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SO(10) MODEL OF GUT, PSEUDO - DIMENSION RULE AND THE DECAY $K_L^0 \to \mu\bar{e}$

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The flavour non-diagonal decay $K_L^0 \to \mu\bar{e}$, predicted by the SO(10) GUT model to occur at the M_C scale, has not been seen in experiments. The limit on the branching ratio is $B(K_L^0 \to \mu\bar{e}) < 3.3 \times 10^{-11}$. In the model, M_C turns out to be undetectably large. Its version based on D parity, however, admits a much smaller value for M_C which is detectable. For this reason, as argued in the literature, $B(K_L^0 \to \mu\bar{e})$ can be pushed to the level $\approx 10^{-12}$ in the context of this version. This argument, as pointed out in this paper, ignores the bearing of the pseudo-dimension rule on the decay concerned which is forbidden and as such suppresssed by this rule. As a necessary outcome of this suppression, $B(K_L^0 \to \mu\bar{e})$ is destined to lie far below the level of 10^{-12} and, accordingly, the prospect of experimental detection of this decay turns out to be much less than what is conventionally expected. This paper, therefore, concludes that the non-observation of this decay does not disrepute the SO(10) GUT model with D parity.

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1. Introduction

The SO(10) model [1-5] of GUT predicts a lot of new physics if its spontaneous symmetry breaking is assumed to proceed via the left-right symmetric path $SU(4)_C \times SU(2)_L \times SU(2)_R$, which is one of the two maximal subgroups of the SO(10) group. This path gives rise to three intermediate mass scales (IMSs), namely M_C, M_{W_R} and M_{Z_R}, lying between the unification scale M_U and M_W scale with $M_U \geq M_C \geq M_{W_R} \geq M_{Z_R} \geq M_W$. The new physics, which occurs in the scenario of the model concerned, owes to these IMSs for its genesis.

The same model, however, predicts the existence of an additional IMS if the discrete symmetry D parity [5-7] (which is a two-element subgroup of the SO(10) group) is explicitly taken into account, provided the symmetry breaking of this model is assumed to take place via the path $D\times$ SU(4)$_C\times$ SU(2)$_L\times$ SU(2)$_R$. The new scale M_D is introduced in such a way that $M_D \geq M_C$. Apart form its important consequences [5–7], its introduction brightens the prospects of experimental verifications of the SO(10)-predictions relating to new physics as the other three IMSs are lowered because M_D is sandwiched between M_U and M_C. In passing, we also note that the SO(10) model fails to populate the region lying between M_U and M_W with new physics if its spontaneous symmetry breaking is assumed to proceed via the left-right asymmetric path described by its other maximal subgroup SU(5) $\times$ U(1).

In this paper we shall be interested in the decay $K_L^0 \to \mu\bar{e}$ which, being a flavour non-diagonal process, is strictly forbidden by the standard model, i.e., SU(3)$_C\times$ SU(2)$_L\times$ U(1)$_Y$. This decay, therefore, is important in its own right as it carries the signature of the new physics associated with the M_C scale at which this decay is predicted to occur by the SO(10) model. As is well known, the M_C scale arises due to spontaneous breaking of SU(4)$_C$ symmetry to SU(3)$_C\times$ U(1)$_{B-L}$ symmetry. The crucial role played by the SU(4)$_C$ symmetry in the context of the decay $K_L^0 \to \mu\bar{e}$ is reflected in the fact that quark-lepton partial unification is described by this symmetry which treats L and as such $(B-L)$ as the fourth colour degree of freedom [8], L and B denoting lepton number and baryon number, respectively. This symmetry admits the conversion of the quarks comprising K_L^0 into the leptons necessary for the occurence of the decay $K_L^0 \to \mu\bar{e}$. This decay is a gauge mediated process, as is evident from the tree level Feynman diagram shown in Fig. 1, underlying this decay. It is also transparent from this diagram that the gauge bosons X must be leptoquark bosons. Furthermore, as this decay is predicted to take place at the M_C scale, the coloured gauge bosons X belong to the set of the fifteen gauge bosons of the SU(4)$_C$ symmetry. These X bosons, carrying electric charge $Q = 2/3$, are SU(2)$_L$ singlets and constitute a colour triplet. As the SU(4)$_C$ is actually responsible for the existence of the decay $K_L^0 \to \mu\bar{e}$, this decay is also predicted by any model in which this symmetry is embedded. It is worthwhile mentioning in this context that this decay is also described by horizontal symmetry [9,10].

In order to prepare the necessary background for our motivation in this paper, we proceed by noting that the following expression for the branching ratio of the decay $K_L^0 \to \mu\bar{e}$ has been obtained [5,6] with the help of the SO(10) GUT model:

$$B(K_L^0 \to \mu\bar{e}) \approx 10^{-8}\left(\frac{100\ \text{TeV}}{M_C(\text{in TeV})}\right)^4 .$$

This expression reveals clearly that the value of the branching ratio of the decay $K_L^0 \to \mu\bar{e}$ is significantly enhanced in the SO(10) GUT model with D parity relative to the same in the context of the SO(10) model without D parity as the value of M_C is much less in the former model compared to its value in the latter model. It is worth mentioning here that the decay considered has not been observed in experiments, but $B(K_L^0 \to \mu\bar{e}) < 3.3\times 10^{-11}$. This empirical fact in turn implies $M_C > 2\times 10^2$ TeV. Keeping these two bounds in mind, and by choosing a suitable value of $M_C(\approx 10^3$ TeV, which is detectable), one can obtain a value for $B(K_L^0 \to \mu\bar{e})$ as high as $\approx 10^{-12}$ within the framework of the SO(10) GUT model with

