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TEMPERATURE DEPENDENCE OF PHYSICAL PSEUDOSCALAR MESON (η, η' and ι) MASSES ALONG WITH THE CONSISTENT MASS MATRIX AND THE UNITARY MATRIX

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Taking three orthonormal basis states and constructing the Lagrangian, we have obtained the unitary transformation matrix and mass squared matrix of η, η' and ι. The temperature dependence of these pseudoscalar meson masses has been studied while getting the value of the mass of one of the basis states η_1 by taking into account the gluon condensate value which yields the pure gluonium mass estimate. Variation of their masses with the temperature has also been studied. The calculations for the radiative decays of these pseudoscalar mesons have also been carried out and the results are encouraging.

1. Introduction

The phase transition in quantumchromodynamics is a well known phenomenon [1,2] and it is argued that the Hamiltonian is chiral invariant at high temperature. Variation with temperature of the masses of the pseudoscalar mesons was studied, and it has been found that η'-mass varies more rapidly with temperature than η-mass, whereas π^0-mass remains almost constant, which indicates that η and η' are mixed states of quarkonium and gluonium, i.e., the variation of the mass of that meson is large with temperature which has large content of gluonium as its constituent.

One should emphasise that Pisarsky and Wilczek [1] had discussed $U_{L-R}(1)$ symmetry restoration at high temperature and discussed the effects for spectrum of pseudoscalar mesons $\pi^0 - \eta - \eta'$. They pointed out that the masses of the pseudoscalar mesons with higher gluonium content decrease rapidly with temperature.

Sen and Gautam [2] have shown that $U_{L-R}(1)$ breaking is associated with the emergence of the gluon enriched meson. They have shown the effect of gluon content for $\pi^0 - \eta - \eta'$ systems in their analysis.

In our approach, described in the following section, we deal with the mass term of the Lagrangian. In the process, the connection of the physical states to the pure gluonium and quarkonium states has been evaluated numerically, and out of a number of solutions obtained, the best choice has been utilised in our analysis. By this mixing process, we construct the mass square matrix.

2. *Theory*

To study the gluon content in η, η' and ι, we start with three orthonormal basis states η_8, η_1 and ι_0, where η_8 is a member of the $SU(3)$ octet, η_1 is $SU(3)$ singlet and ι_0 is two gluon colour singlet state $\mid gg >$. Thus, here the basis states are

$$\begin{aligned} \mid \eta_8 > &= \frac{1}{6} \mid \bar{u}u + \bar{d}d - 2\bar{s}s >, \\ \mid \eta_1 > &= \frac{1}{3} \mid \bar{u}u + \bar{d}d + \bar{s}s >, \\ \mid \iota_0 > &= \mid gg > . \end{aligned} \tag{1}$$

The three physical states are η, η' and ι, which are the mixed states of η_8, η_1 and ι_0. The mixing angles are the Eulerian angles and these states are related by matrix equation,

$$\begin{pmatrix} \eta \\ \eta' \\ \iota \end{pmatrix} = \begin{pmatrix} A & B & C \\ D & E & F \\ K & L & I \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_1 \\ \iota_0 \end{pmatrix}, \tag{2}$$

where

$$A = \cos\alpha\cos\beta\cos\gamma - \sin\alpha\sin\gamma,$$

$$B = \cos\beta\sin\alpha\cos\gamma + \cos\alpha\sin\gamma,$$

$$C = -\sin\beta\cos\gamma,$$

$$D = -\cos\alpha\cos\beta\sin\gamma - \sin\alpha\cos\gamma,$$

$$E = -\sin\alpha\cos\beta\sin\gamma + \cos\alpha\cos\gamma,$$

$$F = \sin\beta\sin\gamma, \tag{3}$$

$$K = \cos\alpha\sin\beta,$$

$$L = \sin\alpha\sin\beta,$$

$$I = \cos\beta.$$

For the physical masses of η, η' and ι, the Lagrangian density is of the form

$$-L_m = m_\eta^2\eta^2 + m_{\eta'}^2\eta'^2 + m_\iota^2\iota^2. \tag{4}$$

Expressing physical states in terms of the basis states from Eq. (2), we find Eq. (4) as

$$-L_m = m_\eta^2(A\eta_8 + B\eta_1 + C\iota_0)^2 + m_{\eta'}^2(D\eta_8 + E\eta_1 + F\iota_0)^2 + m_\iota^2(K\eta_8 + L\eta_1 + I\iota_0)^2, \tag{5}$$

