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MAPPING OF CORRELATION FILTER VALUES OPTIMIZED TO PHASE CONSTRAINED SPATIAL LIGHT MODULATORS

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Correlation filters (CFs) are designed for pattern recognition tasks, taking into account the implementation of possibilities of available spatial light modulators (SLMs). Commercially available SLMs have the amplitude and phase responses that cannot be independently controlled. Furthermore, the phase modulation curves of such SLMs generally cannot cover the desired (0, 2π) interval. Therefore, previous filter designs include the SLM contraints as starting point for designing the CFs. Here we start with previously designed and calculated CFs and propose an optimum mapping of such CF values to the values constrained by the modulation properties of arbitrary SLM, in order to achieve a maximum intensity of the output correlation peak. Optimization procedure, achieved by the variation of a phase shift parameter, is discussed theoretically and numerically.

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1. Introduction

Utilization of electronically addressed spatial light modulators (SLMs), like liquid crystal displays (LCDs), offers various possibilities in realizing complex functions, but leads to implementation problems. For example, commercially available LCDs, driven by a standard video signal and connected to a computer device, can be used as transmissiontype SLMs. Since the commercial LCDs are not supposed to be used as SLMs, their implementation in optical correlators is constrained mainly by the phase modulation capacity, the coupling problems, and the flatness imperfections. The phase mismatching generally reduces the diffraction efficiency of correlation filters and generates a zero-order spot [1], while coupling of the amplitude and phase modulation curves leads to searching for an optimal single parameter curve [2]. On the other hand, using the flattest location on the LCD [3] one can minimize the effects of flatness distortion.

Thus, to use such constrained LCD in optical correlator systems as a correlation filter (CF), one must apply appropriate mapping of the calculated filter data. General solutions for the CF design, which include the correlation metrics, the LCD constraints, and the reference object information, have been reported [4,5]. The optimization of a wide class of correlation metrics is obtained by applying the minimum Euclidean distance principle to the complex optimal CF [6,7]. In the reported work, see for example Refs. 4 to 7, the modulation properties of LCDs were included in the numerical design of the CF. Here, we suppose that the CF data are given independently for a particular LCD and without the knowledge of the reference database or the procedure used in designing the CF. We optimize mapping of such previously designed and calculated CF to the values constrained by the modulation properties of a single parameter LCD that cannot realize the desired $(0, 2\pi)$ phase interval. Theoretical analysis shows that the optimization can be achieved by the variation of a phase shift parameter. Computer simulations show good agreement with the obtained preliminary experimental results [8].

2. A single-parameter SLM with phase mismatching characteristics

In this section, we briefly describe a single-parameter SLM, such as twisted nematic (TN) LCD, capable of amplitude, phase and polarization modulation of the incident linearly polarized laser light. Applications using such type of SLMs have been widely reported [9–12].

The TN LCD basically consists of a stack of birefringent slices, sandwiched between two parallel glass plates. The orientation of the extraordinary axis of a slice is called the molecular director. When no voltage is applied to the stack, the molecular directors form a helicoidal pattern in which the plane of the input polarization is rotated continuously by a twist angle α. When the voltage is applied, the molecules tend to align in the direction of the electric field. The applied electric field thus influence the phase delay of the light transmitted through the LCD, since the effective refractive index of the liquid crystal depends on the angle between the molecules and the direction of incident light. The propagation of linearly polarized light along the twist axis depends on the applied voltage and is described theoretically [13–15] by the use of the Jones calculus. Thus, applying a certain voltage to each pixel of the LCD panel and using two polarizers, one in front of the panel and another (analyzer) behind the panel, permits to control the amplitude and phase of the light emerging from the analyzer.

The complex transmittance of the TN LCD pixel, described as [3]:

$$t_{\rm pix}(V,\psi_1,\psi_2) = \sqrt{I_{\rm pix}(V,\psi_1,\psi_2)}\cdot \exp\left[{\rm i}\,\delta_{\rm pix}(V,\psi_1,\psi_2)\right]\,, \tag{1}$$

is a function of the applied voltage V and the polarizer and analyzer angles ψ_1 and ψ_2, respectively. Since the parameters ψ_1 and ψ_2, once selected, are constant over the entire display, only the parameter V varies from one pixel to another. The applied voltage V is defined by the gray-scale level (GSL) and the settings like brightness and contrast.