D parity, as argued in Ref. 5. The important point to be noted here is that this argument is based on the implicit assumption that the decay $K_L^0 \to \mu\bar{e}$ is favoured by all selection rules relevant for the M_C scale and as such operative in this decay. This is, however, not really the case because this decay is forbidden by the pseudo-dimension rule (PD rule) [11–22] which, for the reason pointed out in Sect. 3 of this paper, is valid for all mass scales. Due to the suppression of the decay $K_L^0 \to \mu\bar{e}$ caused by this rule, its branching ratio is destined to lie much below the expected [5] level of $\approx 10^{-12}$. Our motivation in this paper is to highlight the point that the value of $B(K_L^0 \to \mu\bar{e})$ could be pushed to the level $\approx 10^{-12}$ if this decay were actually a favoured one, as implicitly assumed in the literature [5,6]. One of our intentions in this paper is to emphasize that the prospect of experimental detection of the decay concerned is much less than what is expected from the argument mentioned earlier [5].

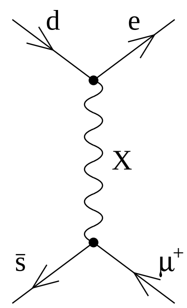

Fig. 1. Tree level Feynman diagram that leads to the decay $K_L^0 \to \mu\bar{e}$ in models with $SU(4)_C$ *unification. X denotes a leptoquark gauge boson of this symmetry.*

This paper is organised as follows. The essential points regarding the pseudo-dimension (PD) rule are discussed in Sect. 2. In Sect. 3, this rule is exploited to demonstrate that the Feynman diagram underlying the decay $K_L^0 \to \mu\bar{e}$ is forbidden. This means that this decay is forbidden by the same rule. This in turn implies that suppression must be witnessed in this decay. The bearing of this suppression on the branching ratio of the decay concerned is discussed in the context of the SO(10) GUT model with *D* parity. The conclusions are given in Sect. 4.

2. Pseudo-dimension rule

In this section we present an overview of the pseudo-dimension rule [11–22] by considering the salient points regarding it. This rule is expressed in terms of the pseudo-dimensions (discussed below) of the fields of particles involved in a decay process. The most striking feature of this rule is that it covers all types of decays (strong, electroweak and superweak decays) for the reason to be pointed out in this section. It is worth mentioning here that this rule is also applicable to production of particles [14–18] through decays of other particles.

To facilitate our discussion on the pseudo-dimensions of fields, we first focus our attention on their canonical dimensions as the former dimensions are related to the latter ones.

As is well known, the canonical dimensions are 1 for all boson fields and 3/2 for all fermion fields. These dimensions, therefore, are related to the statistics obeyed by the fields, but not to their respective actual spins. It is, therefore, not surprising that a non-trivial decay selection rule cannot be framed in terms of the canonical dimensions of the fields of the particles taking part in a decay process, as pointed out in Ref. 13. This difficulty, however, can be bypassed [13] by switching over from the canonical dimensions of fields to their pseudo-dimensions which reduce to the former ones for some special categories of fields. In fact, pseudo-dimensions are assigned to fields by imposing the following requirements on such dimensions.

(i) The pseudo-dimension of a massive field, be it a boson or fermion field, carrying integral scalar quantum numbers and non-zero spin J, must be linearly related to J in such a way that it coincides with the canonical dimension of this field if $J = 1/2$, and becomes larger than the latter dimension if $J > 1/2$;

(ii) the pseudo-dimensions of massless fermion and boson fields and of massive spin-zero boson fields having integral scalar quantum numbers must be identical with their respective canonical dimension;

(iii) the pseudo-dimension of a field carrying fractional scalar quantum numbers must be less than that of a field having integral scalar quantum numbers, provided both fields share the same actual spin.

Denoting the pseudo-dimension of a field by d and making use of the requirements stated above, it is easy to obtain the following formulae [11–22] for d. The derivations [13,18,22] of the formulae are given in Appendix A1.

As stated above, the requirement (i) refers to a massive $(m \neq 0)$ field with $J \neq 0$ which carries integral scalar quantum numbers. This requirement leads to:

$$d = 3J; \qquad J \neq 0, \tag{1}$$

which holds true for all massive boson fields with $J \neq 0$ and all massive fermion fields except the quark fields (which carry fractional scalar quantum numbers) as Eq. (1) is valid for a field having integral scalar quantum numbers.

Also, taking advantage of the requirement (ii) the following formula is valid:

$$d = 3/2. \tag{2}$$

This relation is applicable to all massless fermion fields which carry integral scalar quantum numbers.

Furthermore, the requirement (ii) yields the following formula:

$$d = 1. \tag{3}$$

This formula covers the boson fileds (with integral scalar quantum numbers) the pseudo-dimensions of which are not given by Eq. (1). In fact, this formula holds true for all massless boson fields with $J \neq 0$. Such fields include photon and gluon fields. The same formula is also valid for all massive as well as massless boson fields with $J = 0$. This formula, therefore, is applicable to Higgs fields, scalar lepton fields (which include $\tilde{e}, \tilde{\nu}_e$), Goldstone fields, etc.