where masses of the basis states are given by

$$m_{\eta_8}^2 = m_\eta^2 A^2 + m_{\eta'}^2 D^2 + m_\iota^2 K^2, \tag{6a}$$

$$m_{\eta_1}^2 = m_\eta^2 B^2 + m_{\eta'}^2 E^2 + m_\iota^2 L^2, \tag{6b}$$

$$m_{\iota_0}^2 = m_\eta^2 C^2 + m_{\eta'}^2 F^2 + m_\iota^2 I^2. \tag{6c}$$

Here $m_\eta, m_{\eta'}$ and m_ι are masses of η, η' and ι. We have taken $m_{\eta_8} = 0.571$ GeV [3]. So, we get the required mass square matrix as

$$M^2 = \begin{pmatrix} m_{\eta_8}^2 & m_{81}^2 & m_{8g}^2 \\ m_{81}^2 & m_{\eta_1}^2 & m_{1g}^2 \\ m_{8g}^2 & m_{1g}^2 & m_{\iota_0}^2 \end{pmatrix}. \tag{7}$$

We will later show how the temperature dependence of masses of the pseudoscalar mesons have been incorporated in our analysis. Next, we have tried to find the bound on the masses of m_{η_1} and m_{ι_0} and thus we will be able to get the unitary matrix.

3. Determination of the masses of the η_1 and ι_0 states

Equations (6a) and (6b) give

$$\begin{aligned} m_{\eta_8}^2 + m_{\eta_1}^2 = m_\eta^2 (\cos^2\alpha\, \cos^2\beta\, \cos^2\gamma + \sin^2\alpha\, \sin^2\gamma + \cos^2\beta\, \sin^2\alpha\, \cos^2\gamma \\ + \cos^2\alpha\, \sin^2\gamma) + m_{\eta'}^2 (\cos^2\alpha\, \cos^2\beta\, \sin^2\gamma + \sin^2\alpha\, \cos^2\gamma \\ + \sin^2\alpha\, \cos^2\beta\, \sin^2\gamma + \cos^2\alpha\, \cos^2\gamma) + m_\iota^2 (\cos^2\alpha\, \sin^2\beta + \sin^2\alpha\, \sin^2\beta) \\ = [(m_\eta^2 \cos^2\gamma + m_{\eta'}^2 \sin^2\gamma) - m_\iota^2] \cos^2\beta + [m_\eta^2 \sin^2\gamma + m_{\eta'}^2 \cos^2\gamma + m_\iota^2], \end{aligned} \tag{8}$$

$$\cos^2\beta = \frac{m_{\eta_8}^2 + m_{\eta_1}^2 - (m_\eta^2 \sin^2\gamma + m_{\eta'}^2 \cos^2\gamma) - m_\iota^2}{m_\eta^2 \cos^2\gamma + m_{\eta'}^2 \sin^2\gamma - m_\iota^2}. \tag{9}$$

Assuming both the numerator and denominator to be negative, we get the constraint from Eq. (9)

$$m_{\eta_1} > \ 0.95 \text{ GeV}. \tag{10}$$

Eq. (6c) may be rearranged to give

$$\cos^2\beta = \frac{(m_\eta^2 \cos^2\gamma + m_{\eta'}^2 \sin^2\gamma + m_\iota^2 - m_{\iota_0}^2) - m_\iota^2}{(m_\eta^2 \cos^2\gamma + m_{\eta'}^2 \sin^2\gamma) - m_\iota^2}. \tag{11}$$

Comparing Eqs. (9) and (11), we get

$$m_{\iota_0}^2 = 2.977 - m_{\eta_1}^2. \tag{12}$$

Therefore,

$$m_{\eta_1} < 1.725. \tag{13}$$

From Eqs. (10) and (13), we get bounds on the mass of m_{η_1} as

$$0.95 < m_{\eta_1} < 1.725. \tag{14}$$

From Refs. [4] and [5], we get

$$< 0 \mid \frac{2\pi}{3} G_a^{\mu\nu} G_{\mu\nu}^a \mid 0 >= 0.11 \text{ GeV}^4. \tag{15}$$

The decay constant of the pure gluonium state in terms of the mass of pure gluonium state and gluon condensate G_0 can be estimated from [6]

$$f_\iota = \frac{1}{m_\iota} (\frac{1}{2} b G_0)^{1/2}, \tag{16}$$

where

$$b = \frac{11}{3}N_c - \frac{2}{3}N_f,$$

N_c and N_f being the numbers of colours and flavours, respectively, and

$$G_0 = < 0 \mid G_a^{\mu\nu} G_{\mu\nu}^a \mid 0 > . \tag{17}$$

Eq. (15) provides the values of m_{ι_0} and thus fixes m_{η_1}. In this way, the condensate value provides $m_{\iota_0} = 1.43$ GeV with $f_\iota = 0.2$ GeV [7] and the mass of η_1 turns out to be 0.96 GeV. So, these two values become apparently fixed if gluon condensate is fixed at the value given above (Eq. (15)).

