness. The net impact of government guarantees on bank risk-taking is vague and depends on the dominant effects (Hakenes & Schnabel, 2010). Empirical studies tended to conclude that the moral hazard effect plays a dominant role relative to the franchise value effect, thus offsetting the latter's positive role in risk controlling (Hovakimian & Kane, 2000; Sapienza, 2004; Gropp et al., 2011; Viva et al., 2021).

Despite extensive discussions, the overall impact of government guarantees on the stability of the banking system remains mixed. On one hand, government guarantees play an active role in preventing bank runs caused by large-scale investors' panic, which helps to strengthen the stability of the financial system. König et al. (2014) documented both the decrease in the likelihood of bank run and the increase in the likelihood of sovereign debt defaults due to guarantees. Guarantees are also welfare-improving as they induce banks to improve liquidity provision, which may increase the likelihood of runs though. (Allen et al., 2018). Berger et al. (2020) analysed the effect of the Troubled Assets Relief Program (TRAP) on financial system stability. They found that TARP significantly reduced contributions to systemic risk, but this effect was relatively short-lived and might be reversed in the long run. An important inference showed that moral hazard incentives to take on excessive risk were less likely to be manifested during the crisis when risks were already high, and more likely to be displayed in the more normal times that followed. On the other hand, there are views that government guarantees, while preventing runs, may trigger moral hazard and distort the incentives of banks, and finally lead to a more fragile financial system (see, e.g., Demirgüç-Kunt & Detragiache, 2002; Gropp et al., 2014; Acharya & Mora, 2015). Moreover, government guarantees will lead to undesirable consequences that transform the risk of bank failure into sovereign credit risk, which is a key channel for connecting banking risk to sovereign stability (Leonello, 2018; Izumi, 2020). Anghel et al. (2021) pointed out that the pandemic crisis and government guarantees, responsible for increased levels of financing with public debt and contingent liabilities, contributed to growing fiscal risk in European Union.

Most of the existing theoretical models that examine the impact of external policies on bank risks are static and mainly divided into three types: (a)the bank's NPV maximization model (Keeley, 1990; Gennotte & Pyle, 1991; Park, 1997; Schenck & Thornton, 2016), which assumes that bank managers seek to maximize the NPV of bank assets; (b)profits/payoffs maximization model (Klein, 1971; Dell'Ariccia et al., 2010; Allen et al., 2011; König et al., 2014; Bahaj & Malherbe, 2020); (c)depositors' expected utility maximization model, which assumes that the banking sector is perfectly competitive, thus banks make no profit, such as Leonello (2018) and Allen et al. (2018) using the global-game approach to solve this problem, García-Palacios et al. (2014) and so on.

Thus, the effect of government guarantees on bank risk remains in debate. Moreover, static models are still one of the mainstream methods to solve such complex problems. Currently, most of the theoretical researches on government guarantees concentrated on the broad public guarantee, without distinguishing the capacity and willingness of guarantees. It is even usually assumed that government is of full capacity to fund public subsidies (Diamond & Dybvig, 1983; Cooper & Ross, 2002; Tsafack et al., 2021), which is divorced from reality. Since implicit government guarantees can be regarded as bank contingent assets (Polackova, 1998; Gray et al., 2007), we choose and expand the bank's NPV maximization model to analyse the impact of implicit government guarantees on bank risk. In addition, the government's willingness and capacity to guarantee are jointly incorporated in our model.

3. Modelling net present value of loan portfolios

Our model is based on three assumptions. First, banks invest deposits (funding at a fixed rate r_D) and their capital in risky projects by issuing loans to get fixed income if the project succeeds. The amount available for issuing loans L, i.e., the present value of bank assets, equals to the sum of the bank's capital C and the borrowed amount D. Second, the bank is protected by limited liability, choosing the riskiness of the project and the size of the loan to maximize its NPV, which composed of the value of implicit government guarantee and NPV of loan portfolios. Loan scale is a negatively sloped demand function of the gross interest rate the bank charges on loans, $r_L : L(r_L) = a - br_L$[4]. Capital acts as a reserve or buffer when losses occur (Nehrebecka, 2021). To control the cost of implicit guarantees and the exposure of banks, regulators often impose capital controls in various forms. For simplicity, just

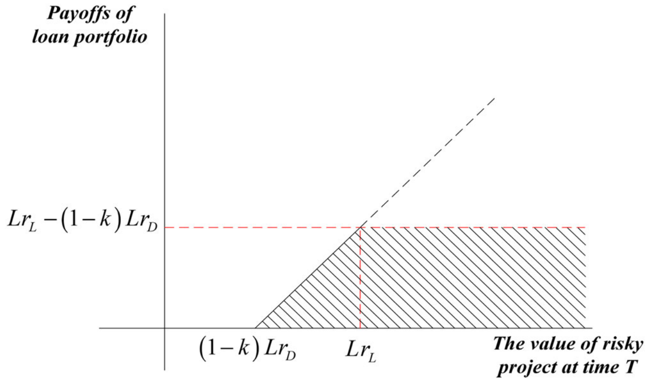

Figure 2. Payoffs of loan portfolios.
Source: drawn by the author.

as capital adequacy ratio, one of the most important capital regulation indicators, we finally assume that the capital of a bank is required to be no less than a fixed capital-to-asset value ratio, k (Gennotte & Pyle, 1991; Park, 1997). Since the cost of capital financing is higher than taking deposits, the above capital constraint is binding, and the corresponding deposit amount equals to $(1-k)L$.

In the absence of guarantees, the bank's NPV is only composed of loan portfolios. In this section, we innovatively proposed a new model on loan portfolios' NPV based on the Option Pricing Theory.[5] In previous literature, the dynamics of bank asset value is usually assumed to be geometric Brownian motion, which grows in a geometric trend and can be infinite in the previous literature (Merton, 1977; Gray et al., 2007; Tsafack et al., 2021). However, banks with loan portfolios as their main asset should have limited returns.

In this article, the NPV of the loan depends on risky project: If the risky project succeeds, the bank will receive a fixed amount; if it fails, there are two cases: (a) its value exceeds the amount bank owed, i.e., $(1-k)Lr_D$, then bank's shareholders will receive the balance; (b) its value is lower than bank's default barrier, which will lead to bank failure. The lending bank has the equivalent of combinations of options on the value of the risky project. The payoffs of the loan portfolio are the trapezoidal shadow areas shown in Figure 2.

The dynamics for the value of the risky project, V, over time can be described by geometric Brownian motion: $dV = \mu V dt + \sigma V dz$, where μ is the drift term, σ is the risky project return volatility, z is the Wiener process which follows a normal distribution with mean 0 and variance t. Thus, the NPV of a loan portfolio could be seen as a portfolio consisting of a unit long call, C_1, striking at $(1-k)Lr_D$ and a unit short call, C_2, striking at Lr_L.

$$C_1 = e^{-rT}E_0^Q \max\left[V_T - (1-k)Lr_D, 0\right] = LN(d_{11}) - (1-k)Lr_D e^{-rT}N(d_{12}), \tag{1}$$

$$C_2 = e^{-rT}E_0^Q \max[V_T - Lr_L, 0] = LN(d_{21}) - Lr_L e^{-rT}N(d_{22}), \tag{2}$$

$$A(L,\sigma) = C_1 - C_2, \tag{3}$$

where Q is the risk-neutral probability measure, A is the NPV of loan portfolios, $d_{12} = \frac{\ln(L/(1-k)Lr_D)+(r-\sigma^2/2)T}{\sigma\sqrt{T}}$, $d_{22} = \frac{\ln(L/Lr_L)+(r-\sigma^2/2)T}{\sigma\sqrt{T}}$, $N(\cdot)$ is the standard normal cumulative density function, $d_{\cdot 1} = d_{\cdot 2} + \sigma\sqrt{T}$.