By exploiting the requirement (iii), one obtains the following relation:

$$d = 1/2 \qquad \text{for a field} \qquad q^{\alpha}_{1/2}, \tag{4}$$

where $q^{\alpha}_{1/2}$ denotes a spin-half field with fractional scalar quantum numbers, α being the colour index. Equation (4) is valid for spin-half quark fields as well as for spin-half fields $D^{\alpha}_{1/2}$ (having fractional scalar quantum numbers) described by some extended versions of the standard model which include the superstring-inspired E_6-based models [24].

Also, the requirement (iii) gives rise to the following relation;

$$d = 1/3 \qquad \text{for a field} \qquad q^{\alpha}_{0}, \tag{5}$$

where q^{α}_{0} denotes a scalar field carrying fractional scalar quantum numbers. Equation (5) is valid for scalar quarks (which are the supersymmetric partners of spin-half quarks) and also for scalar leptoquarks [24] $D^{\alpha}_{0}, (D^{\alpha}_{0})^c$ which are the supersymmetric partners of spin-half field $D^{\alpha}_{1/2}$ mentioned above.

Finally, the requirement (iii) leads to the following formula:

$$d = 2\frac{1}{3} \qquad \text{for a field} \qquad X^{\alpha}_{1}, \tag{6}$$

where X^{α}_{1} denotes a vector field with fractional scalar quantum numbers. This formula, therefore, is applicable to a vector leptoquark [24].

A few remarks are in order here. Equations (1) and (3) reveal that the pseudo-dimensions of boson fields with integral scalar quantum numbers are integral. In sharp contrast to this, the pseudo-dimensions of boson fields with fractional scalar quantum numbers are non-integral, as reflected in Eqs.(5) and (6), and become fractional for such fields with $J = 0$ as evident from Eq. (5). Needless to mention that canonical dimensions for boson fields are invariably integral, whether their scalar quantum numbers are integral or fractional. It is worth noting here that pseudo-dimensions of fermion fields, like their canonical dimensions, are odd-half integral as manifested in Eqs. (1), (2) and (4). It is remarkable that the pseudo-dimension of a spin-half field with fractional scalar quantum numbers happens to be fractional, as evident from Eq. (4). There is hardly any need to emphasize that canonical dimension of a fermion field can never be fractional, even if its scalar quantum numbers are fractional. It follows from our above discussion as well as from Eqs. (1)–(6) that the pseudo-dimensions of fields, in general, are different from their respective canonical dimensions. However, as stressed earlier, the former dimensions reduce to the latter ones for some special categories of fields mentioned in the context of the requirements (i) and (ii). This fact has the obvious implication that pseudo-dimensions of fields admit the interpretation of some kind of its dimension.

We now define the quantities in terms of which the PD rule is stated. In what follows, we will denote by d_u the pseudo-dimension of the field of an unstable particle A undergoing the decays A$\rightarrow$ BC, EFG,...... It may be stressed that the term unstable particle is used here in the most general sense, i.e., it refers to any particle which suffers decays irrespective of the nature of interactions responsible for its decays. Furthermore, we will denote by D the sum of the pseudo-dimensions of the fields of the particles constituting a decay mode of A. Therefore, by definition, $D = d_B + d_C$ for the two-body mode BC, $D = d_E + d_F + d_G$ for the three-body mode EFG and so on. The PD rule is stated in terms of d_u and D defined above. This rule reads [11–22] as follows.

The allowed decays of a particle are governed by one and only one of the two constraints

$$d_u \geq D \tag{7a}$$

and

$$d_u \leq D \tag{7b},$$

where d_u is fixed for a given decaying particle, whereas D can generally take a finite spectrum of discrete values corresponding to the finite number of the decay modes of the particle concerned. As evident from its statement, this rule does not refer to the nature of the interactions responsible for the decays of a particle. This is precisely the reason for its validity in all types of decays (strong, electroweak and superweak decays). Stated differently, this rule is valid at all mass scales. In this connection it may be noted that this rule is expected to have its theoretical basis in a GUT. However, as is well known, the standard model $SU(3)_C\times SU(2)_L\times U(1)_Y$ turns out to be the low energy limit of a GUT. Therefore, this model offers a theoretical basis for this rule when this rule is applied to decays occurring at low energies. These points, although discussed elsewhere [11,18], have been elaborated in Appendix A2.

We now consider the procedure for application of the PD rule to the decays of a given particle. Our first step to accomplish this objective, as implied by the statement of this rule, is to fix the constraint operative in the decays of the particle concerned. This is because the PD rule itself does not specify which one of the two constraints, given by relations (7a) and (7b), holds true for the decaying particle under consideration. This fact, however, does not pose any problem as specification of the constraint appropriate for a given decaying particle can be made by any one of the following two methods which suits our purpose : (i) direct method and (ii) indirect method.