4. Incorporation of temperature in the mass matrix and study of its effect

Our motive is to study the temperature dependence of the masses of the pseudoscalar mesons. For this purpose, in our approach, it is essential to have a knowledge about the values of the angles α, β and γ. We have taken the angle γ as variable and solved for the angles α and β . As already explained in the earlier section, m_{η_1} turns out to be 0.96 GeV. The other unknown parameter which remains to be determined, γ, has also been found. The values of γ in the range of 35^0 to 45^0 can reproduce masses of the η, η' and ι mesons correctly when solved for mass squared matrix. So, the sets of unitary matrices have been chosen corresponding to these values of γ. We have also noted that within this range of γ values, the only choice of $m_{\eta_1} = 0.96$ GeV reproduces the correct masses of η, η' and ι. These sets are given in Table 1. From these results, it is clear that ι is gluonium dominating

TABLE 1. Sets of unitary matrices and values of Eulerian angles.

SET I $\quad \gamma = 40°,\ \alpha = 37.42°,\ \beta = 6.35°$

$$\begin{aligned} \mid \eta >&= 0.91 \mid N > +0.38 \mid S > -0.08 \mid gg > \\ \mid \eta' >&= -0.49 \mid N > +0.83 \mid S > +0.07 \mid gg > \\ \mid \iota >&= 0.11 \mid N > \pm 0.03 \mid S > +0.99 \mid gg > \end{aligned}$$

SET II $\quad \gamma = 45°,\ \alpha = 41.42°,\ \beta = 6.47°$

$$\begin{aligned} \mid \eta >&= 0.84 \mid N > +0.52 \mid S > -0.07 \mid gg > \\ \mid \eta' >&= -0.64 \mid N > +0.76 \mid S > +0.08 \mid gg > \\ \mid \iota >&= 0.11 \mid N > \pm 0.02 \mid S > +0.99 \mid gg > \end{aligned}$$

SET III $\quad \gamma = 35°,\ \alpha = 33.71°,\ \beta = 6.25°$

$$\begin{aligned} \mid \eta >&= 0.99 \mid N > +0.24 \mid S > -0.08 \mid gg > \\ \mid \eta' >&= -0.33 \mid N > +0.89 \mid S > +0.06 \mid gg > \\ \mid \iota >&= 0.10 \mid N > \pm 0.03 \mid S > +0.99 \mid gg > \end{aligned}$$

state though η' is enriched with gluonium, but percentage of gluonium in η' state is less than that of ι state. In order to study the present problem, we use the following mass matrix, which is a generalization of the quarkonium mass matrix of De Rujula, Georgi and Glashow [8] and Isgur [9] and several other authors [10–12]. We have used this to discuss the 0^- $\eta - \eta' - \iota$ mixing. The mass square matrix is

$$M^2 = \begin{pmatrix} m_{88}^2 + \lambda_{88} & \sqrt{2}\lambda_{81} & \sqrt{2}\lambda_{8g} \\ \sqrt{2}\lambda_{81} & m_{11}^2 + \lambda_{11} & \lambda_{1g} \\ \sqrt{2}\lambda_{8g} & \lambda_{1g} & m_g^2 + \lambda_{gg}, \end{pmatrix} \tag{18}$$

which is same as given in Eq. (7), where the parameters $\lambda_{88}, \lambda_{11}$ and λ_{81} denote the interaction effects between (8,1) and $(8,1).\lambda_{8g}, \lambda_{1g}$ are the interaction effects between (8,1) and G. The parameter λ_{gg} denotes the possible interaction between gg. The mass of the pure glueball is denoted m_g. Following Wilczek and Pisarsky [1], the temperature dependence of the mass matrix has been incorporated by adding the temperature dependent term P in the (3,3) elements of the mass matrix given in Eq. (18) related to the anomaly content appearing in the theory. From Sen and Gautam [2], we get the functional form of P by defining $P = C' - C_0$, where $C' = \{\cosh(\beta M_P) - 1\} M_P^2$[2], and $\beta = 1/T$, where T is the temperature and M_P is the pseudoscalar meson mass. For the tuning of the zero temperature, masses of η, η' and ι (i.e., their physical masses) our constant C_0 turns out to be 1.95 GeV^2. The function $[\cosh(M_P\beta - 1)M_P^2]$ decreases with the increase of the temperature (given in Fig. 2) and finally vanishes at high temperature which signifies the effective restoration of the $U_{L-R}(1)$ symmetry. We are now examining the nature of the variation of the masses of η, η' and ι with temperature. Introduction of P gives