Theoretically, the phase-mostly mode is achieved for $\psi_1 \perp \psi_2$ [13]. Then:

$$I_{\rm pix} = 1 - \left(\frac{\pi}{2}\right)^2 \frac{\sin^2\gamma}{\gamma^2}\,, \tag{2}$$

$$\delta_{\rm pix} = \beta + \tan^{-1}\left(\frac{\beta}{\gamma}\tan(\gamma)\right)\,, \tag{3}$$

with

$$\beta = \frac{\pi d}{\lambda}[n_e(\theta) - n_0]\,, \tag{4}$$

$$\gamma = \sqrt{\left(\frac{\pi}{2}\right)^2 + \beta^2}\,, \tag{5}$$

where n_0 and $n_e(\theta)$ are ordinary and extraordinary indices of refraction of the liquid crystal and θ is the tilt angle of the liquid crystal molecules to the electrical field. The dependence of intensity transmittance on β is oscillatory, while the phase delay is approximately linear function of β. In reality, to define the phase-mostly mode, the values $I_{\rm pix}(V)$ and $\delta_{\rm pix}(V)$ as well as the parameters ψ_1 and ψ_2, must be measured experimentally [15]. The phase-mostly mode is used for defining the phase-only filter (POF).

3. *Correlation peak description*

The complex amplitude of the correlation peak in a discrete frequency plane correlator can be described as:

$$A_{\rm cp} = \mathbf{s}^T\mathbf{h} = |\mathbf{s}^T\mathbf{h}|\,\exp({\rm i}\gamma)\,, \tag{6}$$

where the superscript ^{T} denotes the transpose operation, **s** and **h** denote the complex-valued column vectors representing the input and CF signals, respectively, in the Fourier-transform (FT) space, and $\exp({\rm i}\gamma)$ shows the direction of $\mathbf{s}^T\mathbf{h}$ in the complex plane. If we take the positive sense for γ to be in the counterclockwise direction, then the direction of $\mathbf{s}^T\mathbf{h}$ is defined by the angle between the real axis and $\mathbf{s}^T\mathbf{h}$. Note that in general, in the autocorrelation case, we have $\gamma = 0$, i.e., the amplitude of the autocorrelation peak is a real positive number.

The correlation peak intensity $I_{\rm cp}$ is defined as

$$I_{\rm cp} = |A_{\rm cp}|^2 = |\mathbf{s}^T\mathbf{h}|^2 \,. \qquad (7)$$

We suppose that the filter vector $\mathbf{h}$ is formed from the database described by the matrix X, where each column represents one image in the FT space. In the simplest correlation case, $\mathbf{X}$ consists of the one column vector $\mathbf{f}$, and $\mathbf{h}$ is equal to $\mathbf{f}^*$, i.e., we have the classical matched spatial-filter case. In such a case, the autocorrelation $I_{\rm cp}$ is defined as: $I_{\rm ap} = |\mathbf{f}^T\mathbf{h}|^2 = |\mathbf{f}^T\mathbf{f}^*|^2$.

To realize $\mathbf{h}$ optically, we must use SLMs with known modulation properties. Suppose Ω is a complex matrix with L rows and K columns, where each column contains the amplitude and phase values for the K typical situations (like the amplitude-mostly mode or phase-mostly mode). For a single parameter LCD, the phase-mostly mode is the one best suited for implementing CFs.

In the next section, we consider the typical single parameter LCD that cannot realize the desired (0, 2π) phase interval.

4. SLM contraints

The implementation of $\mathbf{h}$, using the known SLM properties, can be described by the mapping operator M,

$$M\{\mathbf{h}\} \rightarrow \mathbf{h}_k \exp(\mathrm{i}\,\Phi_l)\,, \qquad (8)$$

where $\mathbf{h}_k$ is a complex vector constrained by the values of the k^{th} column of the Ω matrix, $0 \le k < K$, and Φ_l is the phase shift parameter,

$$\Phi_l = l\,\frac{2\pi}{L}\,, \qquad (9)$$

where, generally $0 \le l < L$. The parameter Φ_l is introduced because the phase values describing the SLM properties can be arbitrarily shifted within the interval (0, 2π).

From Eqs. (6) and (7), we can see that the maximization of $I_{\rm cp}$ is the same as the maximization of $|A_{\rm cp}|$, and the $|A_{\rm cp}|$ can be written as

$$|A_{\rm cp}| = \Re e\{\mathbf{s}^T\mathbf{h}_k \exp(-\mathrm{i}\gamma)\} \qquad (10)$$

where $\Re e(x)$ denotes the real part of x. Now, the $|A_{\rm cp}|$ is equal

$$|A_{\rm cp}| = \Re e\{\mathbf{s}^T\mathbf{h}_k \exp[\mathrm{i}\,(\Phi_l - \gamma)]\} = |\mathbf{s}^T\mathbf{h}_k|\cos(\Phi_k + \Phi_l - \gamma)\,, \qquad (11)$$

where the phase Φ_k is for an ideal SLM equal to γ. Generally, Φ_k differs from γ due to the phase mismatching between the calculated and realized $\mathbf{h}$ values. From Eq. (11), it is obvious that the variation of the free parameter Φ_l will follow the cosine function with the maximum

$$\Phi_l = \gamma - \Phi_k\,, \tag{12}$$

which thus optimally compensates the deviation of Φ_k from γ. Here, we can conclude that larger Φ_l means larger mismatching between the calculated and realized vectors, $\mathbf{h}$ and $\mathbf{h}_k$, respectively.