The direct method : This method is applicable to a particle for which the most dominant decay mode has an appreciable branching ratio. It makes use of the most dominat decay mode as such a mode must necessarily enjoy the status of an allowed mode from the point of view of the PD rule in order that this rule can have any claim to be a reliable decay selection rule. To illustrate the direct method, we consider $\rho(770)$ - decay for which the most dominant mode is $\rho \to 2\pi$, having a very appreciable branching ratio. For the decaying field ρ, we have $d_u = 3$ which follows from Eq. (1) and for the 2π mode, $D = d_\pi + d_\pi = 1 + 1 = 2$, since $d_\pi = 1$ as evident from Eq. (3). Clearly, the most dominant decay $\rho(d_u = 3) \to 2\pi(D = 2)$ reveals that the constraint operative in ρ-decay must have the form $d_u \geq D$. It is worth noting here that the task of specification of the constraint appropriate for a decaying particle amounts to a fixation of the sign of the inequality appearing in the relevant constraint, as the equality sign occurs in both constraints given by relations (7a) and (7b). In passing, we also note that the direct method runs into difficulty if the most dominant decay mode only indicates the equality sign and thereby fails to fix the sign of the inequality. This difficulty, however, can be bypassed by switching over to the indirect method.

The indirect method : This method is applicable to a decaying particle which belongs to a set of correlated particles having the same actual spin. Such a set of particles falls into either of the two categories: (i) a set of particles (of identical actual spin) belonging to a particular representation of a group (being either a global gauge group like $SU_f(N)$ or a local gauge group like a GUT group), and (ii) a set of particles (of identical actual spin) forming a spectroscopy (such as, for example, ψ- and Υ-spectroscopy).

We first consider the decays of a particle belonging to a set of particles of category (i). For the decays of such a set of particles, the following general conclusion holds (as shown elsewhere [11,18]): the allowed decays of all particles belonging to a particular representation of a group, being either a global or a local gauge group, are described by one and the same constraint. This fact allows to specify the form of the constraint for a given decaying particle without any reference to its decay modes, if we can somehow manage to ascertain the form of the constraint relevant for some other particle, provided both particles belong to the same representation of a group. To illustrate this point, we consider the particles described by a given representation of a local gauge group. To be specific, we consider the gauge bosons belonging to 45-representation of the SO(10) GUT group. These gauge bosons, needless to mention, include photon (γ). We can specify the constraint for γ^* - decay by using the direct method in the context of the decays $\gamma^* \to e^+e^-, \mu^+\mu^-$. For the decaying field γ^*, we have $d_u = 1$ which follows from Eq. (3), and for the e^+e^- mode, $D = d_{e^+} + d_{e^-} = 3/2 + 3/2 = 3$, since $d_{e^+} = d_{e^-} = 3/2$, as evident from Eq. (1). Obviously, $D = 3$ for $\mu^+\mu^-(D = 3)$ mode also. Therefore, the decays $\gamma^*(d_u = 1) \to e^+e^-(D = 3), \mu^+\mu^-(D = 3)$ clearly reveal that the constraint operative in γ^*-decay must have the form $d_u \leq D$. This constraint, for the reason stated above, must hold true for the decays of all other gauge bosons belonging to the above mentioned representation of the SO(10) group.

Finally, we discuss the indirect method for the specification of the constraints relevant for correlated particles of category (ii) which have the same spin. Such particles include vector quarkonia which form a spectroscopy, e.g. ψ- and Υ-spectroscopy. The constraints operative in their decays can be specified by taking advantage of the following facts [14,20] (which can be easily verified by the direct method discussed above):

(i) the allowed decays of vector quarkonia, which are bound resonances, are described by the constraint $d_u \leq D$;

(ii) the allowed decays of vector quarkonia, which are unbound resonances, are described by the constraints

$$d_u \geq D \qquad \text{for} \qquad n = 1, 3, 5, \ldots$$

and

$$d_u \leq D \qquad \text{for} \qquad n = 2, 4, 6 \ldots$$

where $n = 1$ for the first unbound resonance, $n = 2$ for the second unbound resonance and so on. The above mentioned facts can be exploited in fixing the approprate constraints for the members of, for example, the ψ-family. It is easy to check that the decays of $\psi(3.10)$ and $\psi(3.69)$, which are bound resonances, are described by the constraint $d_u \leq D$. On the other hand, the constraints appropriate for the decays of the first ($n = 1$) unbound resonance $\psi(3.77)$ and second ($n = 2$) unbound resonance $\psi(4.04)$ are $d_u \geq D$ and $d_u \leq D$, respectively.

3. PD rule and the decay $K_L^0 \to \mu\bar{e}$

In the previous section, we have stressed the fact that the PD rule is valid in all types of decays (strong, electroweak, superweak decays), irrespective of the nature of interactions responsible for these decays. In other words, this rule is operative at all mass scales, including the M_C scale. This rule, therefore, must be taken into account in order to ascertain the status of the decay $K_L^0 \to \mu\bar{e}$. This decay, as already noted in Sect. 1, is favoured by all familiar selection rules relevant for the M_C scale and as such for the decay concerned. For this reason, the decay under consideration has been treated in literature [5,6] as an allowed one in the perspective of the SO(10) model of GUT. However, as will be demonstrated in this section, this decay is forbidden by the PD rule. This means that the overall status of this decay is that of a forbidden decay and not of a favoured one, as implicitly assumed in the literature [5,6]. Consequently, this decay must undergo suppression. Now, we examine the implications of this suppression.