The first-order conditions for maximizing the NPV of the loan portfolio are:

$$A_L = \left[N(d_{11}) - (1-k)r_D e^{-rT} N(d_{12})\right] - \left[N(d_{21}) + \left(\frac{L}{b} - r_L\right) e^{-rT} N(d_{22})\right] = 0, \tag{4}$$

$$A_\sigma = L\sqrt{T}\left[N'(d_{11}) - N'(d_{21})\right] = 0. \tag{5}$$

where $N'(\cdot)$ is the standard normal probability density function. According to Equation (5), $d_{11} = -d_{21}$, which means d_{21} is smaller than zero at optimum.[6] The second-order derivatives of A are:

$$A_{LL} = -\left[\frac{2}{b} e^{-rT} N(d_{22}) + \frac{L}{b^2 r_L \sigma\sqrt{T}} e^{-rT} N'(d_{22})\right] < 0,$$

$$A_{\sigma\sigma} = L\sqrt{T}\left[\frac{d_{11}d_{12}}{\sigma} N'(d_{11}) - \frac{d_{21}d_{22}}{\sigma} N'(d_{21})\right] < 0, \quad A_{L\sigma} = \frac{d_{21}L}{b\sigma} e^{-rT} N'(d_{22}) < 0.$$

As the loan scale (or risk) increases, the marginal value of the loan declines. We can obtain a local optimum when the value function of loan portfolios satisfies concavity condition: $|J_A| = A_{LL}A_{\sigma\sigma} - A_{L\sigma}^2 > 0$, where $|J_A|$ is Jacobian determinant. We are interested in the effect of the tightening capital constraint on the optimal asset scale, L^*, and risk level σ^*.

Proposition 1. *In the absence of implicit government guarantees, the tighter capital constraint would lead to an increase in asset scale and a decrease in asset risk.*

Proof: See 'Appendix'

As shown in the Appendix, the negative sign of cross partial derivative $A_{L\sigma}$ is a sufficient condition for banks to increase loan scale and decrease asset risk in response to a tightening capital constraint. It means that capital regulation can effectively limit banks' risk-taking without causing any damage to liquidity creation in the absence of implicit government guarantees. As the marginal value of scale increases with the decrease of risk level, it may even motivate banks to expand investment to achieve economies of scale.

4. A static value maximization model

This section develops a static value maximization model with a non-binding capacity constraint based on unified theory. In face of distressed banks, the government has two solutions: (a) taking losses all or partly over (depending on its capacity and willingness to guarantee) and repaying depositors with shareholders receiving nothing;

(b) abandoning the distressed bank leaving creditors to suffer losses. Because of the prevailing perception of implicit guarantees, it has long been believed that the PRC government would take measures to support banks when necessary. To the extent that such implicit guarantees are politically binding, the costs or liabilities they impose on the guarantor are essentially the same as those of explicit guarantees (Merton, 1977). The cost of the guarantor, in turn, is the value of guarantees owned by insured banks. That is, the deposit insurance (seen as an explicit guarantee) pricing model proposed by Merton (1977) can be used to evaluate implicit guarantees.

Unlike Merton's model, we assume that bank asset is a combination of derivatives of the risky project (see Equation (3)). Consequently, the value of implicit guarantees essentially depends on the value of the risky project. The expected losses associated with the outstanding liabilities can be valued as an implicit put option (see Equation (6)), which is triggered if the value of the risky project is insufficient to meet the promised payments.

$$P_1 = e^{-rT} E_0^Q \max\left[(1-k)Lr_D - V_T, 0\right] = (1-k)Lr_D e^{-rT} N(-d_{12}) - LN(-d_{11}), \tag{6}$$

Default occurs when asset value falls below the promised payment, Dr_D. The probability of default equals to

$$\Pr(V_T < Dr_D) = \Pr\left(L \exp\left[(r-\sigma^2)T + \sigma\varepsilon\sqrt{T}\right] < (1-k)Lr_D\right) = \Pr(\varepsilon < -d_{12}), \tag{7}$$

Since $\varepsilon \sim N(0,1)$, the 'risk-neutral' probability of default is $N(-d_{12})$.

It is not recommended that authorities completely absorb all losses in the banking sector at any time, which may cause severer moral hazard. Therefore, it is reasonable to suppose a certain proportion of expected losses covered by the government (Gray et al., 2007), denoted as $\alpha(0 \le \alpha \le 1)$, reflecting its willingness to guarantee. The creditors of the bank suffer from the expected uncovered losses, $(1-\alpha)P_1$, which can be reflected by the CDS spreads. The value of implicit government guarantees (IG) is equal to

$$IG(L,\sigma) = \alpha P_1 = \alpha e^{-rT} E_0^Q \max\left[(1-k)Lr_D - V_T, 0\right]. \tag{8}$$

The bank's optimization problem consists of choosing the original asset scale L and risk level σ to maximize the following objective function

$$f(L,\sigma) = IG(L,\sigma) + A(L,\sigma) = \alpha P_1 + C_1 - C_2. \tag{9}$$

So far, the bank's NPV can be considered a risky project's derivative portfolio. The first-order conditions for an interior maximum are:

$$\begin{aligned} f_L = {} & \alpha\left[(1-k)r_D e^{-rT} N(-d_{12}) - N(-d_{11})\right] + \left[N(d_{11}) - (1-k)r_D e^{-rT} N(d_{12})\right] \\ & - \left[N(d_{21}) + \left(\frac{L}{b} - r_L\right) e^{-rT} N(d_{22})\right] = 0, \end{aligned} \tag{10}$$

$$f_\sigma = \alpha L\sqrt{T}N'(d_{11}) + L\sqrt{T}\left[N'(d_{11}) - N'(d_{21})\right] = 0. \tag{11}$$

Banks invest until the marginal return from implicit government guarantees offsets the marginal NPV of the investment. Equations (10) and (11) show that implicit guarantees result in inefficient risk taking $(A_\sigma < 0)$ and inefficient investment $(A_L < 0)$[7]. With guarantees, banks act like frantic gamblers, motivated to make more investments with higher risk, regardless of their unfavourable consequences once occur only partially borne by the guarantee provider. The NPV of the resulting loans with guarantees must be lower than the optimum without guarantees.

Given an optimal asset risk, there are two different levels of scale available for banks to choose.[8] It is plausible to believe that banks will choose the larger scale because of the 'scale effect'. At this point, the higher the risk of the project, the lower the likelihood of success:

$$\frac{dN(d_{22})}{d\sigma} = -\frac{d_{21}}{\sigma}N'(d_{22})<0. \tag{12}$$

Implicit guarantees from the government are socially inequitable, and the expectation of support exerts an important influence on the banks' management decisions, including the amount of investment and risk taking. Given decreasing willingness (a decrease in α), how the optimal risk and scale vary attracts our attention.

Proposition 2. *When the government is capable to subsidize the whole distressed banks, a decrease in the willingness of implicit guarantee, α, will result in banks' scale shrinkage and less risk taking simultaneously.*

Proof: See 'Appendix'

Essentially, the premier objective of implicit government guarantees is to protect the distressed FI from bankruptcy, to stabilize the financial system. While this proposition presents that the implicit guarantee on deposits and/or debt results in severe moral hazard provided that the government can fulfil the promised amount, which results in unintended asset risk-taking and inefficient investment. These increases might eventually offset or even write off the positive impact of implicit guarantees on banking stability. This problem will be illustrated by numerical examples in Section 6.

In turn, the reduction in implicit guarantee (in this case, only the reduction in willingness) increases the incentives for banks to monitor risks, since they get fewer guarantee subsidies from failures, reducing the resulting asset risk. The sign of the cross partial derivative $f_{L\sigma}$ seriously affects the bank's behavior in responding to the increasing willingness to guarantee. As we conclude from Equation (12), $f_{L\sigma}$ has a positive sign, which means that as willingness increases, the increase in asset risk will stimulate banks to find ways to expand their exposure simultaneously.[9] Conversely, as willingness decreases, the decrease in asset risk will stimulate banks to shrink their loan scale simultaneously. Banks would restrain their earlier excessive asset risk taking and scale expansion to cope with the decline of willingness.