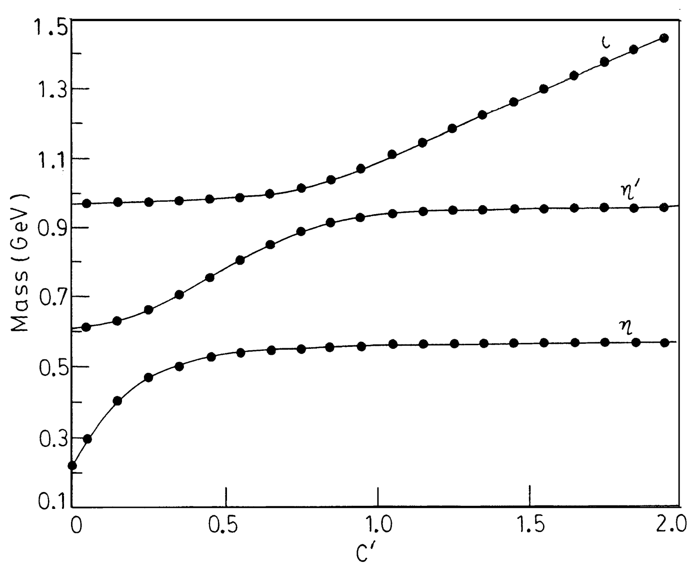

Fig. 1. Variation of pseudoscalar meson masses with C'.

us a clear picture on the variation of these pseudoscalar masses with temperature (Fig.1). We have plotted the masses of the pseudoscalar mesons against C'. We observe that for $C' > 1, \iota$ mass increases much rapidly than the masses of η' and η. On the other hand, when $C' < 1$, increasing rate of $m_{\eta'}$ is quite large. At $C' = 0$, we get $m_{\eta'} = 0.606$ GeV, $m_\eta = 0.213$ GeV and $m_\iota = 0.969$ GeV. Below $C' = 0.5, m_\eta$ decreases rapidly, but m_ι remains nearly constant. At $C' = 1.95$, $m_{\eta'} = 0.958$ GeV, $m_\eta = 0.558$ GeV and $m_\iota = 1.46$ GeV.

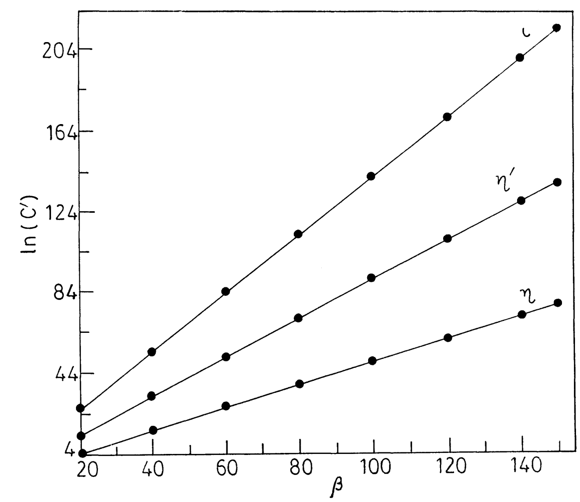

Fig. 2. Variation of C' with temperature.

Now, we observe the variation of the function C' with temperature (Fig. 2). The nature of the graph (Fig. 1) depicts η' as the closest gluon enriched candidate to be identified next to ι amongst the pseudoscalar mesons.

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TEMPERATURNA OVISNOST MASA FIZIČKIH PSEUDOSKALARNIH MEZONA (η, η' i ι) UZ KONZISTENTNU MATRICU MASE I UNITARNU MATRICU

Polazeći od tri ortonormirana osnovna stanja i konstruiranog lagranžijana, izveli smo unitarnu pretvorbenu matricu i matricu kvadratnih masa mezona η, η' i ι. Proučavali smo temperaturnu ovisnost masa tih pseudoskalarnih mezona. Dobili smo vrijednost mase jednog od osnovnih stanja η_1 uzimajući u obzir iznos gluonskog kondenzata koji daje čistu ocjenu mase gluonijuma. Promjene njihove mase s temperaturom smo također proučili. Izračunali smo i radijativni raspad tih pseudoskalarnih mezona i rezultati su ohrabrujući.