For our purpose, we proceed by noting that the decay $K_L^0 \to \mu\bar{e}$ can take place unsuppressed provided the Feynman diagram associated with this decay is not suppressed by any one of the selection rules which are operative in this decay. This in turn necessitates that the vertices of this diagram are not forbidden by any one of the set of the selection rules relevant for this decay. For convenience of further discussion on this point, we recall that the vertices describing the coupling of the leptoquark gauge bosons X of the $SU(4)_C$ to fermions are given in Ref. 2 (for one family of fermions):

$$\pounds_{int} = \frac{g}{\sqrt{2}} X_\mu^\alpha \left[\bar{d}_{L\alpha}\gamma^\mu e_L^- + \bar{d}_{R\alpha}\gamma^\mu e_R^- + \bar{u}_{L\alpha}\gamma^\mu \nu_L + \bar{u}_{R\alpha}\gamma^\mu \nu_R\right] + h.c.$$

where α denotes the colour index and other terms have their usual meaning. Manifestly, the vertices have the form $X^\alpha q_\alpha^c l$, and those appearing in the expression for $h.c.$ have the form $\bar{X}^\alpha q_\alpha l^c$, where the superscript c denotes charge conjugate, and q and l represent a quark and a lepton, respectively. Needless to state that the forms of the vertices remain the same for other two families of fermions. Obviously, the vertices involved in the Feynman diagram shown in Fig. 1 have the forms indicated above. These vertices imply the following transitions

$$X^\alpha \to q_\alpha l^c, \qquad \bar{X}^\alpha \to q_\alpha^c l,$$

which should not be forbidden by any selection rule in order that the vertices concerned may be treated as favoured ones. Otherwise, the Feynman diagram involving these vertices must suffer suppression. The transitions shown above, as is well known, are allowed by all selection rules which are appropriate for the $SU(4)_C$ symmetry. The same transitions, however, are forbidden by the PD rule, as is discussed below.

We now proceed to envisage the status of the transitions $X^{\alpha} \to q_{\alpha} l^{c}$, $\bar{X}^{\alpha} \to q_{\alpha}^{c} l$ in the light of the PD rule. These transitions, needless to emphasize, may be formally treated as the decays of X^{α} into q_{α} and l^{c} and $\bar{X}^{\alpha}$ into q_{α}^{c} and l. In Sect. 2, we have noted that the decays of all gauge bosons of the SO(10) GUT model are described by the constraint $d_u \leq D$. This constraint, therefore, is operative in the decays of the bosons X^{α} and $\bar{X}^{\alpha}$ which belong to the set of 45 gauge bosons of this model. However, before we employ this constraint in X^{α}- and $\bar{X}^{\alpha}$-decay, it is desirable to examine the performance of this constraint in the context of the decays of some other gauge boson which is a member of the set of 45 gauge bosons described by the SO(10) GUT model. To concretize our discussion, we consider Z^0-decay as Z^0 belongs to this set. For the reason stated above, the decays of Z^0 are also described by the constraint $d_u \leq D$. It is worth stressing here that those decay modes of Z^0 which are in conformity with this constraint can enjoy the status of allowed modes according to the PD rule. It is easy to see that the observed leptonic decays $Z^0(d_u = 3) \to e^+e^-(D = 3), \mu^+\mu^-(D = 3), \tau^+\tau^-(D = 3)$ are consistent with this constraint. Furthermore, the empirical fact that Z^0 decays dominantly into hardrons [23] is also correctly described by this constraint. To see this point, we note that the hadronic decays of this vector boson originate from the electroweak vertices $Z^0 q\bar{q}$ which lead to the basic decays $Z^0 \to q\bar{q}$. As these quarks hadronize through the gluon-induced mechanism, the applicaion of the PD rule in the hadronic decays of Z^0 really amounts to the application of this rule to the decays $Z^0 \to q\bar{q}$. In order to evaluate D for the $q\bar{q}$ mode, it is important to remain aware of the fact that Z^0 is a colour singlet and as such $q\bar{q} = q^{\alpha}\bar{q_{\alpha}} + q^{\beta}\bar{q_{\beta}} + q^{\gamma}\bar{q_{\gamma}}$. Clearly, for $q\bar{q}$ mode $D = (1/2+1/2)+(1/2+1/2)+(1/2+1/2) = 3$ since $d_{q\alpha} = d_{\bar{q_{\alpha}}} = 1/2$, etc, as evident from Eq. (4). Therefore, the constraint $d_u \leq D$ is indeed satisfied for the decays $Z^0(d_u = 3) \to q\bar{q}(D = 3)$, which in turn means that the hadronic decays of Z^0 are favoured by the PD rule. The same constraint, however, is violated in the 1-loop decays $Z^0(d_u = 3) \to \pi^o\gamma(D = 1+1 = 2), 2\gamma(D = 1+1 = 2)$ which are, therefore, forbidden according to the PD rule. These decays are experimentally found to be strongly suppressed [23]. Our discussion on Z^0-decays reveals that this decay is correctly described by the constraint $d_u \leq D$ which, we repeat to emphasize, is also valid in the decays of all other gauge bosons of the SO(10) model of GUT.

