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FLUCTUATIONS IN SPIN PRECESSION PHENOMENA DRIVEN BY DISCRETE TIME JUMPS ON A RANDOM GRAPH

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By allowing the states of a spin system to be connected by a graph in discrete time, we demostrate that discrete time ”paths” between vertices (representing the states of the spin system) can induce fluctuations in the probability of the states that in turn generate fluctuations in x spin polarization of a spin system.

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1. Introduction

It is widely agreed upon that, because of the incomplete nature of the quantum theory with its non-causal, nondeterministic, and non-local structure, very primitive foundations are lacking in its formulation [1–3]. The search for a viable ”hidden variable theory” to date has also met with an avalanche of criticism [4,5] and many students of quantum theory have chosen to accept the quantum principles as empirical principles of the sub-atomic world [6]. From a quite another direction, in gravitational physics, it has been demonstrated that because of the fact that the path integral in Euclidean gravity is not bounded from below when space-time develops wormholes and non-trivial topologies at the Planck scale, the continuum of space-time should be replaced by a more fundamental discrete combinatoric structure [7–10]. For both of the above reasons, numerous authors have suggested that both Riemannian geometry and gauge theory should be derived from concepts emerging from discreteness and combinatorics with the guidelines of topology leading to an ultimate structure [11–13]. Wheeler has elaborated on these ideas [14,15] suggesting that a set of points and possible links between them should form the basis of a theory of pregeometry. Finkelstein [16] has developed similar ideas, and Wootters [17], inspired by the notion of a ”Penrose spin network” [18], has developed a theory of space-time based on quantum correlations between fundamental spins in Hilbert space. Somewhat related to these ideas, Caldirola [19,20] long ago proposed the notion of discrete time differences in quantum theory, and Recami [21] has interpreted this as an expression of the fact that in the beginning there was no notion of space-time and only individual particles were the primitive entities, and after quantum correlations and thermal averaging, Minkowski space was born, but fluctuations in the latter universe could still be expected away from Minkowski space-time. We have applied discrete-time quantum theory to electron-spin polarization precession [22], electron-spin resonance [23], to spectral shifts in the hydrogen spectrum [24] and to the internal transitions of ”thought to be” elementary particles [25]. Following these developments, it was recognized that ”discrete time jumps” might be induced by the environment in the spirit of the ”Procrustean principle” expounded by Nanopoulous [26]. The basic idea here is that due to the truncation of non-local string modes, a system (local modes or particles) is in constant interaction with the non-local modes that represent the environment. These ideas lead to an arrow of time in cosmology and to small CPT violations in the K^0 - $\overline{K^0}$ system [27,28]. Encouraged by these environmental effects, we have introduced Markov jump-processes into the physics of quantum spin phenomena and pointed out that a study of the short-time behavior of a spin-precessing particle in a magnetic field might reveal the presence of Markov discrete time jumps [29–32]. In a separate note, we also studied the behavior of a composite particle under the influence of Markov effects and demonstrated that short-time chaotic fluctuations in the spin-precession amplitude might not only reveal discrete time jumps but would also provide us with a window through which to study the composite structure of elementary particles [33].

In the following note, we carry these studies a step further by proposing that quantum jumps between individual spin states are a result of ”walks” on a random graph wherein the vertices represent the spin states and the edges represent the junction between the states. Using principles of combinatorics and graph theory [34,35], we then calculate the fluctuations in the probability of the various spin components induced by ”discrete time walks” between the vertices of the graph (states). Such a model is in accord with a pregeometric picture of space-time and could provide us with a fundamental picture of quantum transitions that relies only on the structure of connected graphs and fundamental probabilities assigned to each edge (link). It is also hoped that such a picture might pave the way to a fresh approach to ”Hidden Variable Theory” where the ”hidden variables” are the inner vertices and edges of a graph with the outer vertices representing the quantum states in the phenomenological world.

2. Discrete time induced transitions and paths on random graphs

To begin the analysis, we briefly review the approach taken in Ref. 30 to study the precession of a spin 1 ($q = -e$) gauge boson in a z-component magnetic field. For the hamiltonian we have

$$H = M_w c^2 + \frac{e}{M_w} S_z B. \tag{1}$$

The eigenstates and eigenvalues are

$$\begin{aligned} \Psi_+ &= U_+, \qquad E_+ = M_w c^2 + \frac{e}{M_w}\hbar B, \\ \Psi_- &= U_-, \qquad E_- = M_w c^2 - \frac{e}{M_w}\hbar B, \\ \Psi_0 &= U_0, \qquad E_0 = M_w c^2. \end{aligned} \tag{2}$$