Moreover, implicit guarantees also influence the optimum value of loan portfolios:

$$A_\alpha = A_L \frac{\partial L^*}{\partial \alpha} + A_\sigma \frac{\partial \sigma^*}{\partial \alpha} < 0. \tag{13}$$

As shown by Equation (13), A decreases as α increases. The NPV of loan portfolios with implicit government guarantees is lower than that without implicit guarantees, indicating that implicit government guarantees will cause deterioration in the quality of bank assets. The excessive increase in risk level and investment scale caused by implicit guarantees may even offset the direct positive effects of implicit government guarantees on financial stability.

The quality of the regulatory environment may reshape the unintended consequence of implicit government guarantees. The awareness of improper incentives brought by government guarantees has led to various prudent regulatory measures (e.g., capital controls) on insured banks. From the regulator's point of view, the effect of capital constraint on banks' risk level and asset scale under this circumstance is of interest.

Proposition 3. *There is a unique critical value of willingness, α_m, such that:*

a. *If the government is relatively willing to provide implicit guarantees ($\alpha_m < \alpha \leq 1$), the bank's asset scale and risk will decrease with a tightening capital control.*
b. *When the willingness to implicit guarantee is at a relatively low level ($0 \leq \alpha < \alpha_m$), if the bank increases its asset risk in response to a tighter capital constraint, its asset scale will increase simultaneously.*

Proof: See 'Appendix'

The proposition establishes that capital constraint under implicit guarantee is potent in controlling asset risk taking and scale expansion of banks only if the willingness is at a relatively high level. Under this circumstance, the capital constraint can effectively limit the excessive scale expansion of banks in comparison to the case of no-guarantee. We can conclude from *Proposition 2* that banks will take excessive risk and expand their asset scale if they are implicitly guaranteed. If authorities want to reduce the asset risk and scale of banks significantly, they can reduce the willingness and tighten the capital constraint simultaneously. However, this combination is invalid when the willingness falls below the threshold, α_m. In this case, the tightening capital constraint may have the opposite effect.

5. Introducing the constraint of guarantee capacity

Although the impact of the government's willingness to support has been discussed in Section 4, the implicit guarantees are supposed to include not only the government's willingness but also their capacity. In this part, we introduce the ceiling on capacity into our basic model and denote it as G. Considering the constraint of government's guarantee capacity, the value of implicit government guarantee is:

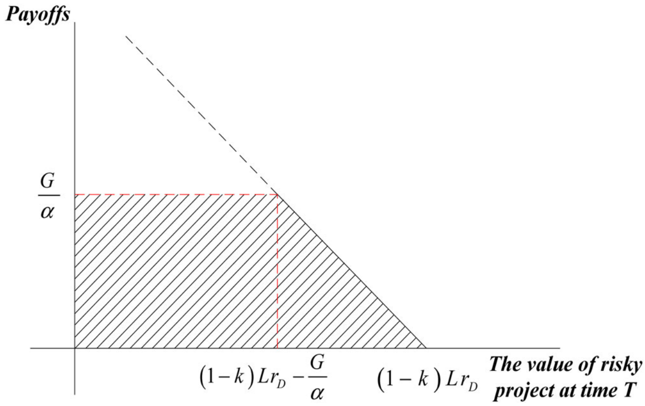

Figure 3. Payoffs of implicit government guarantee under capacity constraint divided by α.
Source: drawn by the author.

$$\begin{aligned} IG &= e^{-rT}E_0^Q \max\left\{\min\left[\alpha\left((1-k)Lr_D - V_T\right), G\right], 0\right\} \\ &= \alpha e^{-rT}E_0^Q \max\left[\min\left((1-k)Lr_D - V_T, G/\alpha\right), 0\right] . \end{aligned} \tag{14}$$

This constraint is not binding when the capacity of implicit guarantee is not less than the maximum subsidy portion, namely, $G \geq \alpha(1-k)Lr_D$. Thus, the value of implicit government guarantees is equal to αP_1 when $\alpha \in [0, G/[(1-k)Lr_D]]$. If the constraint is binding, $\alpha \in (G/[(1-k)Lr_D], 1]$, the payoffs of implicit government guarantee under the constraint of guarantee capacity is trapezoidal shadow area in Figure 3 multiplied by α.

The value of implicit government guarantee under the binding constraint of capacity can be regarded as a portfolio that consists of a unit long put, P_1, and a unit short put, P_3, which strikes on $(1-k)Lr_D - G/\alpha$, multiplied by α.

$$\begin{aligned} P_3 &= e^{-rT}E_0^Q \max\left\{\left[(1-k)Lr_D - G/\alpha\right] - V_T, 0\right\} \\ &= \left[(1-k)Lr_D - G/\alpha\right]e^{-rT}N(-d_{32}) - LN(-d_{31}) , \end{aligned} \tag{15}$$

where $d_{32} = \frac{\ln\left\{L/[(1-k)Lr_D - G/\alpha]\right\} + (r-\sigma^2/2)T}{\sigma\sqrt{T}}$, $d_{31} = d_{32} + \sigma\sqrt{T}$. The value of implicit government guarantees can be expressed as a piecewise continuous function:

$$IG = \begin{cases} \alpha P_1, & 0 \leq \alpha \leq G/\left[(1-k)Lr_D\right] \\ \alpha(P_1 - P_3), & G/\left[(1-k)Lr_D\right] < \alpha \leq 1. \end{cases}$$

Both the government's willingness and capacity affect the value of implicit government guarantees. Hence, we pay special attention to situations where the capacity constraint is binding. Given $G/[(1-k)Lr_D] < \alpha \leq 1$, we calculate the first-order, second-order, and partial derivatives of IG as follows:

$$IG_\alpha = (1-k)Lr_De^{-rT}\left[N(-d_{12}) - N(-d_{32})\right] - L\left[N(-d_{11}) - N(-d_{31})\right] > 0,$$
$$IG_G = e^{-rT}N(-d_{32}) > 0,\ IG_{\alpha\alpha} = -\frac{G^2e^{-rT}N'(-d_{32})}{\sigma\sqrt{T}\alpha^2\left[\alpha(1-k)Lr_D - G\right]} < 0,$$
$$IG_{GG} = -\frac{e^{-rT}N'(-d_{32})}{\sigma\sqrt{T}\left[\alpha(1-k)Lr_D - G\right]} < 0, IG_{G\alpha} = IG_{\alpha G} = \frac{Ge^{-rT}N'(-d_{32})}{\sigma\sqrt{T}\alpha\left[\alpha(1-k)Lr_D - G\right]} > 0.$$

Both willingness and capacity have positive effects on the value of implicit government guarantees, ceteris paribus.[10] Furthermore, their marginal contributions to the value of implicit guarantees decrease gradually. On account of the positive sign of cross partial, marginal contribution of willingness (or capacity) increases as capacity (or willingness) increases. That is, there exists a substitution effect between willingness and capacity.