We are now in a position to investigate the status of the decays $X^{\alpha} \to q_{\alpha} l^{c}$ and $\bar{X}^{\alpha} \to q_{\alpha}^{c} l$ from the viewpoint of the PD rule. According to this rule, only those decay modes of the X^{α}- and $\bar{X}^{\alpha}$-bosons can be treated as favoured ones which are consistent with the constraint $d_u \leq D$ which, as noted earlier, describes the decays of the bosons concerned. For these bosons, which are vector leptoquarks, $d_u = 2\frac{1}{3} \approx 2.33$, which follows from Eq. (6). As these bosons are coloured ones, colour sum is inadmissible for the evaluation of D for the modes $q_{\alpha} l^{c}$ and $q_{\alpha}^{c} l$ (unlike the $q\bar{q}$ mode of Z^0 which is a colour singlet). Manifestly, for the $q_{\alpha} l^{c}$ mode, we have $D = d_{q_{\alpha}} + d_{l^c} = 1/2 + 3/2 = 2$ since $d_{q_{\alpha}} = 1/2$ and $d_{l^c} = 3/2$, as evident from Eqs. (4) and (1), respectively. Obviously, $D = 2$ for the $q_{\alpha}^{c} l$ mode. Consequently, the decays $X^{\alpha}(d_u \approx 2.33) \to q_{\alpha} l^{c}(D = 2)$ and $\bar{X}^{\alpha}(d_u \approx 2.33) \to q_{\alpha}^{c} l(D = 2)$ clearly fail to satisfy the constraint $d_u \leq D$, and as such they are forbidden by the PD rule. The forbidden character of these decays, i.e., the transitions $X^{\alpha} \to q_{\alpha} l^{c}$, $\bar{X}^{\alpha} \to q_{\alpha}^{c} l$ implies that the vertices $X^{\alpha} q_{\alpha} l^{c}, \bar{X}^{\alpha} q_{\alpha}^{c} l$, which cause these transitions, are also forbidden by the same rule. As an outcome of this, the Feynman diagram shown in Fig. 1 involving these vertices must also be forbidden by this rule, which causes suppression of this diagram. Consequently, the process $K_L^0 \to \mu\overline{e}$ associated with this diagram must exhibit suppression.

Finally, we focus our attention to the bearing of the suppression of the decay $K_L^0 \to \mu\overline{e}$ induced by the PD rule in the context of the feasibility of its experimental detection. As already noted in Sect. 1, experiments [23] reveal that $B(K_L^0 \to \mu\overline{e}) < 3.3 \times 10^{-11}$. In the perspective of this empirical fact, as argued in Ref. 5, the theoretical value of $B(K_L^0 \to \mu\overline{e})$ can be pushed to the level 10^{-12} within the framework of the SO(10) GUT model with D parity. This argument tacitly assumes that the decay concerned is not suppressed at all. However, as demonstrated above, this decay is suppressed by the PD rule. This suppression turns out to be too serious. This is because the theoretical value of $B(K_L^0 \to \mu\overline{e})$ happens to be quite small ($\approx 10^{-12}$) even if this decay is assumed to be unsuppressed. There is hardly any need to emphasize that, due to suppression caused by the PD rule, the effective theoretical value of $B(K_L^0 \to \mu\overline{e})$ must be much less than the conventionally expected value [5] of ($\approx 10^{-12}$), and as such, this decay must be far below the level of experimental detection.

It is transparent from the above discussion that the decay $K_L^0 \to \mu\overline{e}$ might have been observed at the level $\approx 10^{-12}$ if this decay were not suppressed by the PD rule. Therefore, its nonobservation is really not worrisome for the SO(10) GUT model with D parity as this empirical fact can be easily accounted for in terms of its suppression induced by the PD rule.

4. Conclusions

In this paper we have shown that the Feynman diagram underlying the SO(10)-predicted decay $K_L^0 \to \mu\overline{e}$ is forbidden by the pseudo-dimension rule, and as such this decay is suppressed. The bearing of the suppression of this decay on its branching ratio has been discussed in the context of the SO(10) GUT model with D parity. Within the framework of this version of the SO(10) model, as argued in the literature, the branching ratio of the decay concerned can be pushed to the level $\approx 10^{-12}$ as experiments indicate that $B(K_L^0 \to \mu\overline{e}) < 3.3 \times 10^{-11}$. This argument is based on the implicit assumption that the decay under consideration is not suppressed by any of the selection rules operative in this decay. This decay, however, is suppressed by the pseudo-dimension rule which forces the branching ratio of this decay to go far below the level of $\approx 10^{-12}$. This paper, therefore, concludes that the prospect of experimental detection of this decay is much less than what is expected naively from the viewpoint of the SO(10) model with D parity. One of the important conclusions of this paper is that the so far non-observation of the decay $K_L^0 \to \mu\overline{e}$ is really not worrisome for the SO(10) model with D parity as this empirical fact can be easily accounted for in terms of the influence of the pseudo-dimension rule on this decay.

Appendix A1: Derivations of the formulae for the pseudo-dimensions of fields

To start with, we consider the requirement (i) stated in Sect. 2 which refers to a massive field carrying integral scalar quantum numbers and non-zero actual spin J. This requirement implies that the pseudo-dimension d can be expressed as

$$d = KJ, \qquad J \neq 0, \tag{A1.1}$$

where K, the constant of proportionality, can be easily determined by taking advantage of the fact that for a spin-half field (with integral scalar quantum numbers) d must be equal to its canonical dimension (which is 3/2). This means that $d = 3/2$ when $J = 1/2$. Also, it follows from Eq. (A1.1) that $d = K/2$ when $J = 1/2$. Obviously, then $K/2 = 3/2$, which in turn leads to $K = 3$. Therefore, Eq.(A1.1) takes the form $d = 3J$ with $J \neq 0$, which is Eq. (1) of Sect. 2.