For the spin 1 operators S_x and S_z, we have

$$S_x = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \qquad S_z = \hbar \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix},$$

with a state initially polarized in the x-direction ($\langle S_x \rangle = \hbar$) of

$$\Psi = \frac{1}{2}\Psi_+ + \frac{1}{\sqrt{2}}\Psi_0 + \frac{1}{2}\Psi_-$$

For the time dependent state that gives ($\langle S_x \rangle_{t=0} = \hbar$) we have

$$\Psi = \begin{pmatrix} \frac{1}{2}\exp(-\frac{iE_+}{\hbar}t) \\ \frac{1}{\sqrt{2}}\exp(-\frac{iE_0}{\hbar}t) \\ \frac{1}{2}\exp(-\frac{iE_-}{\hbar}t) \end{pmatrix}, \tag{3}$$

giving at time t, $\Psi^+ S_x \Psi = \hbar \cos \frac{eB}{M_w} t$. In Ref. 30 we then modified Eq. (3) to read

$$\Psi = \begin{pmatrix} \sqrt{P(+)_n}\ \exp(-\frac{iE_+}{\hbar}t) \\ \sqrt{P(0)_n}\ \exp(-\frac{iE_0}{\hbar}t) \\ \sqrt{P(-)_n}\ \exp(-\frac{iE_-}{\hbar}t) \end{pmatrix}, \tag{4}$$

In Eq. (4), $P(+)_n, P(-)_n$ and $P(0)_n$ represent the probabilities for the three states after n steps generated by a Markov jump process with transition matrix

$$\begin{array}{c} \\ + \\ 0 \\ - \end{array}\begin{array}{c} \begin{array}{ccc} + & 0 & - \end{array} \\ \begin{pmatrix} 1-q-q^2 & q & q^2 \\ p & 1-p-q & q \\ p^2 & p & 1-p-p^2 \end{pmatrix} \end{array}. \tag{5}$$

Here p is the probability of jump from (– to 0) or (0 to +), p^2 is the probability of jump from (– to +), q is the probability of jump from (+ to 0) or (0 to –) and q^2 represents probability of jump from (+ to –)) and initial probabilities are $P(+)_0 = \frac{1}{4}, P(0)_0 = \frac{1}{2}$ and $P(-)_0 = \frac{1}{4}$.

To calculate the x spin polarization, we have using Eq. (4)

$$\Psi^+ S_x \Psi = \frac{\hbar}{\sqrt{2}}(2\sqrt{P(+)_n P(0)_n} + 2\sqrt{P(-)_n P(0)_n})\ \cos \frac{eB}{M_w} t. \tag{6}$$

We now propose an alternative mechanism to calculate $P(+)_n, P(0)_n$ and $P(-)_n$, using random graphs. Let us consider the graphs in Fig. 1.

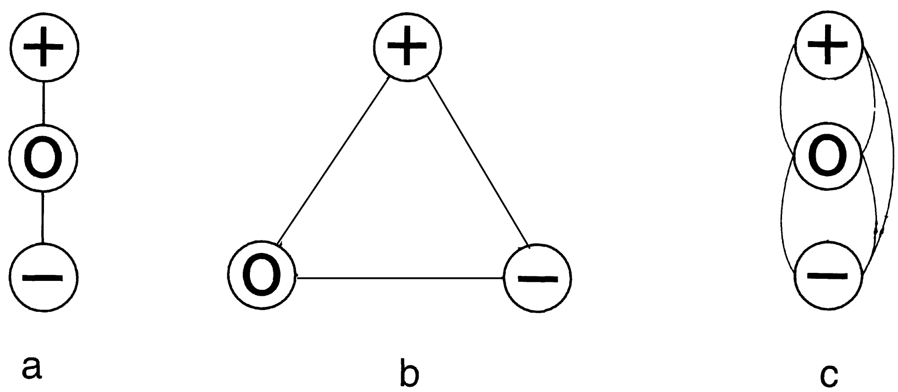

Fig. 1. Random graphs representing the links (edges) between states +, 0 and – of a spin-one system.