The objective function for value maximization is:

$$f(L,\sigma) = IG(L,\sigma) + A(L,\sigma) = \begin{cases} \alpha P_1 + C_1 - C_2, 0 \le \alpha \le G/\left[(1-k)Lr_D\right] \\ \alpha(P_1 - P_3) + C_1 - C_2, G/\left[(1-k)Lr_D\right] < \alpha \le 1. \end{cases} \tag{16}$$

When the capacity constraint is not binding, the first-order conditions for value maximization are the same as Equations (10) and (11). Under this circumstance, implicit guarantee leads to inefficient investment and inefficient asset risk taking, which is the same as that of no capacity constraint. Nonetheless, when the capacity cap takes effect, the first-order conditions of interior optimum are:

$$\begin{aligned} f_L = {} & \alpha\left[(1-k)r_De^{-rT}N(-d_{12}) - N(-d_{11}) - (1-k)r_De^{-rT}N(-d_{32}) + N(-d_{31})\right] \\ & + \left[N(d_{11}) - (1-k)r_De^{-rT}N(d_{12})\right] - \left[N(d_{21}) + \left(\frac{L}{b} - r_L\right)e^{-rT}N(d_{22})\right] = 0, \end{aligned} \tag{17}$$

$$f_\sigma = \alpha L\sqrt{T}\left[N'(d_{11}) - N'(d_{31})\right] + L\sqrt{T}\left[N'(d_{11}) - N'(d_{21})\right] = 0. \tag{18}$$

Similar to the case of non-binding capacity, we conclude that implicit guarantees induce inefficient asset risk taking ($A_\sigma < 0$) at the interior optimum as well.[11] Whether implicit guarantees drive inefficient investment or not is ambiguous.

There are two situations in which capacity declines: (a) A forced decline caused by the economic downturn which is uncontrollable for authorities. Economic operation affects the government's financial situation to a large extent and determines the government's ability to support it. Moreover, the impact of the economic downturn on the financial industry is far greater than implicit guarantees, which are more complex and beyond the scope of this article. (b) A proactive downward adjustment to control banks' risk exposure. The purpose of this decline is to alleviate the unintended consequences of implicit guarantees and to eliminate banks' illusion of being heavily subsidized for their failures free of charge, thus making the bank's operations robust and stable. Furthermore, this decline often occurs with a decline in willingness. The impact of capacity decline on the banks' optimal asset risk and asset scale is of particular concern to authority. We mainly discuss the proactive decline of capacity.

Proposition 4. *If the bank increases scale in response to the decreasing government's capacity to guarantee, G, its asset risk decreases simultaneously, ceteris paribus.*

Proof: See 'Appendix'

This proposition points out that a proactive reduction of capacity stimulates the bank's vigilance against excessive risk taking, thus reducing the risk exposures of banks to mitigate the negative impact of moral hazard. Because this will lead to a decline in the maximal subsidies that distressed banks can obtain from the government given the same willingness, and a decline in implicit guarantee's value simultaneously. An analogous proactive reduction policy was issued at the end of 2017 to address the debt concern on local government. The central government of China announced that the local government's liabilities will no longer be bailed out, which undermines the extensive illusions of insurance. Local government's liabilities can be defined as four types, each of which is combined with two of the following four characteristics: explicit vs. implicit and direct vs. contingent (Polackova, 1998). Implicit government guarantees are regarded as an expected responsibility of government and pertain to the implicit contingent liabilities. The bailout provided by the central government to local governments could be considered as a dual-guarantee, which is the most favourable way to enlarge the capacity in addition to the improvement of the government's financial condition. Such indirect dual-guarantee may eventually be interpreted as an increase in implicit guarantees directly provided by the central government to distressed banks. The withdrawal of the central government's liability bailout to local government may greatly reduce the ability of local governments to support banks in predicament, forcing bankers to be more cautious and reducing the risks borne by banks.

Furthermore, we are also concerned about the effect of willingness on bank risk and scale. But this effect cannot be easily and directly revealed by analytical calculation on the theoretical model. We will clarify it by using numerical calculation in the following section.

Last but not the least, as previous literature indicated, there are still widespread public debates on the role of implicit government guarantees in financial stability. To examine the impact of implicit government guarantees on banking stability, we finally discuss how changes in implicit guarantees affect banks' 'real' default risk—considered as the bank's overall risk. In accordance with Equations (6) and (14), the 'real' expected losses of the bank, denoted as P^R, are equal to

$$P^R = P_1^* - IG^* = e^{-rT} E_0^Q \max\left[D_T^R - V_T, 0\right] = D_T^R e^{-rT} N\left(-d_2^R\right) - L^* N\left(-d_1^R\right). \tag{19}$$

where the superscript '*' represents the optimum. Given the 'real' expected losses, we can derive the 'real risk-neutral' probability of default, $N(-d_2^R)$, where the 'real' distance to default is $d_2^R = \frac{\ln(L^*/D_T^R) + (r - \sigma^{*2}/2)T}{\sigma^* \sqrt{T}}$, $d_1^R = d_2^R + \sigma\sqrt{T}$, D_T^R is the 'real' default barrier. Furthermore, both willingness and capacity affect the 'real' expected default losses of banks through two channels.

$$\frac{dP^R}{d\alpha} = \frac{\partial P^R}{\partial \alpha} + \frac{\partial P^R}{\partial L^*}\frac{\partial L^*}{\partial \alpha} + \frac{\partial P^R}{\partial \sigma^*}\frac{\partial \sigma^*}{\partial \alpha}. \tag{20}$$

$$\frac{dP^R}{dG} = \frac{\partial P^R}{\partial G} + \frac{\partial P^R}{\partial L^*}\frac{\partial L^*}{\partial G} + \frac{\partial P^R}{\partial \sigma^*}\frac{\partial \sigma^*}{\partial G}. \tag{21}$$

The first term of Equation (20) or Equation (21) indicates the direct effect of willingness or capacity, (denote $DE_\alpha = -\frac{\partial P^R}{\partial \alpha}$ and $DE_G = -\frac{\partial P^R}{\partial G}$ as their direct effects on reducing expected losses, respectively), while the last two terms represent the indirect effect (denote $IDE_\alpha = -\left(\frac{\partial P^R}{\partial L^*}\frac{\partial L^*}{\partial \alpha} + \frac{\partial P^R}{\partial \sigma^*}\frac{\partial \sigma^*}{\partial \alpha}\right)$ and $IDE_G = -\left(\frac{\partial P^R}{\partial L^*}\frac{\partial L^*}{\partial G} + \frac{\partial P^R}{\partial \sigma^*}\frac{\partial \sigma^*}{\partial G}\right)$ as indirect effects on reducing expected losses, respectively). On the one hand, they have positive direct effects on reducing the expected losses:

$$DE_\alpha = \begin{cases} P_1^* > 0, 0 \le \alpha \le G/\left[(1-k)L^* r_D\right] \\ P_1^* - P_3^* - \dfrac{G}{\alpha} e^{-rT} N(-d_{32}) > 0, G/\left[(1-k)L^* r_D\right] < \alpha \le 1, \end{cases}$$
$$DE_G = e^{-rT} N(-d_{32}) > 0.$$

On the other hand, they have indirect effects on banks' asset risk and scale. When the capacity constraint is not binding, the indirect impact of willingness on the reduction of P^R is negative:

$$IDE_\alpha = -(1-\alpha)\left[(1-k)r_D e^{-rT} N(-d_{12}) - N(-d_{11})\right]\frac{\partial L^*}{\partial \alpha} - (1-\alpha)L\sqrt{T}N'(d_{11})\frac{\partial \sigma^*}{\partial \alpha} < 0.$$

Therefore, the impact of willingness on P^R definitively depends on the trade-off of these two opposite effects. While the constraint is binding, the sign of these two indirect impacts becomes unpredictable. The combined consequence of direct and indirect effects is vague. Therefore, it may cause unintended consequences of the bank's overall risk increase (refer to the 'real' default risk in this article).