The next problem is addressed to the selection of the input vector $\mathbf{s}$. Optimizations are best realized by measuring the autocorrelation maxima. Since the original data base used for creating CF, as well as the designing procedure, are supposedly unknown, we propose the use of a vector $\mathbf{h}_1$ instead of $\mathbf{s}$,

$$\mathbf{h}_l = F\left\{|F^{-1}(\mathbf{h}^*)|\right\}\,, \tag{13}$$

where $\mathbf{h}_1$ is defined as the FT of the modulus of the inverse FT of $\mathbf{h}^*$.

Combining Eqs. (7), (11) and (13), we obtain

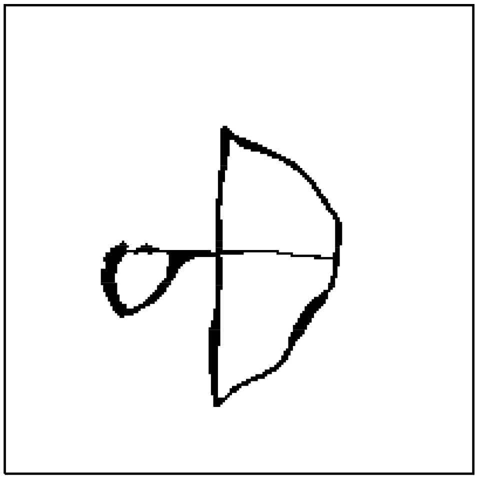
$$I_{\rm cp} = |\mathbf{h}_1^T \mathbf{h}_k|^2 \cos[\Phi_l - (\gamma - \Phi_k)]\,. \tag{14}$$

Thus, the maximum value of $I_{\rm cp}$ can be found by locating a global maximum of the product of two terms on the right-hand side of Eq. (14), and by the variation of the parameters k and Φ_l. Equation (14) shows that different columns of the matrix Ω influence not only a change in the height of the $I_{\rm cp}$, but also change the location of the maximum of the cosine function. The variation of the parameter Φ_l for arbitrary SLM depends on the maximum phase delay $\Phi_{k,\rm max}$, achievable with the selected set of parameters ■, and is in the interval from 0 to $(2\pi - \Phi_{k,\rm max})$.

5. *Numerical simulation*

To represent images and their FTs, we used arrays of the size 256×256, or the corresponding vector dimensions. The phases of the calculated POFs are coded by using 256- or 8-bit GSLs. To demonstrate numerically the optimization procedure described in Sects. 3 and 4, we use an image shown in Fig. 1. The image represents the Glagolitic letter 'key' taken from the Valun plate, then isolated and binarized. The POF was obtained by calculating the FT of the 'key' image, then by calculating its complex conjugate, and finally by extracting the phase of the complex conjugated FT. The phase distribution within the interval $(0 - 2\pi)$ corresponds to the interval (0 – 255) GSLs, as shown in Fig. 2.

We start the optimization procedure by investigating the modulation properties of our SLM. Since we are interested in the phase-mostly operating mode of the SLM, we can assume that the amplitude modulation is constant and that the phase characteristics is linear. However, typical $\Phi_{k,\rm max}$ is for commercial LCDs around 1.4π [1,3,15]. The phase modulation curve for such LCD is shown in Fig. 3. The



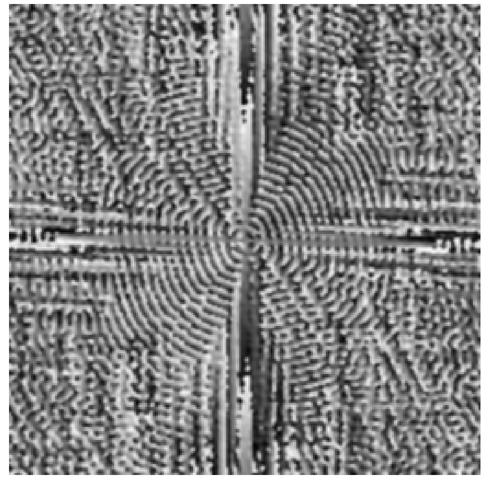


Fig. 1. Binarized image of a Glagolitic letter 'key' taken from the Valun plate.