The adjacency matrices for graphs (a), (b), (c), respectively, are (Refs. 34, 35)

$$\begin{array}{c} \\ + \\ 0 \\ - \end{array}\begin{array}{c} \begin{array}{ccc} + & 0 & - \end{array} \\ \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \end{array} \quad \begin{array}{c} \\ + \\ 0 \\ - \end{array}\begin{array}{c} \begin{array}{ccc} + & 0 & - \end{array} \\ \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \end{array} \quad \begin{array}{c} \\ + \\ 0 \\ - \end{array}\begin{array}{c} \begin{array}{ccc} + & 0 & - \end{array} \\ \begin{pmatrix} 0 & 2 & 1 \\ 2 & 0 & 2 \\ 1 & 2 & 0 \end{pmatrix} \end{array}. \tag{7}$$

(a) (b) (c)

In Fig. 1, the vertices represent the states +, 0, – of the spin 1 system, and the edges represent the links between the vertices (or states). For each matrix M the element i, j of the matrix M^n represents the number of walks from the vertex i to the vertex j in n discrete time steps. For example, for the case (a)

$$M^3 = \begin{array}{c} \\ + \\ 0 \\ - \end{array} \begin{array}{c} \begin{array}{ccc} + & 0 & - \end{array} \\ \begin{pmatrix} 0 & 2 & 0 \\ 2 & 0 & 2 \\ 0 & 2 & 0 \end{pmatrix} \end{array} . \tag{8}$$

For the matrix in Eq. (8), there would be two walks from 0 to + in 3 discrete time steps, etc. We now assign a probability p for each edge and write for the probability of each state (i) after n discrete time steps

$$P_i(n) = P_i(0) - \sum_{j \neq i} P_i(0) p^n k_{ij}(n) + \sum_{j \neq i} P_j(0) p^n k_{ij}(n), \tag{9}$$

($P_i(0)$ are the initial probabilities).

In Eq. (9), the first term on the right is the probability at $n = 0$, the second term represents the ”flow out” of probability from vertex i where $k_{ij}(n)$ represents the number of ”discrete time walks” from i to j in n discrete time steps, p^n represents the probability for n edges or links. The third term on the right of Eq. (9) represents the ”flow in” of probability to vertex i from all of the other vertices. Note $k_{ij}(n)(k_{ij} = k_{ji})$ is given by the ij element of the matrix M^n. To calculate the specific value of $P_i(n)$, we consider case (b) in Fig. 1. The initial probabilities are (note the equivalence of the notation $P(+)_n = P_+(n) \ldots$ throughout)

$$P_+(0) = \frac{1}{4}, P_-(0) = \frac{1}{4} \quad \text{and} \quad P_0(0) = \frac{1}{2}.$$

From Eq. (3), for the case (b) after n steps we have

$$M^n = \begin{pmatrix} \frac{1}{3}2^n + \frac{2}{3}(-1)^n & \frac{1}{3}2^n - \frac{1}{3}(-1)^n & -- \\ -- & -- & -- \\ -- & -- & -- \end{pmatrix} . \tag{10}$$

In Eq. (10), we have symmetry about the diagonal and all the diagonal elements are equal. From Eq. (9), we have after n discrete time steps

$$P_+(n) = \frac{1}{4} + \frac{p^n}{4}\left(\frac{1}{3}2^n - \frac{1}{3}(-1)^n\right), \tag{11}$$

and

$$P_-(n) = \frac{1}{4} + \frac{p^n}{4}\left(\frac{1}{3}2^n - \frac{1}{3}(-1)^n\right), \tag{12}$$

For $P_0(n)$ we have

$$P_0(n) = \frac{1}{2} - \frac{p^n}{2}\left(\frac{1}{3}2^n - \frac{1}{3}(-1)^n\right)^2 2 + \left(\frac{1}{4}+\frac{1}{4}\right)p^n\left(\frac{1}{3}2^n - \frac{1}{3}(-1)^n\right)^2,$$

$$P_0(n) = \frac{1}{2} - \frac{p^n}{2}\left(\frac{1}{3}2^n - \frac{1}{3}(-1)^n\right). \qquad (13)$$

We note from Eq. (4), Eq. (6) and the values for $P_0(n)$, $P_+(n)$ and $P_-(n)$, calculated for the case (b) that

$$\langle S_x \rangle = \hbar\sqrt{1 - p^n\left(\frac{1}{3}2^n - \frac{1}{3}(-1)^n\right)^2}\,\cos\frac{eB}{M_w}t. \qquad (14)$$

For $n \to \infty$ we note that $\langle S_x \rangle$ approaches its usual value without discrete time jumps. We also note from Eqs. (11), (12) and (13) that for all n, $P_0(n) + P_+(n) + P_-(n) = 1$, as it must be to conserve the probability. In Eq. (14), we have simply substituted into Eq. (6) the probabilities calculated from the graph in the case (b) using the combinatoric rules for calculating the probabilities in Eq. (9), based on the number of walks between vertices after n discrete time steps. Each graph in Fig. 1 would generate a different function to be substituted into Eq. (6).