The fact that the influence of willingness and capacity on the bank's overall risk cannot be entirely determined by its impact on the expected losses is worth further explanation. Because they influence the optimal asset risk and scale simultaneously. To clearly understand their impact on the bank's overall risk, we calculate the derivatives of $N(-d_2^R)$ with respect to them:

$$\frac{dN\left(-d_2^R\right)}{d\alpha} = -N'\left(-d_2^R\right)\left[\left(\frac{\partial d_2^R}{\partial L^*} + \frac{\partial d_2^R}{\partial D_T^R}\frac{\partial D_T^R}{\partial L^*}\right)\frac{\partial L^*}{\partial \alpha} + \left(\frac{\partial d_2^R}{\partial \sigma^*} + \frac{\partial d_2^R}{\partial D_T^R}\frac{\partial D_T^R}{\partial \sigma^*}\right)\frac{\partial \sigma^*}{\partial \alpha}\right], \tag{22}$$

$$\frac{dN\left(-d_2^R\right)}{dG} = -N'\left(-d_2^R\right)\left[\left(\frac{\partial d_2^R}{\partial L^*} + \frac{\partial d_2^R}{\partial D_T^R}\frac{\partial D_T^R}{\partial L^*}\right)\frac{\partial L^*}{\partial G} + \left(\frac{\partial d_2^R}{\partial \sigma^*} + \frac{\partial d_2^R}{\partial D_T^R}\frac{\partial D_T^R}{\partial \sigma^*}\right)\frac{\partial \sigma^*}{\partial G}\right]. \tag{23}$$

Both willingness and capacity have indirect impacts on the 'real' barriers and distance to default through optimal asset risk and investment, further affecting the probability of default. In theoretical analysis, their ultimate impacts on the bank's overall risk seem to be uncertain and can be solved by numerical analysis.

6. A numerical example

In this section, some numerical simulations of our model are presented for two purposes. First, these graphical illustrations are used to demonstrate the effects identified in this article. Second, they can reveal other important effects that cannot be determined by analytical calculations.

The initial values of exogenous variables are determined as follows. First, the debt maturity of FIs is standardized to 1 year and the risk-free interest rate is set to 5%. Second, for the linear demand function of loans $L(r_L) = a - br_L$, a and b are set to 100 and 8, respectively, following Dell'Ariccia et al. (2010). Third, we assume that the returns on deposits exceed the risk-free returns, $r_D = 1.055$. Consistent with the theoretical analysis discussed above, our numerical examples are divided into three cases.

6.1. No implicit government guarantees

Figure 4 demonstrates the veracity of *Proposition 1* which claims the effectiveness of capital constraint on reducing the bank's asset risk taking. With a tightening capital control, the proportion of capital is forced to increase, thus prompting banks to supervise, but encouraging banks to achieve optimum by other means—such as scale expansion.

6.2. No capacity constraint (or non-binding capacity constraint)

Consistently with *Proposition 2*, Figure 5 illustrates the positive impact of the government's willingness to implicit guarantee on both asset risk and scale. When the willingness increases to a certain degree, asset risk and scale surge as willingness

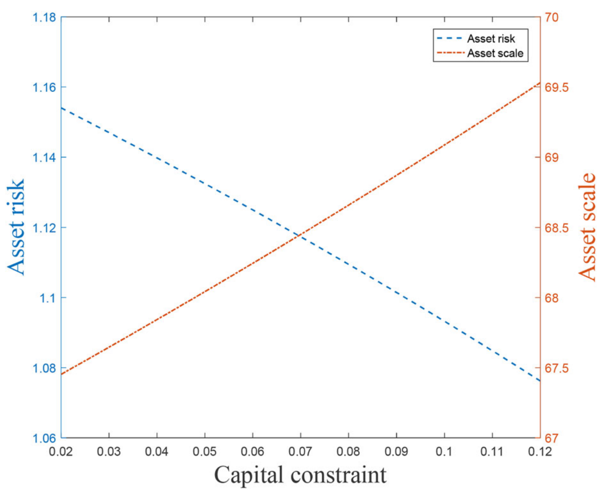

Figure 4. Optimal asset risk and asset scale as functions of capital constraint.
Notes: The changing interval of capital constraint is [2%, 12%].
Source: the results of the numerical calculation, which is calculated by the author.

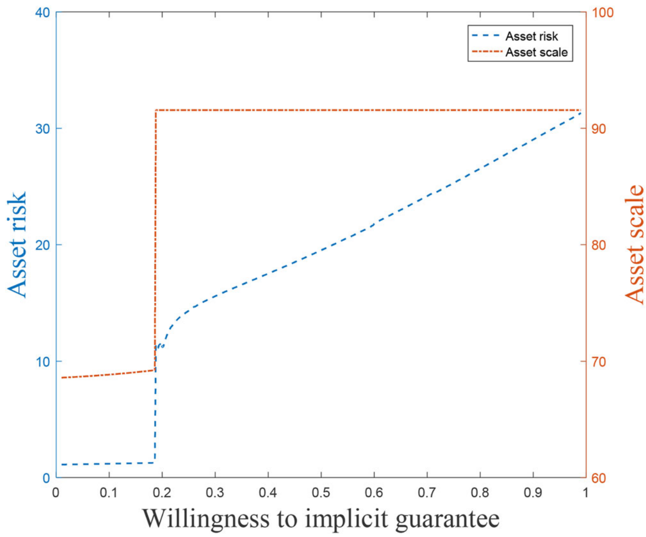

Figure 5. Optimal asset risk and asset scale as functions of the willingness to implicit guarantee.
Notes: Capital constraint, *k*, here is set to 8%[12].
Source: the results of the numerical calculation, which is calculated by the author.

increases. In addition, due to the upper limit of the loan scale, the scale curve tends to be flat with the increase in α. Thus, when the asset scale reaches the limit, excessive asset risk taking is the main unintended consequence of implicit guarantees.

To maximize the value of banks, bankers must trade-off between the value of implicit government guarantees and the NPV of investments. Figure 6 reveals that when the government is able to fulfil the promised subsidy, as α increases, the maximization of bank value will increasingly rely on the *IG*. Bank becomes swamped in risk because of such overdependence. Although the drop in 'real' expected losses seems to delude people into thinking that the bank is becoming safer, the extremely high asset risk (see Figure 5) will pull banks to the brink of default if α is at a relatively high level (see Figure 7). Thus, the gradual removal of implicit guarantee (reducing willingness in this case) is conducive to the bank's stability.

As shown in Figure 8, there is assuredly a threshold of willingness, α_m, corresponding to *Proposition 3*. Bank's risk decreases with a tightening capital control if $\alpha_m<\alpha \leq 1$; but the decrease of scale is inconspicuous because the scale will surge to its upper limit when the level of α is relatively high. Otherwise, both asset scale and risk increase with α, if $0 \leq \alpha<\alpha_m$. On both sides of the threshold, the capital constraint has two opposite consequences. The right-hand side performs effectively in controlling asset risk. While the other is entirely ineffective in reducing asset risk and scale $(0 \leq \alpha < \alpha_m)$. Therefore, other regulatory measures for the effectiveness of risk and (or) scale control must be considered to avoid the adverse effects of ineffective capital constraint.

6.3. Capacity constraint is binding

On the one hand, given an interior optimum, the bank's asset risk decreases as the capacity decreases but increases as the willingness decreases. The reason why the

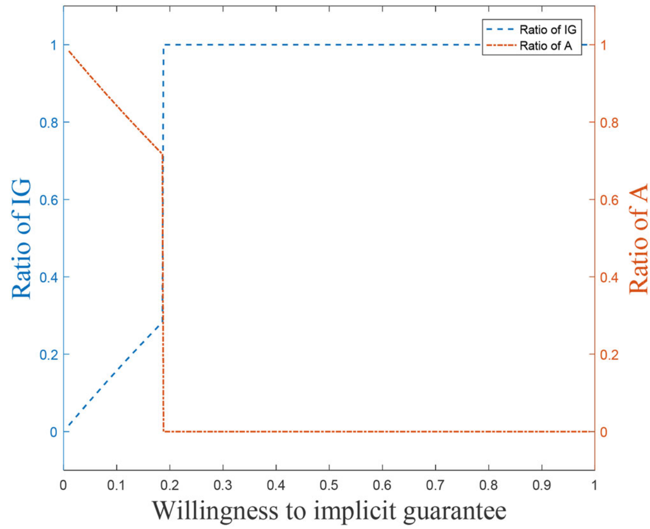

Figure 6. The contribution of implicit government guarantee and loan portfolios to bank value as functions of the willingness to implicit guarantee.
Notes: *Ratio of IG* = *IG/f*, *Ratio of A* = *A/f*. Capital constraint, *k*, here is set to 8%. The changing interval of willingness is [0.01, 1].
Source: the results of the numerical calculation, which is calculated by the author.

