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LETTER TO THE EDITOR

COULD NEUTRINO FLAVOURS BE SUPERPOSITIONS OF MASSIVE, MASSLESS AND TACHYONIC STATES?

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It is suggested that neutrino flavours might result from linear unitary superpositions of mass eigenstates of different types: massive, massless and tachyonic. The number of flavours (three) appears to be prescribed at the onset of the formulation.

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In a previous paper [1], a modified Dirac equation with two mass parameters was discussed. Under the constraint that one of the two parameters be non-vanishing, a 12-spinorial equation was derived [1], which describes three possible mass states of a particle: massive, left-handed massless and tachyonic. This equation may be cast in the form

$$\mathrm{i}\,\mathbf{\Gamma}^{\alpha}\partial_{\alpha}\Psi(x) = \mathbf{M}\Psi(x). \tag{1}$$

Here, units are such that $\hbar = c = 1$. The relevant notation and conventions are spelled out in the following.

(i) Greek (Latin) indices run through the values 0,1,2,3 (1,2,3), unless otherwise indicated. The summation convention is applied to repeated up and down indices.

(ii) The frame of reference $\mathcal{X}$ of the real space-time coordinates $x = \{x^\mu\}$ has pseudoeuclidean metric $g^{\mu\nu} = \mathrm{diag}\{+1, -1, -1, -1\}$. The "space-index" S and the "time-index" T of $\mathcal{X}$ are so defined [1]: $S = 0$ if $s = \{x^k\}$ is a right-handed triplet ($S = 1$ otherwise), and $T = 0$ if $t = x^0$ runs forward ($T = 1$ otherwise).

(iii) The 4×4 Dirac matrices γ^μ (in a fixed chosen representation) obey the usual rules

$$\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2g^{\mu\nu} I, \qquad \text{and} \qquad (\gamma^\mu)^\dagger = \gamma^0 \gamma^\mu \gamma^0, \tag{2}$$

where I is the 4×4 identity matrix. The Dirac matrix $\gamma^5 = \mathrm{i}\, \gamma^0 \gamma^1 \gamma^2 \gamma^3$ is hermitian and unitary, and anticommutes with all γ^μ.

(iv) For any 4×4 Dirac matrix γ, the corresponding 12×12 notation (in 4×4 blocks) is as follows [1]:

$$\Gamma = \mathrm{diag}\{\gamma, \gamma, \gamma\}. \tag{3}$$

The 12-spinorial notation is as follows [1]:

$$\Psi(x) = \mathrm{column}\{\psi_{-1}(x), \psi_0(x), \psi_1(x)\}, \tag{4}$$

where each $\psi_\sigma(x)$ ($\sigma = -1, 0, 1$) is a complex four-spinor.

(v) In 4×4 blocks, the 12×12 matrix $\mathbf{M}$ at the right-hand side of Eq. (1) has the structure [1]

$$\mathbf{M} = \mathrm{diag}\left\{ mI, \frac{m}{2}(I - \epsilon\gamma^5), -m\epsilon\gamma^5 \right\}, \tag{5}$$

where $\epsilon = (-1)^{T+S}$ and $m > 0$.

Equation (1) decouples into three equations on the four-spinors $\psi_\sigma(x)$:

$$\mathrm{i}\, \gamma^\alpha \partial_\alpha \psi_{-1}(x) = m\psi_{-1}(x), \tag{6}$$

$$\mathrm{i}\, \gamma^\alpha \partial_\alpha \psi_0(x) = \frac{m}{2}(I - \epsilon\gamma^5)\psi_0(x), \tag{7}$$

$$\mathrm{i}\, \gamma^\alpha \partial_\alpha \psi_1(x) = -m\epsilon\gamma^5 \psi_1(x). \tag{8}$$

Each spinor $\psi_\sigma(x)$ is a mass eigenstate. That is, an eigenstate of the operator $\mathcal{P}^2 = -\partial_\alpha g^{\alpha\beta} \partial_\beta$ for the eigenvalue $-\sigma m^2$. Thus, if m were the electron mass, Eq. (6) would describe electrons, Eq. (7) would describe massless electron neutrinos and Eq. (8) would describe hypothetical tachyonic electrons. This is how Eq. (1) was meant to be interpreted in Ref. 1. In fact, Eq. (7) was later discussed in more details in Ref. 2, but from the same viewpoint: it does provide a consistent formalism for the study of massless left-handed neutrinos (and this modified massless Dirac equation [1] does not generate a "superfluous" conserved right-handed current).

Since neutrinos (of the three flavours) are regarded as massless by some, massive by others and tachyonic in recent experiments [5], I would like to point out how the aim of Eq. (1) may be changed, in order to accommodate these findings and possibilities. Specifically, Eqs. (6–8) can be re-interpreted as describing three types of neutrinos, each being a mass eigenstate (massive, left-handed massless and tachyonic). Flavour eigenstates of the three known flavours (electron, muon, tau) could result as linear unitary superpositions of the mass eigenstates, in a way that is similar to Pontecorvo's treatment of neutrino oscillations [6]. Thus, if the symbol $\phi_\omega(x)$ denotes the four-spinors representing the flavour eigenstates ($\omega = -1, 0, 1$), one would have

$$\phi_\omega(x) = u_\omega^\sigma \psi_\sigma(x), \tag{9}$$

where u_ω^σ is some $\mathbf{3} \times \mathbf{3}$ unitary matrix of constant coefficients (the "mixing" parameters). In the 12-spinorial notation, this is equivalent to

$$\Phi(x) = \mathbf{U}\Psi(x), \tag{10}$$

with $\mathbf{U}$ being the unitary 12×12 matrix formed by the 4×4 blocks $u_\omega^\sigma I$. The resulting equation of motion for $\Phi(x)$ is as follows:

$$\mathrm{i}\, \Gamma^\alpha \partial_\alpha \Phi(x) = \mathbf{F}\Phi(x), \qquad \text{where} \qquad \mathbf{F} = \mathbf{UMU}^\dagger. \tag{11}$$

In principle, in the framework of Eqs. (9) and (10), each neutrino flavour would have three possible outcomes for measurements of the squared four-momentum: $m^2, 0$ and $-m^2$ (each with a certain probability depending on the values of the relevant mixing coefficients). Also, according to Eq. (11), flavour oscillations should occur during propagation. The experimental input would be needed to determine both the essential mixing parameters and m. On the other hand, the number of flavours appears to be clearly and uniquely prescribed.

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DA LI SU NEUTRINSKI OKUSI LINEARNA KOMBINACIJA MASENIH, BEZMASENIH I TAHIONSKIH STANJA?

Predlaže se da bi neutrinski okusi mogli biti linearna unitarna kombinacija svojstvenih stanja različitih vrsta: masenih, bezmasenih i tahionskih. Čini se da je broj okusa (tri) određenim polaznom formulacijom.