We now consider a two preon composite (Ref. 30) of a spin 1 gauge boson ($q = -1$) with the following basic states of product $1/2 \otimes 1/2$ (α is spin up function and β is spin down function). In Fig. 2, we have the possible graph for connecting edges (links) between vertices (states).

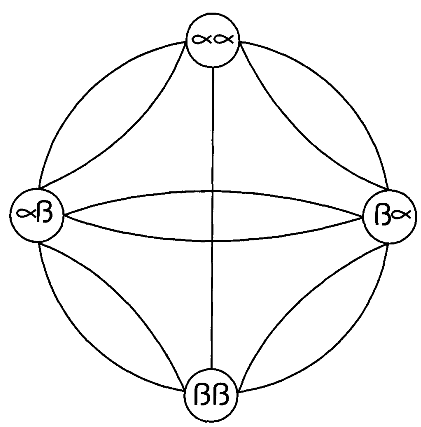

Fig. 2. A graph for connecting links (edges) between vertices (states) of a two-preon composite of a spin 1 gauge boson.

For the adjacency matrix of Fig. 2, for the 4 vertices (states) and 11 edges we have

$$M = \begin{array}{c} \\ \alpha\alpha \\ \alpha\beta \\ \beta\alpha \\ \beta\beta \end{array} \begin{array}{c} \begin{array}{cccc} \alpha\alpha & \alpha\beta & \beta\alpha & \beta\beta \end{array} \\ \left(\begin{array}{cccc} 0 & 2 & 2 & 1 \\ 2 & 0 & 2 & 2 \\ 2 & 2 & 0 & 2 \\ 1 & 2 & 2 & 0 \end{array} \right) \end{array} . \qquad (15)$$

The wave function for the two fermion configurations from (Ref. 30) is (with no discrete-time effects)

$$\Psi = \frac{\alpha\alpha}{2} \exp\left(-\frac{\mathrm{i}E_+}{\hbar}t\right) + \frac{\beta\beta}{2} \exp\left(-\frac{\mathrm{i}E_-}{\hbar}t\right) + \frac{\alpha\beta + \beta\alpha}{2} \exp\left(-\frac{\mathrm{i}E_0}{\hbar}t\right) . \qquad (16)$$

Here the initial probabilities are $P_{++}(0) = P_{--}(0) = P_{+-}(0) = P_{-+}(0) = 1/4$. This gives

$$\langle S_x \rangle = \hbar \cos \frac{eB}{M_w} t.$$

Here $e_p = e/2$ is the preon charge, $m_p = M_w/2$ is the preon mass. We modify Eq. (16) to take into account the modified probabilities shown in Fig. 2. Thus

$$\begin{aligned} \Psi \quad &= \sqrt{P_{++}(n)}\ \exp\left(-\tfrac{\mathrm{i}E_+}{\hbar}t\right)\alpha\alpha + \sqrt{P_{--}(n)}\ \exp\left(-\tfrac{\mathrm{i}E_-}{\hbar}t\right)\beta\beta \qquad (17) \\ &+ \left(\sqrt{P_{+-}(n)}\ \alpha\beta + \sqrt{P_{-+}(n)}\ \beta\alpha\right) \exp\left(-\tfrac{\mathrm{i}E_0}{\hbar}t\right) \end{aligned}$$

Here $P_{+-}(n) = P_{-+}(n), \alpha$ is spin up function and β is spin down function. If we evaluate $\langle S_x \rangle$, using S_x in matrix notation as

$$S_x = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} ,$$

and Eq. (17) for Ψ, we find

$$\langle S_x \rangle = \hbar \left(2\sqrt{P_{++}(n)P_{+-}(n)} + 2\sqrt{P_{--}(n)P_{-+}(n)} \right) \cos \frac{eB}{M_w} t. \qquad (18)$$

From the graph in Fig. 2 we see that since the initial probabilities of all states ($\alpha\alpha, \beta\beta, \alpha\beta$ and $\beta\alpha$) are the same ($P = 1/4$), so when we evaluate Eq. (9), we will just get the discrete time independent initial probabilities. To generate probabilities that depend on n, we would have to skew certain "walks" using different p values. For a composite particle in a z-component magnetic field, we could choose p_1 for walks $\beta\beta \to \alpha\beta, \beta\beta \to \beta\alpha$ and $\beta\beta \to \alpha\alpha$, and p_2 for all other one-step walks between vertices. Such a choice would suggest that in high B fields for $p_1 > p_2$, there is a preferred probability for jumping to higher energy levels. If we label the probabilities of the states as $P_{++} = P_1, P_{+-} = P_2, P_{-+} = P_3$ and $P_{--} = P_4$, we have from Eq. (9)

$$P_4(n) = \frac{1}{4} + \frac{1}{4}\left[\begin{array}{c} -p_1^n\left(k_{42}(n) + k_{43}(n) + k_{41}(n)\right) \\ +p_2^n\left(k_{42}(n) + k_{43}(n) + k_{41}(n)\right) \end{array}\right]$$