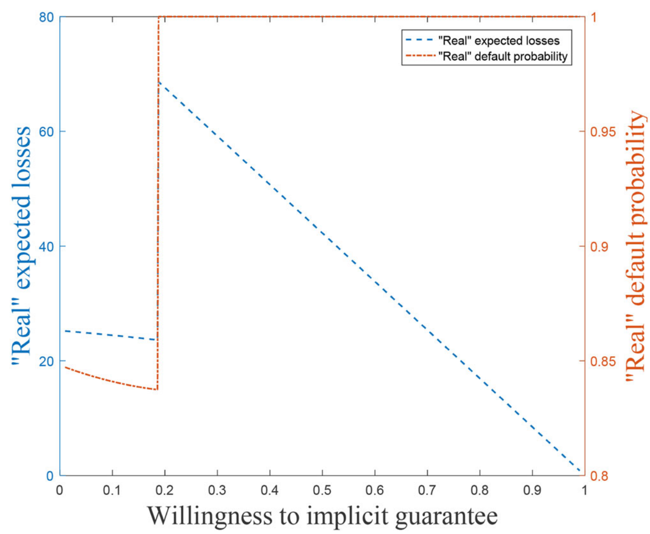

Figure 7. 'Real' expected losses and default probability as functions of the willingness to implicit guarantee (if the capacity constraint is not binding).
Notes: Capital constraint, *k*, here is set to 8%. The changing interval of willingness is [0.01, 1].
Source: the results of the numerical calculation, which is calculated by the author.

increase of willingness does not lead to an increase of risk taking is that once the subsidy exceeds the maximum amount available for the distressed bank, G, the increase in willingness seems like a 'blank check' with no incentives for banks to take risks. But the unwanted asset risk taking remains when these two factors jointly increase

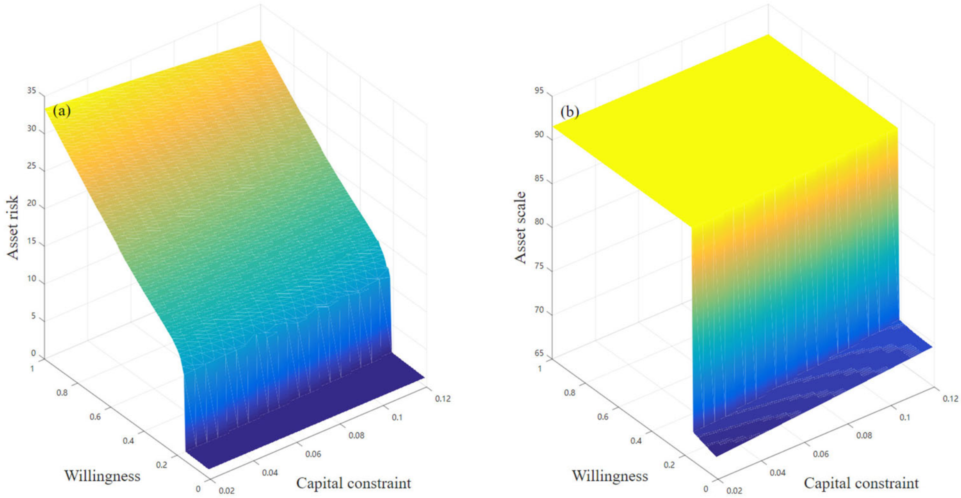

Figure 8. Optimal asset risk and asset scale as functions of willingness and capital constraint.
Notes: The changing interval of capital constraint and willingness are [2%, 12%] and [0.01, 1], respectively.
Source: the results of the numerical calculation, which is calculated by the author.

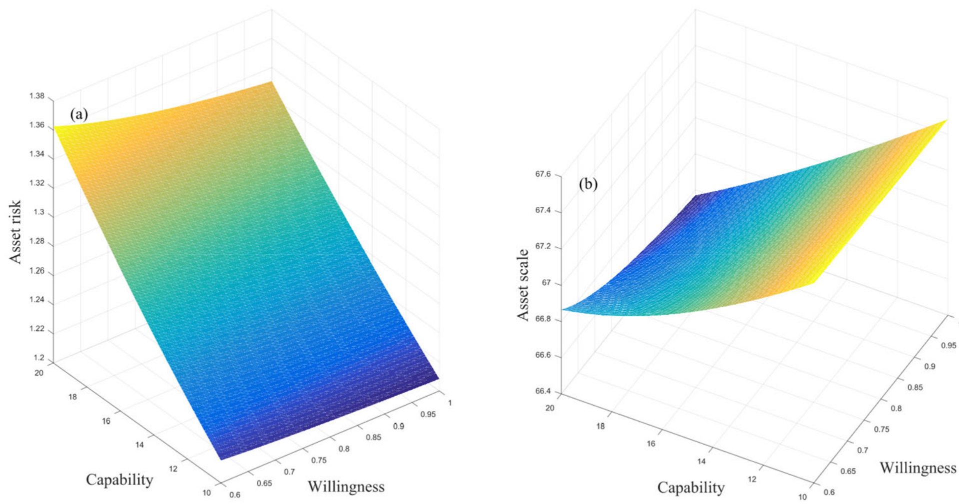

Figure 9. Optimal asset risk and asset scale as functions of willingness and capacity.
Notes: Capital constraint, *k*, here is set to 8%. The changing interval of capacity and willingness are [10, 20] and [0.6, 1], respectively.
Source: the results of the numerical calculation, which is calculated by the author.

(see Figure 9(a)). Similarly, the bank's optimal scale, on the other hand, decreases with the lower capacity (or higher willingness).

The increase in capacity drives the implicit guarantee to contribute more to the value of the bank while the increase in willingness has the opposite effect. As the willingness increases, the contribution of implicit guarantees declines slightly (see Figure 10). Banks seem to be more rational than those without the capacity constraint (see Figure 6), with *IG* accounting for no more than half. That means that the

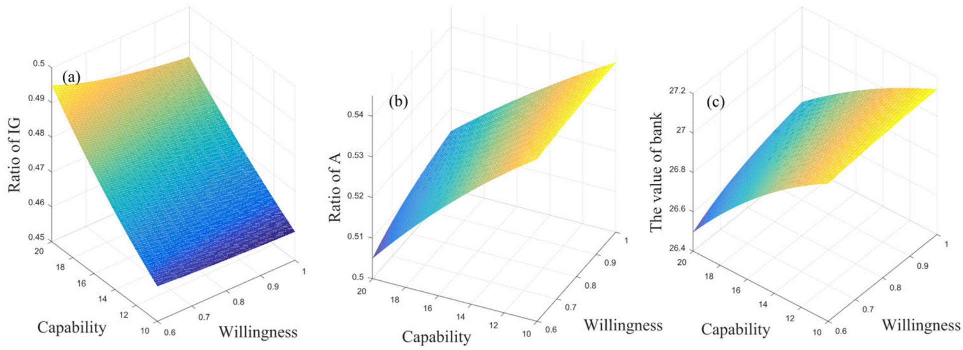

Figure 10. The contribution of implicit government guarantee and loan portfolios to bank value as functions of willingness and capacity.

Notes: Ratio of IG = IG/f, Ratio of A = A/f. Capital constraint, *k*, here is set to 8%. The changing interval of capacity and willingness are [10, 20] and [0.6, 1], respectively.