We now shift our attention to the special categories of fields (having integral scalar quantum numbers) already mentioned in the context of the requirement (ii), stated in Sect. 2. This requirement concerns itself with the equality of the pseudo- dimensions and the corresponding canonical dimensions of these fields. The formulae, given by Eqs. (2) and (3) in Sect. 2, follow automatically from this requirement (as canonical dimensions of fermion and boson fields are 3/2 and 1, respectively).

We now focus our attention on fields having fractional scalar quantum numbers. The pseudo-dimensions of such fields must satisfy the requirement (iii). To begin with, we consider a spin-half field $q^{\alpha}_{\frac{1}{2}}, \alpha$ denoting colour index. Its pseudo-dimension must be less than that of a spin-half field with integral scalar quantum numbers, as implied by the requirement (iii). Bearing in mind that a field cannot be a dimensionless entity (i.e. $d \neq 0$), and that the pseudo-dimension of a spin-half field with integral scalar quantum numbers is 3/2, it is easy to see that d for a spin-half field with fractional scalar quantum numbers must satisfy the relation:

$$0 < d < 3/2. \tag{A1.2}$$

In order to specify the value of d in conformity with this relation, we note that, as evident from Eqs. (1) and (2) of Sect. 2, the pseudo-dimensions of fermion fields are odd-half integer. The above relation clearly reveals that it is still possible to assign an odd-half integer value to d of the fermion field under consideration. In fact, the only odd-half integral value of d consistent with this relation is 1/2. Accordingly, we get:

$$d = 1/2 \qquad \text{for a field} \qquad q^{\alpha}_{\frac{1}{2}}, \tag{A1.3}$$

which is Eq. (4) of Sect. 2. It is easy to see that Eq. (A1.3) can be given the form

$$d = (1/3)(3/2) = (1/3)d_c,$$

where $d_c = 3/2$ is the canonical dimension. This relation can be recast in the following form

$$\frac{d}{d_c} = \frac{1}{3} \qquad \text{for a field} \qquad q^{\alpha}_{\frac{1}{2}}. \tag{A1.3a}$$

The importance of this relation will be transparent from our discussions to follow.

We now turn our attention to a spin-zero field with fractional scalar quantum numbers. For such a field, the requirement (iii) amounts to the following condition on d

$$0 < d < 1. \tag{A1.4}$$

This relation tells us that d must be fractional. However, in order to assign a unique value to d, we require an additional condition. We discuss this point below.

We consider non-composite fermions like electron (e), neutrino (ν_e) and their respective supersymmetric partners, namely scalar electron $(\tilde{e})$ and scalar neutrino $(\tilde{\nu}_e)$. It is worthwhile mentioning here that there is as yet no compelling experimental evidence to justify their compositeness. We proceed by recalling that $d = 3/2$ for e, which follows from Eq. (1) given in Sect. 2. Also, canonical dimension $d_c = 3/2$ for this particle. Therefore, $d/d_c = 1$ for e. For its supersymmetric partners $\tilde{e}$ we have $d = 1$, as reflected in Eq.(3) of Sect. 2. Furthermore, $d_c = 1$ for $\tilde{e}$. Obviously, (d/d_c) remains unchanged if one goes from e to $\tilde{e}$. It is easy to verify that the value of (d/d_c) also does not change if we switch over from ν_e to $\tilde{\nu}_e$. Considering other non-composite fermions and their respective supersymmetric partners, one can easily check that the value of (d/d_c) remains the same if we go from a normal particle to its supersymmetric partners. This conclusion is perfectly general and as such must also hold true for a spin-half field $q^{\alpha}_{1/2}$ with fractional scalar quantum numbers and its supersymmetric partners to be denoted by q^{α}_0 for convenience of writing. Needless to state that the spin-zero field q^{α}_0 carries fractional scalar quantum numbers as both $q^{\alpha}_{1/2}$ and q^{α}_0 share the same set of scalar quantum numbers. We have already shown that $d/d_c = 1/3$ for the field $q^{\alpha}_{1/2}$, as evident from Eq. (A1.3a). Therefore, $d/d_c = 1/3$ also for its supersymmetric partners q^{α}_0 for the reason stated above. Since $d_c = 1$ for the spin-zero field q^{α}_0, we finally get for this field

$$d = 1/3, \tag{A1.5}$$

which is Eq. (5) of Sect. 2.

We now focus our attention on massive vector fields with fractional scalar quantum numbers. For such a field, the requirement (iii), along with the condition $d \neq 0$, implies the following restriction

$$0 < d < 3$$

on d of the field under consideration, 3 being the value of the pseudo-dimension of a massive vector field with integral scalar quantum numbers, as evident from Eq. (1) of Sect. 2. For convenience of further discussions, we shall denote the pseudo-dimension of a massive field having the spin J and integral scalar quantum numbers by d^i_J and that of a (massive) field carrying the same spin J but having fractional scalar quantum numbers by d^f_J. Obviously, d^f_J lags behind d^i_f by the amount $(d^i_J - d^f_J)$; $d^i_J > d^f_J$, as implied by the requirement (iii) stated in Sect. 2. The genesis of the quantity $(d^i_J - d^f_J)$ lies in the difference in the nature of the scalar quantum numbers involved ; d^i_J is related to integral scalar quantum numbers and d^f_J to fractional scalar quantum numbers (J being same in both these cases). It is worth stressing here that the pseudo-dimension of a field is influenced by the nature (i.e., integral or fractional) of its scalar quantum numbers but not by their actual values. For example, both u- and d-quarks have the same value $d = 1/2$, but their electric charges are different. It is to be noted that if we go from d^i_J to d^i_{J+1}, the nature of the scalar quantum numbers remains the same. A similar remark also holds true if we go from d^f_J to d^f_{J+1}. As both d^i_{J+1} and d^f_{J+1} are related to the same spin, the quantities $(d^i_J - d^f_J)$ and $(d^i_{J+1} - d^f_{J+1})$ must have the same value. Therefore, considering massive scalar and vector fields we get