, and also

$$P_2(n) = P_3(n) = \frac{1}{4} + \frac{1}{4}\left(p_1^n - p_2^n\right)k_{42}(n), \qquad (19)$$
$$P_1(n) = \frac{1}{4} + \frac{1}{4}\left(p_1^n - p_2^n\right)k_{41}(n).$$

Here $k_{42} = k_{24}, k_{43} = k_{34}$, etc., and we assume k_{ij} means for n steps. From the adjacency matrix in Eq. (15), $k_{12}(1) = k_{21}(1) = 2,\ k_{14}(1) = k_{41}(1) = 1$, etc.

For M^2 we have

$$M^2 = \begin{pmatrix} 9 & 6 & 6 & 8 \\ 6 & 12 & 8 & 6 \\ 6 & 8 & 12 & 6 \\ 8 & 6 & 6 & 9 \end{pmatrix}$$

and $k_{12}(2) = k_{21}(2) = 6,\ k_{14}(2) = k_{23}(2) = k_{32}(2) = 8$, etc. Here $k_{12}(2)$ means that there are 6 walks between vertex 1 and 2 in two discrete time steps. When the values for $P_1(n), P_2(n) = P_3(n)$ and $P_4(n)$ from Eq. (19) are substituted into Eq. (18) we obtain a formula for the variation of $\langle S_x \rangle$ with n and t for a composite spin 1 gauge boson using the graphs in Fig. 2. We note that all of the $k_{ij}(n)$ values would have to be calculated from M^n which would either involve multiplying out M^n or finding the difference equation for each element and solving as we did for Fig. 1 (case b).

3. Conclusion

In the above analysis, we have demonstrated that small variations of x spin polarization with a discrete index n will be a signal of quantum transitions induced by a random graph connecting quantum states. This approach is an additional mechanism to generate fluctuations in the spin polarization. In Refs. 29 – 33 we studied Markov mechanisms to generate these fluctuations, and the two separate mechanisms will lead to distinct signatures for small time variations of $\langle S_x \rangle$. The idea of a random graph could also be used to discuss a new approach to ”hidden variables” where summation over the inner vertices and inner links could generate the ”dispersion” [36] for the exterior vertices which represent the observable quantum states. Another interesting consequence of the calculation based on Fig. 2 is that if there is no skewing of the ”walks” in the graph of Fig. 2, our analysis would lead to constant probabilities which suggests that a lack of variation of $\langle S_x \rangle$ with n would be evidence of the composite structure of the gauge bosons.

In closing, we mention two final implications of the above analysis. Firstly, in Fig. 2, the states $\alpha\beta$ and $\beta\alpha$ are considered different, and by having more edges connecting then, we might construct a random graph interpretation of the exclusion principle. In this sense the exclusion principle might be a manifestation of the symmetry of a random graph [37–39]. Whatever the origin of the exclusion principle [40], any measurements that reveal the presence of the individual states $\alpha\beta, \beta\alpha$ would suggest that there must be a dynamical mechanism generating symmetry or anti-symmetry for identical particles and thus the principle can be derived. Also, with respect to cosmology, Nagels [41] has derived the three-dimensional "closed" structure of space-time using a random graph of points by assigning different probabilities for their connection and then maximizing the probability. Such a study suggests that the theory of "walks" on a random graph may not only resolve many of the problems of quantum theory, but may lead to a ultimate pregeometric origin of space-time and gravitation.

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FLUKTUACIJE PRI POJAVAMA PRECESIJE SPINA UZROKOVANE DISKRETNIM VREMENSKIM SKOKOVIMA NA NASUMNOM GRAFU

Kada se dozvoli vezanje stanja spinskog sustava grafom u diskretnom vremenu, nalazi se da diskretni vremenski "putevi" medu vrhovima (koji predstavljaju stanja spinskog sustava) mogu izazvati fluktuacije vjerojatnosti stanja, koje pak mogu uzrokovati fluktuacije x komponente spina spinskog sustava.