Fig. 2 (right). The calculated POF values for the image shown in Fig. 1.

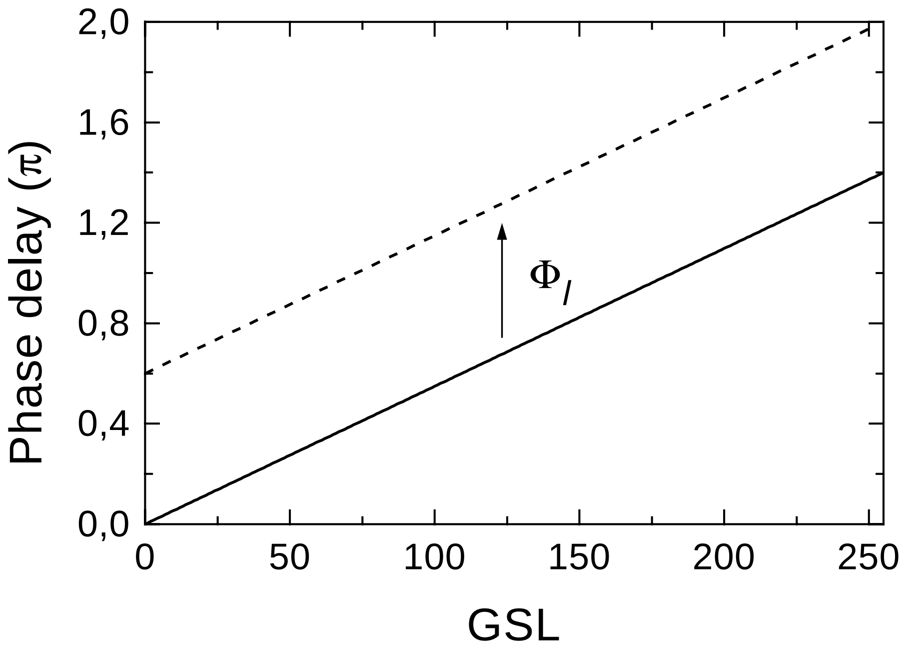


Fig. 3. Phase modulation characteristics of a mismatched SLM in the linear approximation.

solid line shows the phase mismatching, i.e., the lower phase interval ($0 - 1.4\pi$) is covered. Thus, the filter information contained in the interval ($1.4\pi - 2.0\pi$) is lost.

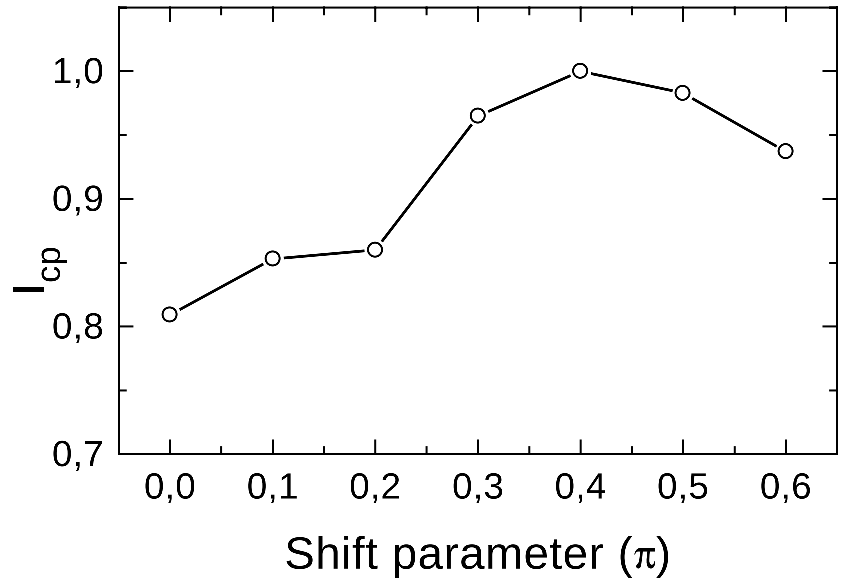


Fig. 4. Correlation peak value obtained by using only a given POF value.