$$d^i_0 - d^f_0 = d^i_1 - d^f_1,$$

which leads to

$$d^f_1 = 2\frac{1}{3}, \qquad \text{(A1.6)}$$

since $d^i_0 = 1, d^f_0 = 1/3$ and $d^i_1 = 3$, as evident from Eqs. (3), (5) and (1), respectively, given in Sect. 2, d^f_1 denoting the pseudo-dimension of a massive vector field with fractional scalar quantum numbers. In passing, we may also note that Eq. (A1.6) is consistent with the restriction $0 < d^f_1 < 3$, as implied by the requirement (iii), already indicated above.

Appendix A2: Theoretical basis of the PD rule

In order to search for theoretical basis of the PD rule, we proceed by recalling the following fact [11,18] mentioned in Sect. 2. The allowed decays of all particles belonging to a particular representation of a gauge group, be it global or local, are described by one and same the constraint. The interactions responsible for all these decays need not be the same and, in fact, generally may be different. This point becomes immediately transparent if we consider the nature of interactions of the favoured decays of the 0^- particles described by octet representation of $SU_f(3)$. As is well known, the decay $\pi^0 \to 2\gamma$ involves strong and electromagnetic interactions, whereas the decays $\pi^+ \to \mu^+\nu_\mu$, $\pi^- \to \mu^-\overline{\nu_\mu}$ occur through weak interactions, which also give rise to the decay $K \to 3\pi$. Also, strong interactions cause the decay $\eta \to 3\pi$. It can be easily checked that all these decays are described by the same constraint $d_u \leq D$. This is apparently surprising as the decays considered above do not involve the same interactions. A similar remark also holds true for the allowed decays of the $3^+/2$ particles (which include Ω^-) belonging to the 10-dimensional representation of $SU_f(3)$, because these decays, as can be easily checked, are described by the same constraint $d_u \geq D$ despite the fact that the interactions responsible for all these decays are not the same. The same remark is also valid if we switch over to a local gauge group like, for example, SO(10) GUT group. We have already noted in Sect. 2 that the allowed decays of all gauge bosons belonging to the 45-representation of SO(10) GUT group are described by one and the same constraint $d_u \leq D$, although the decays of virtual gluons, virtual photons as well as real or virtual weak vector bosons, heavy and superheavy coloured leptoquark vector bosons, respectively, involve strong, electroweak and superweak interactions. However, these apparently surprising facts can be easily reconciled with if we treat these different interactions as the different manifestations of one and the same interaction, as stipulated in GUT. In fact, only in the perspective of a GUT which considers a single interaction, namely the GUT-interaction, the validity of a single constraint in the decays of all particles belonging to a particular representation of a group can be satisfactorily accounted for. Therefore, it is reasonable to assume that the dynamics underlying the PD rule is described by a GUT. This means that the rule concerned finds its theoretical basis in a GUT. We have already noted in Sect. 3 the well-known fact that a GUT model reduces to the standard model $SU(3)_C \times SU(2)_L \times U(1)_Y$ in the low-energy limit. Therefore, the latter model can be considered as the theoretical basis of the PD rule when this rule is applied in decays occuring at low energies. Our above arguments are qualitative. A quantative derivation of the PD rule from a GUT model or from the standard model for low-energy decays is still lacking. It is worthwhile noting here that QCD can qualitatively account for the OZI rule. A quantitative derivation of this rule from QCD has not yet been achieved.

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SO(10) MODEL GUTe, PSEUDO - DIMENZIJSKO PRAVILO I RASPAD $K_L^0 \to \mu \overline{e}$

Raspad $K_L^0 \to \mu\overline{e}$, za koji SO(10) model GUTe predviđa da se dešava na M_C ljestvici, još se nije opazio. Eksperimentalna granica omjera grananja je $B(K_L^0 \to \mu\overline{e}) < 3.3 \times 10^{-11}$. U modelu je M_C prevelik da bi se mogao opaziti. Nove varijante zasnovane na D parnosti, međutim, dozvoljavaju mnogo manju vrijednost M_C koja bi se mogla opaziti. Stoga, prema raspravama u literaturi, $B(K_L^0 \to \mu\overline{e})$ bi se mogao povećati do razine $\approx 10^{-12}$ u okviru tih modela. Ove tvrdnje, kako se pokazuje u radu, zanemaruju pseudo-dimenzijsko pravilo prema kojemu je taj raspad zabranjen, pa je prema tome jako potisnut. Rezultat je zabrane da relativna vjerojatnost $B(K_L^0 \to \mu\overline{e})$ leži daleko ispod 10^{-12}, pa, prema tome, mogućnost eksperimentalnog opažanja tog raspada je daleko ispod onog što se obično očekuje. Zaključak je ovog rada da neopažanje raspravljenog procesa ne obara SO(10) model GUTe s D parnošću.

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