Source: the results of the numerical calculation, which is calculated by the author.

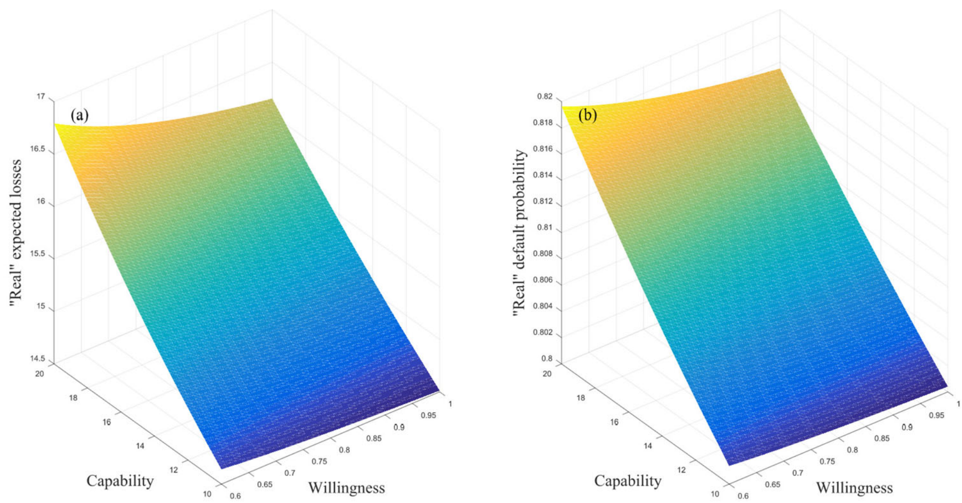

Figure 11. 'Real' expected losses and default probability as functions of the willingness and capacity.

Notes: Capital constraint, *k*, here is set to 8%. The changing interval of capacity and willingness are [10, 20] and [0.6, 1], respectively.

Source: the results of the numerical calculation, which is calculated by the author.

capacity constraint seems to mitigate the negative impact of implicit guarantees to some extent.

As shown in Figure 11, the combined effect of two factors of implicit government guarantee increases the expected losses borne by banks, making banks more vulnerable to failure. Furthermore, the implicit government guarantee does damage the value of bank (see Figure 10(c)). In essence, the primary goal of implicit government guarantees is to protect distressed FIs from failures and stabilize the financial system. However, in any case, implicit guarantees will bring higher bank risks, including asset risk and overall risk. The withdrawal of implicit government guarantees can fundamentally eliminate these undesirable consequences in the long run. Furthermore, it is necessary to remove this expectation of guarantees to restore market discipline.

Conclusion

This article establishes the bank value maximization model in three cases: (a) No implicit government guarantees; (b) Implicit government guarantees without capacity constraints; (c) Implicit government guarantees with a binding capacity constraint. Then, this article examines the impact of implicit government guarantee changes (including changes in willingness and capacity) on bank risks (including asset risks and overall risks) and asset scale.

It is universally acknowledged that the premier objective of implicit government guarantees is to protect distressed FI from failures and to stabilize the financial system. Due to information asymmetry and moral hazard, it brings about the unintended consequences of implicit government guarantees: excessive and inefficient risk taking (including higher asset risk and overall risk) at any cases we considered, inefficient investment, and deterioration in the quality of bank assets in case of non-binding capacity constraint. The main reason we have drawn from the research lies that the introduction of implicit government guarantees makes the value of banks increasingly dependent on *IG* rather than loan portfolios. In other words, banks rely more on their implausible contingent asset other than its veritable asset, which makes their risk decision more radical. Bankers start to find ways to benefit from implicit guarantees rather than loan portfolios, which completely violates the original intention of the implicit guarantees. Violating the market rules of 'survival of the fittest', providing temporary protection to distressed institutions may eventually result in deeper and severer financial instability.

According to our analysis, regulators can choose a tightening capital constraint to contain banks' risk-taking behaviours in the absence of implicit government guarantees. When the guarantee capacity constraint is not binding, the government can reduce its willingness to guarantee, to reduce the bank's asset risk and overall risk, and improve the bank's asset quality. Only if the willingness to implicit guarantee is at a relatively high level $(\alpha_m < \alpha \leq 1)$, the supervisory authorities can reduce bank asset risk to a greater extent by coordinating with higher capital constraints. Otherwise, the capital constraint is invalid. At this time, the supervisory authorities are supposed to choose other tools to control bank risks. Under the condition that the guarantee capacity constraint is binding, the reduction of willingness to guarantee cannot achieve the purpose of reducing the bank risks, and the bank risk-taking must be moderated by reducing the guarantee capacity.

However, a radical change to the perception that guarantees are in place could lead to disruptive withdrawals—such as by short-term repo lenders—and could quickly undermine the solvency of some FIs and corporates. Gradual withdrawal of implicit government guarantees is supposed to be accompanied by stringent financial regulations (not just the capital controls discussed in this article, because sometimes it is ineffective), the establishment of capital buffer and other reforms to insolvency and resolution in a legal framework and so on, to eliminate potential negative effects expected to occur during the withdrawal of implicit government guarantees. This policy coordination and cooperation are worth further study.

Notes

1. See *People's Republic of China: Financial system stability assessment* issued by IMF, (2017).
2. For example, the retail investors of investment products are implicitly compensated until the enactment of the new Regulation of Asset Management.
3. People's Bank of China declared 19 systemically important banks for the first time on October 15, 2021, which were accessed and identified jointly by the People's Bank of China and the China Banking and Insurance Regulatory Commission in accordance with the 'Measures for the Evaluation of Systemically Important Banks'.
4. See Dell'Ariccia *et al.* (2010), in turn, loan rate could be written as a function with a negative slope: $r_L = (a - L)/b$. For banks to make profits, r_L must be larger than r_D.
5. See Black and Scholes (1973) and Merton (1973).
6. Because $r_L > r_D$, d_{11} is always greater than d_{21}.
7. This result is consistent with the effect of deposit insurance—a kind of explicit guarantee, in Gennotte and Pyle (1991).
8. According to Eq. (11), $N'(d_{21})$ is determined because d_{11} is determined by the given risk level. Thus, there are two different levels of loan scale for d_{21} to be less than 0 (a lower level) or larger than 0 (a higher level).
9. To some extent, such investment augments in risky assets multiplies the negative externality of implicit government guarantees on bank behaviours. Thus, the gradual removal of implicit government guarantees is of great necessity.
10. These properties are consistent with empirical results from Antzoulatos and Tsoumas (2014).
11. According to eq. (12), we can obtain $d_{31} > d_{11} > d_{21} > 0$ and $N(d_{31}) > N(d_{11}) > N(d_{21}) > 0$. $N'(\cdot)$ is the probability density function of standard normal distribution and decreases monotonically in $\mathbb{R}$. Thus, we obtain$N'(d_{21}) > N'(d_{11}) > N'(d_{31}) > 0$ ulteriorly.
12. This setting accords to legal capital requirement – Capital adequacy ratio in Basel III.