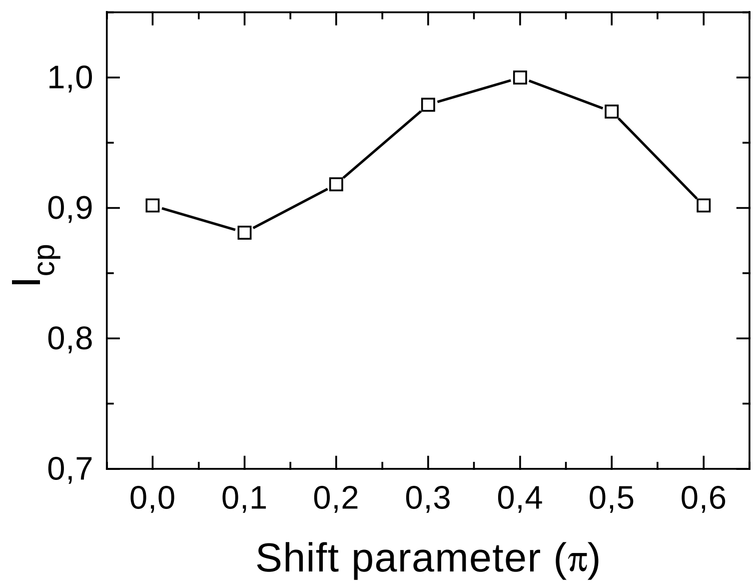


Fig. 5. Correlation peak values obtained by using given POF values and the original image shown in Fig. 1.

By a variation of the parameter Φ_l from 0 to 0.6π, we shift the curve until the upper phase interval (0.6π – 2.0π) is covered, as denoted by the dashed line in Fig. 3. Obviously, the correlation output is influenced by the phase distribution of the calculated POF and by the phase interval defined by the parameter Φ_l.

To demonstrate the variation effects of the parameter Φ_l, we use the calculated POF of Fig. 2. The correlation peak values, calculated by using Eq. (7), with the input described by Eq. (13), are shown in Fig. 4. From Fig. 4, we can see that the maximum $I_{\rm cp}$ value is obtained for the shift parameter $\Phi_l = 0.4\pi$.

To test our optimization procedure, we repeated the calculations of the $I_{\rm cp}$ values as a function of the shift parameter Φ_l for the original 'key' image shown in Fig. 1. These results are shown in Fig. 5, where we can observe similar behaviour to the one shown in Fig. 4.

6. Discussion and conclusions

On one hand, CFs are produced by using sophisticated numerical programs which are designed to satisfy pattern recognition tasks. They are given generally as two-dimensional complex-valued arrays ready for numerical or optical experiments. On the other hand, optical implementation of CFs is not trivial because the commercially available SLMs can display only a single parameter of two-dimensional arrays. Furthermore, the phase modulation curves of SLMs generally cannot cover the desired (0, 2π) interval and are disturbed by the coupled amplitude variations.

In this paper, an optimum mapping of the given CF data to the data constrained by the SLM is proposed. It is based on the possibility to shift the phase coding interval within the (0, 2π) interval and search for the maximum of the output correlation peak values. The example of a calculated POF shows (without using the reference image) that the shifting of the phase curve for 0.4π gives improvement in the correlation peak intensity (to the not shifted curve) by 20%. The test calculations, performed with a reference image (Glagolitic letter 'key'), have shown identical position of the optimum shift parameter, although the obtained improvement was 10% (compare the curves of Figs. 4 and 5). The preliminary experimental measurements with real SLMs [8] support the theoretical and numerical optimizations predicted by our model. Thus, the approach presented here offers an improved utilization of the phase constrained SLMs in hybrid-optical correlators.

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OPTIMIZIRANJE KORELACIJSKIH FILTARA PREMA FAZNIM PROSTORNIM SVJETLOSNIM MODULATORIMA

Na razvoj korelacijskih filtara utječu potrebe raspoznavanja uzoraka kao i mogućnosti upotrebe dostupnih prostornih svjetlosnih modulatora. Tržišno dostupni prostorni svjetlosni modulatori imaju amplitudne i fazne odzive koji se ne mogu neovisno definirati. Štoviše, njihova fazna značajka najčešće ne obuhvaća željeni interval $(0, 2\pi)$. Zbog toga su prijašnji pristupi uzimali u obzir ograničenja prostornih svjetlosnih modulatora kao početnu točku u razvoju korelacijskih filtara. Ovdje polazimo od već izračunatih i razvijenih korelacijskih filtara i predlažemo optimalno preslikavanje tih vrijednosti u vrijednosti dozvoljene modulacijskim svojstvima proizvoljnog prostornog svjetlosnog modulatora, sa ciljem dobivanja maksimalnog intenziteta izlaznog korelacijskog signala. Optimiziranje dobiveno varijacijom parametra faznog pomaka obrađuje se teorijski i numerički.