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Appendix

Proof of Proposition 1. At the optimum, A_L and A_σ are both equal to zero, for any level of k. Their total derivatives with respect to k must therefore be equal to zero as well:

$$\begin{bmatrix} A_{LL} & A_{L\sigma} \\ A_{\sigma L} & A_{\sigma\sigma} \end{bmatrix} \begin{bmatrix} \partial L^*/\partial k \\ \partial\sigma^*/\partial k \end{bmatrix} + \begin{bmatrix} A_{Lk} \\ A_{\sigma k} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \tag{A.1}$$

where

$$A_{Lk} = r_D e^{-rT} N(d_{12}) > 0, \; A_{\sigma k} = -\frac{d_{11}}{(1-k)\sigma} LN'(d_{11}) < 0$$

According to Cramer's rule, comparative static derivatives are equal to

$$\frac{\partial L^*}{\partial k} = \frac{A_{L\sigma}A_{\sigma k} - A_{\sigma\sigma}A_{Lk}}{|J_A|} > 0, \; \frac{\partial \sigma^*}{\partial k} = \frac{A_{L\sigma}A_{Lk} - A_{LL}A_{\sigma k}}{|J_A|} < 0.$$

Proof of Proposition 2. Similar to Equation (A.1), we can obtain a matrix equation of f as follow:

$$\begin{bmatrix} f_{LL} & f_{L\sigma} \\ f_{\sigma L} & f_{\sigma\sigma} \end{bmatrix} \begin{bmatrix} \partial L^*/\partial \alpha \\ \partial\sigma^*/\partial \alpha \end{bmatrix} = \begin{bmatrix} -f_{L\alpha} \\ -f_{\sigma\alpha} \end{bmatrix} \tag{A.2}$$

where the second derivatives of f are shown to be equal to

$$f_{LL} = -\left[\frac{2}{b} e^{-rT} N(d_{22}) + \frac{L}{b^2 r_L \sigma\sqrt{T}} e^{-rT} N'(d_{22})\right] < 0$$
$$f_{\sigma\sigma} = L\sqrt{T}\left[(1+\alpha)\frac{d_{11}d_{12}}{\sigma} N'(d_{11}) - \frac{d_{21}d_{22}}{\sigma} N'(d_{21})\right] \; f_{L\sigma} = \frac{d_{21}L}{b\sigma} e^{-rT} N'(d_{22}) > 0$$
$$f_{L\alpha} = (1-k) r_D e^{-rT} N(-d_{12}) - N(-d_{11}) \geq 0 \; f_{\sigma\alpha} = L\sqrt{T}N'(d_{11}) > 0$$

We assume our objective function to be concave, hence a necessary and sufficient condition for that is $f_{\sigma\sigma} < 0$ and Jacobian determinant $|J_f| > 0$:

$$|J_f| \equiv f_{\sigma\sigma} f_{LL} - f_{\sigma L}^2 > 0 \tag{A.3}$$

The comparative static derivatives are

$$\frac{\partial L^*}{\partial \alpha} = \frac{f_{\sigma\alpha} f_{L\sigma} - f_{L\alpha} f_{\sigma\sigma}}{|J_f|} > 0, \; \frac{\partial \sigma^*}{\partial \alpha} = \frac{f_{L\alpha} f_{L\sigma} - f_{\sigma\alpha} f_{LL}}{|J_f|} > 0$$

If the equilibrium is interior, $|J_f|$ is positive at the optimum and a lower willingness to guarantee leads to lower risk taking and investment.

Proof of Proposition 3. In a similar way, we can obtain a matrix equation of f as follow

$$\begin{bmatrix} f_{LL} & f_{L\sigma} \\ f_{\sigma L} & f_{\sigma\sigma} \end{bmatrix} \begin{bmatrix} \partial L^*/\partial k \\ \partial\sigma^*/\partial k \end{bmatrix} = \begin{bmatrix} -f_{Lk} \\ -f_{\sigma k} \end{bmatrix} \tag{A.4}$$

where

$$f_{Lk} = r_D e^{-rT}\left[(1+\alpha)N(d_{12}) - \alpha\right], f_{\sigma k} = -\frac{(1+\alpha)d_{11}}{(1-k)\sigma} LN'(d_{11})<0$$

When $\alpha = 0$, $f_{Lk}\big|_{\alpha=0} = r_D e^{-rT} N(d_{12})>0$. Differentiating f_{Lk} with respect to α :

$$f_{Lk\alpha} = r_D e^{-rT}\left[N(d_{12}) - 1 - \frac{d_{11}}{\sigma^*}(1+\alpha)N'(d_{12})\frac{\partial \sigma^*}{\partial \alpha}\right]<0.$$

It means that f_{Lk} decreases with the increase of α. Therefore, there is $\alpha_m \in [0,1]$ such that $f_{Lk} = 0$. $f_{Lk}>0$ if $0 \le \alpha<\alpha_m$, $f_{Lk}<0$ if $\alpha_m<\alpha \le 1$.

According to Cramer's rule, comparative static derivatives are equal to

$$\frac{\partial L^*}{\partial k} = \frac{f_{L\sigma}f_{\sigma k} - f_{\sigma\sigma}f_{Lk}}{|J_f|}, \ \frac{\partial \sigma^*}{\partial k} = \frac{f_{L\sigma}f_{Lk} - f_{LL}f_{\sigma k}}{|J_f|}$$

If $\alpha_m<\alpha \le 1$, it is doubtless that the optimal scale and risk would decrease with the tightening of capital constraint ($\frac{\partial L^*}{\partial k}<0$, $\frac{\partial \sigma^*}{\partial k}<0$). From Equation (A.4), while $0 \le \alpha<\alpha_m$, if $\frac{\partial L^*}{\partial k}$ is negative, $\frac{\partial \sigma^*}{\partial k}$ must be negative, too.

Proof of Proposition 4. When the capacity constraint is binding, $\alpha \in (G/[(1-k)Lr_D], 1]$, the total derivatives of f_σ and f_L with respect to G equal to the following matrix equation

$$\begin{bmatrix} f_{LL} & f_{L\sigma} \\ f_{\sigma L} & f_{\sigma\sigma} \end{bmatrix}\begin{bmatrix} \partial L^*/\partial G \\ \partial \sigma^*/\partial G \end{bmatrix} = \begin{bmatrix} -f_{LG} \\ -f_{\sigma G} \end{bmatrix}. \tag{A.5}$$

where the second derivatives of f are equal to

$$\begin{aligned}
f_{LL} &= -e^{-rT}\left[\frac{G^2 N'(d_{32})}{L^2\sigma\sqrt{T}\left[\alpha(1-k)Lr_D - G\right]} + \frac{2}{b}N(d_{22}) + \frac{LN'(d_{22})}{b^2 r_L \sigma\sqrt{T}}\right]<0 \\
f_{\sigma\sigma} &= L\sqrt{T}\left[(1+\alpha)\frac{d_{11}d_{12}}{\sigma}N'(d_{11}) - \frac{d_{21}d_{22}}{\sigma}N'(d_{21}) - \alpha\frac{d_{31}d_{32}}{\sigma}N'(d_{31})\right] \\
f_{L\sigma} &= \frac{d_{21}L}{b\sigma}e^{-rT}N'(d_{22}) - \frac{d_{31}G}{\sigma L}e^{-rT}N'(d_{32}) \\
f_{LG} &= \frac{Ge^{-rT}N'(-d_{32})}{\sigma L\sqrt{T}\left[\alpha(1-k)Lr_D - G\right]} \ge 0, f_{\sigma G} = \frac{\alpha L d_{31} N'(d_{31})}{\sigma\left[\alpha(1-k)Lr_D - G\right]} \ge 0
\end{aligned}$$

A necessary and sufficient condition for our objective function to be concave is $f_{\sigma\sigma}<0$ and Jacobian determinant $|J_f|>0$ (satisfying Equation (A.3)). The comparative static derivatives are:

$$\frac{\partial L^*}{\partial G} = \frac{f_{L\sigma}f_{\sigma G} - f_{\sigma\sigma}f_{LG}}{|J_f|}, \ \frac{\partial \sigma^*}{\partial G} = \frac{f_{L\sigma}f_{LG} - f_{LL}f_{\sigma G}}{|J_f|}$$

A necessary condition for $\frac{\partial L^*}{\partial G}<0$ is $f_{L\sigma}<0$. If $\frac{\partial L^*}{\partial G}$ and $f_{L\sigma}$ are negative, $\frac{\partial \sigma^*}{\partial G}$ must be positive from Equation (A.5) in interior equilibrium. A decreasing capacity to guarantee leads to an increase in the scale of investment and a decrease in risk.